CHAPTER 7

COORDINATE GEOMETRY

(A) Main Concepts and Results

Distance Formula, Section Formula, Area of a Triangle.

- The distance between two points P (x_1, y_1) and Q (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The distance of a point P (x,y) from the origin is $\sqrt{x^2 + y^2}$
- The coordinates of the point P which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$
- The coordinates of the mid-point of the line segment joining the points P (x_1, y_1) and Q (x_2, y_2) are $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
- The area of a triangle with vertices A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) is

  $$\frac{1}{2}\,[x_1\,(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)]$$

  which is non–zero unless the points A, B and C are collinear.

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1: If the distance between the points (2, –2) and (–1, x) is 5, one of the values of x is

(A) –2 (B) 2 (C) –1 (D) 1

Solution : Answer (B)

Sample Question 2: The mid-point of the line segment joining the points A (–2, 8) and B (– 6, – 4) is

(A) (– 4, – 6) (B) (2, 6) (C) (– 4, 2) (D) (4, 2)

Solution : Answer (C)

Sample Question 3: The points A (9, 0), B (9, 6), C (–9, 6) and D (–9, 0) are the vertices of a

(A) square (B) rectangle (C) rhombus (D) trapezium

Solution : Answer (B)

EXERCISE 7.1

Choose the correct answer from the given four options:

1. The distance of the point P (2, 3) from the x-axis is

   (A) 2 (B) 3 (C) 1 (D) 5

2. The distance between the points A (0, 6) and B (0, –2) is

   (A) 6 (B) 8 (C) 4 (D) 2

3. The distance of the point P (–6, 8) from the origin is

   (A) 8 (B) $2\sqrt{7}$ (C) 10 (D) 6

4. The distance between the points (0, 5) and (–5, 0) is

   (A) 5 (B) $5\sqrt{2}$ (C) $2\sqrt{5}$ (D) 10

5. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is

   (A) 5 (B) 3 (C) $\sqrt{34}$ (D) 4

6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

   (A) 5 (B) 12 (C) 11 (D) $7+\sqrt{5}$

7. The area of a triangle with vertices A (3, 0), B (7, 0) and C (8, 4) is

   (A) 14 (B) 28 (C) 8 (D) 6

8. The points (–4, 0), (4, 0), (0, 3) are the vertices of a

   (A) right triangle (B) isosceles triangle

   (C) equilateral triangle (D) scalene triangle

9. The point which divides the line segment joining the points (7, –6) and (3, 4) in ratio 1 : 2 internally lies in the

(A) I quadrant (B) II quadrant

(C) III quadrant (D) IV quadrant

10. The point which lies on the perpendicular bisector of the line segment joining the points A (–2, –5) and B (2, 5) is

(A) (0, 0) (B) (0, 2) (C) (2, 0) (D) (–2, 0)

11. The fourth vertex D of a parallelogram ABCD whose three vertices are A (–2, 3), B (6, 7) and C (8, 3) is

(A) (0, 1) (B) (0, –1) (C) (–1, 0) (D) (1, 0)

12. If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then

(A) $AP = \frac{1}{3} AB$ (B) AP = PB (C) $PB = \frac{1}{3} AB$ (D) $AP = \frac{1}{2} AB$

13. If P $\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q (– 6, 5) and R (– 2, 3), then the value of *a* is

(A) – 4 (B) – 12 (C) 12 (D) – 6

14. The perpendicular bisector of the line segment joining the points A (1, 5) and B (4, 6) cuts the *y*-axis at

(A) (0, 13) (B) (0, –13)

(C) (0, 12) (D) (13, 0)

15. The coordinates of the point which is equidistant from the three vertices of the Δ AOB as shown in the Fig. 7.1 is

(A) (x, y) (B) (y, x)

(C) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (D) $\left(\frac{y}{2}, \frac{x}{2}\right)$

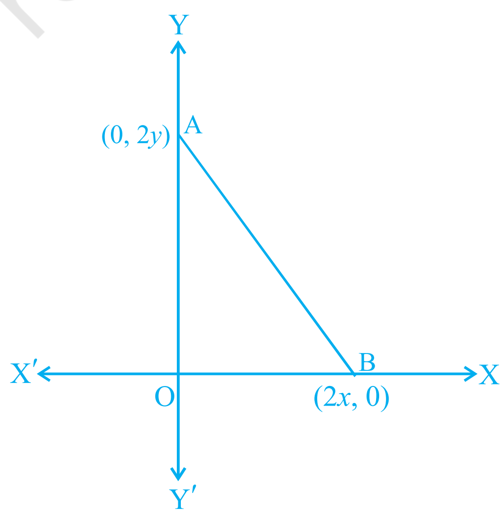

Fig. 7.1

16. A circle drawn with origin as the centre passes through $(\frac{13}{2}, 0)$. The point which does not lie in the interior of the circle is

(A) $\frac{-3}{4},1$ (B) $2,\frac{7}{3}$ (C) $5,\frac{-1}{2}$ (D) $\left(-6,\frac{5}{2}\right)$

17. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively

(A) (0, – 5) and (2, 0) (B) (0, 10) and (– 4, 0)

(C) (0, 4) and (– 10, 0) (D) (0, – 10) and (4, 0)

18. The area of a triangle with vertices $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ is

(A) $(a + b + c)^2$ (B) 0 (C) $a + b + c$ (D) abc

19. If the distance between the points $(4, p)$ and (1, 0) is 5, then the value of p is

(A) 4 only (B) ± 4 (C) – 4 only (D) 0

20. If the points A (1, 2), O (0, 0) and C (a, b) are collinear, then

(A) $a = b$ (B) $a = 2b$ (C) $2a = b$ (D) $a = –b$

(C) Short Answer Questions with Reasoning

State whether the following statements are true or false. Justify your answer.

Sample Question 1 : The points A (–1, 0), B (3, 1), C (2, 2) and D (–2, 1) are the vertices of a parallelogram.

Solution : True. The coordinates of the mid-points of both the diagonals AC and BD are $\frac{1}{2},1$, i.e., the diagonals bisect each other.

Sample Question 2 : The points (4, 5), (7, 6) and (6, 3) are collinear.

Solution : False. Since the area of the triangle formed by the points is 4 sq. units, the points are not collinear.

Sample Question 3 : Point P (0, –7) is the point of intersection of y-axis and perpendicular bisector of line segment joining the points A (–1, 0) and B (7, –6).

Solution : True. P (0, –7) lies on the y -axis. It is at a distance of $\sqrt{50}$ units from both the points (–1, 0) and (7, –6).

EXERCISE 7.2

State whether the following statements are true or false. Justify your answer.

1. Δ ABC with vertices A (–2, 0), B (2, 0) and C (0, 2) is similar to Δ DEF with vertices D (–4, 0) E (4, 0) and F (0, 4).

2. Point P (– 4, 2) lies on the line segment joining the points A (– 4, 6) and B (– 4, – 6).

3. The points (0, 5), (0, –9) and (3, 6) are collinear.

4. Point P (0, 2) is the point of intersection of y–axis and perpendicular bisector of line segment joining the points A (–1, 1) and B (3, 3).

5. Points A (3, 1), B (12, –2) and C (0, 2) cannot be the vertices of a triangle.

6. Points A (4, 3), B (6, 4), C (5, –6) and D (–3, 5) are the vertices of a parallelogram.

7. A circle has its centre at the origin and a point P (5, 0) lies on it. The point Q (6, 8) lies outside the circle.

8. The point A (2, 7) lies on the perpendicular bisector of line segment joining the points P (6, 5) and Q (0, – 4).

9. Point P (5, –3) is one of the two points of trisection of the line segment joining the points A (7, – 2) and B (1, – 5).

10. Points A (–6, 10), B (–4, 6) and C (3, –8) are collinear such that AB = $\frac{2}{9}$AC.

11. The point P (–2, 4) lies on a circle of radius 6 and centre C (3, 5).

12. The points A (–1, –2), B (4, 3), C (2, 5) and D (–3, 0) in that order form a rectangle.

(D) Short Answer Questions

Sample Question 1 : If the mid-point of the line segment joining the points A (3, 4) and B (k, 6) is P (x, y) and $x + y - 10 = 0$, find the value of k.

Solution : Mid-point of the line segment joining A (3, 4) and B (k, 6) = $\frac{3+k}{2}, \frac{4+6}{2}$

$$= \frac{3+k}{2}, 5$$

Then, $\frac{3+k}{2}, 5 = (x, y)$

Therefore, $\frac{3+k}{2} = x$ and $5 = y$.

Since $x + y - 10 = 0$, we have

$$\frac{3+k}{2} + 5 - 10 = 0$$

i.e., $3 + k = 10$

Therefore, $k = 7$.

Sample Question 2 : Find the area of the triangle ABC with A (1, –4) and the mid-points of sides through A being (2, – 1) and (0, – 1).

Solution: Let the coordinates of B and C be (a, b) and (x, y), respectively.

Then , $\left(\frac{1+a}{2}, \frac{-4+b}{2}\right) = (2, -1)$

Therefore, $1 + a = 4,\quad -4 + b = -2$

$a = 3 \qquad b = 2$

Also, $\left(\frac{1+x}{2}, \frac{-4+y}{2}\right) = (0, -1)$

Therefore, $1 + x = 0, \qquad -4 + y = -2$

i.e., $x = -1$ i.e., $y = 2$

The coordinates of the vertices of Δ ABC are A (1, –4), B (3, 2) and C (–1, 2).

$$\text{Area of } \Delta\text{ ABC} = \frac{1}{2}[1(2-2)+3(2+4)-1(-4-2)]$$

$$= \frac{1}{2}[18+6]$$

$$= 12 \text{ sq. units.}$$

Sample Question 3 : Name the type of triangle PQR formed by the points P $\left(\sqrt{2}, \sqrt{2}\right)$, Q $\left(-\sqrt{2}, -\sqrt{2}\right)$ and R $\left(-\sqrt{6}, \sqrt{6}\right)$.

Solution : Using distance formula

$$PQ = \sqrt{\left(\sqrt{2}+\sqrt{2}\right)^2 + \left(\sqrt{2}+\sqrt{2}\right)^2} = \sqrt{\left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = \sqrt{16} = 4$$

$$PR = \sqrt{\left(\sqrt{2}+\sqrt{6}\right)^2 + \left(\sqrt{2}-\sqrt{6}\right)^2} = \sqrt{2 + 6 + 2\sqrt{12} + 2 + 6 - 2\sqrt{12}} = \sqrt{16} = 4$$

$$RQ = \sqrt{\left(-\sqrt{2}+\sqrt{6}\right)^2 + \left(-\sqrt{2}-\sqrt{6}\right)^2} = \sqrt{2+6-2\sqrt{12}+2+6+2\sqrt{12}} = \sqrt{16} = 4$$

Since PQ = PR = RQ = 4, points P, Q, R form an equilateral triangle.

Sample Question 4 : ABCD is a parallelogram with vertices A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3). Find the coordinates of the fourth vertex D in terms of x_1, x_2, x_3, y_1, y_2 and y_3.

Solution: Let the coordinates of D be (x, y). We know that diagonals of a parallelogram bisect each other.

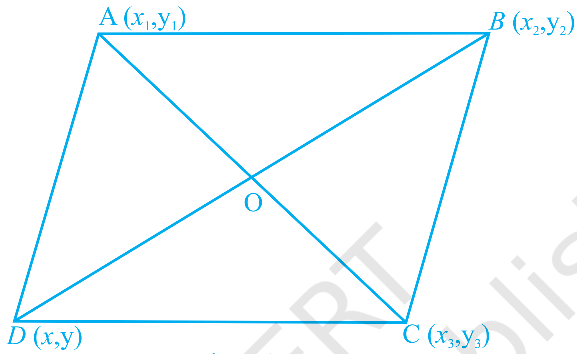

Fig. 7.2

Therefore, mid-point of AC = mid-point of BD $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) = \left(\frac{x_2+x}{2}, \frac{y_2+y}{2}\right)$

i.e., $x_1 + x_3 = x_2 + x$ and $y_1 + y_3 = y_2 + y$

i.e., $x_1 + x_3 - x_2 = x$ and $y_1 + y_3 - y_2 = y$

Thus, the coordinates of D are

$(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$

EXERCISE 7.3

1. Name the type of triangle formed by the points A (–5, 6), B (–4, –2) and C (7, 5).
2. Find the points on the x–axis which are at a distance of $2\sqrt{5}$ from the point (7, –4). How many such points are there?
3. What type of a quadrilateral do the points A (2, –2), B (7, 3), C (11, –1) and D (6, –6) taken in that order, form?
4. Find the value of a, if the distance between the points A (–3, –14) and B $(a, -5)$ is 9 units.
5. Find a point which is equidistant from the points A (–5, 4) and B (–1, 6)? How many such points are there?

6. Find the coordinates of the point Q on the x–axis which lies on the perpendicular bisector of the line segment joining the points A (–5, –2) and B(4, –2). Name the type of triangle formed by the points Q, A and B.

7. Find the value of m if the points (5, 1), (–2, –3) and (8, $2m$) are collinear.

8. If the point A (2, – 4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ.

9. Find the area of the triangle whose vertices are (–8, 4), (–6, 6) and (–3, 9).

10. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and (–1, 7)? Find the coordinates of the point of division.

11. Find the ratio in which the point $P\left(\frac{3}{4},\frac{5}{12}\right)$ divides the line segment joining the points $A\ \frac{1}{2},\frac{3}{2}$ and B (2, –5).

12. If P ($9a$ – 2, –b) divides line segment joining A ($3a$ + 1, –3) and B ($8a$, 5) in the ratio 3 : 1, find the values of a and b.

13. If (a, b) is the mid-point of the line segment joining the points A (10, –6) and B (k, 4) and $a - 2b = 18$, find the value of k and the distance AB.

14. The centre of a circle is ($2a$, a – 7). Find the values of a if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units.

15. The line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1:2 and it lies on the line $3x - 18y + k = 0$. Find the value of k.

16. If $D\left(\frac{-1}{2},\frac{5}{2}\right)$, E (7, 3) and $F\left(\frac{7}{2},\frac{7}{2}\right)$ are the midpoints of sides of Δ ABC, find the area of the Δ ABC.

17. The points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a triangle ABC right angled at B. Find the values of a and hence the area of ΔABC.

18. Find the coordinates of the point R on the line segment joining the points P (–1, 3) and Q (2, 5) such that $PR = \frac{3}{5}PQ$.

19. Find the values of k if the points A (k + 1, $2k$), B ($3k$, $2k$ + 3) and C ($5k$ – 1, $5k$) are collinear.

20. Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points (8, –9) and (2, 1). Also find the coordinates of the point of division.

(E) Long Answer Questions

Sample Question 1 : The mid-points D, E, F of the sides of a triangle ABC are (3, 4), (8, 9) and (6, 7). Find the coordinates of the vertices of the triangle.

Solution : Since D and F are the mid-points of AB and AC, respectively, by mid-point theorem, we can prove that DFEB is a parallelogram. Let the coordinates of B be (x, y).

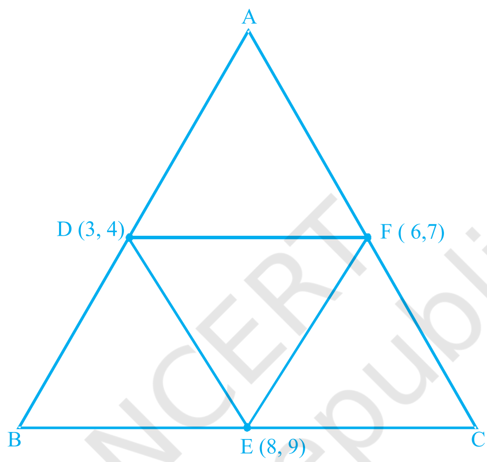

Fig. 7.3

Refer to Sample Question 4 of Section (D) to get

$$x = 3 + 8 - 6 = 5$$

$$y = 4 + 9 - 7 = 6$$

Therefore, B (5, 6) is one of the vertices of the triangle.

Similarly DFCE and DAFE are also parallelograms, and the coordinates of A are (3 + 6 – 8, 4 + 7 – 9) = (1, 2). Coordinates of C are (8 + 6 – 3, 9 + 7 – 4) = (11, 12). Thus, the coordinates of the vertices of the triangle are A (1, 2), B (5,6) and C (11, 12).

EXERCISE 7.4

1. If (– 4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.
2. A (6, 1), B (8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of Δ ADE.

3. The points A (x_1, y_1), B (x_2, y_2) and C $(x_3\ y_3)$ are the vertices of Δ ABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1

(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1

(iv) What are the coordinates of the centroid of the triangle ABC?

4. If the points A (1, –2), B (2, 3) C (a, 2) and D (– 4, –3) form a parallelogram, find the value of a and height of the parallelogram taking AB as base.

5. Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in figure 7.4. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D? If so, what should be his position?

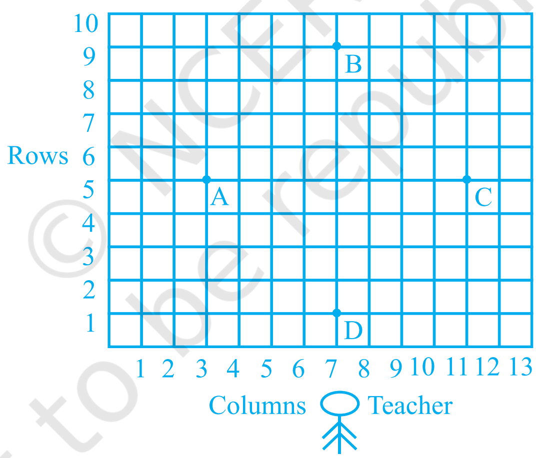

Fig. 7.4

6. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distances covered are in straight lines).

If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.