CHAPTER – 2

EXPONENTS AND POWERS

EXERCISE 2.1

1. Evaluate:

(i) $\left(\frac{3}{5}\right)^{-2}$

(ii) $(-3)^{-3}$

(iii) $\left(\frac{2}{7}\right)^{-4}$

Solution:

(i) $\left(\frac{3}{5}\right)^{-2}$

Let us evaluate the given expression,

By using the formula, $\left(a^{-n} = \frac{1}{a^n}\right)$

So,

$\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{2}$

$= \left(\frac{5}{3}\right) \times \left(\frac{5}{3}\right)$

$= \frac{25}{9}$

(ii) $(-3)^{-3}$

Let us evaluate the given expression,

By using the formula, $\left(a^{-n} = \frac{1}{a^n}\right)$

So,

$(-3)^{-3} = \left(-\frac{1}{3}\right)^3$

$= \left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right)$

$= -\frac{1}{27}$

(iii) $\left(\frac{2}{7}\right)^{-4}$

Let us evaluate the given expression,

By using the formula, $\left(a^{-n} = \frac{1}{a^n}\right)$

So,

$\left(\frac{2}{7}\right)^{-4} = \left(\frac{7}{2}\right)^4$

$= \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right)$

$= \frac{2401}{16}$

2. Simplify:

(i) $[(2)^{-1} + (4)^{-1} + (3)^{-1}]^{-1}$

(ii) $[(4)^{-1} - (5)^{-1}]^2 \times \left(\frac{5}{8}\right)^{-1}$

(iii) $[4^0 + 4^2 - 2^3] \times 3^{-2}$

(iv) $\left[(5)^2 - \left(\frac{1}{4}\right)^{-2}\right] \times \left(\frac{3}{4}\right)^{-2}$

Solution:

(i) $[(2)^{-1} + (4)^{-1} + (3)^{-1}]^{-1}$

Let us simplify the given expression,

$[(2)^{-1} + (4)^{-1} + (3)^{-1}]^{-1} = \left[\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right) + \left(\frac{1}{3}\right)\right]^{-1}$

$= \left[\frac{(6+3+4)}{12}\right]^{-1}$

$= \left[\frac{13}{12}\right]^{-1}$

$= \frac{12}{13}$

(ii) $[(4)^{-1} - (5)^{-1}]^2 \times \left(\frac{5}{8}\right)^{-1}$

Let us simplify the given expression,

$[(4)^{-1} - (5)^{-1}]^2 \times \left(\frac{5}{8}\right)^{-1} = \left[\left(\frac{1}{4}\right) - \left(\frac{1}{5}\right)\right]^2 \times \left(\frac{8}{5}\right)^1$

$= \left[\frac{(5-4)}{20}\right]^2 \times \left(\frac{8}{5}\right)$

$= \left[\frac{1}{20}\right]^2 \times \left(\frac{8}{5}\right)$

$= \left(\frac{1}{20}\right) \times \left(\frac{1}{20}\right) \times \left(\frac{8}{5}\right)$

$= \frac{1}{250}$

(iii) $[4^0 + 4^2 - 2^3] \times 3^{-2}$

Let us simplify the given expression,

$[4^0 + 4^2 - 2^3] \times 3^{-2} = [1 + 16 - 8] \times \left(\frac{1}{3^2}\right)$

$= 9 \times \left(\frac{1}{9}\right)$

$= 1$

(iv) $\left[(5)^2 - \left(\frac{1}{4}\right)^{-2}\right] \times \left(\frac{3}{4}\right)^{-2}$

Let us simplify the given expression,

$\left[(5)^2 - \left(\frac{1}{4}\right)^{-2}\right] \times \left(\frac{3}{4}\right)^{-2} = [25 - (4)^2] \times \left(\frac{4}{3}\right)^2$

$= [25 - 16] \times \left(\frac{16}{9}\right)$

$= 9 \times \left(\frac{16}{9}\right)$

$= 16$

3. Find the multiplicative inverse of the following:

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

(ii) $\left\{\left(-\frac{3}{2}\right)^{-4}\right\}^{\frac{1}{2}}$

(iii) $\left(\frac{5}{7}\right)^{-2} \times \left(\frac{5}{7}\right)^{4} \div \left(\frac{5}{7}\right)^{3}$

Solution:

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Let us simplify to find the multiplicative inverse of the given expression,

$\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}}$

$= \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}}$

$= \left(\frac{2}{3}\right)^{4\times\frac{3}{4}}$

$= \left(\frac{2}{3}\right)^{3}$

$= \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$

$= \frac{8}{27}$

So, the multiplicative inverse of $\frac{8}{27}$ is $\frac{27}{8}$.

(ii) $\left\{\left(-\frac{3}{2}\right)^{-4}\right\}^{\frac{1}{2}}$

Let us simplify to find the multiplicative inverse of the given expression,

$\left\{\left(-\frac{3}{2}\right)^{-4}\right\}^{\frac{1}{2}} = \left(-\frac{3}{2}\right)^{4\times\frac{1}{2}}$

$= \left(-\frac{3}{2}\right)^{-2}$

$= \left(\frac{2}{-3}\right)^{2}$

$= \left(\frac{2}{-3}\right) \times \left(\frac{2}{-3}\right)$

$= \frac{4}{9}$

So, the multiplicative inverse of 4/9 is 9/4.

(iii) $\left(\frac{5}{7}\right)^{-2} \times \left(\frac{5}{7}\right)^{4} \div \left(\frac{5}{7}\right)^{3}$

Let us simplify to find the multiplicative inverse of the given expression,

$\left(\frac{5}{7}\right)^{-2} \times \left(\frac{5}{7}\right)^{4} \div \left(\frac{5}{7}\right)^{3} = \left(\frac{7}{5}\right)^{2} \times \left(\frac{5}{7}\right)^{4} \div \left(\frac{5}{7}\right)^{3}$

$= \left(\frac{7}{5}\right)^{2} \times \left(\frac{5}{7}\right)^{4-3}$

$= \left(\frac{7}{5}\right)^{2} \times \left(\frac{5}{7}\right)$

$= \left(\frac{7}{5}\right) \times \left(\frac{7}{5}\right) \times \left(\frac{5}{7}\right)$

$= \frac{7}{5}$

So, the multiplicative inverse of $\frac{7}{5}$ is $\frac{5}{7}$.

4. (i) Express 16^{-2} as a power with base 2.

(ii) Express 125^{-4} as a power with base 5.

Solution:

(i) Express 16^{-2} as a power with base 2.

Let us simplify,

$(16)^{-2} = (2^4)^{-2}$

$= 2^{-8}$

$= \frac{1}{2^8}$

(ii) Express 125^{-4} as a power with base 5.

Let us simplify,

$(125)^{-4} = (5^3)^{-4}$

$= 5^{-12}$

$= \frac{1}{5^{12}}$

5. Write the following numbers in expanded form using exponents:
(i) 2789.453

(ii) 3007.805

Solution:

(i) 2789.453

The expanded form of

2789.453

$= 2 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 9 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2} + 3 \times 10^{-3}$

(ii) 3007.805

The expanded form of

$3007.805 = 3 \times 10^3 + 7 \times 10^0 + 8 \times 10^{-1} + 5 \times 10^{-3}$

6. Simplify and write in exponential form with positive exponent:

(i) $\left[\left\{\left(\frac{5}{7}\right)^2\right\}^{-1}\right]^{-3}$

(ii) $\left(\frac{2}{7}\right)^2 \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4}$

(iii) $\left(\frac{4}{5}\right)^2 \times 5^4 \times \left(\frac{2}{5}\right)^{-2} \div \left(\frac{5}{2}\right)^{-3}$

(iv) $\frac{[8^{-1}\times 5^{3}]}{2^{-4}}$

Solution:

(i) $\left[\left\{\left(\frac{5}{7}\right)^{2}\right\}^{-1}\right]^{-3}$

Let us simplify,

$\left[\left\{\left(\frac{5}{7}\right)^{2}\right\}^{-1}\right]^{-3} = \left\{\left(\frac{5}{7}\right)^{2}\right\}^{-1\times -3}$

$= \left(\frac{5}{7}\right)^{2\times 3}$

$= \left(\frac{5}{7}\right)^{6}$

The exponential form is $\left(\frac{5}{7}\right)^{6}$.

(ii) $\left(\frac{2}{7}\right)^{2} \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4}$

Let us simplify,

$\left(\frac{2}{7}\right)^{2} \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4} = \left(\frac{2}{7}\right)^{2} \times \left(\frac{2}{7}\right)^{3} \div \left(\frac{7}{5}\right)^{-8}$

$= \left(\frac{2}{7}\right)^{2} \times \left(\frac{2}{7}\right)^{3} \div \left(\frac{5}{7}\right)^{8}$

$= \left(\frac{2^{2}}{7^{2}}\right) \times \left(\frac{2^{3}}{7^{3}}\right) \times \left(\frac{5^{8}}{7^{8}}\right)$

$= \left(\frac{2^{2+3}}{7^{2+3}}\right) \times \left(\frac{5^{8}}{7^{8}}\right)$

$= \left(\frac{2^{5}}{7^{5}}\right) \times \left(\frac{5^{8}}{7^{8}}\right)$

$= \frac{(2^5 \times 5^8)}{7^{5+8}}$

$= \frac{(2^5 \times 5^8)}{7^{13}}$

The exponential form is $\frac{(2^5 \times 5^8)}{7^{13}}$.

(iii) $\left(\frac{4}{5}\right)^2 \times 5^4 \times \left(\frac{2}{5}\right)^{-2} \div \left(\frac{5}{2}\right)^{-3}$

Let us simplify,

$\left(\frac{4}{5}\right)^2 \times 5^4 \times \left(\frac{2}{5}\right)^{-2} \div \left(\frac{5}{2}\right)^{-3} = \left(\frac{(2^2)}{5^2}\right) \times 5^4 \times \left(\frac{2^{-2}}{5^{-2}}\right) \times \left(\frac{2^{-3}}{5^{-3}}\right)$

$= \frac{(2^4 \times 5^4 \times 2^{-2} \times 2^{-3})}{(5^2 \times 5^{-2} \times 5^{-3})}$

$= 2^{4-2-3} \times 5^{4-2+2+3}$

$= 2^{-1} \times 5^7$

$= \frac{5^7}{2^1}$

The exponential form is $\frac{5^7}{2^1}$.

(iv) $\frac{[8^{-1} \times 5^3]}{2^{-4}}$

Let us simplify,

$\frac{[8^{-1} \times 5^3]}{2^{-4}} = \frac{[(2^3)^{-1} \times 5^3]}{2^{-4}}$

$= \frac{[2^3 \times 5^3]}{2^{-4}}$

$= 2^{-3+4} \times 5^3$

$= 2^1 \times 5^3$

$= 2 \times 5 \times 5 \times 5$

$= 250$

The exponential form is 250.

7. Simplify and write the following in exponential form:

(i) $\mathbf{((-2)^3)^2 + 5^{-3} \div 5^{-5} - \left(-\frac{1}{2}\right)^0}$

(ii) $\mathbf{3^{-5} \times 3^2 \div 3^{-6} + (2^2 \times 3)^2 + \left(\frac{2}{3}\right)^{-1} + 2^{-1} + \left(\frac{1}{19}\right)^{-1}}$

Solution:

(i) $((-2)^3)^2 + 5^{-3} \div 5^{-5} - \left(-\frac{1}{2}\right)^0$

Let us simplify,

$((-2)^3)^2 + 5^{-3} \div 5^{-5} - \left(-\frac{1}{2}\right)^0 = (-2)^6 + \frac{1}{5^3} \div \frac{1}{5^5} - 1$

$= 64 + \frac{1}{5^3} \times 5^5 - 1$

$= 64 + 5^{5-3} - 1$

$= 64 + 5^2 - 1$

$= 64 + 25 - 1$

$= 88$

The exponential form is 88.

(ii) $3^{-5} \times 3^2 \div 3^{-6} + (2^2 \times 3)^2 + \left(\frac{2}{3}\right)^{-1} + 2^{-1} + \left(\frac{1}{19}\right)^{-1}$

Let us simplify,

$3^{-5} \times 3^2 \div 3^{-6} + (2^2 \times 3)^2 + \left(\frac{2}{3}\right)^{-1} + 2^{-1} + \left(\frac{1}{19}\right)^{-1} = 3^{-5+2+6} + (2^4 \times 3^2) + \frac{3}{2} + \frac{1}{2} + 19^1$

$= 3^3 + (16 \times 9) + \frac{4}{2} + 19$

$= 27 + 144 + 2 + 19$

$= 192$

The exponential form is 192.

8. Simplify and write in exponential form with negative exponent:

(i) $\mathbf{5^3 \times \left(\frac{4}{5}\right)^3}$

(ii) $\mathbf{\left[\left(\frac{3}{7}\right)^{-2}\right]^{-3}}$

(iii) $\mathbf{\left(\frac{5}{9}\right)^{-2} \times \left(\frac{5}{3}\right)^2 \div \left(\frac{1}{5}\right)^{-2}}$

(iv) $\mathbf{2^{-1}\left[\left(\frac{5}{3}\right)^4 + \left(\frac{3}{5}\right)^{-2}\right] \div \left(\frac{17}{9}\right)}$

(v) $\mathbf{(-7)^3 \times \left(\frac{1}{-7}\right)^{-9} \div (-7)^{10}}$

Solution:

(i) $5^3 \times \left(\frac{4}{5}\right)^3$

Let us simplify,

$5^3 \times \left(\frac{4}{5}\right)^3 = 5^3 \times \left(\frac{4^3}{5^3}\right)$

$= 5^{3-3} \times 4^3$

$= 5^0 \times 4^3$

$= 1 \times 4^3$

$= \left(\frac{1}{4}\right)^{-3}$

The exponential form is $\left(\frac{1}{4}\right)^{-3}$.

(ii) $\left[\left(\frac{3}{7}\right)^{-2}\right]^{-3}$

Let us simplify,

$\left[\left(\frac{3}{7}\right)^{-2}\right]^{-3} = \left(\frac{3}{7}\right)^{-2\times-3}$

$= \left(\frac{3}{7}\right)^{6}$

$= \left(\frac{7}{3}\right)^{-6}$

The exponential form is $\left(\frac{7}{3}\right)^{-6}$.

(iii) $\left(\frac{5}{9}\right)^{-2} \times \left(\frac{5}{3}\right)^{2} \div \left(\frac{1}{5}\right)^{-2}$

Let us simplify,

$\left(\frac{5}{9}\right)^{-2} \times \left(\frac{5}{3}\right)^{2} \div \left(\frac{1}{5}\right)^{-2} = \left(\frac{5^{-2}}{9^{-2}}\right) \times \left(\frac{5^2}{3^2}\right) \div \left(\frac{1}{5^{-2}}\right)$

$= \frac{\left(5^{-2}\times5^2\times5^{-2}\right)}{[(3^2)^{-2}\times3^2]}$

$= \frac{\left(5^{-2+2-2}\right)}{(3^{-4+2})}$

$= \frac{5^{-2}}{3^{-2}}$

$= \left(\frac{5}{3}\right)^{-2}$

The exponential form is $\left(\frac{5}{3}\right)^{-2}$.

(iv) $2^{-1}\left[\left(\frac{5}{3}\right)^{4} + \left(\frac{3}{5}\right)^{-2}\right] \div \left(\frac{17}{9}\right)$

Let us simplify,

$2^{-1}\left[\left(\frac{5}{3}\right)^{4} + \left(\frac{3}{5}\right)^{-2}\right] \div \left(\frac{17}{9}\right) = 2^{-1}\left[\left(\frac{5^4}{3^4}\right) + \left(\frac{3^{-2}}{5^{-2}}\right)\right] \div \left(\frac{17}{9}\right)$

$= 2^{-1}\left[\left(\frac{5^4}{3^4}\right) + \left(\frac{5^2}{3^2}\right)\right] \div \left(\frac{17}{9}\right)$

$= 2^{-1}\left[\left(\frac{625}{81}\right) + \left(\frac{25}{9}\right)\right] \div \left(\frac{17}{9}\right)$

$= \frac{1}{2}\left[\frac{(625 + 225)}{81}\right] \times \left(\frac{9}{17}\right)$

$= \frac{1}{2} \times \left(\frac{850}{81}\right) \times \frac{9}{17}$

$= \frac{25}{9}$

$= \left(\frac{5}{3}\right)^{2}$

$= \left(\frac{3}{5}\right)^{-2}$

The exponential form is $\left(\frac{3}{5}\right)^{-2}$.

(v) $(-7)^3 \times \left(\frac{1}{-7}\right)^{-9} \div (-7)^{10}$

Let us simplify,

$(-7)^3 \times \left(\frac{1}{-7}\right)^{-9} \div (-7)^{10} = (-7)^3 \times (-7)^9 \div (-7)^{10}$

$= (-7)^{3+9-10}$

$= (-7)^2$

$= \left(\frac{1}{-7}\right)^{-2}$

The exponential form is $\left(\frac{1}{-7}\right)^{-2}$.

9. Simplify:

(i) $\frac{(49 \times z^{-3})}{(7^{-3} \times 10 \times z^{-5})}$ **(z ≠ 0)**

(ii) $\frac{(9^3 \times 2^7 \times t^4)}{(3^2 \times 3^4 \times t^2)}$

(iii) $\frac{\left[(3^{-2})^2 \times (5^2)^{-3} \times (-t^{-3})^2\right]}{\left[(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3\right]}$

(iv) $\frac{(2^{-5} \times 15^{-5} \times 500)}{(5^{-6} \times 6^{-5})}$

Solution:

(i) $\frac{(49 \times z^{-3})}{(7-3 \times 10 \times z^{-5})}$ (z ≠ 0)

Let us simplify the given expression,

$\frac{(49 \times z^{-3})}{(7^{-3} \times 10 \times z^{-5})} = \frac{(49 \times z^{-3})}{(7^{-3} \times 10 \times z^{-5})}$

$= \frac{(7^{2+3} \times z^{3+5})}{10}$

$= \frac{(7^5 \times z^2)}{10}$

(ii) $\frac{(9^3 \times 2^7 \times t^4)}{(3^2 \times 3^4 \times t^2)}$

Let us simplify the given expression,

$\frac{(9^3 \times 2^7 \times t^4)}{(3^2 \times 3^4 \times t^2)} = \frac{\left[(3^2)^3 \times (3)^3 \times t^4\right]}{(3^2 \times 3^4 \times t^2)}$

$= \frac{(3^6 \times 3^3 \times t^4)}{(3^2 \times 3^4 \times t^2)}$

$= 3^{6+3-2-4} \times t^{4-2}$

$= 3^3 \times t^2$

$= 27t^2$

(iii) $\frac{\left[(3^{-2})^2 \times (5^2)^{-3} \times (-t^{-3})^2\right]}{[(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3]}$

Let us simplify the given expression,

$\frac{\left[(3^{-2})^2 \times (5^2)^{-3} \times (-t^{-3})^2\right]}{[(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3]} = \frac{[3^{-4} \times 5^{-6} \times t^{-6}]}{[3^{-10} \times 5^{-6} \times t^{-12}]}$

$= 3^{-4+10} \times 5^{-6+6} \times t^{-6+12}$

$= 3^6 \times 5^0 \times t^6$

$= 3^6 \times 1 \times t^6$

$= 729t^6$

(iv) $\frac{(2^{-5} \times 15^{-5} \times 500)}{(5^{-6} \times 6^{-5})}$

Let us simplify the given expression,

$\frac{(2^{-5}\times 15^{-5}\times 500)}{(5^{-6}\times 6^{-5})} = \frac{(2^{-5}\times (3\times 5)^{-5}\times 2^2\times 5^3)}{(5^{-6}\times (2\times 3)^{-5})}$

$= \frac{(2^{-5}\times 3^{-5}\times 5^{-5}\times 2^2\times 5^3)}{(5^{-6}\times 2^{-5}\times 3^{-5})}$

$= 2^{-5+2+5} \times 3^{-5+5} \times 5^{-5+3+6}$

$= 2^2 \times 3^0 \times 5^4$

$= 4 \times 1 \times 625$

$= 2500$

10. By what number should $\left(\frac{3}{-2}\right)^{-3}$ be divided to get $\left(\frac{2}{3}\right)^2$?

Solution:

The required number is $\left(\frac{3}{-2}\right)^{-3} \div \left(\frac{2}{3}\right)^2$

Let us simplify the expression,

$\left(\frac{3}{-2}\right)^{-3} \div \left(\frac{2}{3}\right)^2 = \frac{(3^{-3})}{(-2)^{-3}} \times \frac{(3^2)}{(2^2)}$

$= \frac{[3^{-3}\times 3^2]}{[-2^{-3}\times -2^2]}$

$= \frac{[3^{-3+2}]}{(-2)^{-3+2}}$

$= \frac{3^{-1}}{-2^{-1}}$

$= -\frac{2^1}{3^1}$

$= -\frac{2}{3}$

11. Find the value of m for which $9^m \div 3^{-2} = 9^4$.

Solution:

Let us simplify the expression,

$9^m \div 3^{-2} = 9^4$

$(3^2)^m \div 3^{-2} = (3^2)^4$

$3^{2m} \div 3^{-2} = 3^8$

$3^{2m+2} = 3^8$

Now by comparing the powers, we get

$2m + 2 = 8$

$2m = 8 - 2$

$= 6$

$m = \frac{6}{2}$

$= 3$

12. If $\left(-\frac{5}{7}\right)^{-4} \times \left(-\frac{5}{7}\right)^{12} = \left\{\left(-\frac{5}{7}\right)^3\right\} \times \left(-\frac{5}{7}\right)^{-1}$, find the value of x.

Solution:

Let us simplify the given expression,

$\left(-\frac{5}{7}\right)^{-4} \times \left(-\frac{5}{7}\right)^{12} = \left\{\left(-\frac{5}{7}\right)^3\right\} \times \left(-\frac{5}{7}\right)^{-1}$

$\left(-\frac{5}{7}\right)^{-4} \times \left(-\frac{5}{7}\right)^{12} = \left(-\frac{5}{7}\right)^{3x} \times \left(-\frac{5}{7}\right)^{-1}$

$\left(-\frac{5}{7}\right)^{-4+12} = \left(-\frac{5}{7}\right)^{3x}$

$\left(-\frac{5}{7}\right)^{8} = \left(-\frac{5}{7}\right)^{3x-1}$

Now by comparing the powers, we get

$8 = 3x - 1$

$3x = 8 + 1$

$3x = 9$

$x = \frac{9}{3}$

$= 3$

13. Find x, if $\left(-\frac{2}{3}\right)^{-13} \times \left(\frac{3}{-2}\right)^{8} = \left(-\frac{2}{3}\right)^{-2x+1}$

Solution:

Let us simplify the given expression,

$\left(-\frac{2}{3}\right)^{-13} \times \left(\frac{3}{-2}\right)^{8} = \left(-\frac{2}{3}\right)^{-2x+1}$

$\left(-\frac{2}{3}\right)^{-13} \times \left(\frac{-2}{3}\right)^{-8} = \left(-\frac{2}{3}\right)^{-2x+1}$

$\left(-\frac{2}{3}\right)^{-13-8} = \left(-\frac{2}{3}\right)^{-2x+1}$

$\left(-\frac{2}{3}\right)^{-21} = \left(-\frac{2}{3}\right)^{-2x+1}$

Now by comparing the powers, we get

$-21 = -2x + 1$

$2x = 1 + 21$

$2x = 22$

$x = \frac{22}{2}$

$= 11$

14. (i) If $5^{2x-1} = \frac{1}{(125)^{x-3}}$, find x.

(ii) If $\frac{(9^n \times 3^5 \times 27^3)}{(3 \times 81^4)} = 27$, find n.

Solution:

(i) If $5^{2x-1} = \frac{1}{(125)^{x-3}}$, find x.

Let us simplify the given expression,

$5^{2x-1} = \frac{1}{(5^3)^{x-3}}$

$5^{2x-1} = \frac{1}{5^{3x-9}}$

$5^{2x-1} = 5^{-3x+9}$

Now by comparing the powers, we get

$2x - 1 = -3x + 9$

$2x + 3x = 9 + 1$

$5x = 10$

$x = \frac{10}{5}$

$= 2$

(ii) If $\frac{(9^n \times 3^5 \times 27^3)}{(3 \times 81^4)} = 27$

Let us simplify the given expression,

$$\frac{\left((3^2)^n \times 3^5 \times (3^3)^3\right)}{(3\times(3^4)^4)} = 3^3$$

$[3^{2n} \times 3^5 \times 3^9] = (3\times3^{16}) = 3^3$

$3^{2n+5+9-1-16} = 3^3$

Now by comparing the powers, we get

$2n + 5 + 9 - 1 - 16 = 3$

$2n - 3 = 3$

$2n = 3 + 3$

$2n = 6$

$n = \frac{6}{2}$

$= 3$

EXERCISE 2.2

1. Express the following numbers in standard form:
(i) 0.0000000000085
(ii) 0.000000000000942
(iii) 6020000000000000
(iv) 0.00000000837
Solution:

Let us express the numbers in their standard form,

(i) $0.0000000000085 = 8.5 \times 10^{-12}$

(ii) $0.000000000000942 = 9.42 \times 10^{-13}$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 8.37 \times 10^{-9}$

2. Express the following numbers in usual form:
(i) $\mathbf{3.02 \times 10^{-6}}$
(ii) $\mathbf{1\text{-}007 \times 10^{11}}$
(iii) $\mathbf{5.375 \times 10^{14}}$
(iv) $\mathbf{7.579 \times 10^{-14}}$
Solution:

Let us express the numbers in their usual form,

(i) $3.02 \times 10^{-6} = 0.00000302$

(ii) $1.007 \times 10^{11} = 100700000000$

(iii) $5.375 \times 10^{14} = 5375000000000000$

(iv) $7.579 \times 10^{-14} = 0.000000000000007579$

3. Express the number appearing in the following statements in standard form:

(i) The mass of a proton is 0.000000000000000000000001673 gram.

(ii) The thickness of a piece of paper is 0.0016 cm.

(iii) The diameter of a wire on a computer chip is 0.000003 m.

(iv) A helium atom has a diameter of $\frac{22}{100000000000}$ m.

(v) Mass of a molecule of hydrogen gas is about 0.00000000000000000000334 tons.

(vi) The human body has 1 trillion cells which vary in shapes and sizes.

(vii) The distance from the Earth of the Sun is 149,600,000,000 m.

(viii) The speed of light is 300,000,000 m/sec

(ix) Mass of the Earth is 5,970,000,000,000,000,000,000,000 kg.

(x) Express 3 years in seconds.

(xi) Express 7 hectares in cm^2.

(xii) A sugar factory has annual sales of 3 billion 720 million kilograms of sugar.

Solution:

(i) The mass of a proton is 0.000000000000000000000001673 gram, it is expressed in standard form as 1.673×10^{-24} gram.

(ii) Thickness of a piece of paper in standard form is 0.0016 cm; it is expressed in standard form as 1.6×10^{-3}

(iii) Diameter of a wire on a computer chip is 0.000003 m; it is expressed in standard form as 3.0×10^{-6} m

(iv) A helium atom has a diameter of 22/100000000000 m; it is expressed in standard form as $22 \times 10^{-12} = 2.2 \times 10^{-10}$

(v) Mass of a molecule of hydrogen gas is about 0.00000000000000000000334 tons; it is expressed in standard form as = 3.34×10^{-21} tons

(vi) Human body has 1 trillion of cells which vary in shapes and sizes; it is expressed in standard form as $1,000,000,000,000 = 10^{12}$

(vii) The distance from the Earth of the Sun is expressed in standard form as

$149,600,000,000 \text{ m} = 1.496 \times 10^{11}$

(viii) The speed of light is 300,000,000 m/sec; it is expressed in standard form as

3.0×10^{8} m/sec

(ix) Mass of the Earth is 5,970,000,000,000,000,000,000,000 kg; it is expressed in standard form as 5.97×10^{24} kg

(x) Express 3 years in seconds, it is expressed in standard form as

3 years = 3×365 days

$= 3 \times 365 \times 24$ hours

$= 3 \times 365 \times 24 \times 3600$ seconds

= 1040688000 seconds

$= 1.040688 \times 10^{9}$ seconds

(xi) Express 7 hectares in cm^2, it is expressed in standard form as

7 hectares = $7 \times 10000 \text{ m}^2$

$= 7 \times 10000 \times 100 \times 100\ cm^2$

$= 700000000\ cm^2$

$= 7.0 \times 10^8\ cm^2$

(xii) A sugar factory has annual sales of 3 billion 720 million kilograms of sugar, it is expressed in standard form as

Annual sale of a sugar factory = 3 billion

720 million kilograms sugar = 3,720,000,000 kg = 3.72×10^9 kg

4. Compare the following:

(i) Size of a plant cell to the thickness of a piece of paper.

(ii) Size of a plant cell to the diameter of a wire on a computer chip.

(iii) The thickness of a piece of paper to the diameter of a wire on a computer chip.

Given size of plant cell = 0.00001275 m

Thickness of a piece of paper = 0.0016 cm

Diameter of a wire on a computer chip = 0.000003 m

Solution:

Given:

Size of plant cell= 0.00001275 m = 1.275×10^{-5} m

Thickness of a piece of paper = 0.0016 cm = 1.6×10^{-3} cm

Diameter of a wire on a computer chip = 0.000003 m = 3.0×10^{-6} m

(i) Size of plant cell: thickness of a piece of paper

1.275×10^{-5}: 1.6×10^{-3}

Size of plant cell = $\frac{1.2}{1.6} = \frac{3}{4}$ times of thickness of paper.

(ii) Size of plant cell: diameter of wire on a computer chip

1.275×10^{-5}: 3.0×10^{-6}

12.75: 3.00

Size of plant cell is 4 times of diameter of wire.

(iii) Thickness of a piece of paper: diameter of a wire on a computer chip

1.6×10^{-3}: $3.0 \times 10^{-6} \times 100$ cm

1.6×1000: 300

16.1: 3

Approximately 5 times is the thickness of paper to diameter of wire.

5. The number of red blood cells per cubic millimeter of blood is approximately 5.5 million. If the average body contains 5 liters of blood, what is the total number of red cell in the body? (1 liter = 1,00,000 mm^3)

Solution:

Given:

Red blood per cubic millimeter = 5.5 million = 5.5×10^6

Red blood in 5 liters of blood = $5.5 \times 10^6 \times 5 \times 10^5$ (1 litre = 10^5 mm)

$= 27.5 \times 10^{6+5}$

$= 27.5 \times 10^{11}$

$= 2.75 \times 10 \times 10^{11}$

$= 2.75 \times 10^{12}$

∴ Total number of red blood cells in the body is 2.75×10^{12}.

6. Mass of Mars is 6.42×10^{29} kg and the mass of the sun is 1.99×10^{30} kg. What is the total mass?

Solution:

Given:

Mass of Mars = 6.42×10^{29} kg
and mass of sun = 1.99×10^{30}
Total mass = $6.42 \times 10^{29} + 1.99 \times 10^{30}$
$= 10^{29} (6.42 + 1.99 \times 10)$

$= 10^{29} (6.42 + 19.9)$

$= 26.32 \times 10^{29}$

∴ Total mass of mars is 26.32×10^{29}.

7. A particular star is at a distance of about 8.1×10^{13} km from the Earth. Assuming that the light travels at 3×10^{8} m/sec, find how long does light take from that star to reach the Earth.

Solution:

Given:

Distance between earth and a particular star = 8.1×10^{13} km

Speed of light = 3×10^{8} m/sec.

Time is taken to reach the earth = $\frac{8.1 \times 10^{13}}{3 \times 10^{8}}$

= $2.7 \times 10^{16-8}$

= 2.7×10^{8} sec

∴ Light takes 2.7×10^{8} sec from star to reach the earth.

Mental Maths

Question 1: Fill in the blanks:

(i) Multiplicative inverse of $\left(\frac{-4}{5}\right)^2$ = ……..

(ii) The standard form of 0.000182 = ……

(iii) The standard form of …….. is 0.00000003572.

(iv) Expanded form of 273.92 = ……..

(v) If $(-7)^x \times \left(\frac{1}{-7}\right)^{-3} = 7^8$, then value of x is …….

(vi) If $3^n = 729$, then n = ……

(vii) The value of $(7^0 + 5^0)$ is ……..

Solution:

(i) Multiplicative inverse of $\left(\frac{-4}{5}\right)^2$ or $\frac{(-4)^2}{5^2}$ or $\frac{16}{25}$ is $\frac{25}{16}$

(ii) The standard form of $0.000182 = 1.82 \times 10^{-4}$.

(iii) The standard form of 3.572×10^{-8} is 0.00000003572.

(iv) Expanded form of $273.92 = 2 \times 10^2 + 7 \times 10 + 3 \times 10^0 + 9 \times 10^{-1} + 2 \times 10^{-2}$.

(v) if $(-7)^x \times \left(\frac{1}{-7}\right)^{-3} = 7^8$, then value of x is …

If $(-7)^x \times \left(\frac{1}{-7}\right)^{-3} = 7^8$

$\Rightarrow (-7)^x \times \left(\frac{1}{-7}\right)^{-3} = 7^8$ {8 is even}

$\Rightarrow (-7)^x \times (-7)^{+3} = (-7)^8$

$\Rightarrow (-7)^{x+3} = (-7)^8$

Comparing, we get

$x + 3 = 8$

$\Rightarrow x = 8 - 3 = 5$

$\therefore x = 5$

(vi) If $3^n = 729$, then $n = \cdots$

3	729
3	243
3	81
3	27
3	9
3	3
	1

$3^n = 729 \Rightarrow 3^n = 3^6$

Comparing, $n = 6$

(vii) The value of $(7^0 + 5^0)$ is 1 + 1 = 2.

Question 2: State whether the following statements are true (T) or false (F):

(i) The value of $(-3)^{-2}$ is $\frac{1}{-9}$.

(ii) For negative integers m and n, $a^m \times a^n = a^{m+n}$ $(a \neq 0)$.

(iii) $3^0 \times 4^0 = 12^0$.

(iv) The value of $3^7 - 2^7$ is 1^7.

(v) The standard form of 0.0034 is 3.4×10^3.

(vi) Multiplicative inverse of $(-2)^5$ is $\frac{1}{-32}$.

Solution:

(i) The value of $(-3)^{-2}$ is $\frac{1}{-9}$. (False)

(ii) For negative integers m and n, $a^m \times a^n = a^{m+n}$ $(a \neq 0)$. (True)

(iii) $3^0 \times 4^0 = 12^0$. (True)

$\Rightarrow 1 \times 1 = 1$ ($a^0 = 1$)

(iv) The value of $3^7 - 2^7$ is 1^7. (False)

Correct:

$3^7 - 2^7 = 3 \times 3 \times 3$ ……. 7 times $- 2 \times 2 \times 2$ …. 7 times $\neq 1$

But $1^7 = 1$

(v) the standard form of 0.0034 is 3.4×10^3. (True)

(vi) Multiplicative inverse of $(-2)^5$ is $\frac{1}{-32}$. (True)

Multiplicative inverse of $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times$ ……

$= -32$

$\therefore$ Inverse $= \frac{1}{-32}$.

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 12):

Question 3.

$\left(\frac{1}{2}\right)^{-5}$ is equal to

(a) 32

(b) $\frac{1}{32}$

(c) -32

(d) $\frac{-1}{32}$

Solution:

$\left(\frac{1}{2}\right)^{-5} = (2)^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ (a)

Question 4: The value of $\left[\left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{3}\right)^{-2}\right] \div \left(\frac{1}{5}\right)^{-2}$ is

(a) 0 **(b) -1**

(c) 1 **(d) $\frac{7}{5}$**

Solution:

$$\left[\left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{3}\right)^{-2}\right] \div \left(\frac{1}{5}\right)^{-2}$$

$$= [(3)^2 + (4)^2] \div (5)^2$$

$$= (9 + 16) \div 25$$

$$= 25 \div 25 = 1 \qquad \text{(c)}$$

Question 5: If $2^4 \times 4^3 = 4^x$, then the value of x is

(a) 5

(b) 7

(c) 4

(d) -5

Solution:

$2^4 \times 4^3 = 4^x$

$\Rightarrow 2^4 \times (2^2)^3 = (2^2)^x$

$\Rightarrow 2^4 \times 2^6 = 2^{2x}$

$\Rightarrow 2^{10} = 2^{2x}$

$2x = 10$

$\Rightarrow x = 5$ (a)

Question 6: The value of $\left(\frac{729}{4096}\right)^{\frac{-1}{3}}$ is

(a) $\frac{16}{9}$ **(b) $\frac{-16}{9}$**

(c) $\frac{9}{16}$ **(d) $\frac{-9}{16}$**

Solution:

$$\left(\frac{729}{4096}\right)^{\frac{-1}{3}} = \left(\frac{4096}{729}\right)^{\frac{1}{3}} = \left(\frac{16^3}{9^3}\right)^{\frac{1}{3}}$$

$= \frac{16^{3\times\frac{1}{3}}}{9^{3\times\frac{1}{3}}} = \frac{16}{9}$ (a)

Question 7. $\left(\frac{-4}{3}\right)^{7} \times \left(\frac{-4}{3}\right)^{8} \div \left(\frac{-3}{4}\right)^{-5}$ **is equal to**

(a) $\left(\frac{-4}{3}\right)^{20}$ **(b)** $\left(\frac{-4}{3}\right)^{10}$

(c) $\left(\frac{-3}{4}\right)^{10}$ **(d)** $\left(\frac{-3}{4}\right)^{20}$

Solution:

$\left(\frac{-4}{3}\right)^{7} \times \left(\frac{-4}{3}\right)^{8} \div \left(\frac{-3}{4}\right)^{-5}$

$= \left(\frac{-4}{3}\right)^{7} \times \left(\frac{-4}{3}\right)^{8} \div \left(\frac{4}{-3}\right)^{5}$

$= \left(\frac{-4}{3}\right)^{7+8-5} = \left(\frac{-4}{3}\right)^{10}$

Question 8: Standard form of 0.000000789 is

(a) 78.9×10^{-8}

(b) 789×10^{-9}

(c) 7.89×10^{-7}

(d) None

Solution:

$0.000000789 = 7.89 \times 10^{-7}$ (c)

Question 9: The usual form of 3.67×10^{5} **is**

(a) 3670000

(b) 367000

(c) 36700

(d) 3670
Solution:
$3.67 \times 10^5 = 367000$

Question 10: Multiplicative inverse of $\left(\frac{-2}{3}\right)^4$ is
(a) $\frac{16}{81}$
(b) $\frac{-16}{81}$
(c) $\frac{81}{16}$
(d) $\frac{-81}{16}$
Solution:

Multiplicative inverse of $\left(\frac{-2}{3}\right)^4$ or

$\frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3}$ Or $\frac{16}{81}$ is $\frac{81}{16}$ (c)

Question 11: $2^0 + 3^0 + \left(\frac{1}{4}\right)^0$ is equal to

(a) $\frac{21}{4}$ **(b) 0**

(c) 9 **(d) 3**
Solution:

$2^0 + 3^0 + \left(\frac{1}{4}\right)^0 = 1 + 1 + 1 = 3$ $(\because a^0 = 1)$ (d)

Question 12: If $\left(\frac{12}{13}\right)^4 \times \left(\frac{13}{12}\right)^{-8} = \left(\frac{12}{13}\right)^{2x}$, then the value of x is

(a) -2 **(b) 6**

(c) 2 **(d) -6**

Solution:

$\left(\frac{12}{13}\right)^4 \times \left(\frac{13}{12}\right)^{-8} = \left(\frac{12}{13}\right)^{2x}$

$\Rightarrow \left(\frac{12}{13}\right)^4 \times \left(\frac{12}{13}\right)^8 = \left(\frac{12}{13}\right)^{2x}$

$\Rightarrow \left(\frac{12}{13}\right)^{4+8} = \left(\frac{12}{13}\right)^{2x} \Rightarrow \left(\frac{12}{13}\right)^{12} \times \left(\frac{12}{13}\right)^{2x}$

Comparing, $2x = 12 \Rightarrow x = \frac{12}{2} = 6$

$\therefore x = 6$ (b)

Value Based Questions

Question 1: Anaemia causes due to deficiency of Red Blood cells. The size of a red blood cell is 0.000000007 mm. Express it in standard form. Rahul is suffering from anemia, his doctor adviced him to take healthy food regularly and avoid junk food. Why should we eat healthy food? What values are being promoted?

Solution:

Size of a red blood cell = 0.000000007 mm = 7.0×10^{-9} mm

Deficiency of red blood cells causes anaemia so, in order to keep healthy, healthy food should be taken and avoid junk food.

Higher Order Thinking Skills (HOTS)

Question 1: Find the multiplicative inverse of $(5^0 + 3^0)(5^0 - 3^0)$.

Solution:

$(5^0 + 3^0)(5^0 - 3^0) = (1 + 1)(1 - 1) = 2 \times 0 = 0$
Multiplicative inverse of 0 does not exist.

Question 2: Evaluate $\left(\frac{x^p}{x^q}\right)^{p+q} \times \left(\frac{x^q}{x^r}\right)^{q+r} \times \left(\frac{x^r}{x^p}\right)^{r-p}$

Solution:

$$\left(\frac{x^p}{x^q}\right)^{p+q} \times \left(\frac{x^q}{x^r}\right)^{q+r} \times \left(\frac{x^r}{x^p}\right)^{r-p}$$

$$= (x^{p-q})^{p+q} \times (x^{q-r})^{q+r} \times (x^{r-p})^{r+p}$$

$$= x^{(p-q)(p+q)} \times x^{(q-r)(q+r)} \times x^{(r-p)(r+p)}$$

$$= x^{p^2-q^2} . x^{q^2-r^2} . x^{r^2-p^2}$$

$$= x^{p^2-q^2+q^2-r^2+r^2-p^2} = x^0 = 1$$

Question 3: Evaluate $\frac{x^p}{x^p+x^q} + \frac{1}{x^{p-q}+1}$

Solution:

$$\frac{x^p}{x^p+x^q} + \frac{1}{x^{p-q}+1}$$

$$\frac{x^p}{x^p+x^q} + \frac{1}{\frac{x^p}{x^q}+1}$$

$$\frac{x^p}{x^p+x^q} + \frac{1}{\frac{x^p+x^q}{x^q}}$$

$$\frac{x^p}{x^p+x^q} + \frac{x^p}{x^p+x^q} = \frac{x^p+x^q}{x^p+x^q} = 1$$

Check Your Progress

Question 1: Evaluate:

(i) $\left(\frac{25}{4}\right)^{\frac{-1}{2}}$ **(ii)** $\left(\frac{-3}{7}\right)^{-2}$

Solution:

(i) $\left(\frac{25}{4}\right)^{\frac{-1}{2}} = \left(\frac{4}{25}\right)^{\frac{1}{2}}$

$= \left[\left(\frac{2}{5}\right)^2\right]^{\frac{1}{2}} = \left(\frac{2}{5}\right)^{2\times\frac{1}{2}} = \left(\frac{2}{5}\right)^1 = \frac{2}{5}$

(ii) $\left(\frac{-3}{7}\right)^{-2} = \left(\frac{7}{-3}\right)^2$

$= \frac{7}{-3} \times \frac{7}{-3} = \frac{49}{9}$

Question 2: Simplify:

(i) $\left[\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-3}\right] \div (3)^2$

(ii) $\left[3^3 - \left(\frac{1}{2}\right)^{-3} \times \frac{1}{19}\right]$

(iii) $[2^{-1} + 3^0 + 5^1 + 7^2 + 9^3] \div \left(\frac{2}{3}\right)^{-1}$

Solution:

(i) $\left[\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-3}\right] \div (3)^2$

$= (2^3 + 3^3 + 4^3) \div (3)^2$

$= (8 + 27 + 64) \div 9 = \frac{99}{9} = 11$

(ii) $\left[3^3 - \left(\frac{1}{2}\right)^{-3} \times \frac{1}{19}\right]$

$= [3^3 - (2)^3] \times \frac{1}{19}$

$= (27 - 8) \times \frac{1}{19} = 19 \times \frac{1}{19} = 1$

(iii) $[2^{-1} + 3^0 + 5^1 + 7^2 + 9^3] \div \left(\frac{2}{3}\right)^{-1}$

$= \left(\frac{1}{2} + 1 + 5 + 49 + 729\right) \div \left(\frac{3}{2}\right)$

$= \left(\frac{1}{2} + 784\right) \div \frac{3}{2} = \frac{1569}{2} \div \frac{3}{2}$

$= \frac{1569}{2} \times \frac{2}{3} = \frac{1569}{3} = 523$

Question 3: Find the multiplicative inverse of $\left(\frac{125}{27}\right)^{\frac{-2}{3}}$

Solution:

$\left(\frac{125}{27}\right)^{\frac{-2}{3}} = \left(\frac{27}{125}\right)^{\frac{2}{3}} = \left(\frac{3^3}{5^3}\right)^{\frac{2}{3}}$

$= \frac{3^{3\times\frac{2}{3}}}{5^{3\times\frac{2}{3}}} = \frac{3^2}{5^2} = \frac{9}{25}$

∴ Multiplicative inverse $\frac{9}{25}$ is $\frac{25}{9}$

Question 4: Simplify and write in exponential form:

(i) $\frac{\mathbf{2^{-4}\times 15^{-3}\times 625}}{\mathbf{5^{2}\times 10^{-4}}}$

(ii) $\left(\frac{\mathbf{81}}{\mathbf{16}}\right)^{-\frac{\mathbf{1}}{\mathbf{2}}} \times \left(\frac{\mathbf{2}}{\mathbf{3}}\right)^{\mathbf{2}} \div \left(\frac{\mathbf{3}}{\mathbf{2}}\right)^{\mathbf{-2}}$

Solution:

(i) $\frac{2^{-4}\times 15^{-3}\times 625}{5^{2}\times 10^{-4}}$

5	625
5	125
5	25
5	5
	1

$= \frac{2^{-4}\times (3\times 5)^{-3}\times 5^{4}}{5^{2}\times (5\times 2)^{-4}}$

$= \frac{2^{-4}\times 3^{-3}\times 5^{-3}\times 5^{4}}{5^{2}\times 5^{-4}\times 2^{-4}}$

$= 2^{-4+4} \times 3^{-3} \times 5^{-3+4-5+4}$

$= 2^{0} \times \frac{1}{3^{3}} \times 5^{3} = 1 \times \frac{5^{3}}{3^{3}} = \left(\frac{5}{3}\right)^{3}$

$= \frac{125}{27}$

(ii) $\left(\frac{81}{16}\right)^{-\frac{1}{2}} \times \left(\frac{2}{3}\right)^{2} \div \left(\frac{3}{2}\right)^{-2}$

$= \left(\frac{3^{4}}{2^{4}}\right)^{-\frac{1}{2}} \times \left(\frac{2}{3}\right)^{2} \div \left(\frac{2}{3}\right)^{2}$

$= \left(\frac{3}{2}\right)^{4\times\left(\frac{-1}{2}\right)} \times \left(\frac{2}{3}\right)^{2} \div \left(\frac{2}{3}\right)^{2}$

$= \left(\frac{3}{2}\right)^{-2} \times \left(\frac{2}{3}\right)^{2} \div \left(\frac{2}{3}\right)^{2}$

$= \left(\frac{2}{3}\right)^{2} \times \left(\frac{2}{3}\right)^{2} \div \left(\frac{2}{3}\right)^{2}$

$= \left(\frac{2}{3}\right)^{2+2-2} = \left(\frac{2}{3}\right)^{2} = \frac{4}{9}$

Question 5: Simplify: $\frac{5^{n+2}-6+5^{n+1}}{13\times5^{n}-2\times5^{n+1}}$

Solution:

$\frac{5^{n+2}-6+5^{n+1}}{13\times5^{n}-2\times5^{n+1}}$

$= \frac{5^{n}\times5^{2}-6\times5\times5^{n}}{13\times5^{n}-2\times5\times5^{n}}$

$= \frac{5^{n}(5^{2}-6\times5)}{5^{n}(13-2\times5)}$

$= \frac{25-30}{13-10} = \frac{-5}{3} = -\frac{5}{3}$

Question 6: Prove that $(a + b)^{-1} (a^{-1} + b^{-1}) = (ab)^{-1}$.

Solution:

$(a + b)^{-1} (a^{-1} + b^{-1}) = (ab)^{-1}$

L.H.S. $= (a + b)^{-1} (a^{-1} + b^{-1})$

$= \frac{1}{(a+b)}\left[\frac{1}{a} + \frac{1}{b}\right]$

$= \frac{1}{(a+b)}\left[\frac{(a+b)}{ab}\right]$

$= \frac{1}{ab} = (ab)^{-1} =$ R. H. S.

Hence, Proved L.H.S. = R.H.S.

Question 7: Find the value of x for which

$$\left\{\left(\frac{-2}{7}\right)^{2}\right\}^{x} \times \left(\frac{-7}{2}\right)^{-1} = \frac{-8}{343}$$

Solution:

$$\left\{\left(\frac{-2}{7}\right)^{2}\right\}^{x} \times \left(\frac{-7}{2}\right)^{-1} = \frac{-8}{343}$$

$$= \left(\frac{-2}{7}\right)^{2x} \times \left(-\frac{2}{7}\right) = \frac{(-2)^3}{(7)^3}$$

$$= \left(\frac{-2}{7}\right)^{2x} \times \left(\frac{-2}{7}\right)^{1} = \left(\frac{-2}{7}\right)^{3}$$

$$\Rightarrow \left(\frac{-2}{7}\right)^{2x+1} = \left(\frac{-2}{7}\right)^{3}$$

Comparing, we get,

$$2x + 1 = 3 \Rightarrow 2x = 3 - 1 = 2$$

$$\Rightarrow x = \frac{2}{2} = 1$$

$$x = 1$$

Question 8: If $\frac{2^{-n} \times 8^{2n+1} \times 16^{2n}}{4^{3n}} = 16$, find the value of n.

Solution:

$$\frac{2^{-n} \times 8^{2n+1} \times 16^{2n}}{4^{3n}} = 16$$

$$\Rightarrow \frac{2^{-n} \times \left[(2)^3\right]^{2n+1} \times (2^4)^{2n}}{(2^2)^{3n}} = \frac{1}{2^4}$$

$$\Rightarrow \frac{2^{-n} \times 2^{6n+1} \times (2^4 2^{8n}}{2^{6n}} = \frac{1}{2^4}$$

$$\Rightarrow 2^{-n+6n+3+8n-6n} = 2^{-4}$$

$\Rightarrow 2^{7n+3} = 2^{-4}$

Comparing, we get,

$7n + 3 = -4 \Rightarrow 7n = -4 - 3$

$\Rightarrow 7n = -7 \Rightarrow n = -1$

Hence $n = -1$

Question 9: By what number should $\left[\left(\frac{-5}{2}\right)^3\right]^{-3}$ be multiplied to get $\left(\frac{-2}{5}\right)^5$?

Solution:

product of two numbers = $\left(\frac{-2}{5}\right)^5$

One number = $\left[\left(\frac{-5}{2}\right)^3\right]^{-3}$

Then second number

$= \left(\frac{-2}{5}\right)^5 \div \left[\left(\frac{-5}{2}\right)^3\right]^{-3}$

$= \left(\frac{-2}{5}\right)^5 \div \left[\left(\frac{2}{-5}\right)^3\right]^3$

$= \left(\frac{-2}{5}\right)^5 \div \left(\frac{-2}{5}\right)^9$

$= \left(\frac{-2}{5}\right)^{5-9} = \left(\frac{-2}{5}\right)^4$

$= \frac{(-2)\times(-2)\times(-2)\times(-2)}{5\times5\times5\times5} = \frac{16}{625}$

Question 10: By what number should $\left(\frac{-3}{2}\right)^{-4}$ be divided to get $\left(\frac{-2}{3}\right)^{3}$?

Solution:

Required number = $\left(\frac{-3}{2}\right)^{-4} \div \left(\frac{-2}{3}\right)^{3}$

$= \left(\frac{-2}{3}\right)^{4} \div \left(\frac{-2}{3}\right)^{3} = \left(\frac{-2}{3}\right)^{4-3}$

$= \left(\frac{-2}{3}\right)^{1} = \frac{-2}{3}$

Question 11: Express the following numbers in standard form:
(i) 0.0000000003904
(ii) 12730000000000
Solution:
(i) $0.0000000003904 = 3.904 \times 10^{-10}$
(ii) $12730000000000 = 1.273 \times 10^{13}$

Question 12: Express the following numbers in the usual form:
(i) 5.73×10^{-11}
(ii) 3.895×10^{15}.
Solution:
(i) $5.73 \times 10^{-11} = 0.0000000000573$
(ii) $3.895 \times 10^{15} = 3895000000000000$

Question 13: If $\left(\frac{9}{4}\right)^{-4} \times \left(\frac{2}{3}\right)^{3} = \left(\frac{p}{q}\right)^{11}$, then find the value of $\frac{p}{q}$.

Solution:

$\left(\frac{9}{4}\right)^{-4} \times \left(\frac{2}{3}\right)^{3} = \left(\frac{p}{q}\right)^{11}$

$$\Rightarrow \left[\left(\frac{3}{2}\right)^2\right]^{-4} \times \left(\frac{2}{3}\right)^3 = \left(\frac{p}{q}\right)^{11}$$

$$\Rightarrow \left(\frac{3}{2}\right)^{-8} \times \left(\frac{2}{3}\right)^3 = \left(\frac{p}{q}\right)^{11}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{8} \times \left(\frac{2}{3}\right)^3 = \left(\frac{p}{q}\right)^{11}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{8+3} = \left(\frac{p}{q}\right)^{11} \Rightarrow \left(\frac{2}{3}\right)^{11} = \left(\frac{p}{q}\right)^{11}$$

Comparing, we get

$\frac{p}{q} = \frac{2}{3}$

Question 14: Mass of the Earth is 5.97×10^{24} kg and mass of the Moon is 7.35×10^{22} kg. What is the difference of their masses?
Solution:
Mass of Earth = 5.97×10^{24} kg
and mass of Moon = 7.35×10^{22} kg
∴ Difference = $(5.97 \times 10^{24}) - (7.35 \times 10^{22})$
$= 10^{22} (5.97 \times 10^2 - 7.35)$
$= 10^{22} (597 - 7.35) = 589.65 \times 10^{22}$
$= 5.8965 \times 10^2 \times 10^{22}$
$= 5.8965 \times 10^{24}$