## Short Answer Questions-I (PYQ)

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### [2 Mark]

**Q.1. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $|x| > 1$ in simplest form.**

**Ans.**

$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$

Let $x = \sec\theta \quad \Rightarrow \quad \theta = \sec^{-1} x$

Now, $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta-1}}\right)$

$$= \cot^{-1}\left(\frac{1}{\tan\theta}\right) = \cot^{-1}(\cot\theta) = \theta = \sec^{-1} x$$

**Q.2. Write the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$.**

**Ans.**

$$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \cot^{-1}\left(-\cot\frac{\pi}{6}\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \cot^{-1}\left(\cot\left(\pi - \frac{\pi}{6}\right)\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \cot^{-1}\left(\cot\frac{5\pi}{6}\right)$$

$$= \frac{\pi}{3} - \frac{5\pi}{6} \qquad \left[\because \frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } \frac{5\pi}{6} \in (0, \pi)\right]$$

$$= \frac{2\pi - 5\pi}{6} = -\frac{\pi}{2}$$

**Q.3. What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?**

**Ans.**

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)\qquad\left[\because \frac{2\pi}{3}\notin\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$$

$$=\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

$$=\frac{2\pi}{3}+\frac{\pi}{3}$$

$$=\frac{3\pi}{3}=\pi \qquad \begin{bmatrix}\text{Note: By property of inverse functions}\\ \sin^{-1}(\sin x)=x \quad \text{if} \quad x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\\ \text{and} \quad \cos^{-1}(\cos x)=x \quad \text{if } x\in[0,\pi]\end{bmatrix}$$

**Q.4. Write the principal value of $\tan^{-1}(1)+\cos^{-1}(-12)$.**

**Ans.**

$$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)=\tan^{-1}\left(\tan\frac{\pi}{4}\right)+\cos^{-1}\left(\cos\left(\pi-\frac{\pi}{3}\right)\right)$$

$$=\tan^{-1}\left(\tan\frac{\pi}{4}\right)+\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$$

$$=\frac{\pi}{4}+\frac{2\pi}{3}\qquad\left[\because \frac{\pi}{4}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \text{ and } \frac{2\pi}{3}\in[0,\pi]\right]$$

$$=\frac{3\pi+8\pi}{12}=\frac{11\pi}{12}$$

**Q.5. Write the value of $\tan^{-1}\left[2sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.**

**Ans.**

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\tan^{-1}\left(2\sin\left(2\times\frac{\pi}{6}\right)\right)\qquad\left[\because \cos^{-1}\frac{\sqrt{3}}{2}=\frac{\pi}{6}\right]$$

$$=\tan^{-1}\left(2\sin\frac{\pi}{3}\right)=\tan^{-1}\left(2\times\frac{\sqrt{3}}{2}\right)$$

$$=\tan^{-1}(\sqrt{3})=\frac{\pi}{3}$$

**Q.6. Prove that: $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$**

**Ans.**

$$\text{RHS} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right) \qquad \left[\begin{array}{l}\because \ 0 < x < 1 \\ \Rightarrow \ 0 < \sqrt{x} < 1 \\ \Rightarrow \ \sqrt{x} \geq 0\end{array}\right.$$

$$= \frac{1}{2}.2\tan^{-1}\sqrt{x} = \tan^{-1}\sqrt{x} = \text{LHS}$$

**Q.7. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}, xy < 1$, then write the value of $x + y + xy$.**

**Ans.**

Given $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$

$$\Rightarrow \quad \tan^{-1}\left[\frac{x+y}{1-\text{xy}}\right] = \frac{\pi}{4} \quad [\because \text{xy} < 1]$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{x+y}{1-\text{xy}}\right] = \tan^{-1}1 \quad \Rightarrow \quad \frac{x+y}{1-\text{xy}} = 1 \quad \Rightarrow \quad x + y = 1 - \text{xy}$$

$$\Rightarrow \quad x + y + \text{xy} = 1$$

[**Note:** Principal value branches of sin $x$ and cos $x$ are $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$ respectively.]

## Short Answer Questions-I (OIQ)

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## [2 Mark]

**Q.1. Write the simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$.**

**Ans.**

Let $x = \tan\theta \quad \Rightarrow \quad \theta = \tan^{-1}x$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right] = \tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$$

$$= \tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right] = \tan^{-1}\left[\frac{1-\cos\theta}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right] = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}(\tan^{-1}x)$$

**Q.2. Express in the simplest form:**

$$tan-1\left(\frac{cosx-sinx}{cosx+sinx}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$$

**Ans**.

We have, $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

$$= \tan^{-1}\left(\frac{1 - \tan x}{1 + \sin x}\right)$$
$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - x\right)\right\} = \frac{\pi}{4} - x$$

$$\begin{bmatrix} \because & -\frac{\pi}{4} < x < \frac{\pi}{4} \\ \Rightarrow & \frac{\pi}{4} > -x > -\frac{\pi}{4} \\ \Rightarrow & \frac{\pi}{4} + \frac{\pi}{4} > \frac{\pi}{4} - x > -\frac{\pi}{4} + \frac{\pi}{4} \\ \Rightarrow & \left(\frac{\pi}{4} - x\right) \in \left(0, \frac{\pi}{2}\right) \subset \left(-\frac{\pi}{2}, \frac{\pi}{4}\right) \end{bmatrix}$$

**Q.3. Simplify:** $\mathbf{\tan^{-1}\left(\frac{a+bx}{b-ax}\right), x < \frac{b}{a}}$

**Ans.**

$$\tan^{-1}\left(\frac{a + bx}{b - ax}\right) = \tan^{-1}\left(\frac{\frac{a}{b} + x}{1 - \frac{a}{b}x}\right) = \tan^{-1}\frac{a}{b} + \tan^{-1}x$$

**Q.4. Simplify:** $\mathbf{\tan^{-1}\{1 + \sqrt{x^2} - x\}, x \in R}$

**Ans.**

Let $x = \cot\theta \quad \Rightarrow \quad \theta = \cot^{-1}x$

Now, $\tan^{-1}\{\sqrt{1 + x^2} - x\} = \tan^{-1}\{\sqrt{1 + \cot^2\theta} - \cot\theta\}$

$$= \tan^{-1}\{\operatorname{cosec}\theta - \cot\theta\} = \tan^{-1}\left\{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right\} = \tan^{-1}\left\{\frac{1 - \cos\theta}{\sin\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\} = \tan^{-1}\left\{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right\}$$
$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\cot^{-1}x$$