MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 24

Total Marks : 30

Max. Time : 30 min.

Topics : Limits, Straight Line, Continuity & Derivability, Function, Sequence & Series

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4	**(3 marks, 3 min.)**	**[12, 12]**
Multiple choice objective (no negative marking) Q.5,6	**(5 marks, 4 min.)**	**[10, 8]**
Subjective Questions (no negative marking) Q.7,8	**(4 marks, 5 min.)**	**[8, 10]**

1. $\lim\limits_{x\to 0^+} \dfrac{\left(\frac{\pi}{2}-\cot^{-1}\{x\}\right)x}{\operatorname{sgn}(x)-\cos x}$ (where {.} and sgn(.) denotes fractional part function and signum function respectively) is equal to :

(A) 2 (B) 1 (C) 0 (D) does not exist

2. Let $\lim\limits_{x\to 0} \dfrac{[x]^2}{x^2} = \ell$ and $\lim\limits_{x\to 0} \dfrac{[x^2]}{x^2}$ = m, then

(A) ℓ exists but m does not (B) m exists but ℓ does not

(C) ℓ and m both exist (D) neither ℓ nor m exists

3. Least value of function f(x) = $\dfrac{2\sec^2 x+2\sec x+1}{\sec^2 x+\sec x+5}$ is :

(A) 2 (B) $\frac{1}{5}$ (C) $\frac{2}{19}$ (D) $\frac{5}{7}$

4. Through the centriod of an equilateral triangle a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the distance of P from the base of the triangle. Let h_1 and h_2 be the distance of P from the other two sides of the triangle, then

(A) h is the H.M. of h_1, h_2 (B) h is the G.M. of h_1, h_2

(C) h is the A.M. of h_1, h_2 (D) none of these

5. Given two straight lines x – y – 7 = 0 and x – y + 3 = 0. Equation of a line which divides the distance between them in the ratio 3 : 2 (internally) can be :

(A) x – y – 1 = 0 (B) x – y – 3 = 0 (C) y = x (D) x – y + 1 = 0

6. If f(x) = [x], g(x) = $\begin{cases} 0 & , \quad x \in Z \\ x^2 & , \quad x \in (R - Z) \end{cases}$, then (where [.] is greatest integer function)

(A) $\lim\limits_{x \to 1}$ g(x) exists but g(x) is discontinuous at x = 1

(B) $\lim\limits_{x \to 1}$ f(x) does not exist and f(x) is not continuous at x = 1

(C) gof is continuous function

(D) g(x) is discontinuous at all integer points

7. Let f(x) = cosec 2x + cosec 2^2 x + cosec 2^3 x + ...+ cosec 2^n x , $x \in \left(0, \frac{\pi}{2}\right)$ and g(x) = f(x) + cot 2^n x.

If H(x) = $\begin{cases} (\cos x)^{g(x)} + (\sec x)^{\cos ecx} & \text{if} \quad x > 0 \\ p & \text{if} \quad x = 0 \\ \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x} & \text{if} \quad x < 0 \end{cases}$. Find the value of p, if possible to make the function H(x)

continuous at x = 0.

8. (i) If $\lim\limits_{x \to 0} \dfrac{729^x - 243^x - 81^x + 9^x + 3^x - 1}{x^3}$ = K(ℓn3)^3, then find the value of k.

(ii) If $\lim\limits_{x \to 0} \dfrac{(1 + a^3) + 8e^{\frac{1}{x}}}{1 + (2 + b + b^2)e^{\frac{1}{x}}}$ = 2, where a, b $\in$ R, then find the possible ordered pair (a, b).

Answers Key

1. (A) **2.** (B) **3.** (B) **4.** (C)

5. (A)(B) **6.** (A)(B)(C) **7.** p = 2

8. (i) 6 (ii) (1, 1) and (1, –2)