# Class: XII
# SESSION : 2022-2023
# SUBJECT: Mathematics
# SAMPLE QUESTION PAPER - 1
# with SOLUTION

**Time Allowed: 3 Hours** **Maximum Marks: 80**

***General Instructions :***

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**4 marks each) with sub parts.

## Section A

1. $\int \sqrt{4-x^2}\, dx = ?$ **[1]**

a) None of these
b) $\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C$
c) $x\sqrt{4-x^2} + \sin^{-1}\frac{x}{2} + C$
d) $\frac{1}{2}x\sqrt{4-x^2} - 2\sin^{-1}\frac{x}{2} + C$

2. $[\hat{i} \quad \hat{j} \quad \hat{k}] = ?$ **[1]**

a) 3
b) 1
c) 2
d) 0

3. If P(A) = $\frac{2}{5}$, P(B) = $\frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P\left(\frac{A'}{B'}\right) \cdot P\left(\frac{B'}{A'}\right)$ is equal to **[1]**

a) $\frac{25}{42}$
b) $\frac{5}{6}$
c) $\frac{5}{7}$
d) 1

4. The angle between two lines having direction ratios 1, 1, 2 and $(\sqrt{3}-1)$, $(-\sqrt{3}-1)$, 4 is **[1]**

a) $\frac{\pi}{4}$
b) $\frac{\pi}{3}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{2}$

5. Two numbers are selected at random from integers 1 through 9. If the sum is even, what is the probability that both numbers are odd? **[1]**

a) $\frac{5}{8}$
b) $\frac{1}{6}$
c) $\frac{4}{9}$
d) $\frac{2}{3}$

6. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to **[1]**

a) $\frac{-e^x}{1+x^2} + C$

b) $\frac{e^x}{(1+x^2)} +$ C

c) $\frac{-e^x}{(1+x^2)^2} +$ C

d) $\frac{e^x}{(1+x^2)^2} + C$

7. The direction ratios of two lines are 3, 2, -6 and 1, 2, 2 respectively. The acute angle between these lines is **[1]**

a) $\cos^{-1}\left(\frac{5}{18}\right)$

b) $\cos^{-1}\left(\frac{8}{21}\right)$

c) $\cos^{-1}\left(\frac{5}{21}\right)$

d) $\cos^{-1}\left(\frac{3}{20}\right)$

8. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is **[1]**

a) $20\pi^2$ sq. units

b) $25\pi$ sq. units

c) $20\pi$ sq. units

d) $16\pi^2$ sq. units

9. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle $\theta$, then the value of $|\vec{a} - \vec{b}|$ is **[1]**

a) $2\cos\frac{\theta}{2}$

b) $2\sin\frac{\theta}{2}$

c) $2\cos$

d) $2\sin\theta$

10. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to **[1]**

a) $4\pi^2$ sq units

b) $4\pi$ sq units

c) $2\pi$ sq units

d) $2\sqrt{2}\pi$ sq units

11. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is **[1]**

a) 2

b) $\frac{3}{2}$

c) not defined

d) 4

12. $\int \sin^3(2x + 1)\, dx = ?$ **[1]**

a) $\frac{1}{2}\cos(2x+1) + \frac{1}{3}\cos^3(2x+1) + C$

b) $-\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$

c) $\frac{1}{8}\sin^4(2x+1) + C$

d) None of these

13. If $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and A + A' = I, if the value of $\alpha$ is **[1]**

a)

b)

$\frac{\pi}{6}$ $\frac{\pi}{3}$

c) $\frac{3\pi}{2}$ d) $\pi$

14. Function f(x) = $2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when **[1]**

a) x > 2 b) 1 < x < 2

c) x < 2 d) x > 3

15. Let A be a non-singular square matrix of order 3 × 3. Then |adj A| is equal to **[1]**

a) | A | b) 3| A |

c) $| A |^3$ d) $| A |^2$

16. If A is a 3 × 3 matrix such that |A| =8, then |3 A| equals. **[1]**

a) 8 b) 72

c) 216 d) 24

17. What is the equation of a curve passing through (0, 1) and whose differential equation is given by dy = y tan x dx? **[1]**

a) y = sec x b) y = sin x

c) y = cosec x d) y = cos x

18. Domain of $\cos^{-1}x$ is **[1]**

a) [-1, 0] b) [0, 1]

c) None of these d) [-1, 1]

19. **Assertion (A):** If manufacturer can sell x items at a price of ₹$(5 - \frac{x}{100})$ each. The cost price of x items is ₹$(\frac{x}{5} + 500)$ . Then, the number of items he should sell to earn maximum profit is 240 items. **[1]**
**Reason (R):** The profit for selling x items is given by $\frac{24}{5}x - \frac{x^2}{100}$ - 300.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** The matrix A = $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular. **[1]**
**Reason (R):** A square matrix A is said to be singular, if |A| - 0.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

## Section B

21. Write the cofactor of $a_{12}$ in the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ **[2]**

OR

Solve the system of equations by matrix method
$8x + 4y + 3z = 18$
$2x + y + z = 5$
$x + 2y + z = 5$

22. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$ . **[2]**

23. Verify that $y = (a + bx)e^{2x}$ is the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$. **[2]**

24. Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is 3/5, find the probability that at least one of A and B will solve the problem. **[2]**

25. For what value of $\lambda$ are the vectors $\vec{a}$ and $\vec{b}$ perpendicular to each other? where; $\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 9\hat{j} + 2\hat{k}$ **[2]**

## Section C

26. Find the particular solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$ , given that y = 1, when x = 0. **[3]**

OR

Solve the initial value problem: $(x^2 + 1)\, y' - 2xy = (x^4 + 2x^2 + 1) \cos x$ , $y(0) = 0$

27. Prove $\int_0^{\frac{\pi}{4}} 2\tan^3 x\,dx = 1 - \log 2$ **[3]**

28. Evaluate: $\int \frac{x^2}{(x^4 - x^2 - 12)}dx$. **[3]**

OR

Evaluate: $\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$

29. Show that the points A (1, 2, 7), B (2, 6, 3) and C(3,10, -1) are collinear. **[3]**

OR

Prove using vectors: The quadrilateral obtained by joining midpoints of adjacent sides of a rectangle is a rhombus.

30. If $x = a(1 - \cos^3\theta)$, $y = a\sin^3\theta$, prove that $\frac{d^2y}{dx^2} = \frac{32}{27a}$ at $\theta = \frac{\pi}{6}$ **[3]**

31. Sketch the region bounded by the curve $y = 2x - x^2$ and the x-axis and find it's area. **[3]**

## Section D

32. Minimize Z = x + 2y subject to $2x + y \geqslant 3, x + 2y \geqslant 6, x, y \geqslant 0$ . Show that the minimum of Z occurs at more than two points. **[5]**

33. Show that the function f: $R_0 \to R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where $R_0$ is the set non-zero real numbers. Is the result true, if the domain $R_0$ is replaced by N with co-domain being same as $R_0$? **[5]**

OR

Show that the function f : R → R defined by $f(x) = \frac{x}{x^2+1}, \forall x \in R$ , is neither one-one nor onto.

34. Find the vector equation of the line passing through (1, 2, 3) and parallel to each of the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$ . Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$ . **[5]**

OR

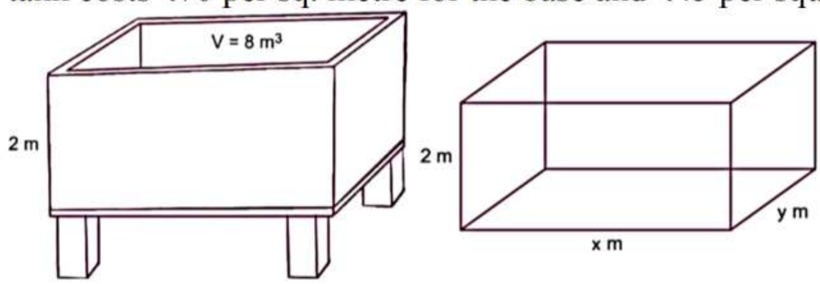
Show that the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$ intersect. Also, find their point intersection.

35. Find $\frac{dy}{dx}$ of the function $x^y + y^x = 1$ **[5]**

## Section E

36. **Read the text carefully and answer the questions:** **[4]**

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 $m^3$ as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.




(i) Express making cost C in terms of length of rectangle base.

(ii) If x and y represent the length and breadth of its rectangular base, then find the relation between the variables.

(iii) Find the value of **x** so that the cost of construction is minimum.

**OR**

Verify by second derivative test that cost is minimum at a critical point.

37. **Read the text carefully and answer the questions:** **[4]**

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.



(i) Write the matrix summarizing sales data of 2019 and 2020.

(ii) Find the matrix summarizing sales data of 2020.

(iii) Find the total number of cars sold in two given years, by each dealer?

**OR**

If each dealer receives a profit of ₹ 50000 on sale of a Hatchback, ₹100000 on sale of a Sedan and ₹200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

38. **Read the text carefully and answer the questions:** **[4]**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

(i) Teacher ask Govind, what is the probability that tickets are drawn by Abhishek, shows a prime number on one ticket and a multiple of 4 on other ticket?

(ii) Teacher ask Girish, what is the probability that tickets drawn by Ankit, shows an even number on first ticket and an odd number on second ticket?

# SOLUTION

## Section A

1. **(b)** $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

**Explanation:** The given integral is $\int\sqrt{4-x^2}dx$

Using $\int\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\int\sqrt{4-x^2}dx=\int\sqrt{2^2-x^2}dx$

$=\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

2. **(b)** 1

**Explanation:** $[\hat{i}\quad\hat{j}\quad\hat{k}]=\hat{i}.(\hat{j}\times\hat{k})=\hat{i}.\hat{i}\quad\ldots(\because\hat{j}\times\hat{k}=\hat{i})=|\hat{i}|^2=1$

3. **(a)** $\frac{25}{42}$

**Explanation:** Here, $P(A)=\frac{2}{5}$, $P(B)=\frac{3}{10}$ and $P(A\cap B)=\frac{1}{5}$

$$P\left(A'/B'\right)=\frac{P\left(A'\cap B'\right)}{P(B')}=\frac{1-P(A\cup B)}{1-P(B)}$$

$$=\frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$$

$$=\frac{1-\left(\frac{2}{5}+\frac{3}{10}-\frac{1}{5}\right)}{1-\frac{3}{10}}$$

$$=\frac{1-\left(\frac{4+3-2}{10}\right)}{\frac{7}{10}}=\frac{1-\frac{1}{2}}{\frac{7}{10}}=\frac{5}{7}$$

And $P\left(B'/A'\right)=\frac{P\left(B'\cap A'\right)}{P\left(A'\right)}=\frac{1-P(A\cup B)}{1-P(A)}$

$$= \frac{1-\frac{1}{2}}{1-\frac{2}{5}} = \frac{1/2}{3/5} = \frac{5}{6}\left[\because P(A \cup B) = \frac{1}{2}\right]$$

$$\therefore P\left(A'/B'\right) \cdot P\left(B'/A'\right) = \frac{5}{7} \cdot \frac{5}{6} = \frac{25}{42}$$

4. **(d)** $\frac{\pi}{2}$

**Explanation:** Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = (\sqrt{3}-1)\hat{i} + (-\sqrt{3}-1)\hat{j} + 4\hat{k}$

$|\vec{a}| = \sqrt{6}, |\vec{b}| = \sqrt{(4-2\sqrt{3}) + (4+2\sqrt{3}) + 16} = 2\sqrt{6}$

$$\cos\alpha = \frac{(\hat{i}+\hat{j}+2\hat{k}) \cdot ((\sqrt{3}-1)\hat{i} + (-\sqrt{3}-1)\hat{j} + 4\hat{k})}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos\alpha = \frac{\sqrt{3}-1-\sqrt{3}-1+8}{12}$$

$$\cos\alpha = \frac{1}{2}$$

$\alpha = 60^\circ$

5. **(a)** $\frac{5}{8}$

**Explanation:** The sum will be even when; both numbers are either even or odd, i.e. for both numbers to be even, the total cases ${}^5C_1 \times {}^4C_1$ (Both the numbers are odd)+${}^4C_1 \times {}^3C_1$ (Both the numbers are even) = 32

The favourable number of cases will be,

Both odd, i.e. selecting numbers from 1, 3, 5, 7, or 9, i.e.

${}^5C_1 \times {}^4C_1 = 20$

Thus, the probability that both numbers are odd will be

$$= \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

$$\Rightarrow \frac{20}{32} = \frac{5}{8}$$

6. **(b)** $\frac{e^x}{\left(1+x^2\right)} + C$

**Explanation:** Given $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$

$$\Rightarrow \int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx = \int e^x\left(\frac{1+x^2-2x}{\left(1+x^2\right)^2}\right)dx$$

$$\Rightarrow \int e^x\left(\frac{1+x^2-2x}{\left(1+x^2\right)^2}\right)dx = \int e^x\left\{\left(\frac{1+x^2}{\left(1+x^2\right)^2}\right)+\left(\frac{-2x}{\left(1+x^2\right)^2}\right)\right\}dx$$

$$= \int e^x\left\{\left(\frac{1}{\left(1+x^2\right)}\right)+\left(\frac{-2x}{\left(1+x^2\right)^2}\right)\right\}dx$$

Now using the property: $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)$

Now in $\int e^x\left\{\left(\frac{1}{\left(1+x^2\right)}\right)+\left(\frac{-2x}{\left(1+x^2\right)^2}\right)\right\}dx$

$$\Rightarrow \ f(x) = \frac{1}{\left(1+x^2\right)}$$

$$\Rightarrow \ f'(x) = \frac{-2x}{\left(1+x^2\right)^2}$$

$$\Rightarrow \int e^x\left\{\left(\frac{1}{\left(1+x^2\right)}\right)+\left(\frac{-2x}{\left(1+x^2\right)^2}\right)\right\}dx = \frac{e^x}{1+x^2}+C$$

$$\Rightarrow \int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx = \frac{e^x}{1+x^2}+C.$$

Which is the required solution.

7. **(c)** $\cos^{-1}\left(\frac{5}{21}\right)$

**Explanation:** Direction ratios are given implies that we can write the parallel vector towards that line, lets consider first parallel vector to be $\vec{a} = 3\hat{i}+2\hat{j}-6\hat{k}$ and second parallel vector be $\vec{b} = \hat{i}+2\hat{j}+2\hat{k}$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{3^2 + 2^2 + (-6)^2}$

$= 7$

$|\vec{b}| = \sqrt{1 + 2^2 + 2^2}$

$= 3$

$$\Rightarrow \cos\alpha = \frac{(3\hat{i} + 2\hat{j} - 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{7 \times 3}$$

$$\Rightarrow \cos\alpha = \frac{3 + 4 - 12}{21}$$

$$\Rightarrow \cos\alpha = \frac{-5}{21}$$

$$\therefore \alpha = \cos^{-1}\left(-\frac{5}{21}\right)$$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant.

$$\alpha = \cos^{-1}\left(\frac{5}{21}\right)$$

8. **(c)** $20\pi$ sq. units

**Explanation:** The area of the standard ellipse is given by ; $\pi ab$. Here, a = 5 and b = 4 Therefore, the area of curve is $\pi(5)(4) = 20\pi$.

9. **(b)** $2\sin\frac{\theta}{2}$

**Explanation:** Given $\vec{a}$ and $\vec{b}$ are unit vectors with inclination is $\theta$

now, $|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}).(\vec{a} - \vec{b}) = |\vec{a}|^2 - \vec{a}.\vec{b} - \vec{b}.\vec{a} + |\vec{b}|^2$

$=1-2(\vec{a}.\vec{b}) + 1$

$=2-2|\vec{a}||\vec{b}|cos\theta$

$=2-2cos\theta$ (where vectors are unit vectors)

$=2(1-\cos\theta)$

$=4sin^2\frac{\theta}{2}$

thus $|\vec{a} - \vec{b}|^2 = 4sin^2\frac{\theta}{2}$

$\therefore |\vec{a} - \vec{b}| = 2\sin\frac{\theta}{2}$

10. **(c)** $2\pi$ sq units

**Explanation:** Since Area = $4\int_0^{\sqrt{2}} \sqrt{2 - x^2}$

$$= 4\left(\frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right)_0^{\sqrt{2}} = 2\pi \text{ sq. units}$$

11. **(a)** 2

**Explanation:** In general terms for a polynomial the degree is the highest power.

The differential equation is $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}=\frac{d^2y}{dx^2}$

Square both the sides

$\Rightarrow \left(1+\left(\frac{dy}{dx}\right)^2\right)^3=\left(\frac{d^2y}{dx^2}\right)^2$

Now for degree to exist the given differential equation must be a polynomial in some differentials.

Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or .... $\frac{d^ny}{dx^n}$

The given differential equation is polynomial in differentials $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

Degree of differential equation is defined as the highest integer power of highest order derivative in the equation.

Here the highest derivative is $\frac{d^2y}{dx^2}$ and there is only one term of highest order derivative in the equation which is $(\frac{d^2y}{dx^2})^2$ whose power is 2 hence degree is 2.

12. **(b)** $-\frac{1}{2}\cos(2x+1)+\frac{1}{6}\cos^3(2x+1)+C$

**Explanation: Formula :-** $\int x^n dx=\frac{x^{n+1}}{n+1}+c; \int\frac{1}{1+x^2}dx=\tan^{-1}x+c$

Therefore ,

$\Rightarrow \int \sin^2(2x+1)\sin(2x+1)dx=\int(1-\cos^2(2x+1))\sin(2x+1)dx$

$=\int \sin(2x+1)dx-\int \cos^2(2x+1)\sin(2x+1)dx$

Put $\cos(2x+1)=t$

$\Rightarrow -2\sin(2x+1)dx=dt$

$I=-\int\frac{dt}{2}-\left(-\frac{1}{2}\right)\int t^2dt$

$=-\frac{1}{2}\int dt+\frac{1}{2}\int t^2dt$

$=-\frac{1}{2}t+\frac{1}{2}\frac{t^3}{3}+c$

$$=-\frac{1}{2}t+\frac{t^3}{6}+c$$

$$=-\frac{1}{2}\cos(2x+1)+\frac{[\cos(2x+1)]^3}{6}+c$$

13. **(b)** $\frac{\pi}{3}$

**Explanation:** Given $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Therefore, $A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

Also given that A + A' = I ...(1)
(Putting the values in equation (1))

$$\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}\cos\alpha+\cos\alpha & -\sin\alpha+\sin\alpha\\ \sin\alpha-\sin\alpha & \cos\alpha+\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

We know the two matrices are equal only when all their corresponding elements or entries are equal i.e. if A = B, then $a_{ij}$ and $b_{ij}$ for all i and j.

This implies,

$2\cos\alpha = 1$

$$\Rightarrow \cos\alpha=\frac{1}{2}$$

$$\Rightarrow \cos\alpha=\cos\frac{\pi}{3}\quad\ldots\left(\because\cos\frac{\pi}{3}=\frac{1}{2}\right)$$

$$\Rightarrow \alpha=\frac{\pi}{3}$$

14. **(b)** 1 < x < 2

**Explanation:** 1 < x < 2

15. **(d)** $|A|^2$

**Explanation:** For a square matrix of order n × n,

We know that $A.\,adjA=|A|I$

Here, n=3

$$\therefore\quad |A.\,adjA|=|A|^n$$

$$|adjA|=|A|^{n-1}$$

So, $|AdjA|=|A|^{3-1}=|A|^2$

16. **(c)** 216

**Explanation:** Given A is a square matrix of order 3 and also |A| = 8

$|3A| = (3)^3 \times |A| = 27 \times 8 = 216$

17. **(a)** y = sec x

**Explanation:** The given differential equation of the curve is,

dy = y tan x dx $\Rightarrow \int \frac{dy}{y} = \int \tan x \cdot dx$ [on integrating]

$\Rightarrow$ log y = log sec x + log C $\Rightarrow$ log y = log C sec x

$\Rightarrow$ y = C sec x ...(i)

Since, the curve passes through the origin (0, 1), then

1 = C sec 0 $\Rightarrow$ C = 1

$\therefore$ Required equation of curve is, y = sec x

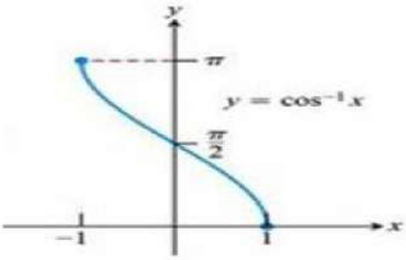
18. **(d)** [-1, 1]

**Explanation:** To Find: The range of $\cos^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \cos^{-1}(x)$ can be obtained from the graph of Y = cos x by interchanging x and y axes.i.e, if a, b is a point on Y = cos x then b, a is the point on the function $y = \cos^{-1}(x)$

Below is the Graph of the range of $\cos^{-1}(x)$




From the graph, it is clear that the domain of $\cos^{-1}(x)$ is [-1, 1]

19. **(c)** A is true but R is false.

**Explanation:** Let S(x) be the selling price of x items and let C(x) be the cost price of x items.

Then, we have

$S(x) = (5 - \frac{x}{100})x = 5x - \frac{x^2}{100}$

and $C(x) = \frac{x}{5} + 500$

Thus, the profit function P(x) is given by

$P(x) = S(x) - C(x) = 5x - \frac{x^2}{100} - \frac{x}{5} - 500$

i.e. $P(x) = \frac{24}{5}x - \frac{x^2}{100} - 500$

On differentiating both sides w.r.t. x, we get

$P'(x) = \frac{24}{5} - \frac{x}{50}$

Now, P'(x) = 0 gives x = 240.

Also, $P'(x) = \frac{-1}{50}$.

So, $P'(240) = \frac{-1}{50} < 0$

Thus, x = 240 is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

20. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** The determinant of the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix} = 8 - 8 = 0$

Hence, A is a singular matrix.

**Section B**

21. Given the matrix is, $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

We need to find the cofactor of $a_{12}$ in the matrix

$$\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$$

Firstly we know what the element at position $a_{12}$ in the matrix is.

$a_{12} = -3$

And as discussed above, the sign at $a_{12}$ is (-).

For cofactor of -3, eliminate first row and second column in the matrix.

Cofactor of $-3 = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix}$

$\Rightarrow$ Cofactor of $-3 = (6 \times -7) - (4 \times 1)$

$\Rightarrow$ Cofactor of $-3 = -42 - 4$

$\Rightarrow$ Cofactor of $-3 = -46$

Since, the sign of cofactor of -3 is (-), then

Cofactor of $-3 = -(-46)$

$\Rightarrow$ Cofactor of $-3 = 46$

Thus, the cofactor of -3 is 46.

OR

Given system of equations

$8x + 4y + 3z = 18$

$2x + y + z = 5$

$x + 2y + z = 5$

The given system can be re- written in matrix form as:

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} X \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

AX = B

Now, $|A| = 8\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$

$= 8(-1) - 4(1) + 3(3)$

$= -8 - 4 + 9$

$= -3$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$C_{11} = (-1)^{1+1}\ 1 - 2 = -1$

$C_{21} = (-1)^{2+1}\ 4 - 6 = 2$

$C_{31} = (-1)^{3+1}\ 4 - 3 = 1$

$C_{12} = (-1)^{1+2}\ 2 - 1 = -1$

$C_{22} = (-1)^{2+1}\ 8 - 3 = 5$

$C_{32} = (-1)^{3+1}\ 8 - 6 = -2$

$C_{13} = (-1)^{1+2}\ 4 - 1 = 3$

$C_{23} = (-1)^{2+1}\ 16 - 4 = -12$

$C_{33} = (-1)^{3+1}\ 8 - 8 = 0$

$$\text{adj}A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}A$$

Now, $X = A^{-1}B = \frac{1}{-3}\begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}\begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$

$$x = \frac{1}{3}\begin{bmatrix} -18+10+5 \\ -18+25-10 \\ 54-60+0 \end{bmatrix}$$

$$x = \frac{1}{-3}\begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Hence, X = 1,Y = 1 and Z = 2

22. We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1).$$

$$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$$

$$\begin{bmatrix} \because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0, \pi) \\ and \tan^{-1}(-x) = -\tan^{-1}x \end{bmatrix}$$

$$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi + 4\pi - 3\pi}{12}$$

$$= \frac{-5\pi + 4\pi}{12} = \frac{-\pi}{12}$$

23. $y = (a + bx)e^{2x}$

Differentiating above equation with respect to x,we get

$$\frac{dy}{dx} = be^{2x} + 2(a + bx)\, e^{2x}$$

On differentiating again w.r.t.x,we get

$$\frac{d^2y}{dx^2} = 2be^{2x} + 2be^{2x} + 4\,(a + bx)\, e^{2x}$$

Now, $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$

$= 2be^{2x} + 2be^{2x} + 4(a + bx)\ e^{2x} - 4be^{2x} - 8(a + bx)\ e^{2x} + 4(a + bx)\ e^{2x}$

$= 0$

$y = (a + bx)\ e^{2x}$ is the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

24. Given : Here probability of A and B that can solve the same problem is given , i.e., P(A) = $\frac{2}{3}$ and P(B) = $\frac{3}{5} \Rightarrow P(\bar{A}) = \frac{1}{3}$ and $P(\bar{B}) = \frac{2}{5}$

Also, A and B are independent . not A and not B are independent.

To Find: atleast one of A and B will solve the problem

Now , P(atleast one of them will solve the problem) = 1 - P(both are unable to solve)

$= 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}) \times P(\bar{B})$

$= 1 - \left(\frac{1}{3} \times \frac{2}{5}\right)$

$= \frac{13}{15}$

25. Since, $\vec{a}$ and $\vec{b}$ are perpendicular

$\therefore \vec{a}.\vec{b} = 0$

$\Rightarrow (\lambda\hat{i} + 2\hat{j} + \bar{k}) \cdot (4\hat{i} - 9\hat{j} + 2\hat{k}) - \mathbf{0}$

$\Rightarrow (\lambda)(4) + (2)(-9) + (1)(2) = 0$

$\Rightarrow 4\lambda - 18 + 2 = 0$

$\Rightarrow 4\lambda - 16 = 0$

$\Rightarrow 4\lambda = 16$

$\Rightarrow \lambda = \frac{16}{4}$

$\Rightarrow \lambda = 4$

**Section C**

26. We have,

$e^x\sqrt{1 - y^2}dx + \frac{y}{x}dy = 0$

$\Rightarrow \quad e^x\sqrt{1 - y^2}dx = \frac{-y}{x}dy$

Therefore,on separating the variables, we get,

$\frac{-y}{\sqrt{1-y^2}}dy = xe^x dx$

Therefore,on integrating both sides, we get,

$\int \frac{-y}{\sqrt{1-y^2}}dy = \int xe^x dx$

On putting $1 - y^2 = t \Rightarrow -ydy = \frac{dt}{2}$ in LHS, we get

$$\int \frac{1}{2\sqrt{t}} dt = \int \underset{III}{xe^x} dx$$

$$\Rightarrow \quad \frac{1}{2}[2\sqrt{t}] = x\int e^x dx - \int \left[\frac{d}{dx}(x)\int e^x dx\right] dx \text{ [using integration by parts]}$$

$$\Rightarrow \sqrt{1 - y^2} = xe^x - \int e^x dx \quad \left[\text{put } t = 1 - y^2\right]$$

$$\Rightarrow \sqrt{1 - y^2} = xe^x - e^x + C \text{ ...(i)}$$

Also, given that y = 1, when x = 0

On putting y = 1 and x = 0 in Eq. (i), we get

$$\sqrt{1 - 1} = 0 - e^0 + C$$

$$\Rightarrow \quad C = 1 \quad \left[\because e^0 = 1\right]$$

On substituting the value of C in Eq. (i), we get

$$\sqrt{1 - y^2} = xe^x - e^x + 1$$

which is the required particular solution of given differential equation.

OR

The given differential equation is:

$(x^2 + 1)\, y' - 2xy = (x^4 + 2x^2 + 1) \cos x$

$$\Rightarrow \frac{dy}{dx} - \frac{2x}{x^2+1} y = (x^2 + 1) \cos x \text{ ...(i)}$$

This is a linear differential equation with $P = \frac{-2x}{x^2+1}$ and $Q = (x^2 + 1) \cos x$

$$\therefore \text{ I.F. } = e^{\int\frac{-2x}{x^2+1} dx} = e^{-\log\left(x^2+1\right)} = (x^2 + 1)^{-1}$$

Multiplying (i) by $\frac{1}{x^2+1}$, we get

$$\frac{1}{x^2+1} \frac{dy}{dx} - \frac{2x}{\left(x^2+1\right)^2} y = \cos x$$

Integrating both sides with respect to x, we get

$$y \times \frac{1}{x^2+1} = \int \cos x \, dx + C$$

$$\Rightarrow \frac{y}{x^2+1} = \sin x + C \text{ ...(ii)}$$

It is given that y (0) = 0 i.e. y = 0 when x = 0

Put x = 0, y = 0 in (ii), we get: C = 0

Put C = 0 in (ii), we get

$$\frac{y}{x^2+1} = \sin x \Rightarrow y = (x^2 + 1)\sin x.$$

27. Given integral is: $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx$

To Prove: $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx = 1 - \log 2$

Let $I = \int_0^{\frac{\pi}{4}} 2\tan^3 x dx$ ...(i)

$= \int_0^{\frac{\pi}{4}} 2 \cdot \tan x \cdot \tan^2 x dx$

$= 2.\int_0^{\frac{\pi}{4}} \tan x \cdot \left(\sec^2 x - 1\right) dx$

$\Rightarrow I = 2\left\{-\int_0^{\frac{\pi}{4}} \tan x dx + \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx\right\}$

$\Rightarrow I = -[2\log\cos x]_0^{\pi/4} + 2. I_1$ ...(ii)

Solving $I_1$:

$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx$

$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx$

Let, tan x = t $\Rightarrow$ $\sec^2 x\, dx = dt$

When x = 0 then t = 0 and when $x = \frac{\pi}{4}$ then t = 1

$\Rightarrow I_1 = \int_0^1 t dt$

$= \left[\frac{t^2}{2}\right]_0^1$

$\Rightarrow I_1 = \frac{1}{2}$

Using this in equation (ii)

$\Rightarrow I = [2\log\cos x]_0^{\pi/4} + 2 \cdot \frac{1}{2}$

$\Rightarrow I = 2\left\{\log\cos\frac{\pi}{4} - \log\cos 0\right\} + 1$

$\Rightarrow I = 2\left\{\log\frac{1}{\sqrt{2}} - \log 1\right\} + 1$

$$\Rightarrow \text{I} = \left\{\log\left(\frac{1}{\sqrt{2}}\right)^2 - \log(1)^2\right\} + 1$$

$\Rightarrow$ I = 1 − log2 + log1

$\Rightarrow I = 1 - \log 2$

Hence Proved.

28. Let, $I = \int \frac{x^2}{\left(x^4 - x^2 - 12\right)} dx$

Using partial fractions,

$$\frac{x^2}{\left(x^4 - x^2 - 12\right)} = \frac{t}{t^2 - t - 12} = \frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3} \text{ .... (1)}$$

Where $t = x^2$

A(t + 3) + B(t - 4) = t

Now put t + 3 = 0

t = -3

A(0) + B(-7) = -3

$B = \frac{3}{7}$

Now put t - 4 = 0

t = 4

A(4 + 3) + B(0) = 4

$A = \frac{4}{7}$

From equation(1)

$$\frac{t}{(t-4)(t+3)} = \frac{4}{7} \times \frac{1}{t-4} + \frac{3}{7} \times \frac{1}{t+3}$$

$$\frac{x^2}{\left(x^2-4\right)\left(x^2+3\right)} = \frac{4}{7} \times \frac{1}{x^2 - 2^2} + \frac{3}{7} \times \frac{1}{x^2 + (\sqrt{3})^2}$$

$$\int \frac{x^2}{\left(x^2-4\right)\left(x^2+3\right)} dx = \frac{4}{7}\int \frac{1}{x^2 - 2^2} dx + \frac{3}{7}\int \frac{1}{x^2 + (\sqrt{3})^2} dx$$

$$= \frac{4}{7} \times \frac{1}{2} \times \frac{1}{2}\log\left|\frac{x-2}{x+2}\right| + \frac{3}{7} \times \frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + c$$

$$= \frac{1}{7}\log\left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7}\tan^{-1}\frac{x}{\sqrt{3}} + c$$

OR

Let the given integral be,

$$I = \int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$$

Let x = a $\tan^2\theta$

dx = 2a $\tan\theta \sec^2\theta \, d\theta$

$$I = (\text{siin}^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}\} \, (2a \tan\theta \sec^2\theta) \, d\theta$$

$$= \int \left(\sin^{-1}\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}\right)(2a \tan\theta \sec^2\theta) \, d\theta$$

$= \int \sin^{-1}(\sin\theta) \, (2a \tan\theta \sec^2\theta) \, d\theta$

$= 2a \int \theta \, (\tan\theta \sec^2\theta) \, d\theta$

$= 2a \, [\theta \int \tan\theta \sec^2\theta \, d\theta - \int(\int \tan\theta \sec^2\theta \, d\theta) \, d\theta]$

$$= 2a \left[\theta\frac{\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}d\theta\right]$$

$$= a\theta \tan^2\theta - \frac{2a}{2}\int (\sec^2\theta - 1) \, d\theta$$

$= a\theta \tan^2\theta - a \tan\theta + a\theta + c$

$$= a\left(\tan^{-1}\sqrt{\frac{x}{a}}\right)\frac{x}{a} - a\sqrt{\frac{x}{a}} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$$

$$I = x \tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$$

29. The given points are A (1, 2, 7), B (2, 6, 3) and C(3,10, - 1) respectively.

$\therefore$ Position vector of point $A = \overrightarrow{OA} = \hat{i} + 2\hat{j} + 7\hat{k}$

Position vector of point $B = \overrightarrow{OB} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

Position vector of point $C = \overrightarrow{OC} = 3\hat{i} + 10\hat{j} - \hat{k}$

Now $\overrightarrow{AB}$ = Position vector of point B – Position vector of point A

$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \left(\hat{i} + 2\hat{j} + 7\hat{k}\right)$

$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \hat{i} - 2\hat{j} - 7\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$…(i)

And $\overrightarrow{AC}$ = Position vector of point C – Position vector of point A

$= 3\hat{i} + 10\hat{j} - \hat{k} - \left(\hat{i} + 2\hat{j} + 7\hat{k}\right)$

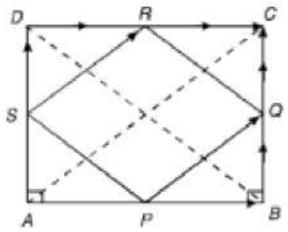
$= 3\hat{i} + 10\hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 7\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k} = 2\left(\hat{i} + 4\hat{j} - 4\hat{k}\right)$...(ii)

$\Rightarrow \overrightarrow{AC} = 2.\overrightarrow{AB}$ [Using eq. (i)]

$\Rightarrow$ Vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel. $\left[\because \vec{a} = mb\right]$

But $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have a common point A and hence they can't be parallel. Thus, the points A, B, C are collinear.

OR




ABCD be rectangle.
Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively,
Now

$\overline{PQ} - P\bar{B} + \overline{BQ} - \frac{1}{2}(\overline{AB} + B\bar{C}) - \frac{1}{2}\overline{AC}$ ....(i)

$\overline{SR} - \overline{SD} + \overrightarrow{DR} - \frac{1}{2}(\overrightarrow{AD} + \overline{DC}) - \frac{1}{2}\overline{AC}$ ....(ii)

From (i) and (ii) we have

$\overrightarrow{PQ} = \overrightarrow{SR}$ i, e. sides PQ and SR are equal and parallel
PQRS is a parallelogram.

$(PQ)^2 = \overline{PQ} \cdot \overline{PQ} = (P\bar{B} + \overline{BQ}) \cdot (\overline{PB} + \overline{BQ}) - PE^2 + BQ^2$.... (iii)

$(PS)^2 - PS \cdot PS - (PA + PS) \cdot (PA + PS) - PA|^2 + AS|^2 - PEP^2 + |BQ|^2$ ....(iv)

From (iii) and (iv) we get,
$(PQ)^2 = (PQ)^2$ i. e. $PQ = PS$
= The adjacent sides of PQRS are equal.
*PQRS* is a rhombus.

30. Given,

x = a (1- $\cos^3 \theta$) ...(i)

y = a $\sin^3 \theta$, ...(ii)

To prove: $\frac{d^2y}{dx^2} = \frac{32}{27a}$ at $\theta = \frac{\pi}{6}$

To find the above we will differentiate the function y wrt x twice.

As $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So, lets first find dy/dx using parametric form and differentiate it again.

$$\frac{dx}{d\theta} = \frac{d}{d\theta}a\left(1 - \cos^3\theta\right) = 3a\cos^2\theta\sin\theta \text{ ...(iii) [using chain rule]}$$

Similarly,

$$\frac{dy}{d\theta} = \frac{d}{d\theta}a\sin^3\theta = 3a\sin^2\theta\cos\theta \text{ ...(iv)}$$

$$\left[\because \frac{d}{dx}\cos x = -\sin x \& \frac{d}{dx}\cos x = \sin x\right]$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta} = \tan\theta$$

Differentiating again w.r.t. x:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan\theta)$$

$$\frac{d^2y}{dx^2} = \sec^2\theta\frac{d\theta}{dx} \text{ ...(v)}$$

[using chain rule and $\frac{d}{dx}\tan x = \sec^2 x$]

From equation (iii)

$$\frac{dx}{d\theta} = 3a\cos^2\theta\sin\theta$$

$$\therefore \frac{d\theta}{dx} = \frac{1}{3a\cos^2\theta\sin\theta}$$

Putting the value in equation ...(v)

$$\frac{d^2y}{dx^2} = \sec^2\theta\frac{1}{3a\cos^2\theta\sin\theta}$$

$$\frac{d^2y}{dx^2} = \frac{1}{3a\cos^4\theta\sin\theta}$$

Put $\theta = \pi/6$

$$\left(\frac{d^2y}{dx^2}\right) \text{at} \left(x = \frac{\pi}{6}\right) = \frac{1}{3a\cos^4\frac{\pi}{6}\sin\frac{\pi}{6}} = \frac{1}{3a\left(\frac{\sqrt{3}}{2}\right)^4\frac{1}{2}}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right) \text{at} \left(x = \frac{\pi}{6}\right) = \frac{32}{27a}$$

Hence proved

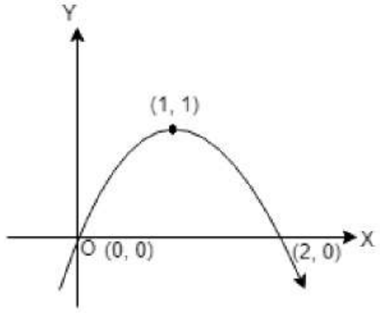
31. The equation of curve is

$y = 2x - x^2$............(1)

$\Rightarrow x^2 - 2x = -y$

$\Rightarrow x^2 - 2x + 1 = -y + 1$

$(x-1)^2 = -(y-1)$,which is a downward parabola with vertex (1,1).




putting y = 0 in (1),we get, $0 = 2x - x^2$,

$\therefore\ x(x-2) = 0$

$x = 0, 2$

$\therefore$ parabola meets x-axis at (0,0),(2,0)

$\therefore$ required area= Area bounded by the curve $y = 2x - x^2$ and the x-axis

$=\int_0^1 y\,dx$

$=\int_0^2 (2x - x^2)dx$

$$=\left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$=\left[4 - \frac{8}{3} - (0-0)\right]$$

$$=\frac{4}{3} sq\ units$$

**Section D**

32. Consider $2x + y \geqslant 3$

Let 2x + y = 3 $\Rightarrow$ y = 3 - 2x

| | | | |
|---|---|---|---|
| x | 0 | 1 | –1 |
| y | 3 | 1 | 5 |

(0, 0) is not contained in the required half plane as (0, 0) does not satisfy the inequation $2x + y \geqslant 3$.

Again $x + 2y \geqslant 6$

Let x + 2y = 6

$$\Rightarrow \frac{x}{6} + \frac{y}{3} = 1$$

Here also (0, 0) does not contain the required half plane. The double shaded region XABY is the solution set. Its corners are A (6, 0) and B (0, 3).

At A (6, 0) Z = 6 + 0 = 6

At B (0, 3) Z = 0 + 2 × 3 = 6

Therefore, at both points the value of Z = 6 which is minimum. In fact at every point on the line AB makes Z = 6 which is also minimum.

33. We observe the following properties of f.

Injectivity: Let x, y $\in$ $R_0$ such that f(x) = f(y). Then,

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f : $R_0 \rightarrow R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of $R_0$ (co-domain) such that f(x) = y. Then,

$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all y $\in$ $R_0$ (co-domain).

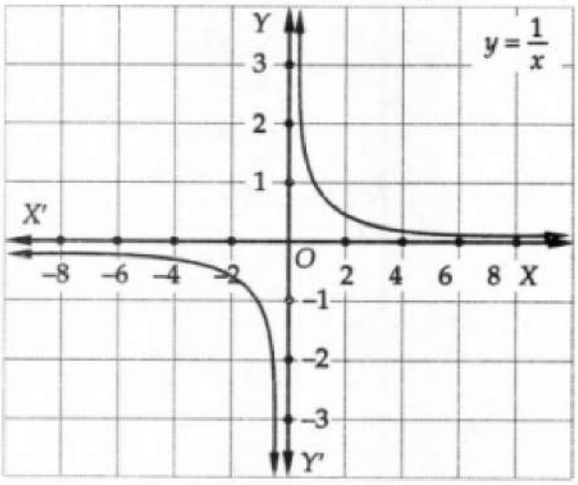
Thus, for each y $\in$ $R_0$ (co-domain) there exits $x = \frac{1}{y} \in R_0$(domain) such that

$f(x) = \frac{1}{x} = y$

So, f : $R_0 \rightarrow R_0$ is onto.

Hence, f: $R_0 \rightarrow R_0$ is one-one onto.

This is also evident from the graph of f(x) as shown in fig.




Let us now consider f : N $\rightarrow$ $R_0$ given by $f(x) = \frac{1}{x}$

For any x, y $\in$ N, we find that

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f: N $\rightarrow$ $R_0$ is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain $R_0$ do not have their pre-image in domain N. So, f: N $\rightarrow$ $R_0$ is not onto.

Thus, f: N $\rightarrow$ $R_0$ is one-one but not onto.

OR

For $x_1, x_2 \in$ R, consider

$f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$

$\Rightarrow x_1x_2^2 + x_1 = x_2x_1^2 + x_2$

$\Rightarrow x_1x_2(x_2 - x_1) = x_2 - x_1$

$\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$

We note that there are point, $x_1$ and $x_2$ with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$ for instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1)\frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$. Hence f is not one-one. Also, f is not onto for if so then for $1 \in R\ \exists\ x \in R$ such that f(x) = 1 which gives $\frac{x}{x^2+1} = 1$. But there is no such x in the domain R, since the equation $x^2 - x + 1 = 0$ does not give any real value of x.

34. Here the equation of two planes are: $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.
Since the line is parallel to the two planes.

$\therefore$ Direction of line $\vec{b} = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$

$= -3\hat{i} + 5\hat{j} + 4\hat{k}$

$\therefore$ Equation of required line is

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$ ........ (i)

Any point on line (i) is $(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$

For this line to intersect the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k})$ we have

$(1 - 3\lambda)2 + (2 + 5\lambda)1 + (3 + 4\lambda)1 = 4$

$\Rightarrow \lambda = 1$

$\therefore$ Point of intersection is (4, -3, -1)

OR

Here,it is given that

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

Here,

$\vec{a}_1 = i + 2\hat{j} + 3\hat{k}$

$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a}_2 = 4\hat{i} + \hat{j}$

$\vec{b}_2 = 5\hat{i} + 2\hat{j} + \hat{k}$

Thus,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(3-8) - \hat{j}(2-20) + \hat{k}(4-15)$

$\therefore \vec{b_1} \times \vec{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$

$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$

$= \sqrt{25 + 324 + 121}$

$= \sqrt{470}$

$\vec{a_2} - \vec{a_1} = (4-1)\hat{i} + (1-2)\hat{j} + (0-3)\hat{k}$

$\therefore \vec{a_2} - \vec{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$

Now,we have

$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k})$

$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$

$= -15 - 18 + 33$

$= 0$

Thus, the distance between the given lines is

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\therefore d = \left| \frac{0}{\sqrt{470}} \right|$$

$\therefore$ d = 0 units

As d = 0

Thus, the given lines intersect each other.

Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.

For that putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$\Rightarrow \vec{L}_1 : x\hat{i} + y\hat{j} + z\hat{k} = (i + 2j + 3\hat{k}) + \lambda(2i + 3\hat{j} + 4\hat{k})$

$\Rightarrow \vec{L}_2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow \vec{L}_1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$

$\Rightarrow \vec{L}_2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$

$$\Rightarrow \vec{L}_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\therefore \vec{L}_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$$

General point on L1 is

$x_1 = 2\lambda + 1, y_1 = 3\lambda + 2, z_1 = 4\lambda + 3$

Suppose, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Thus, point P satisfies the equation of line $\vec{L}_2$.

$$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3-0}{1}$$

$$\therefore \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2}$$

$\Rightarrow 4\lambda - 6 = 15\lambda + 5$

$\Rightarrow 11\lambda = -11$

$\Rightarrow \lambda = -1$

Thus, $x_1 = 2(-1) + 1, y_1 = 3(-1) + 2, z_1 = 4(-1) + 3$

$\Rightarrow x_1 = -1, y_1 = -1, z_1 = -1$

Therefore, point of intersection of given lines is (-1, -1, -1).

35. Given: $x^y + y^x = 1$

Let $y = x^y + y^x = 1$

Let $u = x^y$ and $v = y^x$

Then, $u + v = 1$

$$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$$

For, $u = x^y$

Taking log on both sides, we get

$\log u = \log x^y$

$\Rightarrow \log u = y \cdot \log(x)$

Now, differentiating both sides with respect to x

$$\frac{d}{dx}(\log u) = \frac{d}{dx}[y \cdot \log(x)]$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \left\{y \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(y)\right\}$$

$$\Rightarrow \frac{du}{dx} = u\left[y \cdot \frac{1}{x} + \log x \cdot \left(\frac{dy}{dx}\right)\right]$$

$$\Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x} + \log x \cdot \left(\frac{dy}{dx}\right)\right]$$

For $v = y^x$

Taking log on both sides, we get

$\log v = \log y^x$

$\Rightarrow \log v = x \cdot \log(y)$

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log v) = \frac{d}{dx}[x \cdot \log(y)]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left\{x \cdot \frac{d}{dx}(\log y) + \log y \cdot \frac{d}{dx}x\right\}$$

$$\Rightarrow \frac{dv}{dx} = v\left[x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot \left(\frac{dx}{dx}\right)\right]$$

$$\Rightarrow \frac{dv}{dx} = y^x\left[\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right]$$

because, $\frac{du}{dx} + \frac{dv}{dx} = 0$

So, $x^y\left[\frac{y}{x} + \log x \cdot \left(\frac{dy}{dx}\right)\right] + y^x\left[\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right] = 0$

$$\Rightarrow \left(x^y \log x + xy^{x-1}\right) \cdot \frac{dy}{dx} + \left(yx^{y-1} + y^x \log y\right) = 0$$

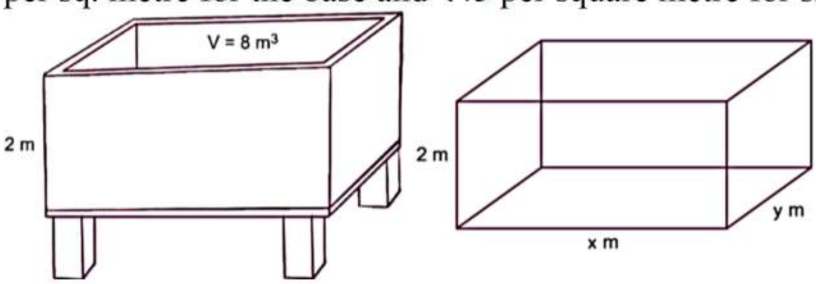
$$\Rightarrow \left(x^y \log x + xy^{x-1}\right) \cdot \frac{dy}{dx} = -\left(yx^{y-1} + y^x \log y\right)$$

$$\frac{dy}{dx} = -\frac{\left(yx^{y-1} + y^x \log y\right)}{\left(x^y \log x + xy^{x-1}\right)}$$

## Section E

36. **Read the text carefully and answer the questions:**

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 $m^3$ as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.




(i) Since 'C' is cost of making tank

$\therefore$ C = 70xy + 45 × 2(2x + 2y)

$\Rightarrow$ C = 70xy + 90(2x + 2y)

$\Rightarrow$ C = 70xy + 180(x + y) [ $\because$ $2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}$ ]

$$\Rightarrow C = 70x \times \frac{4}{x} + 180\left(x + \frac{4}{x}\right)$$

$$\Rightarrow C = 280 + 180\left(x + \frac{4}{x}\right)$$

(ii) $x \cdot y = 4$

Volume of tank = length × breadth × height (Depth)

$8 = x \cdot y \cdot 2$

$\Rightarrow 2xy = 8 \Rightarrow xy = 4$

(iii) For maximum or minimum

$$\frac{dC}{dx} = 0$$

$$\frac{d}{dx}(280 + 180(x + \frac{4}{x})) = 0 \Rightarrow 180\left(1 + 4\left(-\frac{1}{x^2}\right)\right) = 0$$

$$\Rightarrow 180\left(1 - \frac{4}{x^2}\right) = 0 \Rightarrow 1 - \frac{4}{x^2} = 0$$

$$\Rightarrow \frac{4}{x^2} = 1 \Rightarrow x^2 = 4$$

$\Rightarrow x = \pm 2$

$\Rightarrow x = 2$ (length can never be negative)

OR

$$\text{Now, } \frac{d^2C}{dx^2} = 180\left(+\frac{8}{x^3}\right)$$

$$\Rightarrow \left.\frac{d^2C}{dx^2}\right|_{x=2} = 180 \times \frac{8}{8} = 180 = +ve$$

Hence, to minimize C, x = 2m

37. **Read the text carefully and answer the questions:**

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.



(i) *Hatchback* *Sedan* *SUV*

$$\begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}$$

In 2019, dealer A sold 120 Hatchbacks, 50 Sedans and 10 SUV; dealer B sold 100 Hatchbacks, 30 Sedans and 5 SUVs and dealer C sold 90 Hatchbacks, 40 Sedans and 2 SUVs.

∴ Required matrix, say P, is given by

*Hatchback* *Sedan* *SUV*

$$P = \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}$$

In 2020, dealer A sold 300 Hatchbacks, 150 Sedans, 20 SUVs dealer B sold 200 Hatchbacks, 50 sedans, 6 SUVs dealer C sold 100 Hatchbacks, 60 sedans, 5 SUVs.

∴ Required matrix, say Q, is given by

*Hatchback* *Sedan* *SUV*

$$Q = \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}$$

(ii) *Hatchback* *Sedan* *SUV*

$$\begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}$$

In 2020, dealer A sold 300 Hatchback, 150 Sedan, 20 SUV dealer B sold 200 Hatchback, 50 sedan, 6 SUV dealer C sold 100 Hatchback, 60 sedan, 5 SUV.

∴ Required matrix, say Q, is given by

*Hatchback* *Sedan* *SUV*

$$Q = \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}$$

(iii)Total number of cars sold in two given years, by each dealer, is given by

*Hatchback* *Sedan* *SUV*

$$P + Q = \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 120 + 300 & 50 + 150 & 10 + 20 \\ 100 + 200 & 30 + 50 & 5 + 6 \\ 90 + 100 & 40 + 60 & 2 + 5 \end{bmatrix}$$

$$= \begin{matrix} & \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \\ A & 420 & 200 & 30 \\ B & 300 & 80 & 11 \\ C & 190 & 100 & 7 \end{matrix}$$

OR

The amount of profit in 2020 received by each dealer is given by the matrix

$$\begin{matrix} & \textit{Hatchback} & \textit{Sedan} & \textit{SUV} \\ A & 300 & 150 & 20 \\ B & 200 & 50 & 6 \\ C & 100 & 60 & 5 \end{matrix} \begin{bmatrix} 50000 \\ 100000 \\ 200000 \end{bmatrix}$$

$$= \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 15000000 + 15000000 + 4000000 \\ 10000000 + 5000000 + 1200000 \\ 5000000 + 6000000 + 1000000 \end{bmatrix}$$

$$= \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 34000000 \\ 16200000 \\ 12000000 \end{bmatrix}$$

38. **Read the text carefully and answer the questions:**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

(i) Required probability = P(one ticket with prime number and other ticket with a multiple of 4)

$$= 2\left(\frac{15}{50} \times \frac{12}{49}\right) = \frac{36}{245}$$

(ii) P(First ticket shows an even number and second ticket shows an odd number)

$$= \frac{25}{50} \times \frac{25}{49} = \frac{25}{98}$$