Progressions and Series

Tip 1

- Progressions and Series is one of the important topics for CAT and significant number of questions appear in the examination from this section every year.

- Some of the questions from this section can be very tough and time consuming while the others can be very easy.

- The trick to ace this section is to quickly figure out whether a question is solvable or not and not waste time on very difficult questions.

Tip 2

- Some of the questions in this section can be answered by ruling out wrong choices among the options available. This method will both save time and improve accuracy.

- There are many shortcuts which will be of vital importance in answering this section.

- This formula sheet contains an exhaustive list of various formulas and shortcuts.

Tip 3

There are 3 standard types of progressions

- Arithmetic Progression
- Geometric Progression
- Harmonic Progression

Tip 4

Arithmetic progression (A.P)

- If the sum or difference between any two consecutive terms is constant then the terms are said to be in A.P (Example: 2,5,8,11 or a, a+d, a+2d, a+3d…)
- If 'a' is the first term and 'd' is the common difference then the general 'n' term is $T_n = a + (n-1)d$
- Sum of first 'n' terms in A.P = $\frac{n}{2}[2a+(n-1)d]$
- Number of terms in A.P = $\frac{\text{Last term} - \text{First term}}{\text{Common difference}} + 1$
- Sum of all terms of an A.P $= \frac{n}{2}[\text{First term} + \text{Last term}]$

Tip 5

Properties of A.P

If a, b , c, d,…. are in A.P and ‘k’ is a constant then

- a-k, b-k, c-k,… will also be in A.P
- ak, bk, ck,…will also be in A.P
- a/k, b/k, c/k will also be in A.P

Tip 6

Geometric Progression

- If in a succession of numbers the ratio of any term and the previous term is constant then that numbers are said to be in Geometric Progression.

- Ex :1, 3, 9, 27 or a, ar, ar^2, ar^3

- The general expression of an G.P, $T_n = ar^{n-1}$(where a is the first terms and 'r' is the common ratio)

- Sum of 'n' terms in G.P, Sn = $\frac{a(1-r^n)}{1-r}$ (If r<1) or $\frac{a(r^n-1)}{r-1}$ (If r>1)

Tip 7

Properties of G.P

If a, b , c, d,…. are in G.P and ‘k’ is a constant then

1. ak, bk, ck,…will also be in G.P
2. a/k, b/k, c/k will also be in G.P

Sum of term of infinite series in G.P, $S_\infty = \frac{a}{1-r}$ $(-1 < r < 1)$

Tip 8

Harmonic Progression

- If a, b, c, d,……are unequal numbers then they are said to be in H.P if 1/a, 1/b, 1/c,……are in A.P

- The ‘n’ term in H.P is 1/(nth term in A.P)

Properties of H.P :

If a, b, c, d,…are in H.P, then

$$a+d > b+c$$

$$ad > bc$$

Tip 9 & 10

Arithmetic Geometric Series

- A series will be in arithmetic geometricseries if each of its term is formed by product of the corresponding terms of an A.P and G.P.

- The general form of A.G.P series is a, (a+d)r, (a+2d)r^2 , ...

- Sum of 'n' terms of A.G.P series

$$S_n = \frac{a - [a + (n-1)d]r^n}{1 - r} + \frac{dr(1 - r^{n-1})}{(1-r)^2} (r \neq 1)$$

- Sum of infinite terms of A.G.P series

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} (|r| < 1)$$

Tip 11

Standard Series

- The sum of first ‘n’ natural numbers $= \frac{n(n+1)}{2}$
- The sum of squares of first ‘n’ natural numbers $= \frac{n(n+1)(2n+1)}{6}$
- The sum of cubes of first ‘n’ natural numbers $= \left\{\frac{n(n+1)}{2}\right\}^2$
- The sum of first ‘n’ odd natural numbers $= n^2$
- The sum of first ‘n’ even natural numbers $= n(n+1)$
- In any series, if the sum of first n terms is given by S_n, then the n^{th} term $T_n = S_n - S_{n-1}$

Tip 12

Arithmetic mean

- The arithmetic mean = $\frac{\text{Sum of all the terms}}{\text{Number of Terms}}$

- If two number A and B are in A.P then arithmetic mean = $\frac{a+b}{2}$

Tip 13

Arithmetic mean

- Inserting ‘n’ means between two numbers a and b
- The total terms will become n+2, a is the first term and b is the last term
- Then the common difference d = $\frac{b-a}{n+1}$
- The last term b = a+(n+1)d
- The final series is a, a+d, a+2d,….

Tip 14

Geometric Mean

- If a, b, c,… n terms are in G.P then G.M = $\sqrt[n]{a \times b \times c \times \cdots n \text{ terms}}$
- If two numbers a, b are in G.P then their G.M = $\sqrt{a \times b}$

Tip 15

Geometric Mean

- Inserting ‘n’ means between two quantities a and b with common ration ‘r’
- Then the number of terms are n+2 and a, b are the first and last terms
- $r^{n+1} = \frac{b}{a}$ or $r = \frac{\sqrt[n+1]{b}}{a}$
- The final series is a, ar, ar^2,…

Tip 16

Harmonic Mean

- If a, b, c, d,.. are the given numbers in H.P then the Harmonic mean of ‘n’ terms = $\frac{\text{Number of terms}}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\cdots..}$

- If two numbers a and b are in H.P then the Harmonic mean = $\frac{2ab}{a+b}$

Tip 17

Relationship between AM, GM and HM for two numbers a and b,

- AM = $\frac{a+b}{2}$
- G.M = $\sqrt{a \times b}$
- H.M = $\frac{2ab}{a+b}$
- G.M = $\sqrt{AM \times HM}$
- A.M ≥ G.M ≥ H.M