Short Answer Type Questions – II

[3 marks]

Que 1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the length of PQ.

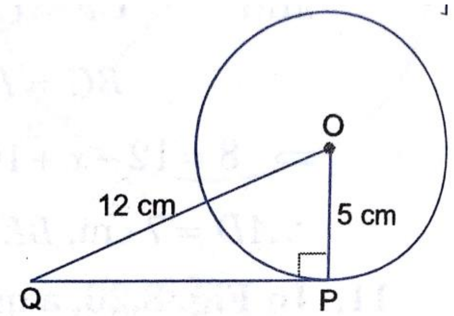

Fig. 8.30

Sol. We have, $\angle OPQ = 90°$

OQ = 12 cm and OP = 5 cm

∴ By Pythagoras Theorem

$OQ^2 = OP^2 + QP^2 \Rightarrow 12^2 = 5^2 + QP^2$

$\Rightarrow QP^2 = 144 - 25 = 119 \Rightarrow QP = \sqrt{119}\, cm$

Que 2. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.

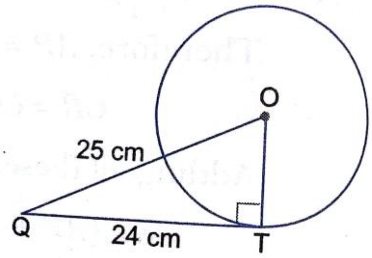

Fig. 8.31

Sol. Let QT be the tangent and OT be the radius of circle. Therefore

$OT \perp QT \quad i.e., \angle OTQ = 90°$

and OQ = 25 cm and QT = 24 cm

Now, by Pythagoras Theorem, we have

$OQ^2 = QT^2 + OT^2 \Rightarrow 25^2 = 24^2 + OT^2$

$\Rightarrow OT^2 = 25^2 - 24^2 = 625 – 576$

$OT^2 = 49 \quad \therefore \quad OT = 7\, cm$

Que 3. In Fig. 8.32, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then find $\angle PTQ$.

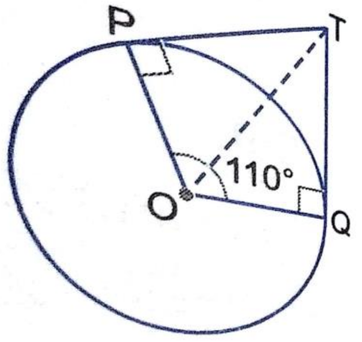

Fig. 8.32

Sol. Since TP and TQ are the tangents to the circle with centre O

So, OP ⊥ PT and OQ ⊥ QT

⇒ $\angle OPT = 90°, \angle OQT = 90°\ and\ \angle POQ = 110°$

So, in quadrilateral OPTQ, we have

$\angle POQ + \angle OPT + \angle PTQ + \angle TQO = 360°$

⇒ $110° + 90° + \angle PTQ + 90° = 360°$ ⇒ $\angle PTQ + 290° = 360°$

∴ $\angle PTQ = 360° - 290°$ ⇒ $\angle PTQ = 70°$

Que 4. Prove that the tangent drawn at the ends of a diameter of a circle are parallel.

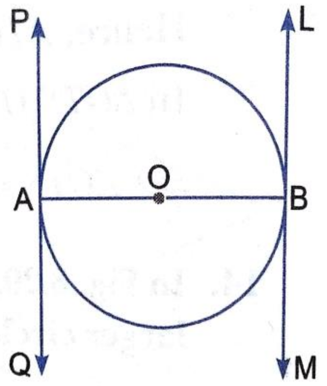

Fig. 8.33

Sol. Let AB be the diameter of the given circle with centre O, and two tangents PQ and LM are drawn at the end of diameter AB respectively.

Now, since the tangent at a point to a circle is perpendicular to the radius through the point of contact.

Therefore, $OA \perp PQ\ and\ OB \perp LM$

i.e., $AB \perp PQ\ and\ also\ AB \perp LM$

⇒ $\angle BAQ = \angle ABL\ (each\ 90°)$

∴ PQ || LM (∵ $\angle BAQ$ and $\angle ABL$ are alternate angles)

Que 5. If tangent PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then find $\angle POA$.

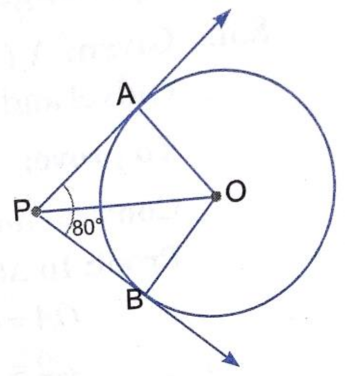

Fig. 8.34

Sol. ∵ PA and PB are tangents to a circle with centre O.

$\therefore OA \perp AP \text{ and } OB \perp PB$

i.e., $\angle APB = 80°,\ \angle OAP = 90° \text{ and } \angle OBP = 90°$

Now, in quadrilateral OAPB, we have

$\angle APB + \angle PBO + \angle OAP = 360°$

$\Rightarrow \quad 80° + 90° + \angle BOA + 90° = 360°$

$\Rightarrow \quad 260° + \angle BOA = 360°$

$\therefore \quad \angle BOA = 360° - 260° \qquad \Rightarrow \quad \angle BOA = 110°$

Now, in $\Delta POA \text{ and } \Delta POB, we\ have$

OP = OP (Common)

OA = OB (Radii of the same circle)

$\angle OAP = \angle OBP = 90°$

$\therefore\ \Delta POA \cong \Delta POB$ (RHS congruence condition)

$\Rightarrow\ \angle POA = \angle POB$ (CPCT)

Now, $\angle POA = \frac{1}{2} \times \angle BOA = \frac{1}{2} \times 100 = 50°$

Que 6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm.

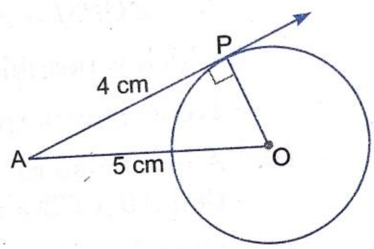

Fig. 8.35

Sol. Let O be the centre and P be the point of contact.

Since tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OPA = 90°$

Now, in right $\Delta OPA,$ we have

$OA^2 = OP^2 + PA^2$ [By Pythagoras Theorem]

$5^2 = OP^2 + 4^2$ $\Rightarrow \; 25 = OP^2 + 16$

$\Rightarrow \; OP^2 = 25 - 16 = 9$ $\therefore \; OP = 3\, cm$

Hence, the radius of the circle is 3 cm.

Que 7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

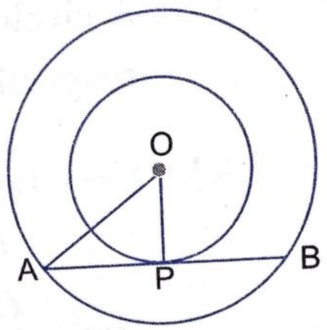

Fig. 8.36

Sol. Let O be the common centre of two concentric circles and let AB be a chord of larger circle touching the smaller circle at P. Join OP.

Since OP is the radius of the smaller circle and AB is tangent to this circle at P.

$\therefore \quad OP \perp AB$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle bisects the chord.

Therefore, AP = BP

In right $\Delta APO, we\ have$

$OA^2 = AP^2 + OP^2$

$\Rightarrow \quad 5^2 = AP^2 + 3^2$ $\Rightarrow \; 25 - 9 = AP^2$

$\Rightarrow \quad AP^2 = 16$ $\Rightarrow \; AP = 4$

Now, AB = 2. AP = 2 × 4 = 8 [$\because \; AP = PB$]

Hence, the length of the chord of the larger circle which touches the smaller circle is 8 cm.