CHAPTER 03

Friction

3.1 Introduction to Friction

When two bodies are in contact, tangential forces acts between bodies called friction forces, will always developed if one attempts to move one bodies with respect to the other. There is a maximum value of friction force and will not prevent motion if sufficiently large forces are applied.

The friction forces mainly due to surface roughness of the bodies in contact. These forces will be very high for very rough surface and can be decrease by making smooth surface.

The friction force oppose the motion of the body and it always acts tangentially along the common surface of contact between the two bodies.

Consider a body on plane as shown in figure 3.1. If F is the force applied to the body to move it toward right over the plane, then an opposing force R_f will act in the opposite direction i.e., towards left and at contact surface as shown in figure 3.1.

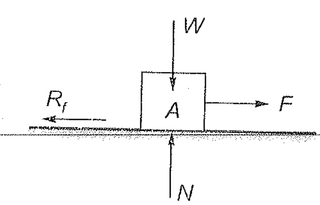


Fig. 3.1

If the magnitude of applied force is greater than R_f then only it is is possible to move body towards right.

$$F > R_f$$

If we start to increase the force F from 1 N, 2 N, 3 N then the body will not be able to move because R_f will also increase correspondingly and will balance F. A stage will reach when $R_f \neq F$ because R_f has a maximum value R_{fl} is known as limiting friction force (R_{fl}) and if F is continuously increased and F being greater than R_{fl}. As the body has been set in motion, the magnitude of friction force drops to a lower value R_{fk} is known as kinetic friction force. This is because there is less interpenetrating between the irregularities of surface in contact when these surface move with respect to each other. The value of R_{fk} is constant and always less than R_{fl}.

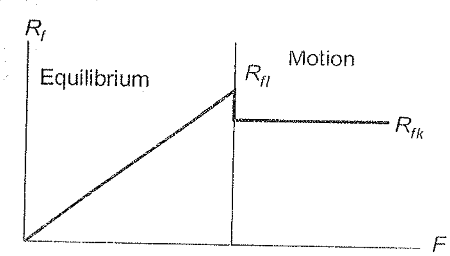


Fig. 3.2

3.2 Dry Friction and Fluid Friction

Fluid friction occur when the adjacent layers in a fluid are moving at different velocities. This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between fluid layers. Fluid friction is treated in the study of fluid mechanics.

Dry friction occur when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide as discussed in part 3.1. This type of friction is also called Coulomb friction. In this chapter we will study only this friction.

3.3 Static and Dynamic Friction

When an external force is applied to a body, it does not move, because frictional force balance the external force applied. This external force is less then R_{fl} therefore, the frictional force experienced by the body is known as static friction as body remains static because of this frictional force and is equal to the applied external force.

Whenever two bodies are in relative motion, the friction resisting force experienced by them is known of dynamic or kinetic friction.

3.4 Laws of Static Friction

The laws of static friction are as follows:

1. The frictional force always acts in a direction opposite to that in which the body tends to move.
2. The limiting frictional force is directly proportional to the normal reaction between the two contact surfaces.
3. The frictional force depends upon the nature of the surfaces in contact.
4. The frictional force is independent of the area and shape of the contacting surfaces.

3.5 Coefficient of Friction

The limiting friction force R_{fl} is directly proportional to the normal reaction between two bodies in contact as shown in figure 3.1.

$$R_{fl} \propto N$$
$$R_{fl} = \mu_s R$$

where μ_s is known as coefficient of static friction.

Thus $$\mu_s = \frac{R_{fl}}{R}$$

The coefficient of static friction is defined as the ratio of limiting friction force to the normal reaction between two bodies in contact.

The coefficient of kinetics friction is given by

$$\mu_k = \frac{R_{fk}}{R}$$

where R_{fk} (kinetic friction force) is little less than R_{fl} in magnitude as discussed earlier. Generally we denote μ_s as μ unless specified.

3.6 Angle of Friction (ϕ)

Consider a body A which is just about to move on plane under external applied force F as shown in figure 3.3. The W is the weight of the body, N is the normal force and R_{fl} is the limiting frictional force.

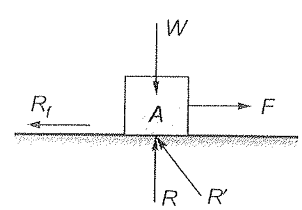


Fig. 3.3

The resultant force of N and R_{fl} may be represented by R' in magnitude and direction as shown in the figure. The ϕ is the angle between R' and N and it is known as angle of friction. The resultant force R' is given by

$$N' = \sqrt{N^2 + R_{fl}^2}$$

and its direction with respect to normal is given by

$$\tan \phi = \frac{R_{fl}}{N}$$

but $$\mu_s = \frac{R_{fl}}{N}$$

Thus $$\phi = \tan^{-1}(\mu_s)$$

This indicates that the friction coefficient is tangent of angle of friction.

3.7 Angle of Repose

The maximum angle of inclination of an inclined plane with horizontal for which a body lying on the inclined plane will not slide due to its weight is known as angle of repose.

Consider a body of weight W resting on a rough plane inclined at an angle α as shown in figure 3.4. Consider the limiting equilibirum condition and resolving forces:

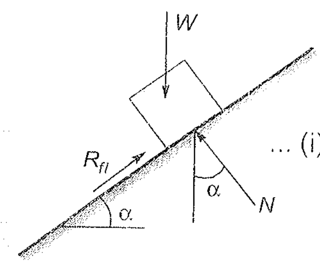


Fig. 3.4

Normal to the plane $W \cos \alpha = N$ ... (i)

Along the plane $W \sin \alpha = R_{fl}$

∴ $R_{fl} = \mu N$

∴ $W \sin \alpha = \mu N$ ... (ii)

Dividing equation (ii) by (i) we get

$$\tan \alpha = \mu$$

But we know that, $\mu = \tan \phi$

ϕ = angle of friction

Therefore it can be stated that angle of repose is equal to the angle of friction.

3.8 Wedge

A wedge is usually of a triangular or trapezoidal in cross-section. The tapered surface of the wedge can help in small adjustments in the position of a body or to apply large force. A friction force is always present along the tapered surface of the m, wedge where it is in contact with another body.

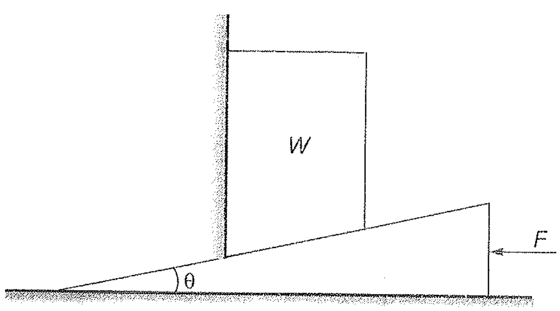


Fig. 3.5

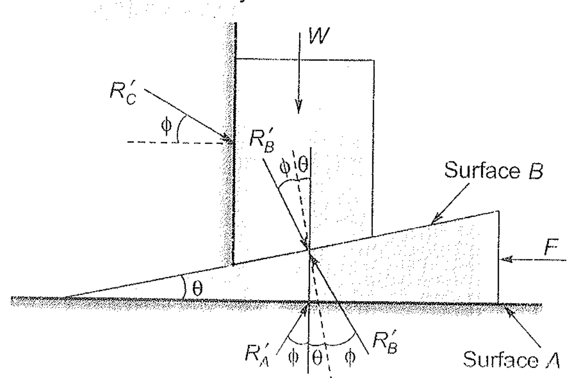
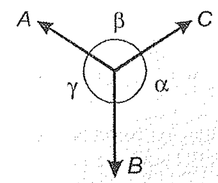


Fig. 3.6

Figure 3.6 shows a wedge used to lift a large weight W with external applied force F perpendicular to weight. The force F required to lift the wedge is calculated from the equilibrium condition of the forces on the weight and wedge. The resultant force as shown in figure 3.7 are inclined at angle ϕ from their respective normal are in the direction to oppose the motion. From the free body diagram we write the force equilibrium condition and the force F is determined. Lami's theorem is convenient if forces are three in numbers.

NOTE

Lami's theorem states that if three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two forces. considering three forces *A*, *B*, *C* acting on a particle or rigid body making angle α, β and γ with each other as shown in figure then



$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

The angle between the force vectors is taken when all the three vectors are emerging from the point.

3.9 Rolling Friction

Consider a wheel having a weight *W* and radius *r* is pulled out of depression as shown in figure 3.8 by applying force *F* horizontally. Considering the equilibrium condition of the wheel, and taking the moments of the forces about the point *O* is zero, where *R* is the reaction at the point *O* on the wheel.

When a body rolls over a surface, the places of contact get deformed and a slight bump is formed. To rotate the body, the bump caused in front has to be constantly overcome, its like constantly climbing over an incline plane. Also the adhesive forces between the two surfaces have to be overcome contently.

Taking $AB \approx r$; we can write

$$F \cdot r = W \cdot L \text{ as } OA = L$$

The horizontal component of *R* is $R\cos\theta$ and for equilibrium condition

$$F = R\cos\theta$$

This component $R\cos\theta$ is called rolling resistance and distance *L* is known as coefficient of rolling resistance. It may be noted that the rolling motion is caused by a couple by the rolling force *F* and equal amount sliding friction $R\cos\theta$ in the opposite direction acting at the point of contact *O* between the body and the surface on which the body rolls.

The rolling friction is always less then the sliding friction as it is only at a point instead of at surface.

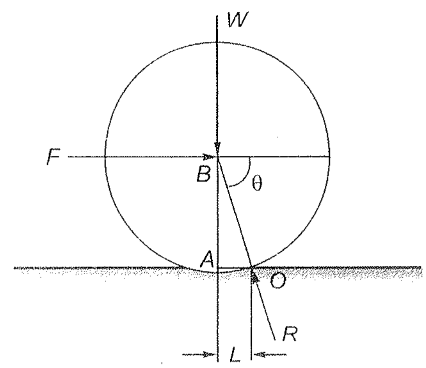


Fig. 3.7

3.10 Belt Friction

In a belt drive system the power from the driver pulley to driven pulley is transmitted by means of a belt wrapped over them. Thus the transmission of power from one pulley to another accomplished only because of existence of friction between the belt and the pulley.

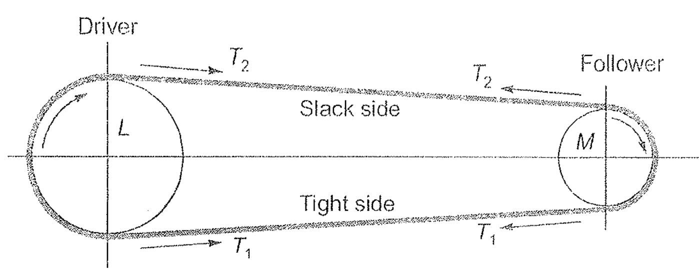


Fig. 3.8

Figure 3.9 shows a driving pulley *L* and the driven pulley *M*. The driving pulley pulls the belt from one side and delivers the same to the others. It is thus obvious, that the tension in the former side is more than in the later side as shown in figure 3.9. The ratio of tension developed in the belt on either sides of pulley is an exponential function of coefficient of friction μ and angle of lap θ of belt on the pulley.

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

3.11 Two Mating Blocks

Block *A* of weight W_1 rests on a block *B* of weight W_2, which is lying on floor, as shown in the Fig. 3.9. Upper block *A* is connected to the wall through a string *CD* and therefore restricted from movement. Let us take μ_1 as coefficient of friction between blocks *A* and *B* and μ_2, as coefficient of friction between block *B* and floor.

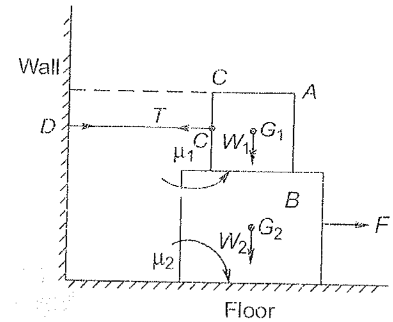


Fig. 3.9

A force *F* is applied on block *B* and let us find F_{min} required for movement of block *B*. Let us draw *free body diagrams for both blocks as shown in* Fig. 3.10. Movement of block B is resist by friction force along the top and bottom surface.

Block A, For equilibrium

$$T = \mu_1 N_1 \qquad ...(1)$$

$$N_1 = W_1 \qquad ...(2)$$

Block B. For equilibrium

$$F = \mu_1 N_1 + \mu_2 N_2 \qquad ...(3)$$

$$N_1 + W_2 = N_2 \qquad ...(4)$$

From these four equation values of required force F_{min} can be determined.

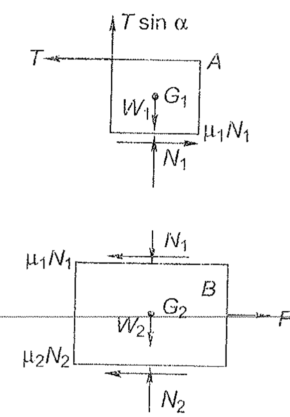


Fig. 3.10

3.12 Friction in Wheels

There are two types of driven wheels as follows:

(*i*) Wheels attached to axle i.e., wheels being pulled by a force.

(*ii*) Wheels attached to engine of an automobile *i.e.*, power is transmitted to the wheels of an automobile, which become the driving wheels and the other pair becomes the driven wheels.

(*i*) **Wheels attached to axle:** A wheel of radius *R* is being pulled by a force *F* over a rough surface, as shown in Fig. 3.11. The coefficient of friction between wheel and surface is μ.

The friction force, $\quad R_f = \mu N$

and $\quad N = W$

$\Rightarrow \quad R_f = \mu W$

Force F applied at axis *O*, cannot provide turning moment about axis *O*. Only friction force is responsible from rotation of wheel.

Turning moment at the centre of the wheel

$$T = R_f = \mu WR$$

If surface is smooth, $\mu = 0$, $T = 0$, the wheel will not rotate.

The wheel rotates in the clockwise direction and wheel moves towards right.

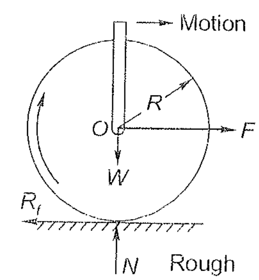


Fig. 3.11

(*ii*) Wheels attached to engine shaft of an automobile: Consider a *rear wheel drive* vehicle as shown in Fig. 3.12, travelling towards right as shown. Driving torque on rear wheels is T.

The engine torque T, tends to rotate the wheels in the clockwise direction. (*i*) If the road is smooth, $\mu = 0$, *the wheels will simply rotate clockwise about their axis but the axle does not move forward*, remains stationery and the vehicle does not move. This phenomenon is sometimes observed when the car is started on a *slushy ground*.

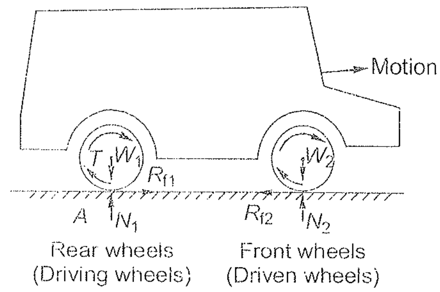


Fig. 3.12

However the road is generally rough and a frictional force R_{f1} acts on the outer surface of wheel and responsible for horizontal movement of wheel and vehicle, if R is the radius of the wheel, then frictional torque, $T_f = R_{f1} \times R$, (where R_{f1} is in the opposite direction at the point of contact, wheel tends to rotate and tends to slip towards left at contact point).

This frictional force R_{f1}, becomes the propelling force on the vehicle in the direction of motion. Frictional force R_{f11} at contact point A can be replaced by a anticlockwise couple $R_{f1} \times R$ and a force R_{f1} at axle axis O as shown in Fig. 3.13(a).

Now the *driven wheels* (front wheels) *are not driven by the engine but are driven by the vehicle as the vehicle is propelled by the engine through the driving wheels* (rear wheels). Friction on driven wheels will be towards left as shown in Fig. 3.13(b), and this provides clockwise turning moment $F_2 \times R$ on the front wheels. Normally $F_2 < F_1$.

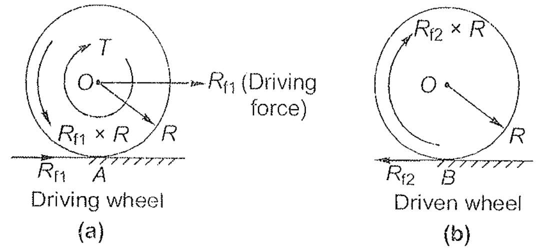


Fig. 3.13

Example 3.1 A block of weight 100 N is lying on a horizontal plane having a coefficient of friction as 0.25. Determine the magnitude of the force, which can move the body, while acting at an angle of 30° with the horizontal.

Solution:

As per problem statement the configuration is shown in figure (a). When force F is applied friction force oppose the motion up to a extent and this force is equal to μR as shown in figure (b). Consider the point when block is just about to move. At that point block is in equilibrium under all these forces.

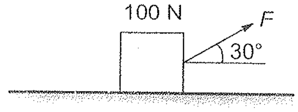


Fig. (a)

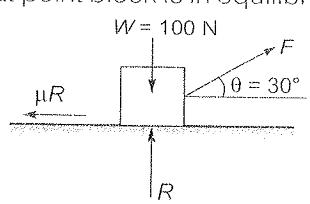


Fig. (b)

Resolving all forces horizontally $\quad W = R + F\sin\theta$

or $\quad R = W - F\sin\theta \quad \ldots$ (i)

Resolving all forces vertically

$$R_F = F\cos\theta$$

but $R_F = \mu R$

Thus $\mu R = F\cos\theta$ ...(ii)

Therefore from equation (ii)

$$\mu(W - F\sin\theta) = F\cos\theta$$

$$\mu W = F(\cos\theta + \mu\sin\theta)$$

$$F = \frac{\mu W}{\cos\theta + \mu\sin\theta}$$

Here $\mu = 0.25,\ \theta = 30°$ and $W = 100$ N

Substituting the values

$$F = \frac{0.25 \times 100}{\cos 30° + 0.25\sin 30°} = 25.22 \text{ N}$$

Example 3.2 A block of weight 12 kN rest on a horizontal floor. A man pulls the block through a string which makes angle α with horizontal. Determine the minimum force required to pull the block. Assume $\mu = 1/\sqrt{3}$.

Solution:

As per problem statement the configuration is shown in figure (a). When force F is applied friction force oppose the motion up to a extent and this force is equal to μR as shown in figure (b). Consider the point when block is just about to move. At that point block is in equilibrium under all these forces.

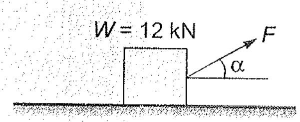


Fig. (a)

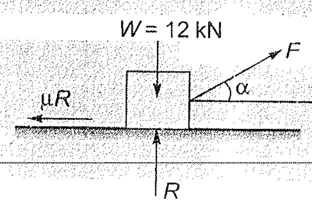


Fig. (b)

Resolving all forces vertically $W = R + F\sin\alpha$

or $R = W - F\sin\alpha$ ...(i)

Resolving forces horizontally

$$\mu R = F\cos\alpha \quad \text{...(ii)}$$

From equation (i) and (ii) we get

$$\mu(W - F\sin\alpha) = F\cos\alpha$$

or $F(\cos\alpha + \mu\sin\alpha) = \mu W$

or $$F = \frac{\mu W}{\cos\alpha + \mu\sin\alpha}$$

Let $$\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$$

Then $$F = \frac{W\sin\phi}{\cos\phi\cos\alpha + \sin\phi\sin\alpha} = \frac{W\sin\phi}{\cos(\alpha - \phi)}$$

F will be minimum when $\cos(\alpha - \phi)$ is maximum.

i.e. $\cos(\alpha - \phi) = 1$

or $\alpha - \phi = 0$ or $\alpha = \phi$

Here $\tan\phi = \mu = \frac{1}{\sqrt{3}}$

or $\phi = 30°$

At $\alpha = 30°$ the value of minimum force is

$$F = \frac{W\sin 30°}{1} = \frac{W}{2}$$

Thus $F_{min} = \frac{12}{2} = 6 \text{ kN}$

Example 3.3 A block of 50 N weight rests in limiting equilibrium on a rough inclined plane whose slope is 30°. The plane is raised to slope of 60°. Determine the force required acting along the plane to support the block.

Solution:

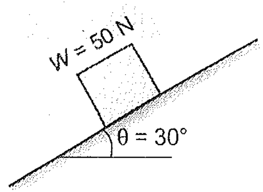


Fig. (a)

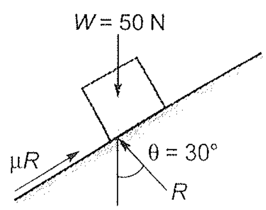


Fig. (b)

The figure (a) shows the situation when the slope is 30° and body is in limiting equilibrium and figure (b) shows the forces acting on the block in that situation. In this situation component of weight along the plane try to moves down the block and friction force oppose the motion.

For equilibrium

$$R = W\cos\theta$$

and $\mu R = \text{W} \sin\theta$

or $\mu W\cos\theta = W\sin\theta$

or $\mu = \tan\theta$

It shows that in the condition of limiting equilibrium the coefficient of friction is equal to slope. This is also called angle of repose or angle of friction.

We have $\theta = 30°$

thus $\mu = \tan 30° = \frac{1}{\sqrt{3}}$

Now consider the case when slope is 60° and block is in equilibrium under applied force as shown in figure (c). All force acting on block is shown in figure (d). Here force F just support the block and block is about to move down. So friction force is in up direction along the plane.

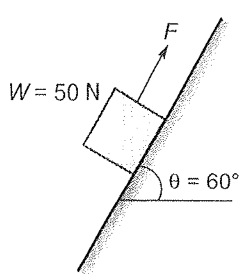


Fig. (c)

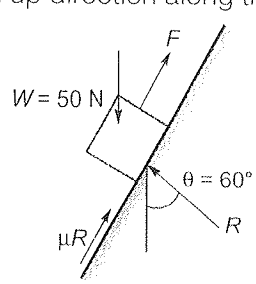


Fig. (d)

For equilibrium

$$W\cos 60° = R$$

and $$F + \mu R = W\sin 60°$$

or $$F = W(\sin 60° - \mu\cos 60°)$$

Here $$W = 50\text{ N}$$

$$\mu = \frac{1}{\sqrt{3}}$$

Thus $$F = 50\left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}\cdot\frac{1}{2}\right) = \frac{50}{\sqrt{3}} = 28.87\text{ N}$$

Example 3.4 A weight W is supported by friction on a plane inclined at an angle α to the horizontal. Prove that the minimum horizontal force required to move up along the plane is $W\tan 2\alpha$.

Solution:

As per problem statement the configuration is shown in figure (a). Here the force F is just sufficient to move the weight upward along the plane. In this situation the friction force will act downward along the plane. Figure (b) shows the all forces acting on the weight.

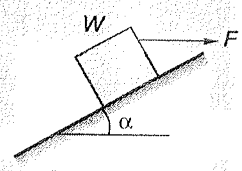


Fig. (a)

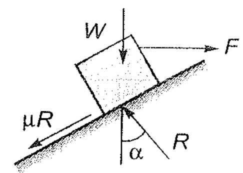


Fig. (b)

Resolving the force normal to plane

$$R = W\cos\alpha + F\sin\alpha \qquad \text{...(i)}$$

Resolving the forces along the plane

$$F\cos\alpha = \mu R + W\sin\alpha \qquad \text{... (ii)}$$

Substituting the value of R from equation (i) in equation (ii) we get

$$F\cos\alpha = \mu(W\cos\alpha + F\sin\alpha) + W\sin\alpha$$

or $$F(\cos\alpha - \mu\sin\alpha) = W(\mu\cos\alpha + \sin\alpha)$$

or $$F = \frac{W(\sin\alpha + \mu\cos\alpha)}{(\cos\alpha - \mu\sin\alpha)}$$

if weight is supported by only friction then

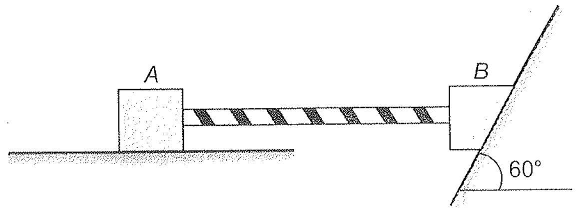
$$\mu = \tan\alpha$$

Therefore $$F = \frac{W(\sin\alpha + \tan\alpha\cos\alpha)}{(\cos\alpha - \tan\alpha\sin\alpha)}$$

or $$F = W\left(\frac{\sin\alpha\cos\alpha + \sin\alpha\cos\alpha}{\cos\alpha^2 - \sin\alpha^2}\right) = W\frac{\sin 2\alpha}{\cos 2\alpha}$$

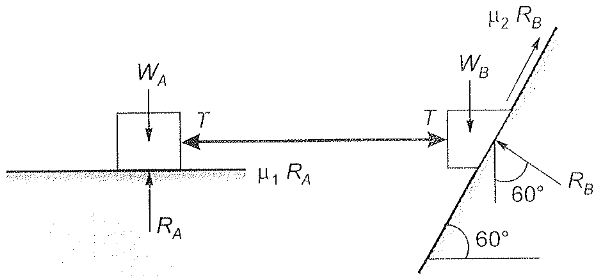
$$= W\tan 2\alpha$$ Hence Proved.

Example 3.5 The block A and B, connected by a horizontal rod and frictionless hinges are supported on two rough planes as shown in figure. The coefficient of friction between block A and the horizontal surface is 0.3 and between B and the inclined surface is 0.4. If the block B weighs 1 kN, what is the minimum weight of block A, that will hold the system in equilibrium.



Solution:

At the point of equilibrium the block *B* is just about to move down. Thus friction will act in the upward direction along the plane as shown in figure.



Now consider the equilibrium of block *A*. The thrust force in rod will try to move block *A* along $-x$ direction. Thus friction force will act in $+x$ direction.

Thus
$$R_A = W_A$$
$$T = \mu_1 R_A = \mu_1 W_A$$

Now consider the equilibrium of block *B*.

Resolving force normal to the plane.

$$R_B = T\sin 60° + W_B \cos 60°$$

Substituting $T = \mu_1 W_A$ we get
$$R_B = \mu_1 W_A \sin 60° + W_B \cos 60°$$

Resolving force along the plane

$$\mu_2 R_B + T\cos 60° = W_B \sin 60°$$

Substituting the value of R_B, T and angle we get

$$\mu_2\left(\mu_1 W_A \frac{\sqrt{3}}{2} + W_B \frac{1}{2}\right) + \mu_1 W_A \frac{1}{2} = W_B \frac{\sqrt{3}}{2}$$

or
$$\mu_1\mu_2 W_A\sqrt{3} + \mu_2 W_B + \mu_1 W_A = W_B\sqrt{3}$$

Substituting the following values

$$\mu_1 = 0.3,\ \mu_2 = 0.4 \text{ and } W_B = 1 \text{ kN}$$

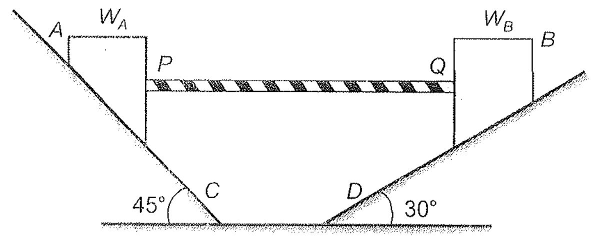
$$0.3\times 0.4\times W_B\sqrt{3} + 0.4\times 1 + 0.3W_A = \sqrt{3}$$

$$0.208W_A + 0.3W_A = \sqrt{3} - 0.4$$

or
$$0.508W_A = 1.33$$

$$W_A = \frac{1.33}{0.508} = 2.62 \text{ kN}$$

Example 3.6 Two loads, W_A and W_B resting on two inclined rough planes *AC* and *BD* are connected by a horizontal bar *PQ* as shown in figure. If the W_A is equal to 100 N, then determine the maximum and minimum values of W_B for which the equilibrium can exist. Take angle of friction for both the plane as 20°.



Solution:

Consider the case first for the maximum value of W_B. In this case load W_B will be at the point of sliding downwards where as the load W_A will be at the point sliding upwards. The friction force acts in the opposite direction of motion and resultant force is inclined at an angle $\phi = 20°$ from normal to plane as shown in figure (a) for both weight.

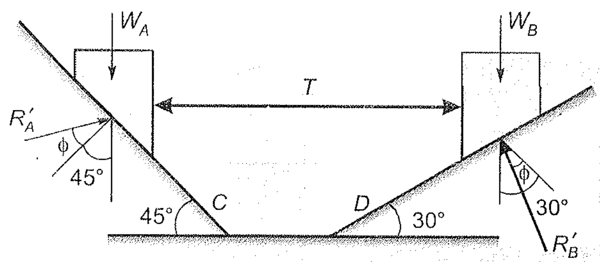


Fig. (a)

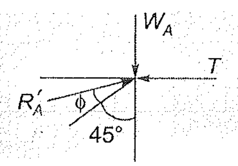


Fig. (b)

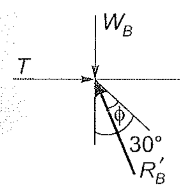


Fig. (c)

Applying Lami's theorem for weight A (figure (b))

$$\frac{T}{\sin(180° - 45° - \phi)} = \frac{R_A}{\sin 90°} = \frac{W_A}{\sin(90° + 45° + \phi)}$$

or $$\frac{T}{\sin(45° + \phi)} = R_A = \frac{W_A}{\cos(45° + \phi)}$$

or $$T = W_A \frac{\sin(45° + \phi)}{\cos(45° + \phi)}$$

or $$T = W_A \tan(45° + \phi) \qquad \ldots(i)$$

Applying Lami's theorem for weight B (figure (c))

$$\frac{T}{\sin(180° - 30° + \phi)} = \frac{R_B}{\sin 90°} = \frac{W_B}{\sin(90° + 30° - \phi)}$$

or $$\frac{T}{\sin(30° - \phi)} = R_B = \frac{W_B}{\cos(30° - \phi)}$$

or $$T = W_B \frac{\sin(30° - \phi)}{\cos(30° - \phi)}$$

or $$T = W_B \tan(30° - \phi) \qquad \ldots(ii)$$

Since the thrust in the bars obtained in equation (i) and (ii) is the same, therefore equating both equation we get

$$W_A \tan(45° + \phi) = W_B (\tan 30° - \phi)$$

or $$W_B = W_A \frac{\tan(45° + \phi)}{\tan(30° - \phi)} \qquad \ldots(iii)$$

This is maximum weight so denoting by max

$$W_{B\max} = W_A \frac{\tan(45° + \phi)}{\tan(30° - \phi)}$$

Here $\quad W_A = 100 \text{ N} \quad \text{and} \quad \phi = 20°$

Thus
$$W_{B\max} = 100\frac{\tan(45° + 20)}{\tan(30° - 20)} = 100\frac{\tan 65°}{\tan 10°}$$

or
$$W_{B\max} = 100\frac{2.144}{0.176} = 1218 \text{ N}$$

In the case of minimum value the load W_B will be at the point of sliding upwards where s the load W_A will be at the point of sliding downwards. In this situation direction of friction force change to opposite as in case of previous for both load as shown in figure (d).

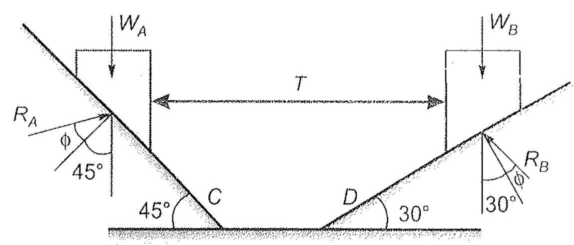


Fig. (d)

We can get the equation for this case by substituting $\phi \to -\phi$. Thus substituting $\phi \to -\phi$ in equation (iii) we get

$$W_{B\min} = W_A\frac{\tan(45° - \phi)}{\tan(30° + \phi)} = 100\frac{\tan(45° - 20°)}{\tan(30° + 20°)}$$

or
$$W_{B\min} = 100\frac{\tan 25°}{\tan 50°} = 100\frac{0.466}{1.191} = 39.1 \text{ N}$$

Thus minimum force in 39.1 N

Example 3.7 A ladder of 10 m length and 4 kN weight is placed against a smooth vertical wall with its lower end 6 m from the wall. In this position the ladder is just about to slip. Determine the coefficient of friction between the ladder and floor and the frictional force acting on ladder.

Solution:

As per problem statement the configuration is shown in figure. (a)

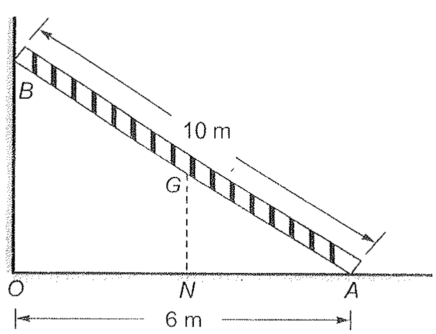


Fig. (a)

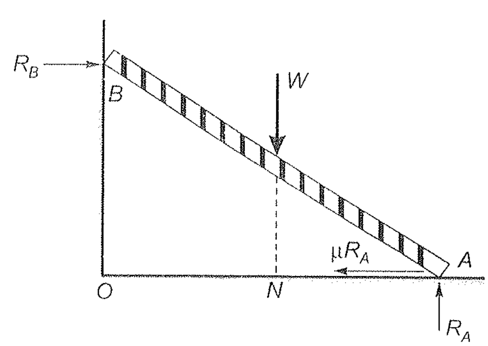


Fig. (b)

From geometry of figure (a)

$$BO = \sqrt{10^2 - 6^2} = 8 \text{ m}$$

$$AN = \frac{6}{2} = 3 \text{ m}$$

Since ladder is a just about to slip so friction force acts in the opposite direction of motion. All force acting on ladder has been shown in figure (b) and ladder is in equilibrium under these forces. Since vertical wall is smooth so there will be no friction force wall and end B of ladder.

Resolving forces horizontally, $\quad R_B = \mu R_A = R_F \quad$ ...(i)

Resolving forces vertically, $\quad R_A = W \quad$ ... (ii)

Taking moment about point A we get

or

$$R_B \times BO = W \times AN$$

$$R_B \times 8 = W \times 3$$

$$R_B = \frac{3W}{8} \quad \text{... (iii)}$$

From equation (i), (ii) and (3) we get,

$$\frac{3W}{8} = \mu W$$

or

$$\mu = \frac{3}{8} = 0.375$$

Friction force is equal to $\quad \mu R_A = R_F = R_B = \frac{3W}{8} = \frac{3 \times 4}{8} = 1.5 \text{ kN}$

Example 3.8 A ladder of 6 m length and 60 N weight rests on a horizontal ground and against a smooth vertical wall at an angle of 20° with the vertical. When a man of 50 N stands on a ring 2 m from the foot of the ladder, it is on the point of slipping. Determine the coefficient of friction between the ladder and ground.

Solution:

As per problem statement configuration is shown in figure (a). Here man is at point C. From geometry of figure (a) we get

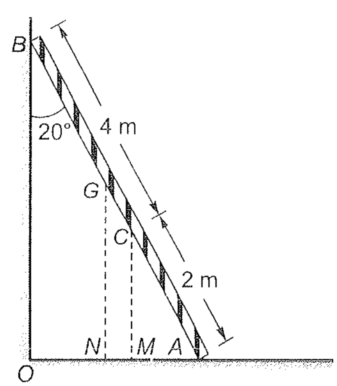


Fig. (a)

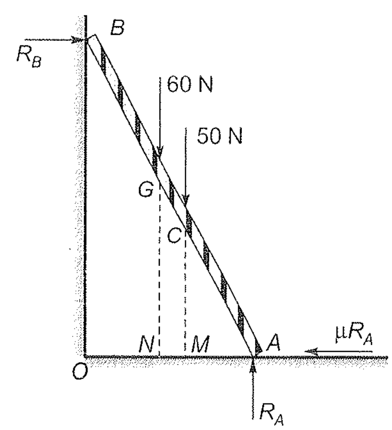


Fig. (b)

$$AB = 6 \text{ m} \qquad AG = 3 \text{ m } (G\text{: centre of gravity})$$
$$AC = 2 \text{ m} \qquad ABO = 20°$$
$$BAO = 70° \qquad AM = AC \cos 70° = 2 \cos 70°$$
$$AN = AG \cos 70° = 3 \cos 70° \qquad AO = AB \cos 70° = 6 \cos 70°$$
$$BO = AB \sin 70° = 3 \sin 70°$$

Since ladder is a just about to slip so friction force acts in the opposite direction of motion. All force acting on ladder has been shown in figure (b) and ladder is in equilibrium under these forces.

Resolving forces vertically $\quad R_A = 50 + 60 = 110 \text{ N} \quad$ ... (i)

Resolving forces horizontally $\quad R_B = \mu R_A = \mu 110 \quad$ ... (ii)

Taking the moment about point A we get

$$R_B \times BO = 60 \times AN + 50 \times AM$$

or

$$R_B \times 6 \sin 70° = 60 \times 3 \cos 70° + 50 \times 2 \cos 70°$$
$$6 R_B \tan 70° = 280$$
$$R_B = \frac{280}{6 \tan 70°} = 17 \text{ N} \quad \text{... (iii)}$$

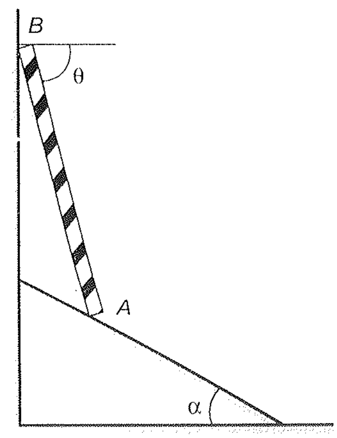
From equation (ii) and (iii) we get

$$110\mu = 17$$
$$\mu = \frac{17}{110} = 0.154$$

Example 3.9 **A uniform ladder of weight W rests with one end against a rough inclined plane of angle α with the horizontal and other end against a smooth vertical wall as shown in figure. If the ladder be at θ with the horizontal and is in limiting equilibrium, prove that.**

$$2 \tan \theta = \cot (\phi - \alpha)$$

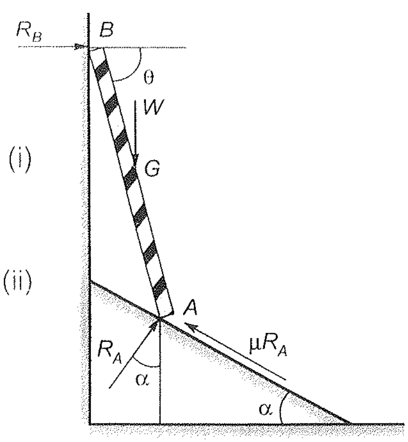
Where ϕ is the angle of limiting friction between the ladder and the inclined plane.



Solution:

Let μ be coefficient of friction between inclined plane and ladder where $\mu = \tan \phi$.

Figure show the all forces acti.g on ladder.



Ladder is in equilibrium under all these forces.

Resolving forces vertically

$$W = R_A \cos \alpha + \mu R_A \sin \alpha$$

or

$$W = R_A (\cos \alpha + \mu \sin \alpha) \quad \text{... (i)}$$

Resolving forces horizontally

$$R_A \sin \alpha + R_B = \mu R_A \cos \alpha$$

or

$$R_B = R_A (\mu \cos \alpha - \sin \alpha) \quad \text{... (ii)}$$

Taking moment about point A we get

$$W \times \frac{L}{2} \cos \theta = R_B \times L \sin \theta$$

or $$R_B = \frac{W}{2\tan\theta} \quad \text{... (iii)}$$

Eliminating R_A and R_B from equation (ii) and (iii) we get

$$2\tan\theta = \frac{\cos\alpha + \mu\sin\alpha}{\mu\cos\alpha - \sin\alpha}$$

Substituting $$\mu = \tan\phi$$

$$2\tan\theta = \frac{\cos\alpha + \tan\phi\sin\alpha}{\tan\phi\cos\alpha - \sin\alpha}$$

or $$2\tan\theta = \frac{\cos\phi\cos\alpha + \sin\phi\sin\alpha}{\sin\phi\cos\alpha - \sin\alpha\cos\phi} = \frac{\cos(\phi-\alpha)}{\sin(\phi-\alpha)}$$

or $$2\tan\theta = \cot(\phi - \alpha)$$

Hence proved

Example 3.10 A block weighing 500 N, over lying a 10° wedge on a horizontal floor and leaning against a vertical wall, is to be raised by applying a horizontal force to the wedge. If the coefficient of friction between all the surface in contact to be 0.3, determine the minimum horizontal force required to raise the block.

Solution:

As per problem statement the configuration is shown in figure (a).

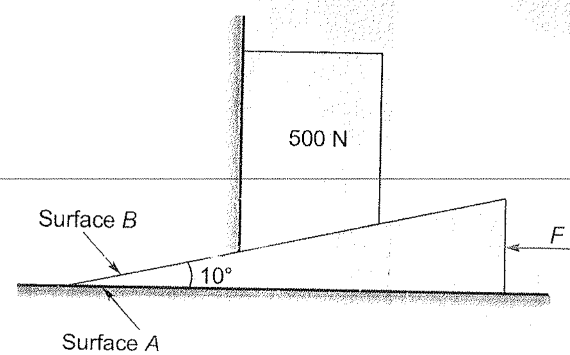


Fig. (a)

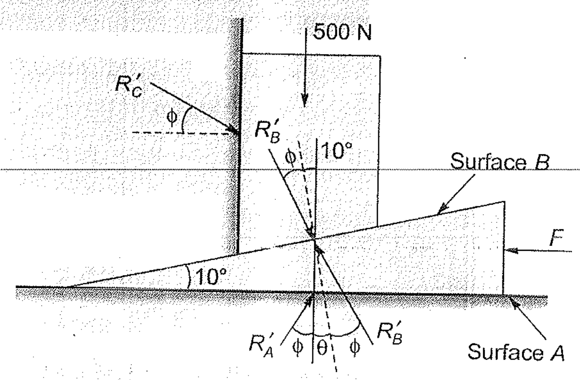


Fig. (b)

Here $$\mu = .0.3 = \tan\phi$$

or $$\phi = \tan^{-1} 0.3 = 16.7°$$

Here friction force acts in opposite direction of the motion. Thus normal resultant force will be inclined 16.7° from the normal to plane due to friction. Figure (b) show the all forces acting on system.

Now consider the equilibrium of block A.

It is in equilibrium under the action of the following force as shown in figure (c).

1. It's weight of 500 N acting downward.
2. Resultant force on surface C, R'_C
3. Resultant force on surface B, R'_B

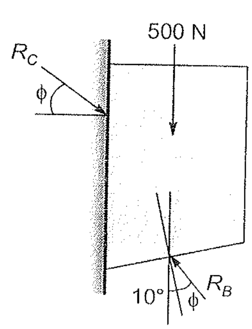


Fig. (c)

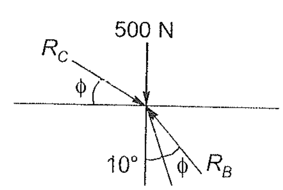


Fig. (d)

Applying Lami's theorem (figure (d))

$$\frac{R_B'}{\sin(90^\circ - \phi)} = \frac{R_C'}{\sin(180^\circ - 10^\circ - \phi)} = \frac{500}{\sin(90^\circ + 10^\circ + 2\phi)}$$

$$\frac{R_B'}{\cos\phi} = \frac{R_C'}{\sin(10^\circ + \phi)} = \frac{500}{\cos(10^\circ + 2\phi)}$$

Thus
$$\frac{R_B'}{\cos 16.7^\circ} = \frac{R_B'}{\sin 26.7^\circ} = \frac{500}{\cos 43.4^\circ}$$

$$R_B' = 500\frac{\cos 16.7^\circ}{\cos 43.4^\circ} = 659.14 \text{ N}$$

Now consider the equilibrium of wedge. It is in equilibrium under following forces (figure (e))

1. Reaction R_B' inclined angle ϕ with normal to plane.
2. Reaction R_A' inclined ϕ from vertical.
3. Force F.

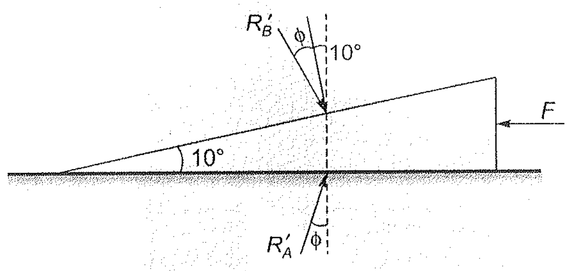


Fig. (e)

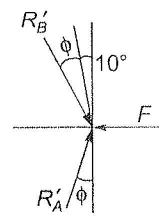


Fig. (f)

Using Lami's theorem (figure (f))

$$\frac{R_B'}{\sin(90^\circ + \phi)} = \frac{R_A'}{\sin(90^\circ + \phi + 10^\circ)} = \frac{F}{\sin(180^\circ - (2\phi + 10^\circ))}$$

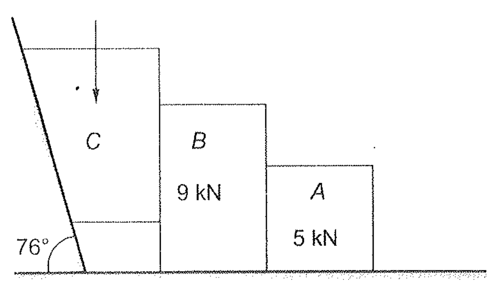
or
$$\frac{R_B'}{\cos\phi} = \frac{R_A'}{\cos(\phi + 10^\circ)} = \frac{F}{\sin(2\phi + 10^\circ)}$$

Substituting the value of R_B' and ϕ

$$\frac{659.14}{\cos 16.7} = \frac{F}{\sin(33.4^\circ + 10^\circ)}$$

$$F = 659.14\frac{\sin 43.4}{\cos 16.7} = 472.83 \text{ N}$$

Example 3.11 Determine the required force P to be applied on wedge A for its impending motion down the plane as shown in figure. Assume the coefficient of friction of 0.3 between all the contact surfaces.



Solution:

Here
$$\mu = 0.3 = \tan\phi$$

Thus angle friction $\phi = \tan^{-1} 0.3 = 16.7^\circ$

Friction force acts at all point of contact and reaction will be inclined angle ϕ with normal to plane. Figure (a) shows all forces acting on system.

Now consider the equilibrium of block A. Figure (b) shows all forces acting on this block and figure (c) is the force diagram.

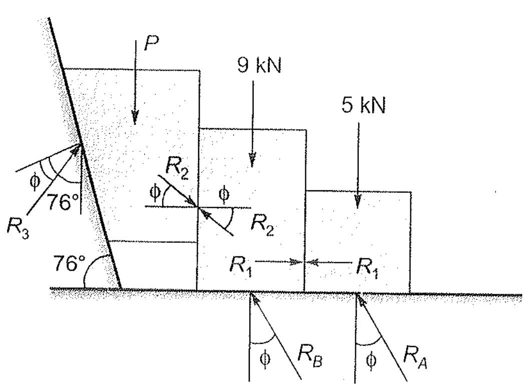


Fig. (a)

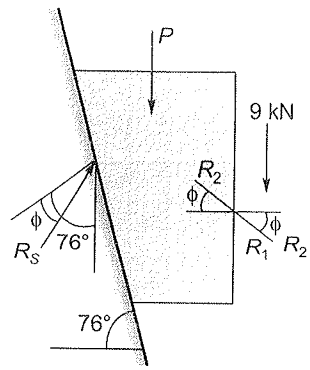


Fig. (b)

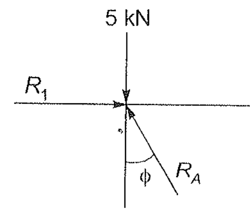


Fig. (c)

Applying Lami's theorem

$$\frac{R_1}{\sin(180^\circ - \phi)} = \frac{R_A}{\sin 90^\circ} = \frac{5}{\sin(90^\circ + \phi)}$$

or

$$\frac{R_1}{\sin\phi} = R_A = \frac{5}{\cos\phi}$$

or

$$R_A = \frac{5}{\cos\phi} = \frac{5}{\cos 16.7^\circ} = 5.22 \text{ kN}$$

$$R_1 = 5\frac{\sin\phi}{\cos\phi} = 5 \tan 16.7^\circ = 1.5 \text{ kN}$$

Now consider equilibrium of block B. Figure (d) shows all forces acting on this block and figure (e) is the force diagram. Because of four forces we will use method of resolution.

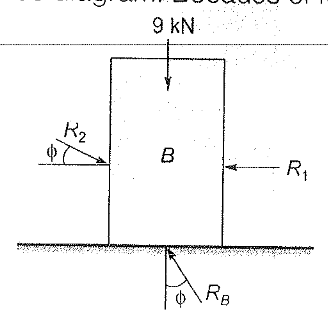


Fig. (d)

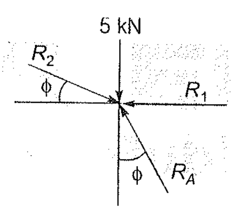


Fig. (e)

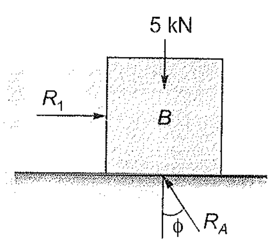


Fig. (f)

Resolving forces vertically

$$9 + R_2 \sin\phi = R_1 + R_B \cos\phi \quad \text{... (i)}$$

Resolving forces horizontally

$$R_2 \cos\phi = R_B \sin\phi$$

Dividing equation (i) by (ii) we get

$$\frac{R_2 \cos\phi - R_1}{R_2 \sin\phi + 9} = \tan\phi$$

Substituting values of ϕ and R_1

$$\frac{R_2 \times 0.958 - 1.5}{R_2 \times 0.287 + 9} = 0.3$$

or

$$0.958\, R_2 - 1.5 = 0.086\, R_2 + 2.7$$

or $$0.872\,R_2 = 4.2$$

or $$R_2 = 4.82 \text{ kN}$$

Now consider the equilibrium of wedge *C*. Figure (f) shows all forces acting on this block and figure (g) is the force diagram.

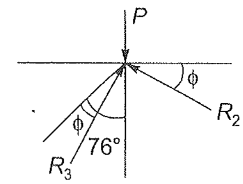


Fig. (g)

Applying Lami's theorem

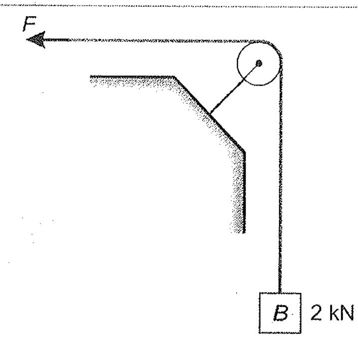
$$\frac{R_3}{\sin(90^\circ + \phi)} = \frac{R_2}{\sin(180^\circ - 70^\circ + \phi)} = \frac{P}{\sin(90^\circ + 70^\circ - \phi - \phi)}$$

or $$\frac{R_3}{\cos\phi} = \frac{R_2}{\sin(70^\circ - \phi)} = \frac{P}{\cos(70^\circ - 2\phi)}$$

Substituting the values of R_2 and ϕ, $$P = R_2 = \frac{\cos(70^\circ - 2\phi)}{\sin(70^\circ - \phi)}$$

or $$P = 4.82 \times \frac{\cos 36.6^\circ}{\sin 53.3} = 4.83 \text{ kN}$$

Example 3.12 **Determine the range of force F for which it will keep the load as shown in figure in equilibrium. Coefficient of friction between rope and pulley is 0.2.**



Solution:

Figure (a) shows the all forces acting on system. The range of the force F for which it will kept the load in equilibrium may either tend to move the pulley in the direction of applied force or in opposite to the direction of applied force F. Let range of applied force be from F_{min} to F_{max}.

Consider the situation first when system tends to move in the direction of applied force i.e. F is F_{max}.

$$T_2 = F_{max}, \text{ and } T_1 = 2 \text{ kN}$$

In this case $$\frac{T_2}{T_1} = e^{\mu\theta}$$

or $$\frac{F_{max}}{2} = e^{0.2\theta}$$

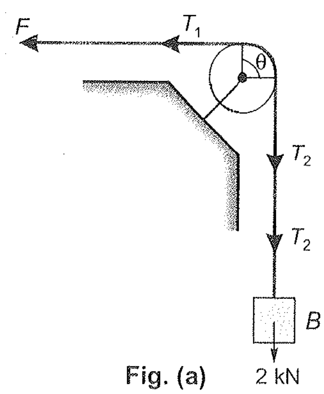


Fig. (a)

Consider the pulley shown in figure (a). It can be easily seen that the angle of lap of belt over the pulley is 90°

Thus $$F_{max} = 2e^{\mu\times\frac{\pi}{2}} = 2e^{0.2\times\frac{\pi}{2}} = 2.74 \text{ kN}$$

Now consider the case when system tends to move in the direction of 2 kN i.e. when F is F_{min}.

In this case $$T_1 = 2 \text{ kN},\ T_2 = F_{min} \text{ and } T_1 > T_2$$

Thus $$\frac{T_1}{T_2} = e^{\mu\theta}$$

or $$\frac{2}{F_{min}} = e^{0.2\times\frac{\pi}{2}}$$

$$F_{min} = \frac{2}{e^{-0.1\pi}} = 1.46 \text{ kN}$$

Objective Brain Teasers

Q.1 Arrange the following contacting surfaces in increasing order of coefficient of static friction

(i) metal on metal–I

(ii) rubber on concrete–II

(iii) metal on stone–III

(iv) stone on stone–IV.

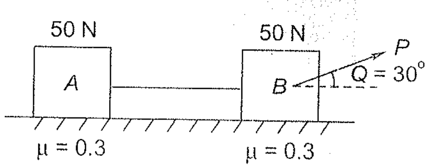
(a) I III IV II

(b) I II III IV

(c) II I III IV

(d) III I II IV

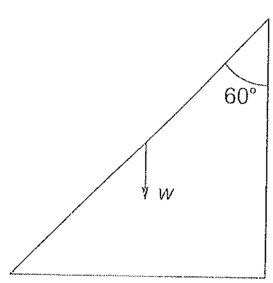
Q.2 Two blocks *A* and *B* are connected through a string. Weight of each block is 50 N. Coefficient of friction between each block and floor is 0.3. An inclined force *P* is applied to start sliding of the system. What is the minimum value of *P* required?



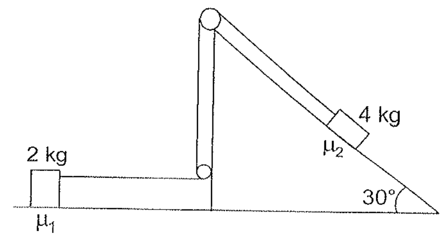
(a) 34.64 N (b) 31.21 N

(c) 29.53 N (d) None of these

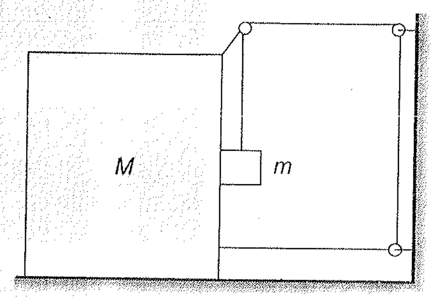
Q.3 A uniform ladder of weight 100 N leans against a smooth vertical wall making an angle of 60° with it. The other end rests on rough horizontal floor. The friction force that the floor exerts on ladder is __________ N.



Q.4 If the tension in the string in the figure is 16 N and the acceleration of each block is 0.5 m/s^2. The value of friction coefficient μ_2 is ______________. (Take g = 10 m/s^2)



Q.5 Find the acceleration of the block of mass *M* in the situation of the figure. The coefficient of friction between two blocks is μ_1 and that between the bigger block and ground is μ_2.



(a) $\dfrac{\mu_1 mg}{\mu_2 Mg}$

(b) $\dfrac{2M+\mu_2 m}{M+m(\mu_1-\mu_2)}\times g$

(c) $\dfrac{(2m-\mu_2(M+m))\times g}{M+m(5+2(\mu_1-\mu_2))}$

(d) $\dfrac{\mu_2(M+m)}{m(\mu_1-\mu_2)}\times g$

ANSWERS

1. (a) 2. (c) 5. (c)

Hints & Explanation

1. (a)

Metal on metal	0.15–0.6
Rubber on concrete	0.6–0.95
Metal on stone	0.3–0.7
Stone on stone	0.4–0.7

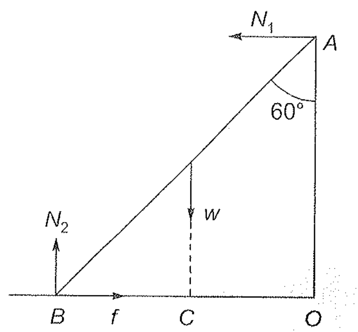
2. (c)

$15 + 15 - 0.15P = 0.866P$

$0.866 + 0.15P = 30$

$$P = \frac{30}{1.016} = 29.53 \text{ N}.$$

3. (86.6)(85 to 87)



Now $\quad N_1 = f$

$\quad N_2 = W$

Taking torque about B,

$$N_1 \times OA = W(CB)$$

$$N_1 \times OA = W\left(\frac{AB}{2}\right) \times \sin 60°$$

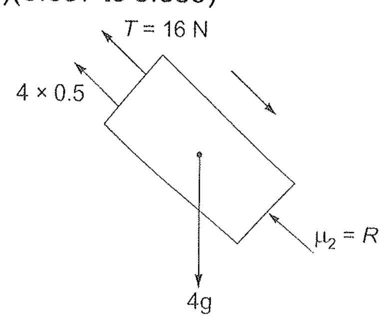
$$\Rightarrow \quad N_1 \times AB\cos 60° = \frac{W}{2}(AB) \sin 60°$$

$$\Rightarrow \quad N_1 = \frac{W}{2}\tan 60°$$

$$f = N_1 = \frac{100}{2} \times \tan 60°$$

$$= 86.6 \text{ N}$$

4. (0.077)(0.057 to 0.059)



$$\Rightarrow \quad \mu_2 R + 4 \times 0.5 + 16 - 4g\sin 30° = 0$$

$$\Rightarrow \quad \mu_2 20\sqrt{3} + 2 + 16 - 20 = 0$$

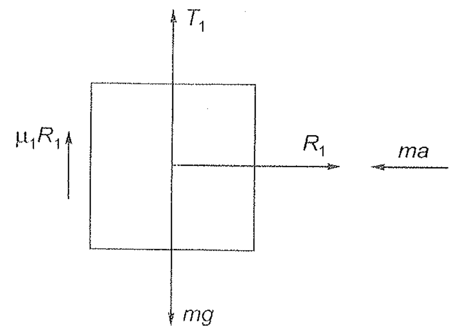
$$\Rightarrow \mu_2 = \frac{2}{20\sqrt{3}} = 0.077$$

5. (c)

Let a be acceleration of M towards right.

∴ Block m must go down with acceleration 2a

For mass m body.



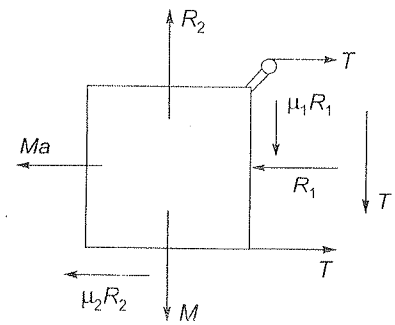
(as block m also move right with acceleration a)

Now, $\quad R_1 = ma \quad$ ...(1)

Again $2ma + T - mg + \mu_1 R_1 = 0$

$\Rightarrow \quad T = mg - (2 + \mu_1)ma \quad$ ...(2)

Consider bigger block.



$\Rightarrow T + \mu_1 R_1 + Mg - R_2 = 0$

$\Rightarrow \quad R_2 = T + \mu_1 ma + Mg$

$\quad = (mg - 2ma - \mu_1 ma)$

$\quad + \mu_1 ma + Mg$

$\Rightarrow \quad R_2 = Mg + mg - 2ma \quad$ ...(3)

Again from FBD-2

$\Rightarrow 2T - R_1 - Ma - \mu_2 R_2 = 0$

$\quad 2T = R_1 + Ma + \mu_2 R_2$

$\Rightarrow 2T = ma + Ma + \mu_2(Mg + mg - 2\,ma) \quad$ ...(4)

Equation (4) and $2 \times (2)$

$\Rightarrow \quad (M + m)a + \mu_2(Mg + mg - 2ma)$

$\quad = 2mg - 2(2 + \mu_1)ma$

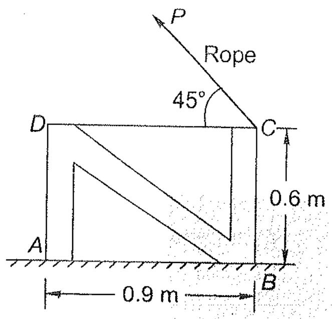
$a(M + m - 2\mu_2\, m + 4m + 2\mu_1 m)$

$\quad = 2\,mg - \mu_2(M + m)g$

$$\Rightarrow \quad a = \frac{(2m - \mu_2(M+m))g}{M + m(5 + 2(\mu_1 - \mu_2))}$$

Student's Assignments

Q.1 A packing crate of mass 45 kg is pulled by a rope as shown in figure. If the coefficient of friction between crate and rope is 0.3, determine (*a*) the magnitude of pull *P* required to move the crate, (*b*) whether the crate will slide or tip.



Q.2 A bar of weight 800 N, 8 m long rests on a stationary support *A* and on a roller support at *B*, as shown in figure. The static coefficient of friction at *A* is 0.4, while the dynamic coefficient of friction of roller support *B* is 0.20. *If support B is moved at constant speed towards left*, how far does it move before the bar begins to move.