H.C.F. and L.C.M. of Polynomials

INTRODUCTION

We have already learnt in Chapter 2 how to find the greatest common divisor (G.C.D.) or highest common factor (H.C.F.) and least common multiple (L.C.M.) of two integers. In this chapter, we will study how to find the G.C.D. and L.C.M. of polynomials which have integral coefficients.

Divisor

A polynomial $d(x)$ is said to be a divisor of polynomial $p(x)$ if $d(x)$ is a factor of $p(x)$, i.e., $p(x)$ can be written as

$p(x) = d(x)\, q(x)$,

where $q(x)$ is a polynomial.

For example, $(x - 2)$ is a divisor of the polynomial $(x - 2)^3 (x + 3)$.

Common Divisor

A polynomial $d(x)$ is said to be a common divisor of the polynomials $p(x)$ and $q(x)$, if $d(x)$ is a factor of each of $p(x)$ and $q(x)$.

For example, $(x + 4)$ is a common divisor of the polynomials $(x + 4)^3 (x - 2) (x + 3)$ and $(x + 4) (x - 2)^3 (x + 5)$.

G.C.D. (H.C.F) of Two Polynomials

The G.C.D. of two polynomials $p(x)$ and $q(x)$ is the common divisor which has highest degree among all common divisors and which has the highest degree term coefficient as positive.

Illustration 1 Find the G.C.D. of $(3x - 2) (4x + 3)$; $(3x - 2)^2 (2x + 5)$

Solution: Here we find that $(3x - 2)$ is a polynomial which is a common divisor and has highest degree among all common divisors. Further, the coefficient of the highest degree term $(3x)$ is 3 which is positive. Hence, $(3x - 2)$ is the G.C.D. of the given polynomial.

G.C.D. by Factorization Method

Step 1 Resolve the given polynomials $p(x)$ and $q(x)$ in the complete factored form.

Step 2 Find the G.C.D. of the numerical factors of $p(x)$ and $q(x)$.

Step 3 Find the factors of highest degree common to the two polynomials $p(x)$ and $q(x)$.

Step 4 The product of all such common factors and the G.C.D. of the numerical factors is the G.C.D. of the two given polynomials $p(x)$ and $q(x)$.

Illustration 2 Find the G.C.D. of $4 + 9x - 9x^2$ and $9x^2 - 24x + 16$

Solution: We have the factorization

$$\begin{aligned} p(x) &= 4 + 9x - 9x^2 = -(9x^2 - 9x - 4) \\ &= -(9x^2 - 12x + 3x - 4) \\ &= -(3x(3x - 4) + 1(3x - 4)) \\ &= -(3x + 1)(3x - 4) \\ q(x) &= 9x^2 - 24x + 16 = (3x - 4)^2 \end{aligned}$$

$\because$ G.C.D. of numerical factors = 1

and the highest degree common divisor = $(3x - 4)$,

$\therefore$ required G.C.D. = $(3x - 4)$.

Illustration 3 Find the G.C.D. of $8(x^4 + x^3 + x^2)$ and $20(x^3 - 1)$

Solution: Here $p(x) = 8(x^4 + x^3 + x^2) = 2^3 . x^2 . (x^2 + x + 1)$

$q(x) = 20(x^3 - 1) = 2^2 . 5 . (x - 1) . (x^2 + x + 1)$.

$\because$ G.C.D. of numerical factors = 2^2

and the highest degree common divisor = $x^2 + x + 1$,

$$\therefore \text{ required G.C.D.} = 2^2 (x^2 + x + 1) = 4(x^2 + x + 1).$$

L.C.M. of Two Polynomials

We know that if a and b are two natural numbers, the product of a and b is equal to the product of their G.C.D. and L.C.M., i.e.,

$a \times b$ = (G.C.D. or H.C.F. of a and b). (L.C.M. of a and b)

or, $$\text{L.C.M. of } a \text{ and } b = \frac{a \times b}{\text{G.C.D. of } a \text{ and } b}$$

Similarly, if $p(x)$ and $q(x)$ are two polynomials, then

$$\text{L.C.M. of } p(x) \text{ and } q(x) = \frac{p(x) \times q(x)}{\text{G.C.D. of } p(x) \text{ and } q(x)}$$

Thus, L.C.M. of two polynomials

$$= \frac{\text{Product of two polynomials}}{\text{G.C.D. of the two polynomials}}$$

Note:

L.C.M. of two or more given polynomials is a polynomial of smallest degree which is divided by each one of the given polynomials.

L.C.M. by Factorization Method

Step 1 Resolve the given polynomials $p(x)$ and $q(x)$ in the complete factored form.

Step 2 The required L.C.M. is the product of each factor of $p(x)$ and $q(x)$ and if a factor is common, we take that factor which has the highest degree in $p(x)$ or $q(x)$.

Illustration 4 Find the L.C.M. of the polynomials

$(x + 2)^2 (x - 1)(x + 4)^2$

and, $(x + 4)^3 (x + 2)(x + 7)$

Solution: We have, $p(x) = (x + 2)^2 (x - 1)(x + 4)^2$

$q(x) = (x + 4)^3 (x + 2)(x + 7)$

Take the highest powers of factors common to both $p(x)$ and $q(x)$ and remaining terms for L.C.M.

$\therefore$ L.C.M. $= (x + 4)^3 (x + 2)^2 (x - 1)(x + 7)$

Illustration 5 Find the L.C.M. of the polynomials

$(2x^2 - 3x - 2)$ and $(x^3 - 4x^2 + 4x)$

Solution: We have, $p(x) = 2x^2 - 3x - 2$

$= (x - 2)(2x + 1)$

$q(x) = x^3 - 4x^2 + 4x = x(x^2 - 4x + 4) = x(x - 2)^2$

$\therefore$ H.C.F. $= (x - 2)$

Hence, $$\text{L.C.M.} = \frac{p(x)\cdot q(x)}{\text{H.C.F.}} = \frac{x(x-2)^3.(2x+1)}{(x-2)}$$

$= x(x - 2)^2 (2x + 1).$

OR

Taking the highest powers of factors common to both $p(x)$ and $q(x)$ and remaining terms for L.C.M., we have

L.C.M. $= x(x - 2)^2 (2x + 1)$

Practice Exercises

Difficulty Level-1
(Based on Memory)

1. Find the G.C.D. of $22x(x + 1)^2$; $36x^2 (2x^2 + 3x + 1)$.

(a) $2x(x + 1)$ (b) $3x(x + 1)$
(c) $x(x + 1)$ (d) None of these

2. Find the G.C.D. of $16 - 4x^2$; $x^2 + x - 6$.

(a) $x - 2$ (b) $x - 3$
(c) $x - 4$ (d) None of these

3. For what value of a, the G.C.D. of $x^2 - 2x - 24$ and $x^2 - ax - 6$ is $(x - 6)$?

(a) 7 (b) 5
(c) 9 (d) None of these

4. Determine the H.C.F. of $50\ a^2b^2c^3$, $80\ ab^3c^2$ and $120a^2b^3c$.

(a) $20a^2b^2c^3$ (b) $10ab^2c$
(c) $80ab^2c$ (d) $10ab^2c^2$

5. What will be the H.C.F. of $(a - b)(a - 2b)$, $(a - 2b)(a - 3b)$ and $(a - 3b)(a - 4b)$?

(a) $(a - 2b)$ (b) $a - 3b$
(c) $a - 4b$ (d) None of these

6. If $x + k$ is the H.C.F. of $x^2 + ax + b$ and $x^2 + cx + d$, the value of k is:

(a) $\frac{b+d}{a+c}$ (b) $\frac{a+b}{c+d}$
(c) $\frac{a-b}{c-d}$ (d) $\frac{b-d}{a-c}$

7. Find the G.C.D. of:

$$(2x-7)(3x+4); (2x-7)^2(x+3).$$

(a) $(2x-5)$ (b) $(2x-9)$
(c) $(2x-7)$ (d) None of these

8. Find the G.C.D. of the polynomials $(x^2-9)(x-3)$ and $x^2 + 6x+9$.

(a) $(x+3)$ (b) $(x+5)$
(c) $(x-3)$ (d) None of these

9. The L.C.M. and H.C.F. of two polynomials $P(x)$ and $Q(x)$ are $36x^3(x+a)(x^3-a^3)$ and $x^2(x-a)$, respectively. If $P(x) = 4x^2(x^2-a^2)$, find $Q(x)$.

(a) $9x^3(x^3-a^3)$
(b) $6x^3(x^3-a^3)$
(c) $9x^3(x^3+a^3)$
(d) None of these

10. The H.C.F. of x^3-y^3, $x^4+x^2y^2+y^4$ and $x^3y^2+x^2y^3+xy^4$ will be:

(a) $x^4+x^2y^2+y^4$ (b) x^2+xy+y^2
(c) x^2-xy+y^2 (d) $x(x+y)$

11. If $(x-4)$ is the H.C.F. of x^2-x-12 and (x^2-mx-8), the value of m is:

(a) 0 (b) 1
(c) 2 (d) 6

12. Find the G.C.D. of $3+13x-30x^2$; $25x^2-30x+9$.

(a) $7x-4$ (b) $5x-3$
(c) $6x-5$ (d) None of these

13. Find the L.C.M. of the polynomials:

$$(x+3)^2(x-2)(x+1)^2; (x+1)^3(x+3)(x+4).$$

(a) $(x+3)(x+1)^2(x+4)$
(b) $(x+3)^2(x+1)(x-2)$
(c) $(x+3)^2(x+1)^3(x-2)(x+4)$
(d) None of these

14. Find the L.C.M. of the polynomials:

$$2x^2-3x-2; x^3-4x^2+4x.$$

(a) $x(x-2)^2(2x+1)$
(b) $x(x-2)(2x+1)^2$
(c) $x(x-2)(2x+1)$
(d) None of these

15. Find the G.C.D. of $8(x^3-x^2+x)$; $28(x^3+1)$.

(a) $6(x^2+x-1)$ (b) $4(x^2-x+1)$
(c) $8(x^2+2x-1)$ (d) None of these

16. Find the G.C.D. of $4x^4+y^4$, $2x^3-xy^2-y^3$ and $2x^2+2xy+y^2$.

(a) $2x^2+2xy+y^2$
(b) $2x^3+4xy+y^2$
(c) $3x^2+2xy+y^2$
(d) None of these

17. Find the G.C.D. of $(x+4)^2(x-3)^2$ and $(x-1)(x+4)(x-3)^2$.

(a) $(x+3)(x+9)^2$
(b) $(x+4)(x-3)^3$
(c) $(x+4)(x-3)^2$
(d) None of these

18. Find the L.C.M. of the polynomials:

$$16-4x^2; x^2+x-6.$$

(a) $-4(x^2-4)(x+3)$
(b) $6(x^2-4)(x+4)$
(c) $8(x^2-6)(x+3)$
(d) None of these

19. Find the G.C.D. of x^2-4 and x^3-5x+6.

(a) $x-3$ (b) $x-2$
(c) $x+4$ (d) None of these

20. The H.C.F. (Highest Common Factor) of two polynomials is $(y-7)$ and their L.C.M. is $y^3-10y^2+11y+70$. If one of the polynomials is $y^2-5y-14$, find the other.

(a) $y^2-12y+35$
(b) $y^2-8y+35$
(c) $y^2-14y+45$
(d) None of these

21. If $x-4$ is the G.C.D. of x^2-x-12 and x^2-mx-8, find the value of m.

(a) 4 (b) 6
(c) 2 (d) None of these

22. Find the G.C.D. of the polynomials:

$$(x-2)^2(x+3)(x-4); (x-2)(x+2)(x-5).$$

(a) $(x-4)$ (b) $(x-6)$
(c) $(x-2)$ (d) None of these

23. For what value of a, the G.C.D. of $x^2-2x-24$ and x^2-ax-6 is $(x-6)$?

(a) 7 (b) 5
(c) 9 (d) None of these

24. L.C.M. and H.C.F. of two polynomials $p(x)$ and $q(x)$ are $36x^3(x+a)(x^3-a^3)$ and $x^2(x-a)$, respectively. If $p(x) = 4x^2(x^2-a^2)$, find $q(x)$.

(a) $12x^3(x^3-a^3)$

(b) $6x^3(x^3-a^3)$

(c) $9x^3(x^3-a^3)$

(d) None of these

25. If $(x-a)$ is the G.C.D. of x^2-x-6 and $x^2+3x-18$, find the value of a.

(a) 3 (b) 6

(c) 9 (d) None of these

26. G.C.D. and L.C.M. of two polynomials $p(x)$ and $q(x)$ are $x(x+a)$ and $12x^2(x+a)(x^2-a^2)$, respectively. If $p(x) = 4x(x+a)^2$, find $q(x)$.

(a) $3x^2(x^2-a^2)$

(b) $5x^2(x^3-a^3)$

(c) $4x^2(x^2-a^2)$

(d) None of these

27. Find the G.C.D. of $8(x^4-16)$ and $12(x^3-8)$.

(a) $6(x-2)$ (b) $4(x-2)$

(c) $8(x-2)$ (d) None of these

28. Find the L.C.M. of the polynomials $(x+3)(-6x^2+5x+4)$; $(2x^2+7x+3)(x+3)$.

(a) $-(x+3)^2(3x-4)(2x+1)$

(b) $(x+3)^2(3x-4)(2x+1)$

(c) $(x+3)^2(3x+4)(2x+1)$

(d) None of these

29. Find the G.C.D. of the polynomials $36x^2-49$ and $6x^2-25x+21$.

(a) $8x-9$ (b) $9x-5$

(c) $6x-7$ (d) None of these

30. Find the L.C.M. of the polynomials:

$$30x^2+13x-3;\ 25x^2-30x+9.$$

(a) $-(5x-3)^2(5x+3)(6x-1)$

(b) $(5x-3)^2(5x+3)(6x-1)$

(c) $(5x+3)^2(6x-1)$

(d) None of these

31. Find the G.C.D. of the polynomials $6x^2+11x$ and $2x^2+x-3$.

(a) $4x+5$ (b) $2x-3$

(c) $2x+3$ (d) None of these

32. H.C.F. of two expressions p and q is 1. Their L.C.M. is:

(a) $(p+q)$ (b) $(p-q)$

(c) pq (d) $\frac{1}{pq}$

33. H.C.F. of $(2x^2-4x)$, $(3x^4-12x^2)$ and $(2x^5-2x^4-4x^3)$ is:

(a) $2x(x+2)$ (b) $2x(2-x)$

(c) $2x(x-2)$ (d) $x(x-2)$

34. The product of two non-zero expressions is $(x+y+z)p^3$. If their H.C.F. is p^2, their L.C.M. is:

(a) $(x+y)p$ (b) $(y+2)p$

(c) $(z+x)p$ (d) $(x+y+z)p$

35. If $(x-1)$ is the H.C.F. of x^2-1 and $px^2-q(x+1)$, then:

(a) $p=2q$ (b) $q=2p$

(c) $3p=2q$ (d) $2p=3q$

36. L.C.M. of (x^2-y^2), (x^3-y^3), $(x^3-x^2y-xy^2+y^3)$ is:

(a) $(x+y)(x-y)(x^2+y^2+xy)$

(b) $(x+y)(x-y)^2(x^2+y^2+xy)$

(c) $(x+y)(x-y)^2(x^2+y^2-xy)$

(d) $(x+y)^2(x-y)^2$

Answer Keys

Difficulty Level-1

1. (a)	2. (a)	3. (b)	4. (b)	5. (d)	6. (d)	7. (c)	8. (d)	9. (a)	10. (b)	11. (c)	12. (b)	13. (c)
14. (a)	15. (b)	16. (a)	17. (c)	18. (a)	19. (b)	20. (a)	21. (c)	22. (c)	23. (b)	24. (c)	25. (a)	26. (a)
27. (b)	28. (a)	29. (c)	30. (b)	31. (c)	32. (c)	33. (d)	34. (d)	35. (a)	36. (b)			

Explanatory Answers

Difficulty Level-1

1. (*a*) Here, $p(x) = 22x(x+1)^2$

$$q(x) = 36x^2(2x^2 + 3x + 1)$$
$$= 36x^2(2x^2 + 2x + x + 1)$$
$$= 36x^2[2x(x+1) + 1(x+1)]$$
$$= 36x^2(2x+1)(x+1)$$

$\therefore$ G.C.D. of numerical factors = 2

and the highest degree common divisor = $x(x+1)$

Hence, required G.C.D. = $2x(x+1)$.

2. (*a*)

3. (*b*) Obviously, $(x-6)$ divides both $x^2 - 2x - 24$ and $x^2 - ax - 6$

So, $x = 6$ must make each polynomial zero

$\therefore$ $(6)^2 - 2 \times 6 - 24 = (6)^2 - 6a - 6$

or, $6a = 30$ or $a = 5$.

4. (*b*)
$$50a^2b^2c^3 = 5^2 \times 2 \times a^2 \times b^2 \times c^3$$
$$80ab^3c^2 = 5 \times 2^4 \times a \times b^3 \times c^3$$
$$120a^2b^3c = 3 \times 5 \times 2^3 \times a^2 \times b^3 \times c$$

$\therefore$ Required H.C.F. $= 5 \times 2 \times a \times b^2 \times c$
$= 10ab^2c$.

5. (*d*) Since no factor is common to three given expressions, 1 will be a common factor and hence H.C.F. = 1.

6. (*d*) Since $x + k$ is the H.C.F. it will divide each one of the given expressions. So, $x = -k$ will make each one zero.

$\therefore$ $k^2 - ak + b = 0$, $k^2 - ck + d = 0$

So, $k^2 - a\,k + b = k^2 - ck + d$

or, $k = \dfrac{b-d}{a-c}$.

7. (*c*) Here we find that $(2x - 7)$ is a polynomial which is a common divisor and has highest degree among all common divisors. Further, the coefficient of the highest degree term $(2x)$ is 2 which is positive. Hence, $(2x - 7)$ is the G.C.D. of the given polynomial.

8. (*d*) Let, $p(x) = (x^2 - 9)(x-3) = (x^2 - 3^2)(x-3)$
$= (x+3)(x-3)^2$

and, $q(x) = x^2 + 6x + 9$
$= x^2 + 2x \times 3 + 3^2 = (x+3)^2$

The highest degree common divisor of the given polynomials is $(x+3)$

$\therefore$ G.C.D. is $(x+3)$.

9. (*a*) L.C.M. $= 36x^3(x+a)(x^3 - a^3)$
$= 36x^3(x+a)(x-a)(x^2 + a^2 + ax)$

H.C.F. $= x^2(x-a)$

$P(x) = 4x^2(x^2 - a^2) = 4x^2(x-a)(x+a)$

$$\therefore Q(x) = \frac{\text{(L.C.M.)(H.C.F.)}}{P(x)}$$

$$= \frac{36x^3(x+a)(x-a)(x^2+a^2+ax)x^2(x-a)}{4x^2(x-a)(x+a)}$$

$$= 9x^3(x^2 + a^2 + ax)(x-a) = 9x^3(x^3 - a^3).$$

10. (*b*)
$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$
$$x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2$$
$$= (x^2 + y^2)^2 - (xy)^2$$
$$= (x^2 + xy + y^2)(x^2 - xy + y^2)$$

and, $x^3y^2 + x^2y^3 + xy^4$
$= xy^2(x^2 + xy + y^2)$

$\therefore$ Required H.C.F. $= x^2 + xy + y^2$.

11. (*c*) Clearly, $x = 4$ will make each one of the given expressions zero. So, $16 - 4m - 8 = 0$ or, $m = 2$.

12. (*b*) Here,

$$p(x) = 3 + 13x - 30x^2 = 3 + 18x - 5x - 30x^2$$
$$= 3(1 + 6x) - 5x(1 + 6x)$$
$$= (3 - 5x)(1 + 6x)$$
$$= -(5x - 3)(1 + 6x)$$
$$q(x) = 25x^2 - 30x + 9 = (5x - 3)^2$$

$\therefore$ G.C.D. of numerical factors = 1 and highest degree of common divisor = $(5x - 3)$.

13. (*c*) $p(x) = (x+3)^2 (x-2)(x+1)^2$

$q(x) = (x+1)^3 (x+3)(x+4)$

$\therefore$ L.C.M. $= (x+3)^2 (x+1)^3 (x-2)(x+4)$.

14. (*a*) We have

$p(x) = 2x^2 - 3x - 2 = 2x^2 - 4x + x - 2$

$= 2x(x-2) + 1(x-2)$

$= (2x+1)(x-2)$

$q(x) = x^3 - 4x^2 + 4x$

$= x(x^2 - 4x + 4) = x(x-2)^2$

$\therefore$ L.C.M. $= x(x-2)^2 (2x+1)$.

15. (*b*) We have the factorization

$p(x) = 8(x^3 - x^2 + x) = 2^3 . x . (x^2 - x + 1)$

$q(x) = 28(x^3 + 1)$

$= 2^2 . 7 . (x+1)(x^2 - x + 1)$

$\therefore$ G.C.D. of numerical factors $= 2^2$

and the highest degree common divisor

$= x^2 - x + 1$

Therefore, required

G.C.D. $= 2^2 (x^2 - x + 1)$

$= 4(x^2 - x + 1)$.

16. (*a*) 1st expression $= (2x^2 + y^2)^2 - (2xy)^2$

$= (2x^2 + y^2 + 2xy)(2x^2 + y^2 - 2xy)$

2nd expression $= (2x^3 - 2y^3) - y^2 (x - y)$

$= 2(x-y)(x^2 + xy + y^2) - y^2 (x-y)$

$= (x-y)(2x^2 + 2xy + 2y^2 - y^2)$

$= (x-y)(2x^2 + 2xy + y^2)$

Hence, G.C.D. $= 2x^2 + 2xy + y^2$.

17. (*c*) Let, $p(x) = (x+4)^2 (x-3)^2$

and, $q(x) = (x-1)(x+4)(x-3)^2$

The highest degree common divisor is

$(x+4)(x-3)^2$

$\therefore$ The G.C.D. of given polynomial is

$(x+4)(x-3)^2$.

18. (*a*) We have

$p(x) = 16 - 4x^2 = 4(4 - x^2)$

$= 4(2-x)(2+x) = -4(x-2)(x+2)$

$q(x) = x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= x(x+3) - 2(x+3) = (x+3)(x-2)$

$\therefore$ L.C.M. $= -4(x-2)(x+2)(x+3)$

$= -4(x^2 - 4)(x+3)$.

19. (*b*) Let, $p(x) = x^2 - 4 = x^2 - 2^2$

$= (x+2)(x-2)$

and, $q(x) = x^2 - 5x + 6 = x^2 - 2x - 3x + 6$

$= x(x-2) - 3(x-2) = (x-2)(x-3)$

The highest degree common divisor is $x - 2$

$\therefore$ G.C.D. of $p(x)$ and $q(x)$ is $x - 2$.

20. (*a*) H.C.F. $= (y-7)$

L.C.M. $= y^3 - 10y^2 + 11y + 70$

$p(x) = y^2 - 5y - 14$

$q(x) = ?$

L.C.M. of two polynomials

$$= \frac{\text{Ist Polynomial} \times \text{IInd Polynomial}}{\text{H.C.F. of two polynomials}}$$

$$\therefore \quad \text{L.C.M.} = \frac{p(x)\, q(x)}{\text{H.C.F.}}$$

$$y^3 - 10y^2 + 11y + 70 = \frac{(y^2 - 5y - 14) \times q(x)}{(y-7)}$$

$$\therefore \quad q(x) = \frac{(y-7)(y^3 - 10y^2 + 11y + 70)}{(y^2 - 5y - 14)}$$

$= (y-7)(y-5)$

$= y^2 - 12y + 35$.

21. (*c*) H.C.F. $= (x-4)$

$p(x) = x^2 - x - 12 = (x-4)(x+3)$

$q(x) = x^2 - mx - 8$

As $(x-4)$ is common in $p(x)$ and $q(x)$. Hence, $x - 4$ should be a factor of $x^2 - mx - 8$

Thus, putting $(x-4) = 0$ in $q(x)$, we get (Remainder theorem)

$q(x) = x^2 - mx - 8$

$q(4) = 4^2 - m \times 4 - 8 = 0$

$\Rightarrow 16 - 4m - 8 = 0$

$\Rightarrow \quad m = 2$.

22. (*c*) Let, $p(x) = (x-2)^2 (x+3)(x-4)$

and, $q(x) = (x-2)(x+2)(x-5)$

the highest degree common divisor of the given polynomials is $x - 2$.

$\therefore$ The G.C.D. is $x - 2$.

23. (*b*) Here, $p(x) = x^2 - 2x - 24$

and, $q(x) = x^2 - ax - 6$

Since $(x - 6)$ is the G.C.D. of $p(x)$ and $q(x)$, $(x - 6)$ is a factor of $p(x)$ and $q(x)$ both

$\Rightarrow \quad p(6) = q(6)$

$\Rightarrow 36 - 2 \times 6 - 24 = 36 - a \times 6 - 6$

$\Rightarrow \quad a = 5.$

24. (*c*) We know that

$p(x) \times q(x) = \text{L.C.M.} \times \text{H.C.F.}$

$4x^2(x^2 - a^2) \times q(x) = 36x^3(x + a)(x^3 - a^3)x^2(x - a)$

$$\Rightarrow \quad q(x) = \frac{36x^5(x^2 - a^2)(x^3 - a^3)}{4x^2(x^2 - a^2)}$$

$= 9x^3(x^3 - a^3).$

25. (*a*) Let, $p(x) = x^2 - x - 6$

and, $q(x) = x^2 + 3x - 18$

Since $(x - a)$ is the G.C.D. of $p(x)$ and $q(x)$, $(x - a)$ is a divisor of $p(x)$ and $q(x)$ or $(x - a)$ is a factor of $p(x)$ and $q(x)$ both.

$\Rightarrow \quad p(a) = 0$ and $q(a) = 0$

$\Rightarrow \quad p(a) = q(a)$

$\Rightarrow \quad a^2 - a - 6 = a^2 + 3a - 18$

$\Rightarrow \quad 4a = 12 \Rightarrow a = 3.$

26. (*a*)
$$q(x) = \frac{\text{L.C.M.} \times \text{H.C.F.}}{p(x)}$$

$$= \frac{12x^2(x + a)(x^2 - a^2)x(x + a)}{4x(x + a)^2}$$

$= 3x^2(x^2 - a^2).$

27. (*b*) $p(x) = 8(x^4 - 16)$

$= 4 \times 2(x^2 + 4)(x + 2)(x - 2)$

$q(x) = 12(x^3 - 8)$

$= 4 \times 3(x - 2)(x^2 + 2x + 4)$

Hence, G.C.D. $= 4(x - 2).$

28. (*a*) $p(x) = (x + 3)(-6x^2 + 5x + 4)$

$= (x + 3)(-6x^2 + 8x - 3x + 4)$

$= -(x + 3)(3x - 4)(2x + 1)$

$q(x) = (2x^2 + 7x + 3)(x + 3)$

$= (2x + 1)(x + 3)(x + 3)$

$\therefore$ L.C.M. $= -(x + 3)^2(3x - 4)(2x + 1).$

29. (*c*) $p(x) = 36x^2 - 49$

$= (6x)^2 - (7)^2 = (6x + 7)(6x - 7)$

$q(x) = 6x^2 - 25x + 21$

$= 6x^2 - 18x - 7x + 21$

$= 6x(x - 3) - 7(x - 3)$

$= (6x - 7)(x - 3)$

$\therefore$ G.C.D. $= (6x - 7).$

30. (*b*) $30x^2 + 13x - 3 = 30x^2 + 18x - 5x - 3$

$= 6x(5x + 3) - 1(5x + 3)$

$= (5x + 3)(6x - 1)$

$q(x) = 25x^2 - 30x + 9$

$= 25x^2 - 15x - 15x + 9$

$= 5x(5x - 3) - 3(5x - 3)$

$= (5x - 3)^2$

L.C.M. $= (5x - 3)^2(5x + 3)(6x - 1).$

31. (*c*) $p(x) = 6x^2 + 11x + 3$

$= 6x^2 + 9x + 2x + 3$

$= 3x(2x + 3) + 1(2x + 3)$

$= (2x + 3)(3x + 1)$

$q(x) = 2x^2 + x - 3$

$= 2x^2 + 3x - 2x - 3$

$= (2x + 3)(x - 1)$

$\therefore$ G.C.D. $= (2x + 3).$

32. (*c*)
$$\text{L.C.M.} = \frac{\text{Product of expressions}}{\text{H.C.F.}}$$

$$= \frac{pq}{1} = pq.$$

33. (*d*) $2x^2 - 4x = 2x(x - 2)$

$3x^4 - 12x^2 = 3x^2(x^2 - 4)$

$= 3x^2(x - 2)(x + 2)$

$2x^5 - 2x^4 - 4x^3 = 2x^3(x^2 - x - 2)$

$= 2x^3(x - 2)(x + 1)$

$\therefore$ H.C.F. $= x(x - 2).$

34. (*d*) L.C.M. $= \frac{\text{Product}}{\text{H.C.F.}} = \frac{(x+y+z)p^3}{p^2}$

$= (x+y+z)p$

35. (*a*) Since $(x-1)$ is the H.C.F., it will divide each one of the given expressions. So, $x = -1$ will make each one zero

$\therefore \quad p \times 1^2 - q(1+1) = 0$ or $p = 2q$.

36. (*b*) $x^2 - y^2 = (x-y)(x+y)$,

$x^3 - y^3 = (x-y)(x^2+xy+y^2)$,

$x^3 - x^2y - xy^2 + y^3 = x^2(x-y) - y^2(x-y)$

$= (x-y)(x^2-y^2)$

$= (x-y)^2(x+y)$

$\therefore \quad$ L.C.M. $= (x-y)^2(x+y)(x^2+y^2+xy)$.