Chapter 2

Network Theorems

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Superposition theorem
- Thevenin's theorem
- Norton's theorem
- Maximum power transfer theorem
- Reciprocity theorem
- Millman's theorem
- Tellegen's theorem
- Substitution theorem
- Compensation theorem

Superposition Theorem

Statement: Whenever a linear bilateral circuit is excited by more than one independent sources. The total response is the algebraic sum of individual responses due to all independent sources.

Steps to Apply Superposition Theorem

Step 1: Select a single source acting alone short the other voltage sources and open the current sources (deactivate).

Step 2: Find the current through or the voltage across the required element due to the sources under consideration.

Step 3: Repeat the above steps for all the independent sources.

Step4: Add all the individual effects produced by individual sources, to obtain the total current through or voltage across the element.

Notes:

1. Dependent sources are never deactivated.
2. When an independent voltage source is deactivated, it is set to zero.

   $\Rightarrow$ replaced by short circuit

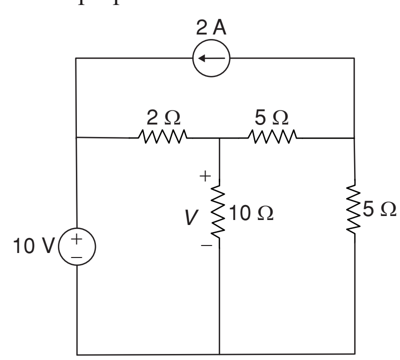
3. When an Independent current source is deactivated, it is set to zero.

   $\Rightarrow$ replaced by open circuit.

   $\therefore\ I = 0, \quad \Rightarrow \quad$ open circuit.

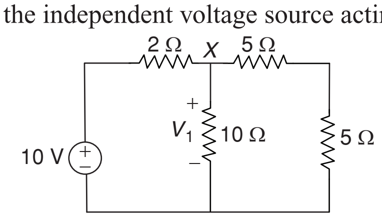
Solved Examples

Examples 1: Use superposition to find $V = ?$



Solution:

(i) Consider the independent voltage source acting alone:



Apply KCL (Kirchoff's Current Law) at node X:

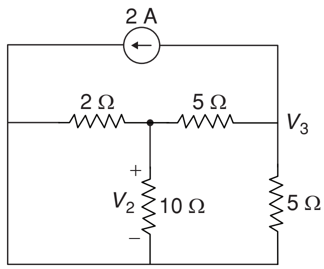
$$\frac{V_1 - 10}{2} + \frac{V_1}{10} + \frac{V_1}{10} = 0$$

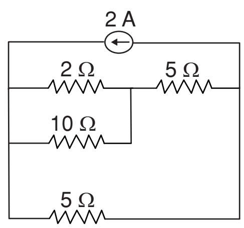
$$5(V_1 - 10) + 2V_1 = 0$$

$$7V_1 = 50$$

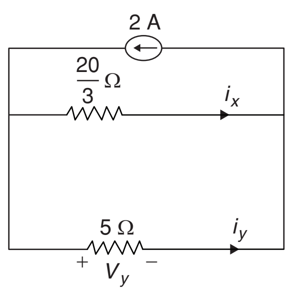
$$V_1 = \frac{50}{7}\text{ V}$$

(ii) Consider the independent current source acting alone:





$\Longrightarrow$



$$i_x = \frac{2\times 5}{\frac{20}{3}+5} = \frac{10\times 3}{35}$$

$$= \frac{30}{35} = \frac{6}{7}\text{ A}$$

$$i_y = 2 - \frac{6}{7}$$

$$= \frac{8}{7}\text{ A}$$

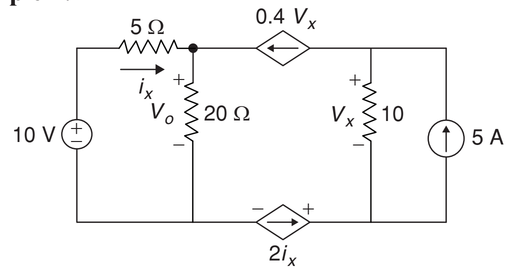
$$V_y = \frac{5\times 8}{7} = \frac{40}{7}\text{ V}$$

$$V_2 = V_{10\,\Omega} = \frac{40}{7} - 5\times\frac{6}{7}$$

$$= \frac{10}{7}\text{ V}$$

$$V = V_1 + V_2 = \frac{50}{7} + \frac{10}{7} = \frac{60}{7}\text{ V}$$

Example 2:



The value of V_o is

(A) −8 V (B) 16 V

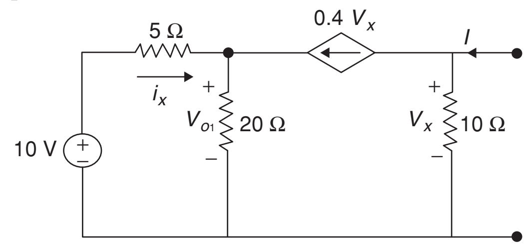
(C) −16 V (D) 24 V

Solution:

(i) Activate independent voltage source only

∴ Independent current source deactivated

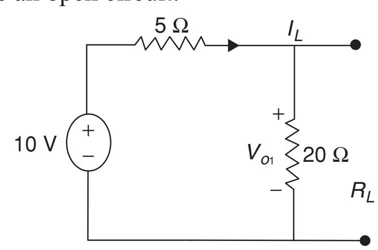
i.e., open circuit



$V_x = -10 \times 0.4\,V_x$

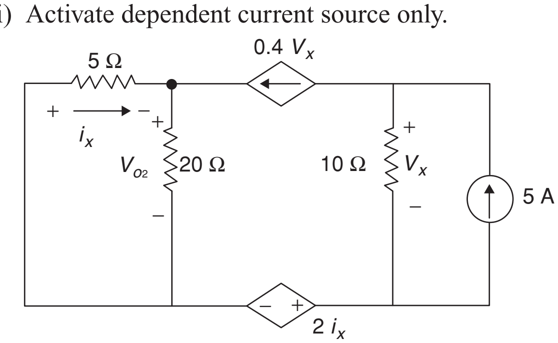
$V_x = -4V_x \quad \Rightarrow V_x = 0$

$\Rightarrow$ $V_x = 0$, so dependent current $0.4V_x$ equal to zero, so it acts like an open circuit.



$$V_{01} = 10 \times \frac{20}{25} = 8\text{ V}$$

(ii) Activate dependent current source only.



KCL at node A,

$$\frac{V_{o2}}{20} + \frac{V_{o2}}{5} 0.4\,V_x = 0$$

$$5\,V_{o2} = 8\,V_x$$

At node *B*

$$5 = \frac{V_x}{10} + 0.4\,V_x$$

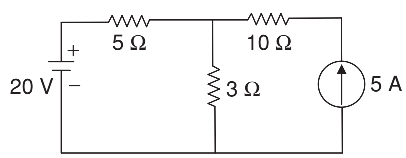
$$50 = V_x + 4\,V_x$$

$$V_x = 10\text{ V}$$

$$V_{o2} = \frac{80}{5} = 16\text{ V}$$

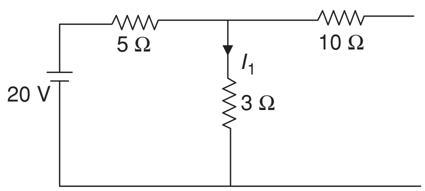
$$\therefore\ V_o = V_{o1} + V_{o2}$$
$$= 8 + 16 = 24\text{ V}$$

Example 3:



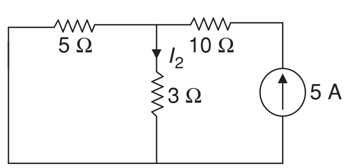
Find the current through the 3 Ω resistor using superposition theorem.

Solution:
Considering 20 V source alone, i.e., 5 A current source is open circuited



$$I_1 = \frac{20}{5+3} = \frac{20}{8} = 2.5\text{ A}$$

Considering 5 A source alone, i.e., 20 V voltage source is short circuited.

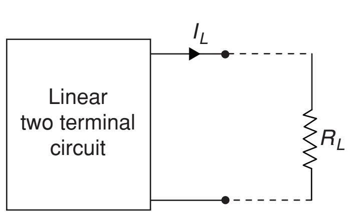


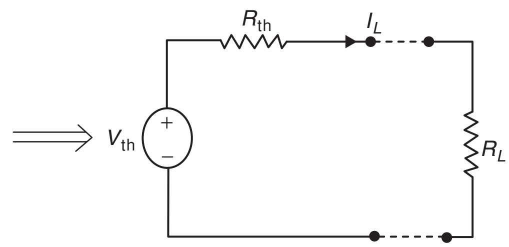
$$I_2 = 5 \times \frac{5}{5+3}$$

$$I_2 = 3.125\text{ A}$$

$$I = I_1 + I_2 = 2.5 + 3.125 = 5.625\text{ A}$$

Thevenin's Theorem





Statement: Any two terminal bilateral linear circuit can be replaced by an equivalent circuit consisting of a Thevenin's voltage source and Thevenin's series resistor. Thevenin's voltage source is the open circuit voltage across the terminals and Thevenin's resistance is the equivalent resistance across the terminals.

Note:

1. Circuit consisting only independent sources; V_{th}, R_{th} are calculated conventionally.
2. A circuit consisting of both dependent and independent sources; V_{th}, I_{sc}

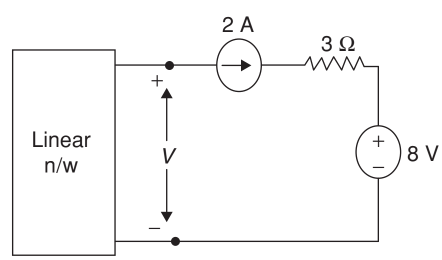
$$\Rightarrow \quad R_{th} = \frac{V_{th}}{I_{sc}}\ \Omega$$

3. A circuit consisting only dependent sources.

$$R_{th} = \frac{V_{DC}}{I_{DC}}\ \Omega$$

$$V_{th} = 0\text{ V and } I_{th} = 0\text{ A}$$

Examples 4: Consider the following circuit



The linear network contains only DC sources and resistances. The value of V is

(A) 14 V (B) 2 V
(C) 8 V (D) indeterminate

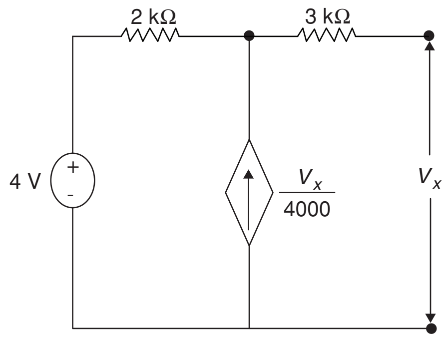
Solution: (D)
From the given circuit

$$V = V_{2A} + 3 \times 2 + 8$$

$$V = V_{2A} + 14$$

But voltage across current source is unknown.
So, $V = 14$ + any value $\Rightarrow$ indeterminate.

Example 5: Obtain the Thevenin's equivalent of the network below:



(A) $V_{th} = 0$ V, $R_{th} = 5$ kΩ
(B) $V_{th} = 2.4$ V, $R_{th} = 1.2$ kΩ
(C) $V_{th} = 8$ V, $R_{th} = 10$ kΩ
(D) None of the above

Solution: (C)
Under open circuit condition,

$$I_{3\Omega} = 0$$

∴ The node equation is given by,

$$\frac{V_x - 4}{2\text{ k}\Omega} - \frac{V_x}{4000} = 0$$

$$\frac{V_x - 4}{2\text{ k}\Omega} - \frac{V_x}{4000}$$

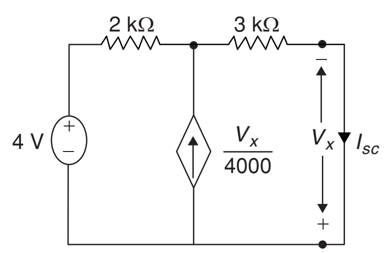
$$2(V_X - 4) = V_x$$

$$\Rightarrow V_X = V_{th} = 8\text{ V}$$

But we know,

$$R_{th} = \frac{V_{th}}{I_{sc}}\text{ case}$$

Compute the short circuit current



But, $V_x = 0$ from the above circuit

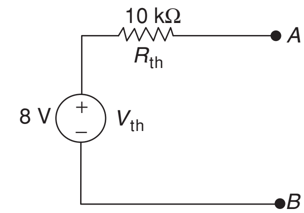
∴ dependent current source $\frac{V_x}{4000} = 0$

⇒ open circuit

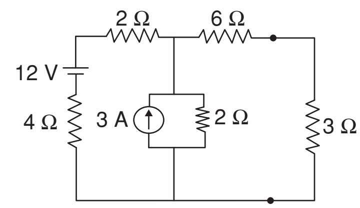
$$\text{i.e., } I_{sc} = \frac{4}{(2+3)}\text{ mA} = 0.8\text{ mA}$$

$$\therefore \quad R_{th} = \frac{V_{th}}{I_{sc}} = \frac{8}{0.8}\text{ k}\Omega = 10\text{ k}\Omega$$

∴ Thevenin's equivalent is

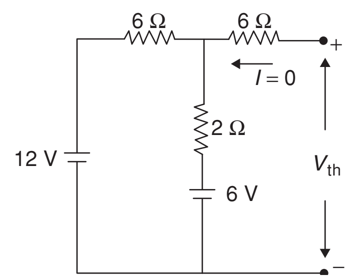


Example 6: Find the current flowing through the 3 Ω resistor.



(A) 1 A
(B) 3 A
(C) 5 A
(D) 0.714 A

Solution: (D)
Applying source transformation to current source it becomes.



Apply nodal Analysis at node A

$$\frac{V_{th} - 12}{6} + \frac{V_{th} - 6}{2} = 0$$

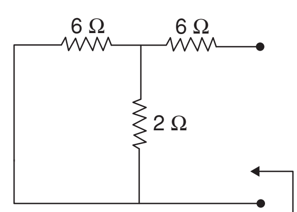
$$V_{th} - 12 + 3\,(V_{th} - 6) = 0$$

$$4V_{th} = 12 + 18$$

$$V_{th} = \frac{30}{4} = 7.5\text{ V}$$

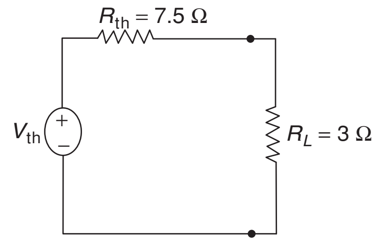
R_{th}:

(i) All independent voltage sources are short circuited current sources are open circuit.
∴ it becomes



$$\begin{aligned} R_{th} &= 6 + 6||2 \\ &= 6 + 1.5 \\ &= 7.5\ \Omega \end{aligned}$$

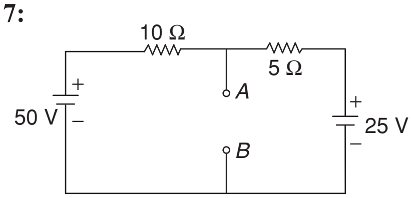
The Thevenin's equivalent network is shown below:



$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{7.5}{7.5+3}$$

$$I_L = 0.71428 \text{ A.}$$

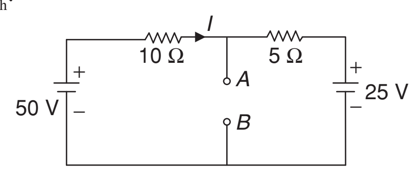
Example 7:



Determine the Thevenin's equivalent circuit across *AB* for the above network shown in figure.

Solution:

To find V_{th}:



$$I = \frac{50-25}{10+5}$$

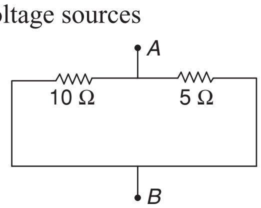
$$I = \frac{25}{15} = 1.67 \text{ A}$$

$$V_{AB} = 1.67 \times 5 + 25$$

$$V_{th} = 33.35 \text{ V}$$

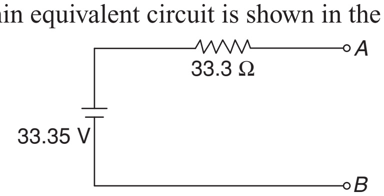
To find R_{th}:

Deactivate the voltage sources

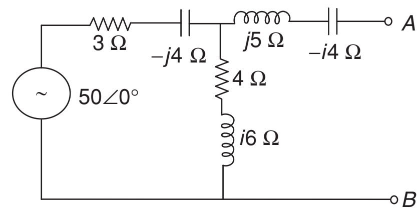


$$R_{th} = 10 \,||\, 5 = 3.33\ \Omega$$

The Thevenin equivalent circuit is shown in the figure:



Example 8: For the circuit shown in figure, determine the Thevenin's equivalent between the output terminals.



Solution:

$$V_{th} = 50\angle 0° \times \frac{(4+j6)}{(4+j6)+(3-j4)}$$

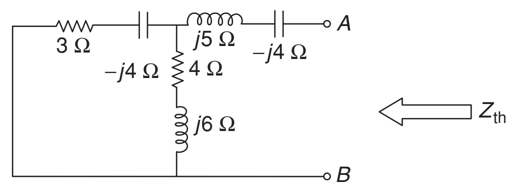
$$= 50\angle 0° \times \frac{(4+j6)}{7+j2}$$

$$= 50\angle 0° \times \frac{7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$V_{th} = 49.5\angle 40.35° \text{V}$$

To find Z_{th}:

Short circuit the source $50\angle 0°$



$$Z_{th} = (j5 - j4) + \frac{(3-j4)(4+j6)}{(3-j4)+(4+j6)}$$

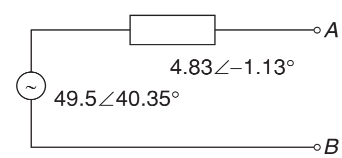
$$= j1 + \frac{5\angle 53.13° \times 7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$= j1 + 4.95\angle -12.78°$$

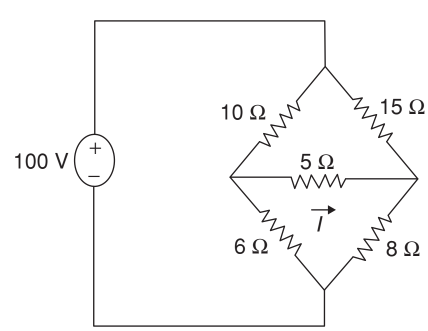
$$= j1 + 4.83 - j1.095$$

$$= 4.83 - j0.095$$

$$Z_{th} = 4.83\angle -1.13°$$



Example 9: Find the current through the 5Ω using Thevenin's Theorem,



(A) $I = 0.194$ A (B) $1 = 0.3$ A
(C) $I = \text{O}$ A (D) None of the above

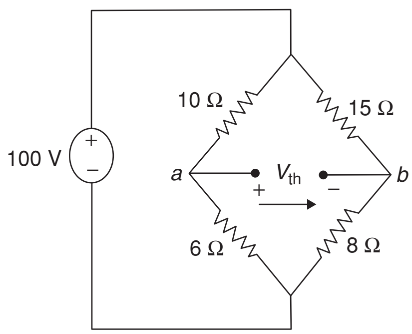
Solution: (A)

Balanced condition

$R_1 R_4 = R_2 R_3$

$10 \times 8 \neq 6 \times 15$

$\therefore$ The bridge is unbalanced so current flowing through the 5 Ω resistor is not zero.



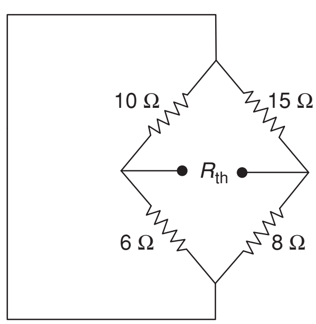
$$V_{th} = V_{ab} = V_a - V_b$$

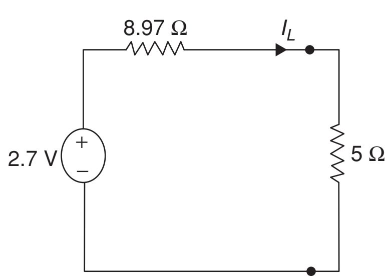
$$V_a = \frac{100 \times 6}{6+10} = 37.5 \text{ V}$$

$$V_b = 100 \times \frac{8}{15+8} = 34.78 \text{ V}$$

$$V_{th} = 37.5 - 34.78 = 2.717 \text{ V}$$

R_{th}:

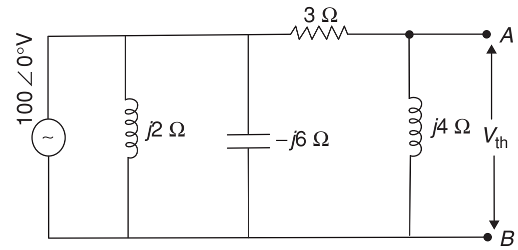




$$R_{th} = (10 \parallel 6) + (15 \parallel 8)$$
$$= 3.75 + 5.21 \approx 8.97 \ \Omega$$

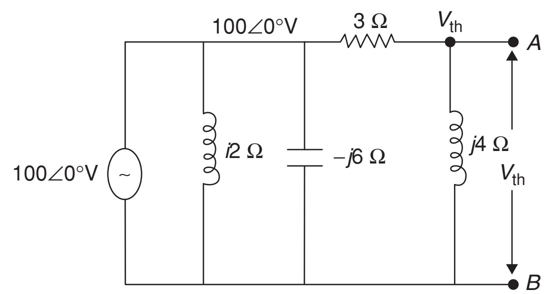
$$I_L = \frac{2.717}{13.97} = 0.194 \text{ A.}$$

Example 10: The Thevenin's equivalent voltage V_{th} across the terminal A and B of the network shown in the figure is given by



(A) $j16(3 - j4)$ (B) $j16(3 + 4)$
(C) $16(3 + j4)$ (D) $16(3 - j4)$

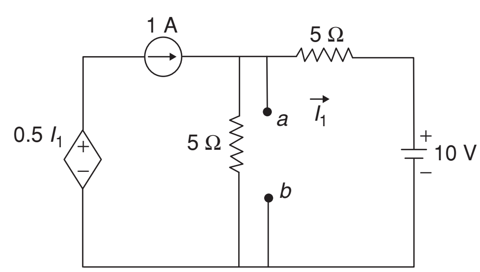
Solution: (A)



$$V_{th} = V_{j4} = 100 \times \frac{j4}{j4+3}$$

$$= \frac{100 \times 4(3 - j4)}{25} = j16(3 - j4).$$

Example 11: For circuit shown in the figure, Thevenin's voltage and Thevenin's equivalent resistance at terminals $a - b$ is



(A) 5 V and 2 Ω (B) 7.5 V and 2.5 Ω
(C) 4 V and 2 Ω (D) 3 V and 2.5 Ω

Solution: (B)

$$V_{th} = V_{ab}$$

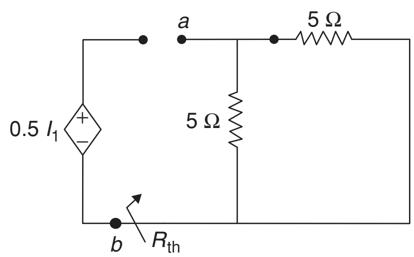
For V_{th}:

$$\frac{V_{th} - 10}{5} + \frac{V_{th}}{5} - 1 = 0$$

$$2V_{th} - 10 - 5 = 0$$

$$V_{th} = 7.5 \text{ V.}$$

For R_{th}: Deactivate the independent sources

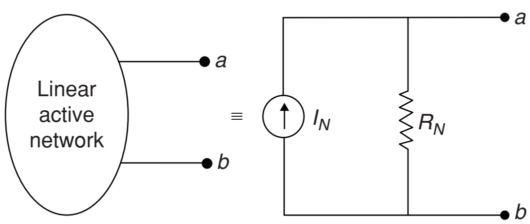


$$R_{th} = R_{ab} = 5||5 = 2.5\ \Omega$$

$$\therefore\ V_{th} = 7.5\text{ V and } R_{th} = 2.5\ \Omega$$

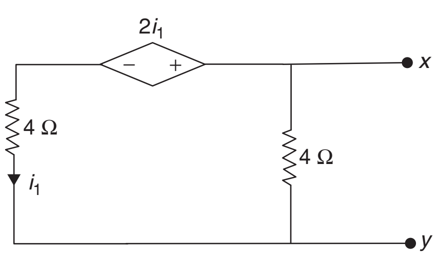
Norton's Theorem

A one port linear, active, resistive network, which contains one or more voltage or current sources can be replaced by a single current source in parallel with a resistance.



$$I_n = \frac{V_{th}}{R_{th}} \text{ and } R_N = R_{th}$$

Example 12:



The Norton's equivalent of the network is

(A) $I_{SC} = 0,\ R_N = 1.5\ \Omega$

(B) $V_{th} = 0\text{ V},\ R_{th} = \frac{4}{3}\ \Omega$

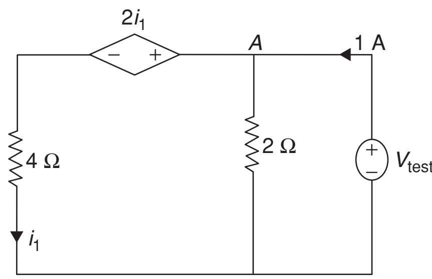
(C) $I_{SC} = 0,\ R_N = \frac{4}{3}\ \Omega$

(D) None of the above

Solution: (A)

The network does not have any active independent sources,

$I_{sc} = 0$



Applying KCL at node A,

$$1\text{ A} = \frac{V_{test}}{2} + i_1$$

But $$i_1 = \frac{V_{test} - 2i_1}{4}$$

$$4i_1 + 2i_1 = V_{test}$$

$$i_1 = \frac{V_{test}}{6}\text{ A}$$

$$1\text{ A} = \frac{V_{test}}{2} + \frac{V_{test}}{6}$$

$$6 = 3V_{test} + V_{test}$$

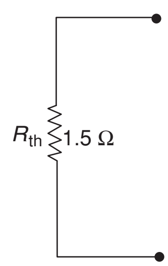
$$V_{test} = 1.5\text{ V}$$

$$R_{th} = \frac{V_{test}}{1\text{ A}} = 1.5\ \Omega$$

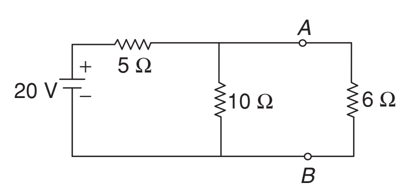
Example 13: For the above circuit find the Thevenin's equivalent circuit.

Solution:

We know, $R_{th} = R_N = 1.5\ \Omega$, so the network does not have any active independent sources. Hence, $V_{th} = 0$ V

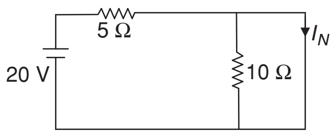


Example 14:



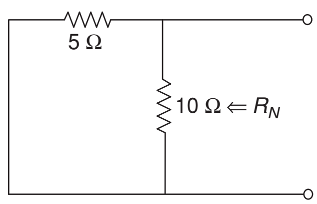
Using Norton's theorem, find the current through the 6 Ω load.

Solution:
To find I_N, short circuit load terminals,



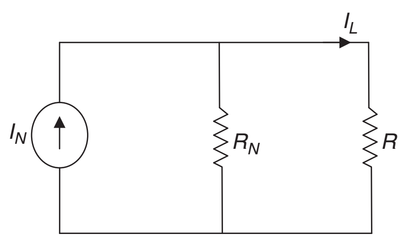
$$I_N = \frac{20}{5} = 4 \text{ A}$$

To find R_N, remove load, short circuit the voltage source 20 V



$$R_N = 5 \| 10 = \frac{5 \times 10}{5 + 10} = 3.33\ \Omega$$

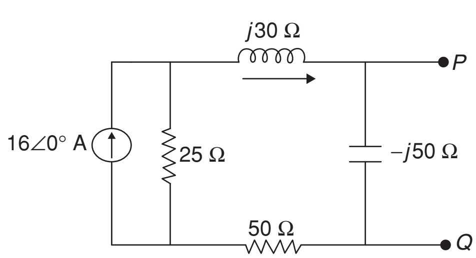
The Norton equivalent circuit is



$$I_L = I_N \frac{R_N}{R_N + R_L} = 4 \times \frac{3.33}{3.33 + 6}$$

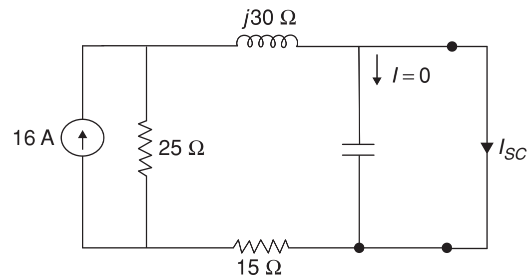
$$I_L = 1.43 \text{ A}$$

Example 15: In the circuit shown below, the Norton's equivalent current, with respect to the terminals P and Q is



(A) $(6.4 + j4.8)$ A
(B) $(16 + j0)$ A
(C) $(6.4 - j4.8)$ A
(D) $(6.56 - j7.87)$ A

Solution: (C)
For Norton equivalent current short circuiting the terminals PQ



$$I_{sc} = 16\angle 0° \times \frac{25}{25 + 15 + j30}$$

$$= \frac{25}{40 + j30} \times 16\angle 0°$$

$$= \frac{25 \times 16}{\sqrt{(40)^2 + (30)^2}} \cdot \frac{1}{\angle 36.86°}$$

$$= 8\angle -36.86°$$

$$= 8\,[\cos 36.86° - j\sin 36.86°]$$

$$I_{sc} = (6.4 - j4.8) \text{ A.}$$

Maximum Power Transfer Theorem

This theorem is used to find the value of load resistance for which there would be maximum amount of power transfer from source to load.

Statement: A resistance load connected to a DC network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.

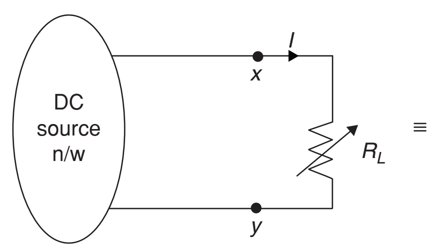


Figure 1 Load connected to the DC source network

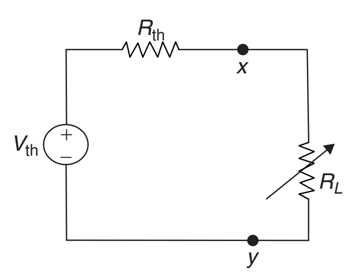
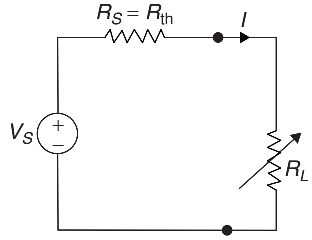


Figure 2 Equivalent network

⇒ This theorem is applicable only for linear networks
⇒ This theorem is applicable only when load is variable

Case 1: Load is variable resistance R_L



$$I = \frac{V_S}{R_S + R_L}$$

$$P_L = V \cdot I = I^2 \cdot R_L \text{ W}$$

Max power deliver to R_L is

$$P_L = \frac{V_s^2}{(R_S + R_L)^2} \cdot R_L \text{ W}$$

To determine the value of R_L for maximum power transferred to the load.

$$\therefore \quad \frac{dP_L}{dR_L} \cdot = 0$$

$$\frac{dP_L}{dR_L} \cdot = \frac{v_s^2[(R_S + R_L)^2 \cdot 1 - 2R_L(R_S + R_L)]}{(R_S + R_L)^2} = 0$$

$$R_s^2 + R_L^2 + 2R_s R_L - 2R_L \cdot R_s - 2R_L^2 = 0$$

$$\boxed{R_s = R_L}$$

for max power transfer condition.

$$P_{\text{max}} = V_s^2 \cdot \frac{R_s}{(R_s + R_s)^2}$$

$$\boxed{P_{\text{max}} = \frac{V_s^2}{4R_s} \text{ W}}$$

$$P_{\text{total}} = I^2 R_S + I^2 R_L$$

$$= 2I^2 R_s \quad \Rightarrow \quad \frac{V_s^2}{2R_s} \text{ W}$$

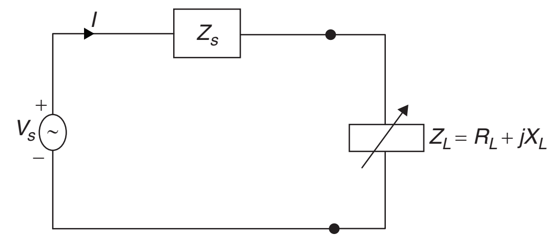
$$\text{Efficiency of MPT} \, \eta = \frac{\text{usefull power (loadpower)}}{\text{Total power}}$$

$$\eta\% = \frac{\dfrac{V_s^2}{4R_s}}{\dfrac{V_s^2}{2R_s}} \times 100$$

$$\eta_{\text{max}} = 50\%$$

$\Rightarrow$ That is, the efficiency of a circuit at Maximum power transfer condition is 50% only.

Case 2: Load is variable Impedance Z_L and source impedance Z_s.



$$I = \frac{V_S}{Z_L + Z_S} = \frac{V_S}{(R_S + jX_S) + (R_L + jX_L)}$$

$$P_L = |I^2| R_L (W)$$

$$|I| = \frac{V_S}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}} \text{ A}$$

$$\boxed{P_L = \frac{V_S^2 \cdot R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}}$$

1. Only R_L is variable:

   For maximum power transfer $\dfrac{dP_L}{dR_L} = 0$

   $\therefore$ Condition for MPT

$$R_L = \sqrt{R_S^2 + (X_S + X_L)^2} \ \Omega$$

$$\boxed{\therefore \quad P_{\text{max}} = \frac{P_L}{R_L} = \sqrt{R_S^2 + (X_S + X_L)^2}}$$

2. Only 'X_L' is variable:

   For MPT $\dfrac{dP_L}{dX_L} = 0$

   Condition for maximum power transfer

   $\Rightarrow \quad X_L = -X_S$

   $\Rightarrow \quad \boxed{X_L + X_S = 0}$

3. Both R_L and X_L are varied simultaneously:

   In this case consider the above two conditions.

$$\therefore Z_L = R_L + jX_L$$
$$= R_s - jX_s = Z_s^*$$

$$\therefore \quad \boxed{Z_L = Z_S^* \ \Omega}$$

$$P_{\text{max}} = P_L \text{ at } R_L = R_s \text{ and at } X_L = -X_S$$

$$P_{\text{max}} = \frac{P_L}{Z_L} = Z_s^*$$

$$P_{\text{max}} = \frac{V_s^2}{4R_L} = \frac{V_s^2}{4R_s} \text{ W}$$

Note:

In $Z_L = R_L + jX_L$

If $X_L = 0$

Then $R_L = \sqrt{R_S^2 + (X_S + X_L)^2}\ \Omega$

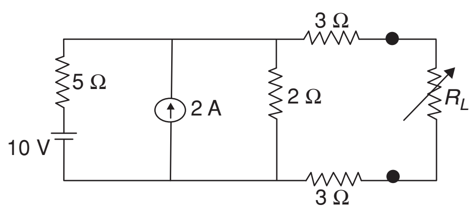
Subjected to $X_L = 0$ above

$$R_L = \sqrt{R_s^2 + X_s^2}\ \Omega$$

$$\boxed{\therefore\ \ R_L = |Z_S|\,\Omega}$$

$$P_{max} = P_L \text{ at } R_L = |Z_s|\,\Omega$$

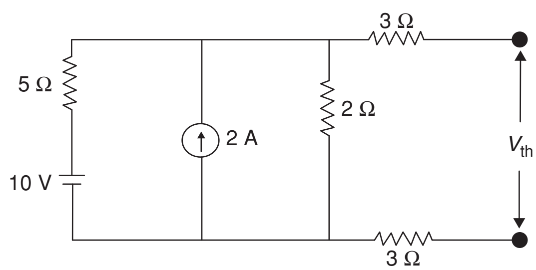
Example 16: Find the maximum power that can be transferred to R_L



Solution:

Find the Thevenin's equivalent circuit

(i) V_{th}:



Applying nodal Analysis,

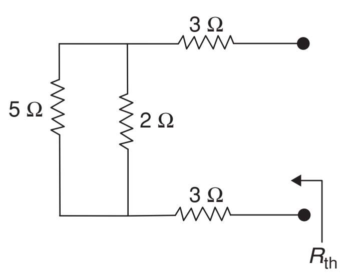
$$\frac{V_{th} - 10}{5} + \frac{V_{th}}{2} - 2 = 0$$

$$2(V_{th} - 10) + 5\,V_{th} = 20$$

$$7V_{th} = 40$$

$$V_{th} = \frac{40}{7}\text{ V}$$

(ii) R_{th}:



$$R_{th} = (5\|2) + 3 + 3$$

$$= 6 + \frac{10}{7}$$

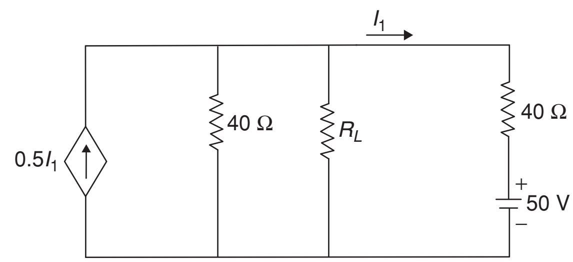
$$= 7.423\ \Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{36.65}{29.69}$$

$$= 1.099\text{ W}$$

$$\approx 1\text{ W}$$

Example 17: In the network of the figure, the maximum power is delivered to R_L if its value is

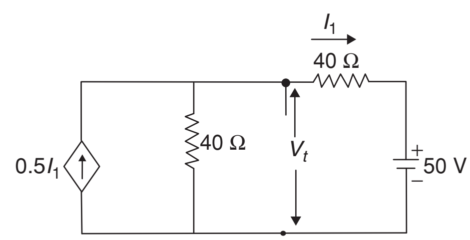


(A) $R_L = 16.66\ \Omega$ (B) $R_L = 20\ \Omega$
(C) $R_L = 26.66\ \Omega$ (D) None of the above

Solution:

$$R_{th} = R_L = \frac{V_{th}}{I_{sc}}$$

V_{th}:



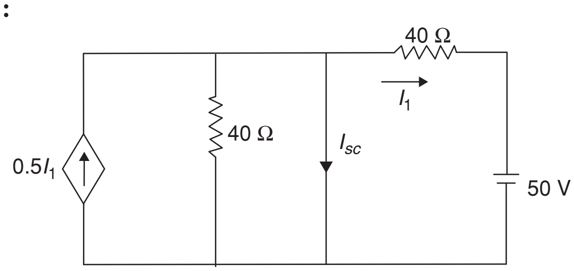
$$\frac{V_{th} - 50}{40} + \frac{V_{th}}{40} = 0.5\left[\frac{V_{th} - 50}{40}\right]$$

$$\frac{1}{2}\left[\frac{V_{th} - 50}{40}\right] + \frac{V_{th}}{40} = 0$$

$$V_{th} - 50 + 2\,V_{th} = 0$$

$$3V_{th} = 50 \quad \Rightarrow \quad V_{th} = \frac{50}{3}\text{ V}$$

I_{sc}:



$$I_{SC} = -0.5I_1$$

$$= -0.5\left[\frac{0-50}{40}\right]$$

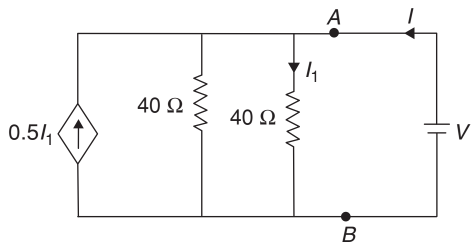
$$= 0.625\text{ A}$$

$$\therefore \quad R_{th} = \frac{V_{th}}{I_{sc}} = \frac{50}{3 \times 0.625}\ \Omega$$

$$= 26.66\ \Omega$$

2nd method: For maximum power delivered to R_L open circuit R_L.

R_{th} Across AB.



All independent sources deactivated

KCL at Node A

$$0.5I_1 + I = \frac{V}{40} + \frac{V}{40}$$

But $$I_1 = \frac{V}{40}$$

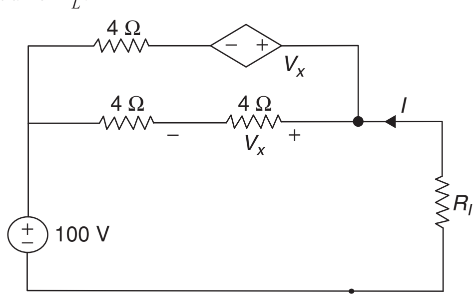
$$0.5\frac{V}{40} + I = \frac{2V}{40}$$

$$I = \frac{1.5\ V}{40}$$

$$\frac{V}{I} = \frac{40}{1.5} = 26.66\ \Omega$$

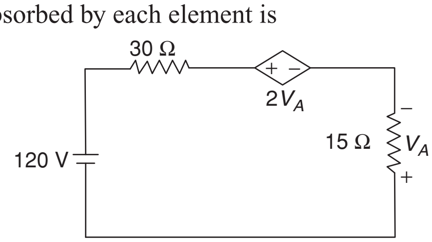
Solution:

In the circuit shown, what value of R_L maximizes the power delivered to R_L?



Note: The value of R_L maximizes the power delivered to the load equal to Thevenin's resistance find the Thevenin's resistance by using case 3.

Example 18: In the circuit shown in the below figure, the power absorbed by each element is



Solution:

Applying KVL for the loop

$$120 = 30I + 2V_A = +\ 15I$$

$$V_A = -15I$$

$$120 = 30\ I - 2V_A + 15\ I = 0$$

$$I = 8\ \text{A}$$

$$P_{120V} = V \cdot I = -8 \times 120 = -960\ \text{W}$$

Delivered by 960 W

$$P_{30\Omega} = 30 \times 8^2 = 1920\ \text{W} \Rightarrow \text{absorbed}$$

$$P_{2VA} = +\ 8 \times (-2 \times 15 \times 8) = -1920\ \text{W}$$

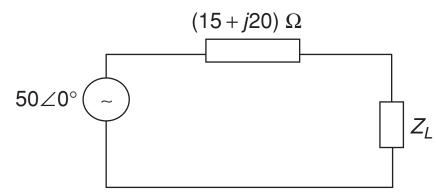
$$P_{15\Omega} = 82 \times 15 = 960\ \text{W}$$

$$\text{Total power} = -960 + 1920 + 960 - 1920$$

$$= 0\ \text{W.}$$

$\therefore$ Total power delivered = total power absorbed.

Example 19:



For the circuit shown in figure, find the value of load impedance for which the source delivers maximum power. Calculate the value of the maximum power

(A) 41.33 W (B) 50 W

(C) 75 W (D) −30 W

Solution: (A)

For maximum power transfer

$$Z_L = Z_S^*$$

$$Z_L = (15 - j20)\ \Omega$$

When $Z_L = (15 - j20)\ \Omega$, the current passing through the circuit is

$$I = \frac{V_S}{Z_S + Z_L} = \frac{50\angle 0^\circ}{15 + j20 + 15 - j20}$$

$$I = \frac{50\angle 0^\circ}{30\angle 0^\circ} = 1.66\angle 0^\circ$$

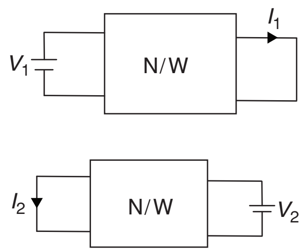
The maximum power transferred to load is

$$P_{max} = I^2 R_L$$

$$= (1.66)^2 \times 15 = 41.33\ \text{W}$$

Reciprocity Theorem

Statement: In a linear, passive, bilateral and time invariant network, the ratio of output to input (source) is constant even though the source is interchanged from input terminals to output terminals.



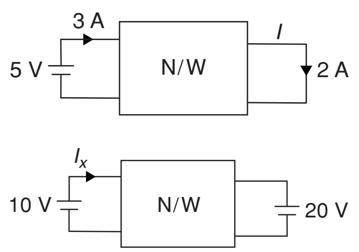
i.e., $\dfrac{I}{V}$ = constant

$$\Rightarrow \quad \frac{I_1}{V_1} = \frac{I_2}{V_2}$$

Note: The presence of the dependent sources makes the network active and hence the reciprocity theorem is not applicable.

$\therefore$ Reciprocity theorem is not applicable to active networks.

Example 20:



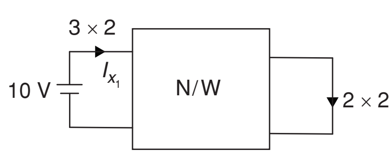
The network contains only resistances. Use the data given in Figure 1 and find the current I_x in Figure 2.

(A) 2 A (B) 6 A
(C) −2 A (D) 1 A

Solution: (C)
From the Reciprocity theorem

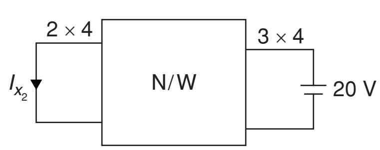
$$\frac{I_1}{V_1} = \frac{I_2}{V_2} = \text{constant}$$

(i) Consider that the 10 V is activated



$I_{x_1} = 6$ A.

(ii) Consider 20 V source is activated



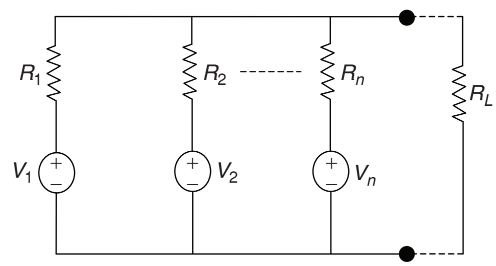
$\therefore I_{x_2} = 8$ A

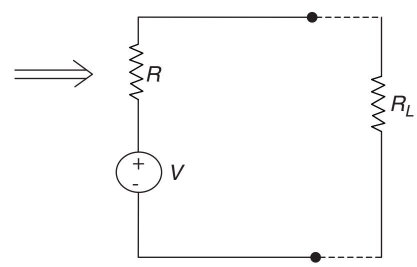
$\therefore$ By using Superposition theorem

$I_x = I_{x_1} - I_{x_2}$
$= 6 - 8 = -2$ A

Millman's Theorem

Statement: When a number of voltage sources ($V_1, V_2 \ldots V_n$) are in parallel having Internal resistances ($R_1, R_2, \ldots R_n$) respectively. The arrangement can be replaced by a single equivalent voltage source V in series with an equivalent series resistance R as given below.





As per Millman's Theorem

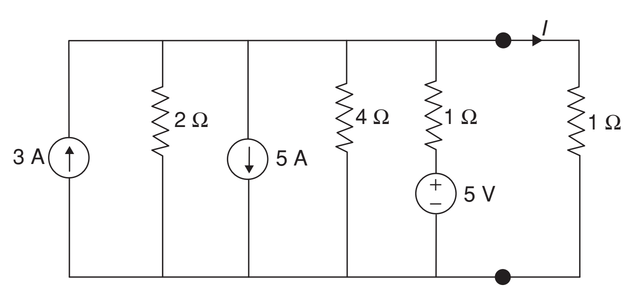
$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \cdots \pm V_n G_n}{G_1 + G_2 + \cdots + G_n}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots + G_n}$$

i.e., $$\boxed{V = \frac{\sum_{k=1}^{n} V_K G_k}{\sum_{k=1}^{n} G_k}}$$

$\Rightarrow$ This theorem is applicable to only linear networks.

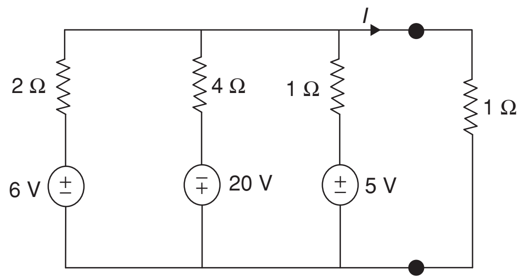
Examples 21: Find the current through the 1 Ω resistor using Millman's Theorem.

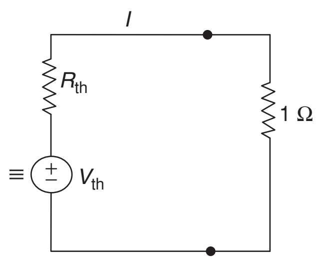


(A) $I = 3$ A (B) $I = 1.09$ A
(C) $I = 7$ A (D) $I = -1$ A

Solution: (B)

Converting current source to an equivalent voltage source





$$I = \frac{V_{th}}{R_{th}+1}\text{ A}$$

$$\Rightarrow \quad V_{th} = \frac{\Sigma V_k G_k}{G_k}$$

$$V_{th} = \frac{\frac{6}{2}-\frac{20}{4}+\frac{5}{1}}{\frac{1}{2}+\frac{1}{4}+\frac{1}{1}}$$

$$= \frac{3-5+5}{0.5+0.25+1} = \frac{3}{1.75} = 1.714\text{ V}$$

$$R = \frac{1}{\frac{1}{2}+\frac{1}{4}+\frac{1}{1}} = \frac{1}{1.75} = 0.5714\ \Omega$$

$$\Rightarrow \quad I = \frac{1.714}{1+0.57} = 1.09\text{ A.}$$

Tellegen's Theorem

Statement: In an arbitary network, the algebraic sum of powers at any given instant is zero. That is, the power delivered by some elements is equal to power absorbed by other elements present in the network.

$$\therefore \quad \sum_{j=1}^{n} V_j \times i_j = 0$$

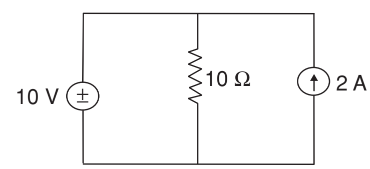
Where n = Total number of branches

Notes:

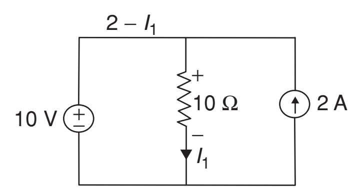
1. When current enters at the positive terminals of an element, then that element will absorbs the power, otherwise it will delivers the power.
2. Sources can deliver power or absorb power, whereas the passive elements will always absorb power since the current will enter at the positive terminal in the respective R, L, C's.

Properties: This Theorem is independent of the nature of the elements.

Example 22: For the circuit shown in figure, verify the Tellengen's Theorem.



Solution:



$$I_1 = \frac{10}{10} = 1$$

$\therefore \ P_{10\Omega} = V \cdot I = I^2 \cdot R$

$= 10$ W (absorbed)

$V_{2A} = 10$ V

$\Rightarrow \ P_{2A} = -2 \times 10 = -20$ W (delivered)

$I_{10V} = 2 - 1 = 1$ A

$\Rightarrow \ P_{10V} = 10 \times 1 = 10$ W (absorbed)

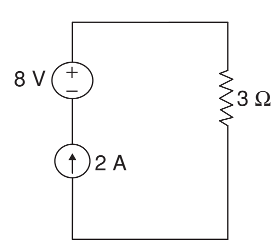
$\therefore \ \Sigma V \cdot I = 0$

Total absorbed power = Total delivered power

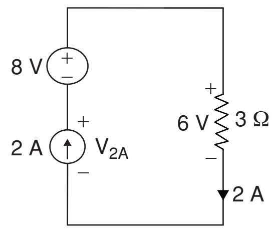
$$P_{2A} + P_{10V} + P_{10\Omega} = 0$$

∴ Tellegen's Theorem is verified.

Example 23:



Solution:
Verify Tellengen's Theorem?



$$V_{2A} = 6 - 8 = -2\text{ V}$$
$$P_{8V} = V \cdot I = -8 \times 2 = -16\text{ W (delivers)}$$
$$P_{2A} = 2 \times 2 = 4\text{ W (absorbed)}$$
$$P_{3\Omega} = \text{V} \cdot I = 12\text{ W (absorbed)}$$

$\therefore \sum \text{power} = 0$

$-16 + 4 + 12 = 0$

$\therefore$ Tellegen's Theorem is verified.

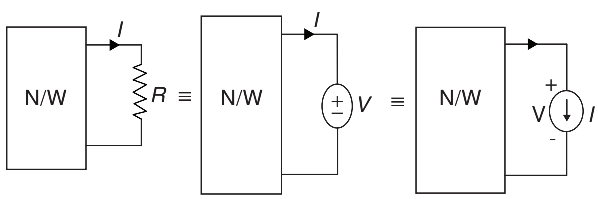
Substitution Theorem

The voltage across the current through any branch of a DC bilateral network can be replaced by any combination of elements that will make the same voltage across and current though the chosen branch.

In a linear network any passive element can be equivalently substituted by an ideal voltage source or an ideal current source provided all the other branch current and voltages are kept constant.

Any branch in a linear network can be substituted by a different branch without disturbing the voltage and currents in the entire network provided the new branch has the same set of terminal voltage and currents as the original network.

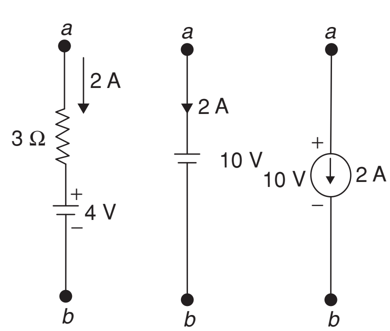
⇒ This theorem is applicable for any LTI and bilateral networks.



Where $V = R \cdot I$

$P = V \cdot I = I^2 R(W)$

Example:



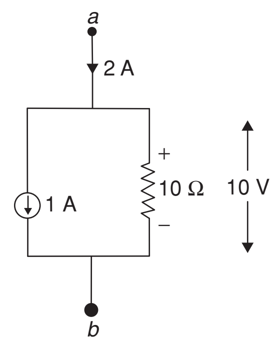


Figure 3 Equivalent circuits of branch $a - b$

Compensation Theorem

Statement: In an LTI network when the resistance 'R' of an uncoupled branch, carrying a current (I), is changed by ΔR, the current in all the branches would change and can be obtained by assuming that an ideal voltage source of (V_s) has been connected in series with ($R + \Delta R$) when all other sources in the network are replaced by their internal resistances $V_s = I \cdot \Delta R$.

This theorem is useful in determining the current and voltage changes in circuit element when the value of its impedance is changed.

Example: Bridge and potentiometer circuit.

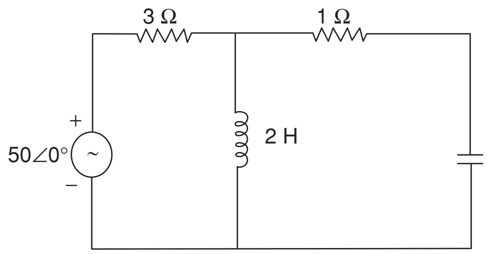
Duality of circuits Two linear circuits are said to be duals of one another if they are described by the same characteristic equations with dual quantities interchanged.

Network and its dual are same only with respect to the performance, but the elements and connects point of view are not equal.

Table 1 *Dual pairs*

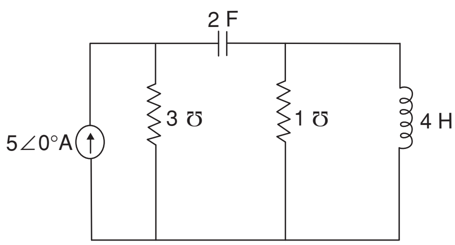
R	↔	G
L	↔	C
Z	↔	Y
V	↔	I
Voltage Source	↔	Current Source
KCL	↔	KVL
Star	↔	Delta
Node	↔	Mesh
Series	↔	Parallel
Open circuit	↔	Short circuit
$L \times \frac{di(t)}{dt}$	↔	$C \cdot \frac{dv(t)}{dt}$
$\frac{1}{c}\int i(t)dt$	↔	$\frac{1}{L}\int v(t) \cdot dt$
Thevenin's	↔	Norton's
$R \cdot i(t)$	↔	$G \cdot V(t)$

Example 24:

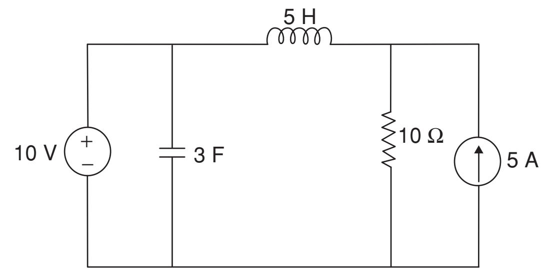


The dual of the network is

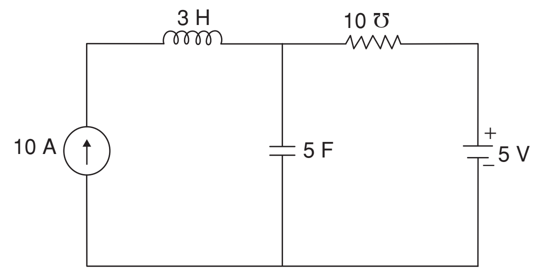
Solution:
Dual of the given network,
$V \leftrightarrow I$, $R \leftrightarrow G$; $L \leftrightarrow C$ and series $\leftrightarrow$ parallel



Example 25: Obtain the dual of the network shown in figure below.



Solution:
Dual of the above network
Series $\leftrightarrow$ parallel, $V \leftrightarrow I$, $L \leftrightarrow C$, $R \leftrightarrow G$.

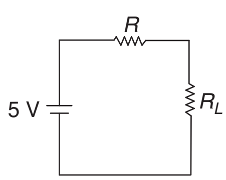


Exercises

Practice Problems 1

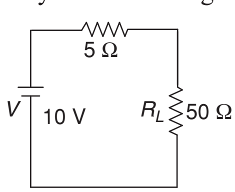
Directions for questions 1 to 28: Select the correct alternative from the given choices.

1. The maximum power transferred to the load in the circuit is given as 0.5 W. Get the values of R and R_L.



(A) 15 Ω, 10 Ω (B) 12.5 Ω, 12.5 Ω
(C) 10 Ω, 15 Ω (D) 10 kΩ, 10 kΩ

2. Find the efficiency of the circuit given for $R_L = 50\ \Omega$

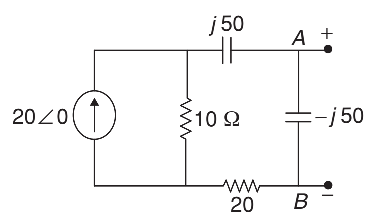


(A) 99% (B) 91%
(C) 80% (D) 87%

3. Current in the circuit is given by the equation $i(t) = 10\cos(20\pi t + 50)$ and the impedance of the load is given as $Z_L = 5 + j3$. Find the average power delivered to the load.

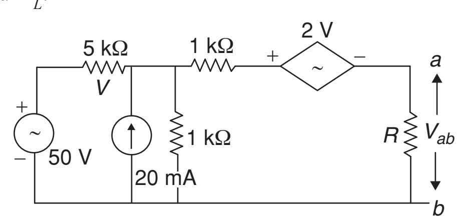
(A) 353.5 W (B) 291.5 W
(C) 250 W (D) 176.7 W

4. In the circuit shown below the Norton equivalent current in amps across $A - B$ is



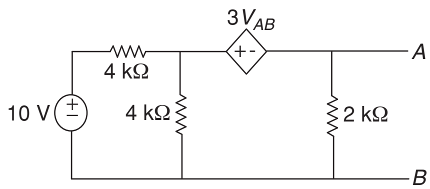
(A) $19.45 + j3.24$ (B) $6.48 - j1.08$
(C) $12.97 - j2.16$ (D) $20 + j0$

5. Find the Thevenin equivalent voltage external to the load R_L.



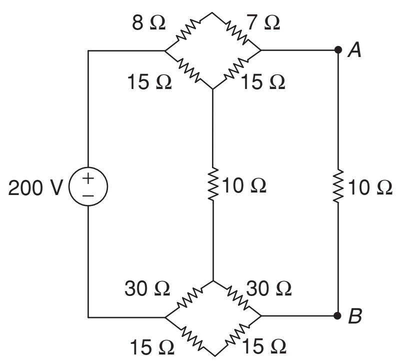
(A) 25 V (B) 50 V
(C) 49 V (D) 45 V

6. Find the Thevenin's resistance associated with the circuit.



(A) 1 kΩ (B) 0.45 kΩ
(C) 2 kΩ (D) 0.22 kΩ

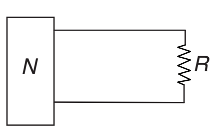
Common Data for Questions 7 and 8: Select the correct alternative from the given choices.



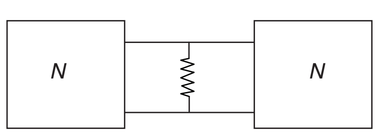
7. Find Thevenin's equivalent voltage of the circuit.
(A) 100 V (B) 120 V
(C) 125 V (D) 150 V

8. The resistance across $A - B$ is 10 Ω. Find the current through the 10 Ω resistor.
(A) 5.1 A (B) 6.45 A
(C) 3.35 A (D) 13.9 A

9. When a resistor R is fed from an electrical network 'N' consumes a power of 'P' W as shown in Figure (a). If an identical network is added as shown in Figure (b) the power consumed by R will be ______.



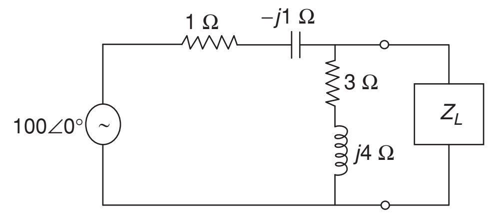
(a)



(b)

(A) 2 P (B) P
(C) $\dfrac{P}{2}$ (D) between P and 4 P

Common Data for Questions 10 and 11: Select the correct alternative from the given choices.

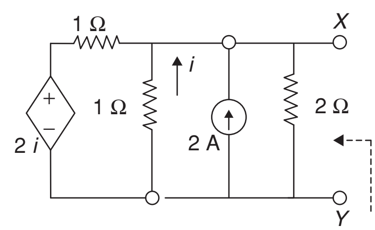


10. Find the value of Z_L at which maximum power is transferred to Z_L.
(A) $(1.24 - j0.676)$ Ω (B) $(1.24 + j0.676)$ Ω
(C) 1.31 Ω (D) 1.24 Ω

11. The maximum power transferred is
(A) 201.6 W (B) 617 W
(C) 2016 W (D) 6170 W

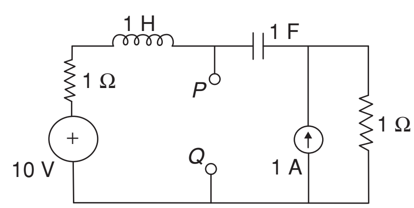
Common Data for Questions 12 to 15: Select the correct alternative from the given choices.

12. For the Circuit shown in the figure, the Thevenin voltage and resistance looking into $x - y$ are ______.



(A) 4/3V, 2 Ω (B) 4V, 2/3 Ω
(C) 4/3V, 2/3 Ω (D) 4V, 2 Ω

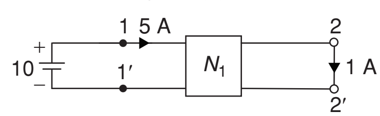
13. The Thevenin equivalent impedance Z_{th} between the nodes P and Q in the following circuit is ______.



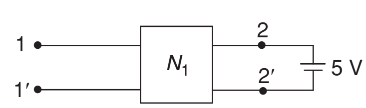
(A) 1 (B) $1 + S + \dfrac{1}{S}$

(C) $2 + S + \dfrac{1}{S}$ (D) $\dfrac{S^2 + S + 1}{S^2 + 2S + 1}$

14. The short circuit test of a 2 port π network is shown in Figure (a). The voltage across the terminals 11′ in the network shown in Figure (b) will be



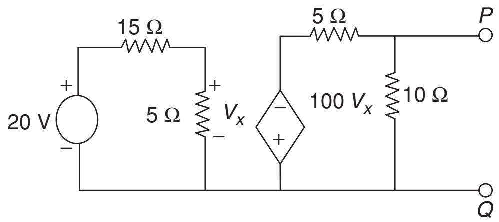
(a)



(b)

(A) 2 V (B) 5 V
(C) 10 V (D) 1 V

15. The Norton's equivalent circuit at terminals PQ has a current source and a Norton's resistance of ______.

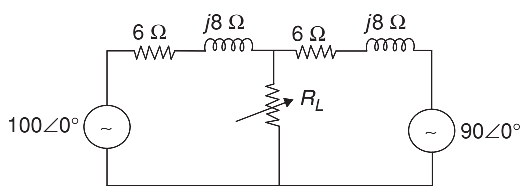


(A) 10 A, 3.33 Ω (B) 100 A, $\frac{85}{12}$ Ω

(C) 100 A, 3.33 Ω (D) 500 A, 3.33 Ω

Common Data for Questions 16 and 17: Select the correct alternative from the given choices.

16.



In the circuit shown in figure, under the maximum power transfer condition, the value of R_L is _____.

(A) 5 Ω (B) 20 Ω

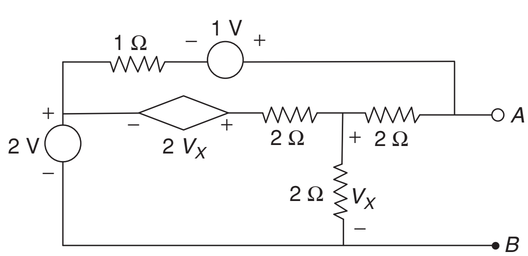
(C) $\frac{25}{3}$ Ω (D) 6 Ω

17. The power absorbed by R_L at maximum power transfer condition is ______.

(A) 1000 W (B) 500 W
(C) 625 W (D) 2000 W

Common Data for Questions 18 to 22: Select the correct alternative from the given choices.

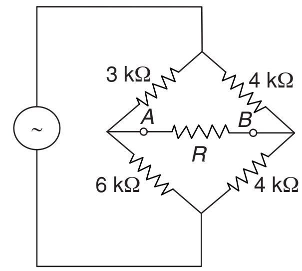
18. The Thevenin voltage at the terminals *AB* of the network shown in the figure is



(A) 4 V (B) 2 V

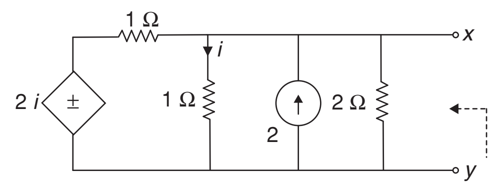
(C) $\frac{3}{2}$ V (D) $\frac{1}{2}$ V

19. The value of the resistance *R*, connected across the terminals *A* and *B*, which will absorb the maximum power is



(A) 4 kΩ (B) 4.11 kΩ
(C) 8 kΩ (D) 9 kΩ

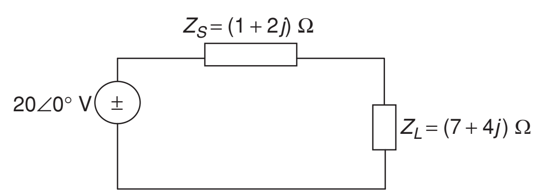
20. For the circuit shown in figure, the Thevenin voltage and resistance looking into $X - Y$ are



(A) $\frac{4}{3}$ V, 2 Ω (B) 4 V, $\frac{2}{3}$ Ω

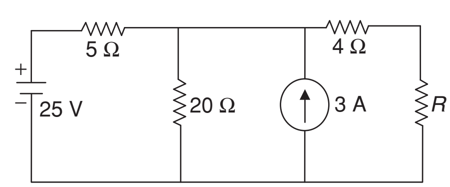
(C) $\frac{4}{3}$ V, $\frac{2}{3}$ Ω (D) 4 V, 2 Ω

21. An AC source of RMS voltage 20 V with internal impedance $Z_S = (1 + 2j)$ Ω feeds a load of impedance $Z_L = (7 + 4j)$ Ω shown in the figure. The reactive power consumed by the load is



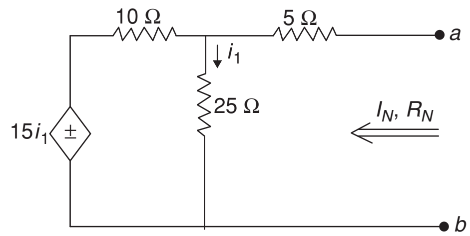
(A) 8 VAR (B) 16 VAR
(C) 28 VAR (D) 35 VAR

22. The value of *R* (in Ohms) required for maximum power transfer in the network shown in the figure is



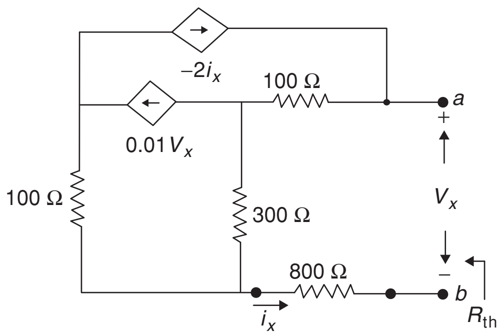
(A) 2 (B) 4
(C) 8 (D) 16

23. For the following circuit the value of i_N and R_N are



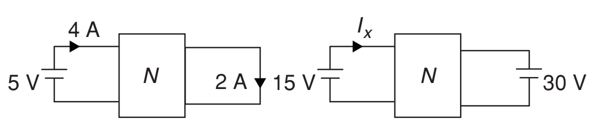
(A) 2 A, 12 Ω (B) 0 A, 20 Ω
(C) 0.5 A, 20 Ω (D) 0 A, 12 Ω

24. For the circuit shown in figure below the values of R_{th} and V_{th} are



(A) 0 V, 100 Ω (B) 0 V, 1200 Ω
(C) 5 V, 100 Ω (D) None of the above

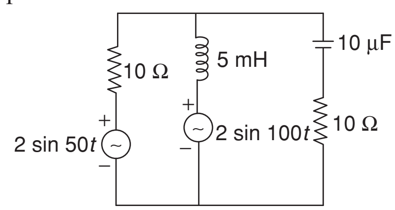
25. Consider the following circuits



(a) (b)

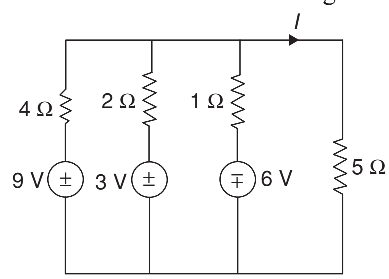
The network '*N*' contains only resistances. Use the data given in Figure (a) and find the current i in Figure (b)
(A) 0 A (B) 12 A
(C) –6 A (D) 6 A

26. In the circuit shown in figure, which one of the following theorem can be more conveniently used to evaluate the responses in the 10 Ω resistors.



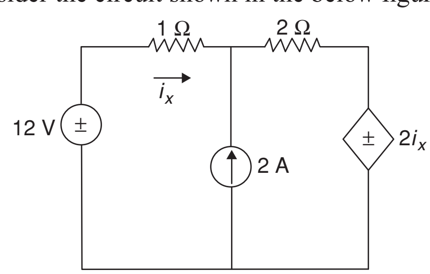
(A) Thevenin's Theorem
(B) M P T T (Maximum Power Transfer Theorem)
(C) Milliman's Theorem
(D) Superposition Theorem

27. Consider the network shown in the figure below:



The current I is
(A) 0.23 A (B) –0.23 A
(C) 2.25 A (D) –0.5 A

28. Consider the circuit shown in the below figure:

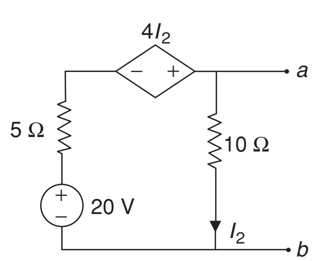


The current i_x would be
(A) $i_x = 1.6$ A (B) $i_x = 1.3$ A
(C) $i_x = -1.5$ A (D) $i_x = 0.8$ A

Practice Problems 2

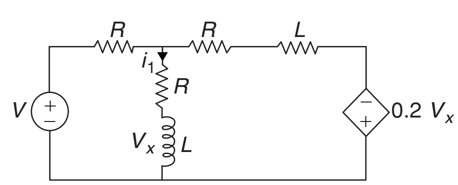
Directions for questions 1 to 16: Select the correct alternative from the given choices.

1. Find the Thevenin's equivalent of the Circuit given below:



(A) $V_{th} = 20$ V, $R_{th} = 3.3$ Ω
(B) $V_{th} = 16$ V, $R_{th} = 5$ Ω
(C) $V_{th} = 20$ V, $R_{th} = 5$ Ω
(D) $V_{th} = 4$ V, $R_{th} = 10$ Ω

2. Find the state equation for the circuit given.



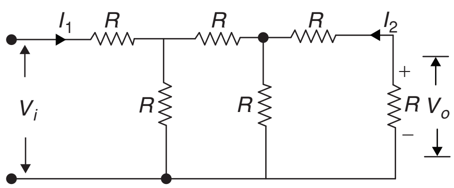
(A) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R + .5\text{ V}$

(B) $L\dfrac{di_2}{dt} = -0.7V_x + 1.5i_2R - .5\text{ V}$

(C) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R + .5\text{ V}$

(D) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R - .5\text{ V}$

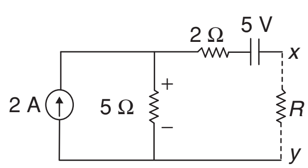
3. Find the transfer function of the network shown.



(A) $\dfrac{V_0}{V_i} = \dfrac{1}{5}$ (B) $\dfrac{V_0}{V_i} = \dfrac{1}{13}$

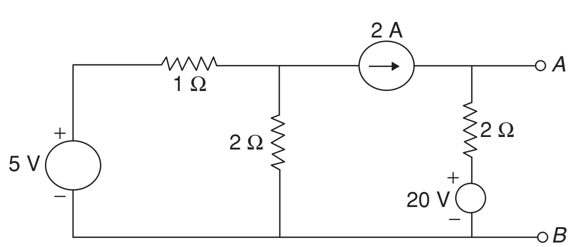
(C) $\dfrac{V_0}{V_i} = 0$ (D) $\dfrac{V_0}{V_i} = \dfrac{-1}{13}$

4. A network is shown below with an unknown load R. Find the value of R so that maximum power is delivered to the load.



(A) 5 Ω (B) 7 Ω
(C) 1.43 Ω (D) 2 Ω

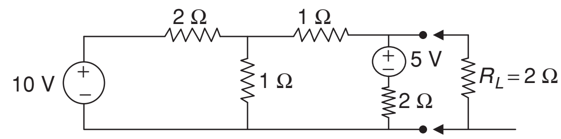
5. The Thevenin's resistance across the terminals AB of the figure is ______.



(A) $\dfrac{2}{3}\,\Omega$ (B) 2 Ω

(C) $\dfrac{7}{3}\,\Omega$ (D) 5 Ω

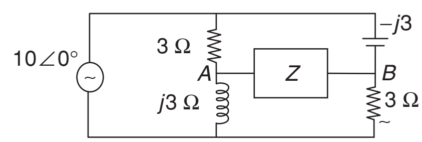
6. In the circuit shown in the figure, the power consumed by R_L is



(A) $\dfrac{25}{16}$ W (B) $\dfrac{25}{3}$ W

(C) $\dfrac{25}{8}$ W (D) $\dfrac{5}{2}$ W

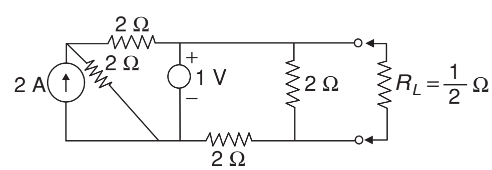
7. In the circuit shown in the given figure the Thevenin impedance between terminals A and B is ______.



(A) 6 Ω (B) 3 Ω

(C) $\dfrac{3}{2} - \dfrac{j3}{2}$ (D) $6 + j6$

8. In the circuit shown in the figure, the current through resistance R_L is ______.



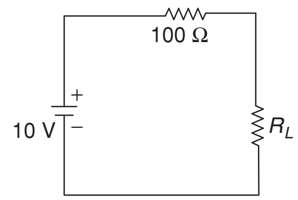
(A) $\dfrac{2}{3}$ A (B) $\dfrac{3}{2}$ A

(C) $\dfrac{4}{3}$ A (D) $\dfrac{1}{3}$ A

9. A source of angular frequency 1 rad/sec has source impedance consisting of 1 Ω resistance in series with 1 H inductance. The load that will obtain the maximum power transfer is
(A) 1 Ω resistance
(B) 1 Ω resistance in parallel with 1 H inductance
(C) 1 Ω resistance in series with 1 F capacitor
(D) 1 Ω resistance in parallel with 1 F capacitor

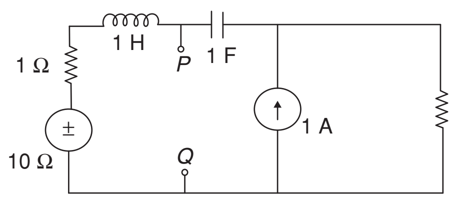
10. Superposition theorem is NOT applicable to networks containing
(A) non linear elements
(B) dependent voltage sources
(C) dependent current sources
(D) transformers

11. The maximum power that can be transferred to the load resistor R_L from the voltage source in the figure is



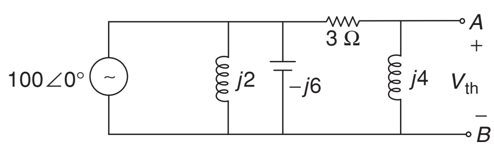
(A) 1 W (B) 10 W
(C) 0.25 W (D) 0.5 W

12. The Thevenin equivalent impedance Z_{th} between the nodes P and Q in the following circuit is



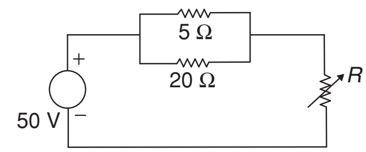
(A) 1 (B) $1+s+\frac{1}{s}$
(C) $2+s+\frac{1}{s}$ (D) $\frac{s^2+s+1}{s^2+2s+1}$

13. The Thevenin equivalent voltage V_{th} appearing between the terminals A and B of the network shown in the figure is given by



(A) $j16\,(3-j4)$ (B) $j16\,(3+j4)$
(C) $16\,(3+j4)$ (D) $16\,(3-j4)$

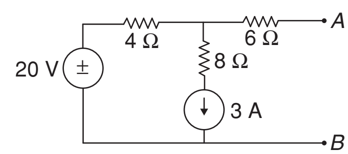
14.



In the circuit shown, the adjustable resistor R is set such that the power in the 5 Ω resistor is 20 W. The value of R is

(A) 6 Ω (B) 25 Ω
(C) 4 Ω (D) 16 Ω

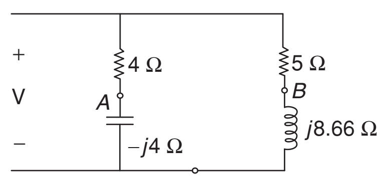
15.



The Norton equivalent of the above circuit is

(A) $I_N = 8$ A, $R_N = 10\ \Omega$
(B) $I_N = 0.8$ A $R_N = 10\ \Omega$
(C) $I_N = 3$ A $R_N = 8\ \Omega$
(D) $I_N = 8$ A $R_N = 3\ \Omega$

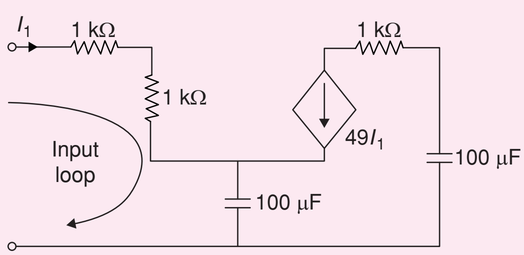
16. In the circuit shown in the below figure, $V_{AB} = 48.3\angle 30°$. The applied voltage V is



(A) $40\angle 90°$ (B) $100\angle 130°$
(C) $50\angle 135°$ (D) $100\angle 135°$

Previous Years' Questions

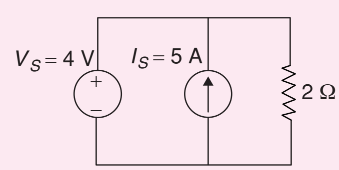
1. The equivalent capacitance of the input loop of the circuit shown is **[2009]**



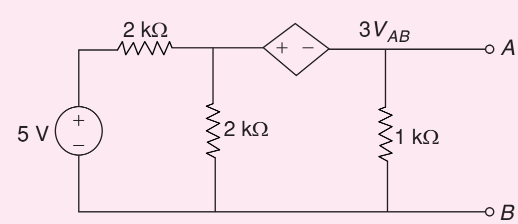
(A) 2 μF (B) 100 μF
(C) 200 μF (D) 4 μF

2. For the circuit shown, find out the current flowing through the 2 Ω resistance. Also identify the changes to be made to double the current through the 2 Ω resistance **[2009]**

(A) (5 A; Put $V_s = 20$ V) (B) (2 A; Put $V_s = 8$ V)
(C) (5 A; Put $I_s = 10$ A) (D) (7 A; Put $I_s = 12$ A)



Common Data for Questions 3 and 4:



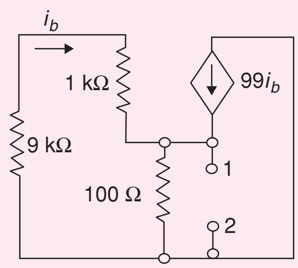
3. For the circuit given above, the Thevenin's resistance across the terminals A and B is **[2009]**

(A) 0.5 kΩ (B) 0.2 kΩ
(C) 1 kΩ (D) 0.11 kΩ

4. For the circuit given above, the Thevenin's voltage across the terminals A and B is **[2009]**

(A) 1.25 V (B) 0.25 V
(C) 1 V (D) 0.5 V

5. The impedance looking into nodes 1 and 2 in the given circuit is **[2012]**

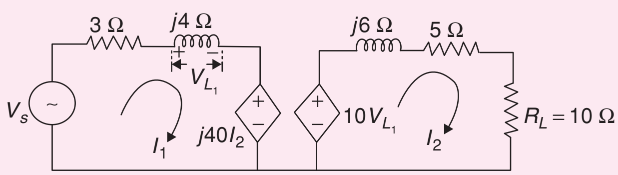


(A) 50 Ω (B) 100 Ω
(C) 5 kΩ (D) 10.1 kΩ

6. A source $v_s(t) = V \cos 100\,\pi t$ has an internal impedance of $(4 + j3)$ Ω. If a purely resistive load connected to this source has to extract the maximum power out of the source, its value is Ω should be **[2013]**

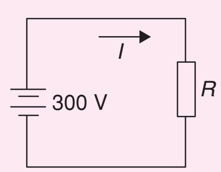
(A) 3 (B) 4
(C) 5 (D) 7

7. In the circuit shown below, if the source voltage $V_s = 100\angle 53.13^{\circ}$ V then the Thevenin's equivalent voltage in Volts as seen by the load resistance R_L is **[2013]**

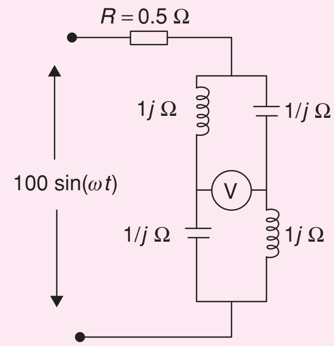


(A) $100\angle 90^{\circ}$ (B) $800\angle 0^{\circ}$
(C) $800\angle 90^{\circ}$ (D) $100\angle 60^{\circ}$

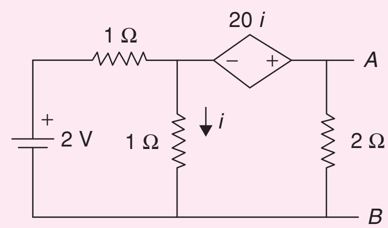
8. In the figure, the value of resistor R is $\left(25 + \dfrac{I}{2}\right)$ ohms, where I is the current in amperes. The current I is ______. **[2014]**



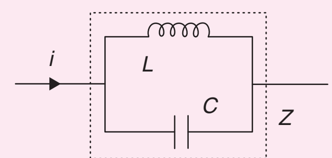
9. The reading of the voltmeter (rms) in Volts, for the circuit shown in the figure is________ **[2014]**



10. For the given circuit, the Thevenin equivalent is to be determined. The Thevenin voltage, V_{Th} (in Volt), seen from terminal AB is _______. **[2015]**

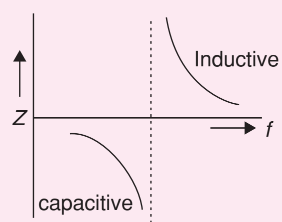


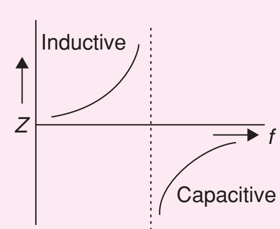
11. An inductor is connected in parallel with a capacitor as shown in the figure.



As the frequency of current i is increased, the impedance (Z) of the network varies as **[2015]**

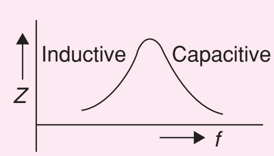
(A)



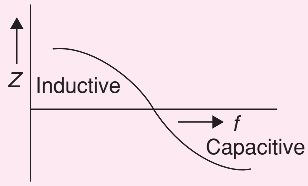


(B)

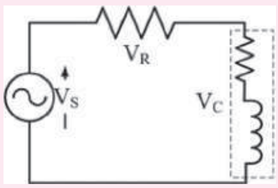
(C)



(D)



12. A resistance and a coil are connected in series and supplied form a single phase, 100V, 50Hz ac source as shown in the figure below. The rms values of possible voltages across the resistance (V_R) and coil (V_C) respectively, in volts, are **[2016]**



(A) 65, 35 (B) 50, 50
(C) 60, 90 (D) 60, 80

13. The voltage (V) and current (A) across a load are as follows.

$v(t) = 100\text{Sin}(\omega t)$,

$I(t) = 10\text{Sin}(\omega t - 60°) + 2\text{Sin}(3\omega t) + 5\text{Sin}(5\omega t)$

The average power consumed by the load, in W is ________. **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** B	**3.** C	**4.** B	**5.** A	**6.** D	**7.** C	**8.** B	**9.** D	**10.** B
11. C	**12.** D	**13.** A	**14.** D	**15.** C	**16.** A	**17.** B	**18.** A	**19.** A	**20.** D
21. A	**22.** C	**23.** B	**24.** A	**25.** A	**26.** C	**27.** B	**28.** A		

Practice Problems 2

1. A	**2.** A	**3.** B	**4.** B	**5.** B	**6.** C	**7.** B	**8.** D	**9.** C	**10.** A
11. C	**12.** A	**13.** A	**14.** D	**15.** B	**16.** C				

Previous Years' Questions

1. A	**2.** B	**3.** B	**4.** D	**5.** A	**6.** C	**7.** C	**8.** 10	**9.** 141.42
10. 3.36 V		**11.** B	**12.** D	**13.** 250				