Short Answer Questions

[2 Mark]

Q.1. Write the integrating factor of the following differential equation:

$$(1+y^2)+(2\text{xy}-\cot y)\frac{\text{dy}}{\text{dx}}=0$$

Ans.

$$(1+y^2)+(2\text{xy}-\cot y)\frac{\text{dy}}{\text{dx}}=0$$

$$\Rightarrow \quad (2\text{xy}-\cot y)\frac{\text{dy}}{\text{dx}}=-(1+y^2)$$

$$\Rightarrow \quad \frac{\text{dy}}{\text{dx}}=-\frac{1+y^2}{2\text{xy}-\cot y}$$

$$\Rightarrow \quad \frac{\text{dx}}{\text{dy}}=-\frac{(2\text{xy}-\cot y)}{1+y^2}$$

$$\Rightarrow \quad \frac{\text{dx}}{\text{dy}}+\frac{2y}{1+y^2}.x=\frac{\cot y}{1+y^2}$$

It is in the form $\frac{\text{dx}}{\text{dy}}+\text{Px}=Q$, where P and Q are function of y

$$\Rightarrow \quad \text{IF}=e^{\int p\ \text{dy}}=e^{\int\frac{2y}{1+y^2}\text{dy}}=e^{\log|1+y^2|}=1+y^2$$

Q.2. Write the general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$

Ans.

We have, $\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \quad \frac{dy}{y} = \frac{dx}{x}$

Integrating both sides, we get

$$\log|y| = \log|x| + \log|C|$$

$\Rightarrow \quad |y| = |xC|$

$\Rightarrow \quad y = Cx$

Q.3. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Ans.

Given $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$\Rightarrow \quad \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

Integrating both sides we get

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$\Rightarrow \quad \tan^{-1} y = \tan^{-1} x + C$

Q.4. Find the general solution of differential equation.

$$\frac{dy}{dx} = \frac{1 - \cos x}{1+\cos x}$$

Ans.

Given $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{1-\cos x}{1+\cos x}$

$\Rightarrow \quad \frac{\mathrm{dy}}{\mathrm{dx}}=\frac{2 \sin ^2 \frac{x}{2}}{2 \cos ^2 \frac{x}{2}}$

$\Rightarrow \quad \frac{\mathrm{dy}}{\mathrm{dx}}=\tan ^2 \frac{x}{2}$

$\Rightarrow \quad \mathrm{dy}=\left(\sec ^2 \frac{x}{2}-1\right) \mathrm{dx}$

Integrating both sides

$\Rightarrow \quad \int \mathrm{dy}=\int \sec ^2 x / 2 \mathrm{dx}-\int \mathrm{dx}$

$\Rightarrow \quad y=2 \tan \frac{x}{2}-x+C$

Q.5. Solve the differential equation

$\frac{\mathrm{dy}}{\mathrm{dx}}=\left(1+x^2\right)\left(1+y^2\right)$

Ans.

Given $\frac{\mathrm{dy}}{\mathrm{dx}}=\left(1+x^2\right)\left(1+y^2\right)$

$\Rightarrow \quad \frac{\mathrm{dy}}{1+y^2}=\left(1+x^2\right) \mathrm{dx}$

$\Rightarrow \quad \int \frac{\mathrm{dy}}{1+y^2}=\int\left(1+x^2\right) \mathrm{dx}$

$\Rightarrow \quad \tan ^{-1} y=x+\frac{x^3}{3}+C$

Q.6. Solve the differential equation.

$\mathbf{y} \log \mathbf{y}\, \mathbf{dx} - \mathbf{xdy} = 0$

Ans.

Given $y \log y \, dx - x dy = 0$]

$\Rightarrow \quad y \log y \, dx = x dy$

$\Rightarrow \quad \frac{dx}{x} = \frac{dy}{y \log y}$

$\Rightarrow \quad \int \frac{dy}{y \log y} = \int \frac{dx}{x}$

Let $\log y = t$

$\Rightarrow \quad \frac{1}{y} dy = dt$

$\Rightarrow \quad \int \frac{dt}{t} = \log x + C_1$

$\Rightarrow \quad \log t = \log x + \log C \qquad$ [where $C_1 = \log C$]

$\Rightarrow \quad t = Cx$

$\Rightarrow \quad \log y = Cx$

$\Rightarrow \quad y = e^{Cx}$

Q.7. Solve differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$.

Ans.

Given $\frac{\mathrm{dx}}{x}+\frac{\mathrm{dy}}{y}=0$

$\Rightarrow \quad \frac{\mathrm{dy}}{y}=-\frac{\mathrm{dx}}{x}$

Integrating both sides, we get

$$\int \frac{\mathrm{dy}}{y}=-\int \frac{\mathrm{dx}}{x}$$

$\Rightarrow \quad \log y = -\log x + \log C$

$\Rightarrow \quad \log y + \log x = \log C$

$\Rightarrow \quad \log xy = \log C$

$\Rightarrow \quad xy = C$