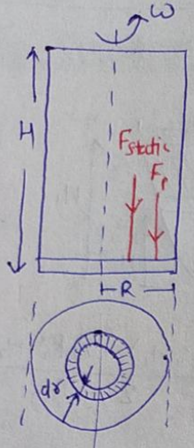
Lecture (12) (Pb - 37)

7/4/19



closed vessel

Forced vortex

Determine F_{Bottom}?

Soln $F_{Bottom} = F_{static} + F_p$

for F_p

$$\frac{dP}{dr} = \frac{\rho . V^2}{r} \quad \{V = \omega . r\}$$

$$\frac{dP}{dr} = \rho \omega^2 r$$

Int it

$$\int_0^P dP = \rho . \omega^2 \int_0^r r . dr, \quad P = \frac{\rho \omega^2 r^2}{2}$$

$$dF_p = P . dA = \frac{\rho \omega^2 r^2}{2} [2\pi r . dr]$$

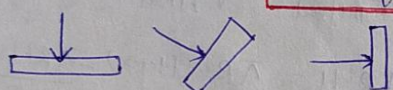
$$= \pi \rho . \omega^2 . r^3 . dr$$

Int it $F_p = \frac{\pi \rho \omega^2 . R^4}{4}$

By eq ①

$$F_{Bottom} = \rho . g H (\pi R^2) + \frac{\pi (\rho \omega^2 . R^4)}{4}$$

$$= \boxed{\rho (\pi R^2) \left\{ g H + \frac{\omega^2 R^2}{4} \right\}}$$



$$\frac{dP}{dh} = \rho g$$

Dimensional analysis (Highly scoring topic) (chapter 5)

① Aim:- To develope a functional relationship

② Modal and simlitude study :-

① To develope a functional relationship

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Step ① $T = f^n (L, g)$

Step ② $T = K . L^a . g^b$

Step ③ $K = 2\pi$ (experimental in lab)

Rayleigh method → $X = f^n (x_1, x_2, x_3)$

X: dependent variable; x_1, x_2, x_3: independent variable

$X = K . x_1^a . x_2^b . x_3^c$ — a, b, c: arbitrary power

Dimension of LHS = Dimension of R.H.S

(Pb) $F_D = f^n [D, V, \rho, \mu]$

Soln = $F_D = K . D^a . V^b \rho^c . \mu^d$ Dimension: $MLT^{-2} = L^a [LT^{-1}]^b [ML^{-3}]^c [ML^{-1}T^{-1}]^d$

$$MLT^{-2} = M^{c+d} \times L^{a+b-3c-d} . T^{-b-d}$$

$c + d = 1, \quad c = 1 - d$

$-b - d = -2, \quad b = 2 - d$

$a + b \cancel{+} - 3c - d = 1$

$a + 2 - d - 3 + 3d - d = 1 \rightarrow \boxed{a = 2d}$

$$F_D = K \cdot D^{2-d} \cdot V^{2-d} \cdot \rho^{1-d} \cdot \mu^d$$

$$F_D = K D^2 . V^2 \rho \left(\frac{\mu}{\rho . V . D}\right)^d$$

Extra:

$$\frac{F_D}{D^2 V^2 \rho} = K\left[\frac{\mu}{\rho V . D}\right]^d$$

(dimensionless) (π_1) (dimensionless) (π_2)

$\pi_1 = K \pi_2^d$

or $\pi_1 = K_1 f^n(\pi_2)$

or $\pi_2 = K_2 . f^n(\pi_1)$

or $f^n(\pi_1, \pi_2) = 0$

(Buckingham's π method)

Buckingham's π method:

$f^n(\pi_1, \pi_2, \pi_3 \text{----} \pi_{m-n}) = 0$

π term = Dimensionless term, No. of π terms $= m-n$

m = total No. of variables

[Dependent variables + Independent variables]

n = No. of Fundamental dimension involved in the Problem

In general M, L, T. $= \boxed{3 = n}$

1. Geometric variable (L) → D, H, L
2. Kinematic variable (T) → V, a, ω, ν
3. Dynamic variable (M) → ρ, μ

How to write a π - term

π = [set of Repeating variables]. varible

or

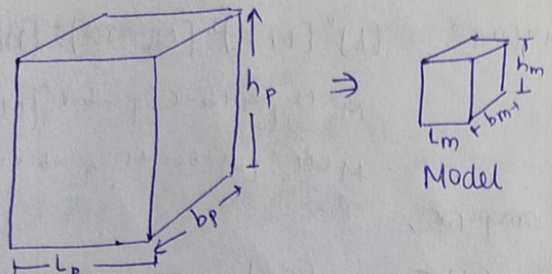
π [G.V (L) . K.V. (T) D.V (M)] variable

(Pb) $F_D = f^n [D, V, \rho, \mu]$

Derive that $\boxed{F_D = K . D^2 . V^2 . \rho f^n \left[\frac{\mu}{\rho . V . D}\right]}$ (Not in shorts notes)

If the ~~design~~ derive expression is given then see the more common varible to form a set of repeating variables in π term

① Geometric similarity In this each and every dimension of Prototype is scaled in the same ratio



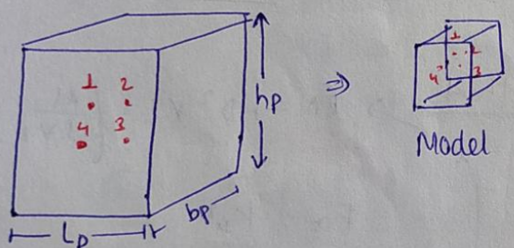
Model

Prototype

$$\frac{L_m}{L_p} = \frac{b_m}{b_p} = \frac{h_m}{h_p} = L_r$$

Zoom in, zoom out — length scale ratio

① Kinematic similarity → the ratio of velocity of accⁿ at the corresp^nd points of model and Prototype should be same (both in magnitude and dirⁿ)



Model

Prototype

$$\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = \cdots\cdots V_r$$

V_r ↓ velocity scale ratio

$$\boxed{V_r = \frac{L_r}{T_r}}$$ ← time ratio scale

$$\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} = \cdots\cdots a_r$$

a_r ← acceleⁿ scale ratio

***

$$\boxed{a_r = \frac{V_r}{T_r} = \frac{L_r}{T_r^2}}$$

② Dynamic similarity → the ratio of different forces and the corresponding point of model and Prototype should be same

→ Force scale ratio

$$\frac{(F_v)_m}{(F_v)_p} = \frac{(F_P)_m}{(F_P)_p} = \frac{(F_G)_m}{(F_G)_p} = \frac{(F_\sigma)_m}{(F_\sigma)_p} = \frac{(F_K)_m}{(F_K)_p} = F_r$$

F_v — Re
F_p — Eu
F_G — Fr
F_σ — We
F_K — Ma
→ F_i

$$F_i = F_v + F_p + F_G + F_\sigma + F_K$$

$$(F_i)_m = F_r\,[F_i]_p$$

$$\frac{(F_i)_m}{(F_i)_p} = F_r$$

$$\frac{(F_v)_m}{(F_v)_p} = \frac{(F_i)_m}{(F_i)_p}$$

$$\left[\frac{F_i}{F_v}\right]_p = \left[\frac{F_i}{F_v}\right]_m$$

$$(Re)_p = (Re)_m$$

Inertia $= ma \approx \rho \forall . \frac{V}{T} \approx \rho L^3 . \frac{V}{T} = \rho L^2 . \frac{L}{T} . V = \rho L^2 . V^2$

Viscous force τA

$= \mu . \frac{V}{L} . L^2 = \mu . V . L$

Pressure force (F_P) $= P . A = P . L^2$

Gravity force (F_G) $= mg = \rho L^3 . g$

Surface tension force $(F_\sigma) = \sigma L$

Compressibility force (F_K) $= K . A = K . L^2$

Dimensionless Nums

① Reynolds NO.

$Re = \frac{F_{inertia}}{F_{viscous}}$

$= \frac{\rho L^2 V^2}{\mu . V . L}$

$Re = \frac{\rho . V . L}{\mu}$

$\boxed{Re = \frac{\rho . V . L}{\mu}}$

Application →

① Pipe flow, flow around, submarine and aeroplane]

$F_r = \frac{V}{\sqrt{Lg}}$

Euler NO :- (E_u)

$$E_u = \sqrt{\frac{F_{inertia}}{F_{pressure}}} = \sqrt{\frac{\rho . \cancel{L^2} . V^2}{P . \cancel{L^2}}} = \boxed{E_u = \frac{V}{\sqrt{\frac{P}{\rho}}}}$$

Sometime *** ESE

$E_u = \frac{\Delta P}{\rho V^2}$

$E_u = \frac{\Delta P}{\frac{1}{2} \rho . V^2}$

Application

① Flow through pipe, Flow over submerged body

Froude NO. (Fr)

$$Fr = \sqrt{\frac{F_{inertia}}{F_{gravity}}} = \sqrt{\frac{\rho L^2 V^2}{\rho L^3 . g}} \quad \boxed{Fr = \frac{V}{\sqrt{Lg}}}$$

Application -

① River flow, open channel flow
② spill ways
③ surface wave motion created by boat

Important (Gate)

(4) Weber No. [We] : $We = \sqrt{\frac{F_{inertia}}{F_\sigma}} = \sqrt{\frac{\rho L^2 . V^2}{\sigma . L}} = \boxed{\frac{V}{\sqrt{\frac{\sigma}{L \cdot \rho}}}}$

Application-

(1) capillary tube flow

(5) Mach No : [Ma] $= \sqrt{\frac{F_i}{F_k}} = \sqrt{\frac{\rho L^2 V^2}{K . L^2}} = \boxed{\frac{V}{\sqrt{\frac{K}{\rho}}}}$ → c (velocity of sound in the medium)

Application-

(1) compressible flow :- aeroplane and Projectile (like missile) through air and supersonic air

(1) $\boxed{Re = \frac{\rho . V . L}{\mu}}$ $\boxed{Fr = \frac{V}{\sqrt{Lg}}}$ same units K or P or $\frac{\sigma}{L}$

(2) $\boxed{Ma = \frac{V}{\sqrt{\frac{K}{\rho}}}}$ $\boxed{Eu = \frac{V}{\sqrt{\frac{P}{\rho}}}}$ $\boxed{We = \frac{V}{\sqrt{\frac{\sigma}{L . \rho}}}}$

Different model law-

1. Reynold's model law $(Re)_m = (Re)_p$
2. Euler's model law $(Eu)_m = (Eu)_p$
3. Froude's model law $(Fr)_m = (Fr)_p$
4. Weber's model law $(We)_m = (We)_p$
5. Mach's model law $(Me)_m = (Me)_p$

Note

***Fluid flow models are usually designed to account for one most dominant Force and occassionly for 2 D Flow.

(Pb) (7) $L_r = \frac{1}{9}$, $Q_p = 1000 \, m^3/s$

$Q_m = ?$

Solm: $Q_r = A_r V_r$

$Q_r = (L_r)^2 \sqrt{L_r}$

$Q_r = (L_r)^{2.5}$

$$\frac{Q_m}{Q_p} = \left[\frac{1}{9}\right]^{2.5} \quad \frac{Q_m}{1000} = \left[\frac{1}{9}\right]^{2.5} \quad \boxed{Q_m = 4.11\, m^3/s} \text{ Ans}$$

Froude law

$$[Fr]_m = [Fr]_p$$

$$\left[\frac{V}{\sqrt{\cancel{g}L}}\right]_m = \left[\frac{V}{\sqrt{\cancel{g}L}}\right]_p$$

$$\frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

$$\boxed{V_r = \sqrt{L_r}}$$

Pb 13

$$(Re)_m = (Re)_p$$

$$\left(\frac{\rho . V . L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$$

$$\rho_r V_r . L_r = \mu_r \quad \text{---} (1)$$

$$[Ma]_m = [Ma]_p$$

$$\left[\frac{V}{\sqrt{\frac{K}{\rho}}}\right]_m = \left[\frac{V}{\sqrt{\frac{K}{\rho}}}\right]_p$$

$$V_r = \sqrt{\frac{K_r}{\rho_r}} \quad \text{---} (2)$$

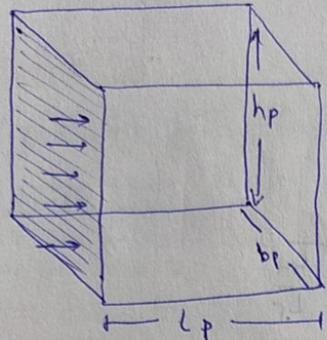
$$\rho_r \cancel{\times} \sqrt{\frac{K_r}{\rho_r}} . L_r = \mu_r$$

$$\sqrt{K_r \rho_r} . L_r = \mu_r$$

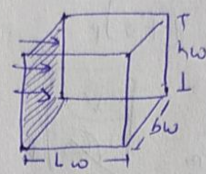
$$\boxed{L_r = \frac{\mu_r}{\sqrt{K_r \rho_r}}}$$

Distorted model Theory:

In making model of a prototype all the dimension are scaled with the same scale ratio (L_R) when the different dimension are scale with different scale ratio then the model formed is not real replica (mirror image) of prototype these type of model comes in the category of distored models.



Prototype



Model

$\frac{L_m}{L_p} = \frac{b_m}{b_p} = (L_r)_H \leftarrow$ Horizontal scale ratio

$\left[\frac{V_m}{V_p} = V_r \text{ or } \frac{\sqrt{m}}{\sqrt{p}} = \sqrt{r} \right]$

$\frac{h_m}{h_p} = (L_r)_v \longleftarrow$ vertical scale ratio

① Area scale Ratio: (A_r)

$A_r = \frac{b_m}{b_p} \cdot \frac{h_m}{h_p}$

$\boxed{A_r = (L_r)_H \, (L_r)_v}$

② Velocity scale Ratio (V_r)

$(F_r)_m = (F_r)_p$

$\left[\frac{V}{\sqrt{h.g}}\right]_m = \left[\frac{V}{\sqrt{hg}}\right]_p$

$\frac{V_m}{V_p} = \sqrt{\frac{h_m}{h_p}}$

$V_r = \sqrt{(L_r)_v}$

③ Discharge scale (Q_r)

$Q_r = A_r V_r$

$Q_r = (L_r)_H (L_r)_v \sqrt{(L_r)_v}$

$Q_r = (L_r)_H \cdot (L_r)_v^{1.5}$

Pb 9

$\frac{0.1}{Q_p} = \left(\frac{1}{100}\right)\left(\frac{1}{100}\right)^{1.5}$

$\boxed{Q_p = 10^5 \, m^3/s}$

Fluid Kinematic discuss →

Pb-29 Eqn of streamline

$xy = 1$

$x.dy + ydx = 0$

$xdy = -ydx$

$\left[\frac{dy}{dx} = -\frac{y}{x}\right]$

at (1,1). $\left(\frac{dy}{dx}\right) = -1$

Pb 32 $\phi = \ln(x^2+y^2)$

$\psi = ?$

$U = -\frac{\partial \psi}{\partial y} = -\frac{\partial \phi}{\partial x}$

$\frac{\partial \psi}{\partial y} = \frac{\partial}{\partial x} \ln(x^2+y^2) = \frac{2x}{x^2+y^2}$

Int it

$\psi = \frac{2x}{x^2} \times \frac{1}{x(\frac{1}{x})} \tan^{-1}\left(\frac{y}{x}\right) + c$

$\psi = 2\tan^{-1}\left(\frac{y}{x}\right) + c$

Pb (33) $U = Ay + B$

At $y = 0,\ U = 0,\ B = 0$

At $y = 1\,m,\ U = 75\,m/s$

$75 = A(1)$

$A = 75$

then, $U = 75y$

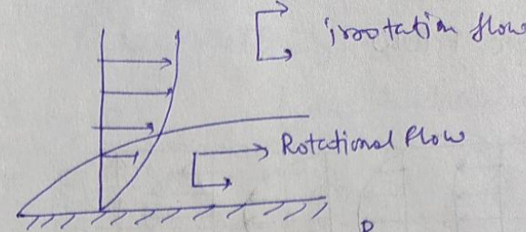
$-\dfrac{\partial \psi}{\partial y} = 75y$

and it $\left[\psi = 37.5y^2 + c^0\right]$

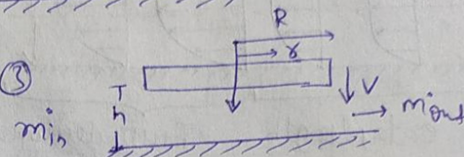
(40) $U = -\dfrac{\partial \phi}{\partial x}$

$V = -\dfrac{\partial \phi}{\partial y}$ $\quad |\vec{V}| = \sqrt{U^2+V^2}$

Pb (39) Inviscid Region of Fluid Flow (fric = 0)



T (3)



$\dot{m}_{in} = \dot{m}_{out}$

$\rho(\pi r^2)V = \rho.(2\pi r h).V_r$

$V.r = 2h.V_r$

$V_r = \dfrac{V.r}{2h}$