Exercise 2.5

Chapter 2 Derivatives Exercise 2.5 1E

Formula: If $y = f(u)$ and $u = g(x)$ are both differentiable functions then

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.

Given function $y = \sqrt[3]{1+4x}$

We expressed y as $y = (f \circ g)(x) = f(g(x))$ where $f(u) = \sqrt[3]{u}$ and

$g(x) = 1 + 4x = u$

Since $f'(u) = \frac{1}{3}u^{\frac{-2}{3}} = \frac{1}{3}\frac{1}{\sqrt[3]{u^2}}$ and $u' = g'(x) = 4$

$\therefore \quad y' = f'(g(x)).g'(x)$

$= \frac{1}{3}\frac{1}{\sqrt[3]{(1+4x)^2}}4 = \frac{4}{3}\frac{1}{\sqrt[3]{(1+4x)^2}}$

$$\boxed{y' = \frac{4}{3}\frac{1}{\sqrt[3]{(1+4x)^2}}}$$

Chapter 2 Derivatives Exercise 2.5 2E

Formula: If $y = f(u)$ and $u = g(x)$ are both differentiable functions then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

given function $y = (2x^3 + 5)^4$

We expressed y as $y = (f \circ g)(x) = f(g(x))$ where $f(u) = u^4$ and

$g(x) = 2x^3 + 5 = u$

Since $f'(u) = 4u^3$ and $g'(x) = u' = 6x^2$

$\therefore \quad y' = f'(g(x)).g'(x)$

$= 4(2x^3 + 5)^3 6x^2$

$$\boxed{\begin{aligned} \therefore\ y' &= 4(2x^3+5)^3 6x^2 \\ &= 24x^2(2x^3+5)^3 \end{aligned}}$$

Chapter 2 Derivatives Exercise 2.5 3E

The measure of rate of change of a quantity with respect to some other quantity is called as the derivative and the method to determine the derivative is called differentiation.

Take $y = f(u)$ and $u = g(x)$ are both differentiable functions then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.

Consider the function: $y = \tan \pi x$

Express the function y as shown below:

$$\begin{aligned} y &= (f \circ g)(x) \\ &= f(g(x)) \end{aligned}$$

Take the functions as shown below:

$$\begin{aligned} g(x) &= u \\ &= \pi x \\ f(u) &= \tan u \\ y &= \tan \pi x \end{aligned}$$

Consider the derivative of $f'(u)$:

$$\begin{aligned} f'(u) &= \frac{dy}{du} \\ &= \frac{d}{du}(\tan u) \\ &= \sec^2 u \\ &= \sec^2 \pi x \end{aligned}$$

Consider the derivative of u:

$$\begin{aligned} u' &= g'(x) \\ &= \pi \end{aligned}$$

Use the chain rule to differentiate the composite function *y*:

$$\begin{aligned} y' &= \frac{dy}{du} \times \frac{du}{dx} \\ &= f'(g(x)) \times g'(x) \\ &= (\sec^2 \pi x) \times \pi \\ &= \pi \sec^2 \pi x \end{aligned}$$

Hence, the final expression is $\boxed{y' = \pi \sec^2 \pi x}$.

Chapter 2 Derivatives Exercise 2.5 4E

Let $y = \sin(\cot x)$

Formula: If $y = f(u)$ and $u = g(x)$ are both differentiable functions then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

we expresses y as $y = (f \circ g)(x) = f(g(x))$ where $f(u) = \sin u$ and $g(x) = u = \cot x$.

Since $f'(u) = \cos u$ and $g'(x) = u' = -\csc^2 x$

$$\begin{aligned} \therefore \quad y' = \frac{dy}{du}.\frac{du}{dx} &= f'(g(x))g'(x) \\ &= \cos(\cot x).(-\csc^2 x) \\ &= -\cos(\cot x)\csc^2 x \end{aligned}$$

$\boxed{\therefore\ y' = -\cos(\cot x)\csc^2 x}$

Chapter 2 Derivatives Exercise 2.5 5E

Here $y = \sqrt{\sin x}$

If $f(x) = \sqrt{x}$ and $g(x) = \sin x$, then

$$y = (f \circ g)(x)$$

$$\Rightarrow \frac{dy}{dx} = (f \circ g)'(x)$$

$$= f'(g(x)).g'(x)$$

Since $f'(x) = \frac{1}{2\sqrt{x}}$ and $g'(x) = \cos x$, we have

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin x}}.\cos x$$

$$= \frac{\cos x}{2\sqrt{\sin x}}$$

Chapter 2 Derivatives Exercise 2.5 6E

Here $y = \sin\sqrt{x}$

If $f(x) = \sin x$ and $g(x) = \sqrt{x}$, then $y = (f \circ g)(x)$

$$\Rightarrow \frac{dy}{dx} = (f \circ g)'(x)$$

$$= f'(g(x)).g'(x)$$

Since $f'(x) = \cos x$ and $g(x) = \frac{1}{2\sqrt{x}}$, we have

$$\frac{dy}{dx} = \cos(\sqrt{x}).\frac{1}{2\sqrt{x}}$$

$$= \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

Chapter 2 Derivatives Exercise 2.5 7E

Consider the following function:

$$F(x) = (x^4 + 3x^2 - 2)^5$$

The objective is to find the derivative of the function by using the Chain rule.

The Chain Rule of differentiation is stated as follows:

If *n* is any real number and $u = g(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

Let $u = x^4 + 3x^2 - 2$ and $n = 5$

The derivative of above function can be found by applying the chain rule as shown below:

$$F'(x) = \frac{d}{dx}\left[(x^4 + 3x^2 - 2)^5\right] \quad \text{Apply chain rule}$$

$$= 5\left[(x^4 + 3x^2 - 2)^{5-1}\right]\frac{d}{dx}(x^4 + 3x^2 - 2) \quad \frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

$$= 5(x^4 + 3x^2 - 2)^4(4x^3 + 6x) \quad \text{Use sum rule}$$

$$= 5(x^4 + 3x^2 - 2)^4 2x(2x^2 + 3)$$

$$= \boxed{10x(x^4 + 3x^2 - 2)^4(2x^2 + 3)}$$

Hence, the derivative of the function is $\boxed{10x(x^4 + 3x^2 - 2)^4(2x^2 + 3)}$.

Chapter 2 Derivatives Exercise 2.5 8E

Consider the function,

$F(x)=(4x-x^2)^{100}$.

The object is to find the derivative of the above function.

Differentiate $F(x)=(4x-x^2)^{100}$ on both sides with respect to x.

$$\frac{d}{dx}[F(x)]$$
$$=\frac{d}{dx}(4x-x^2)^{100}$$
$$=100(4x-x^2)^{99}\frac{d}{dx}(4x-x^2) \text{ Use } \frac{d}{dx}(x^n)=nx^{n-1}$$
$$=100(4x-x^2)^{99}\left[4\frac{d}{dx}(x)-\frac{d}{dx}(x^2)\right] \text{Use } \frac{d}{dx}(f(x)-g(x))=\frac{d}{dx}(f(x))-\frac{d}{dx}(g(x))$$
$$=100(4x-x^2)^{99}(4-2x)$$

Therefore, the derivative of the function $F(x)=(4x-x^2)^{100}$ is,

$$\frac{dF(x)}{dx}=\boxed{100(4x-x^2)^{99}(4-2x)}.$$

Chapter 2 Derivatives Exercise 2.5 9E

Let $F(x)=(1-2x)^{\frac{1}{2}}$

We know that $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}.g'(x)$

$$\therefore \quad F'(x)=\frac{d}{dx}(1-2x)^{\frac{1}{2}}$$
$$=\frac{1}{2}[(1-2x)]^{\frac{1}{2}-1}.\frac{d}{dx}(1-2x)$$
$$=\frac{1}{2}[1-2x]^{\frac{-1}{2}}(-2)$$
$$=-(1-2x)^{\frac{-1}{2}}$$
$$=\frac{-1}{\sqrt{1-2x}}$$
$$\boxed{\therefore F'(x)=\frac{-1}{\sqrt{1-2x}}}$$

Chapter 2 Derivatives Exercise 2.5 10E

Let $f(x)=\frac{1}{(1+\sec x)^2}$

We know that $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}.g'(x)$

$$\therefore \quad f'(x)=\frac{d}{dx}(1+\sec x)^{-2}$$
$$=-2(1+\sec x)^{-3}\frac{d}{dx}(1+\sec x)$$
$$=\frac{-2}{(1+\sec x)^3}[0+\sec x\tan x]$$
$$=\frac{-2(\sec x\tan x)}{(1+\sec x)^3}$$
$$\boxed{\therefore f'(x)=\frac{-2(\sec x\tan x)}{(1+\sec x)^3}}$$

Chapter 2 Derivatives Exercise 2.5 10E

Let $f(z)=\frac{1}{z^2+1}=(1+z^2)^{-1}$

We know that $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}.g'(x)$

$$\therefore\ f'(z)=\frac{d}{dz}(1+z^2)^{-1}$$

$$=(-1)(1+z^2)^{-2}\frac{d}{dz}(1+z^2)$$

$$=\frac{-1}{(1+z^2)^2}(2z)$$

$$=\frac{-2z}{(1+z^2)^2}$$

$$\boxed{\therefore\ f'(z)=\frac{-2z}{(1+z^2)^2}}$$

Chapter 2 Derivatives Exercise 2.5 12E

Here $f(t)=\sqrt[3]{1+\tan t}=(1+\tan t)^{1/3}$

Using chain rule, we have

$$f'(t)=\frac{1}{3}(1+\tan t)^{-2/3}.(1+\tan t)'$$

$$=\frac{1}{3}(1+\tan t)^{-2/3}(\sec^2 t)$$

$$=\frac{(\sec^2 t)(1+\tan t)^{-2/3}}{3}$$

$$=\frac{(\sec^2 t)}{3\sqrt[3]{(1+\tan t)^2}}$$

Chapter 2 Derivatives Exercise 2.5 13E

Consider the function,

$$y=\cos(a^3+x^3)$$

The objective is to find the derivative of the function.

To find the derivative, use the following formula,

$$\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$$

Differentiate both sides of the function $y=\cos(a^3+x^3)$ with respect to x, to obtain,

$$\frac{dy}{dx}=\frac{d}{dx}\cos(a^3+x^3)$$

$$=-\sin(a^3+x^3)\frac{d}{dx}(a^3+x^3)$$

$$=-\sin(a^3+x^3)\left(\frac{d}{dx}(a^3)+\frac{d}{dx}(x^3)\right)$$

$=-\sin(a^3+x^3)(0+3x^2)$ Since a^3 is constant.

$$=-\sin(a^3+x^3)(3x^2)$$

Therefore, the derivative of the function $y=\cos(a^3+x^3)$ is $\boxed{-3x^2\sin(a^3+x^3)}$.

Chapter 2 Derivatives Exercise 2.5 13E

Consider the following function

$$y = a^3 + \cos^3 x$$
$$\frac{dy}{dx} = \frac{d}{dx}\left(a^3 + \cos^3 x\right)$$

Recall the sum rule

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Take $f(x) = a^3, g(x) = \cos^3 x$

$$\frac{d}{dx}\left(a^3 + \cos^3 x\right) = \frac{d}{dx}\left(a^3\right) + \frac{d}{dx}\left(\cos^3 x\right)$$

Recall that derivative of constant function is zero

$$\frac{d}{dx}(c) = 0$$

Since a is constant, the function $f(x) = a^3$ is also constant function

$$\frac{d}{dx}\left(a^3\right) = 0$$

Recall the formula the power rule combined with the chain rule

If n is any real number and $u = h(x)$ is differentiable then

$$\frac{d}{dx}\left[h(x)\right]^n = n\left[h(x)\right]^{n-1} \cdot h'(x)$$

Take $h(x) = \cos x, n = 3$

$$\begin{aligned}\frac{d}{dx}\left(\cos^3 x\right) &= 3(\cos x)^{n-1} \cdot \frac{d}{dx}(\cos x)\\ &= 3\cos^2 x \cdot \frac{d}{dx}(\cos x)\\ &= 3\cos^2 x \cdot (-\sin x)\\ &= -3\sin x \cos^2 x\end{aligned}$$

$$\frac{d}{dx}\left(a^3 + \cos^3 x\right) = 0 + \left(-3\sin x \cos^2 x\right)$$

$$\boxed{\frac{d}{dx}\left(a^3 + \cos^3 x\right) = -3\sin x \cos^2 x}$$

Chapter 2 Derivatives Exercise 2.5 15E

Consider the function $y = x\sec kx$.

Differentiate the function with respect to x.

$$\frac{d}{dx}(y) = \frac{d}{dx}(x\sec kx)$$

Let $f(x) = x, g(x) = \sec kx$

$$\frac{d}{dx}(y) = x\frac{d}{dx}(\sec kx) + \sec kx\frac{d}{dx}(x)$$

Since $\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\frac{d}{dx}\left(g(x)\right) + g(x)\frac{d}{dx}\left(f(x)\right)$

Proceed as follows:

$$\frac{d}{dx}(y) = x\sec(kx)\tan(kx)\frac{d}{dx}(kx) + \sec kx \cdot 1$$

Since $\frac{d}{dx}(\sec ax) = \sec(ax)\tan(ax)\frac{d}{dx}(ax)$

$= x\sec(kx)\tan(kx)\cdot k\frac{d}{dx}(x) + \sec kx \cdot 1$ Since $\frac{d}{dx}(kx) = k\frac{d}{dx}(x)$

$= x\sec(kx)\tan(kx)\cdot k + \sec kx$ Since $\frac{d}{dx}(x) = 1$

$= \sec(kx)(kx\tan(kx)+1)$

Therefore, the differentiated result is as follows:

$$y' = \boxed{\sec(kx)(kx\tan(kx)+1)}.$$

Chapter 2 Derivatives Exercise 2.5 16E

Given function $y = 3\cot(n\theta)$

Differentiate y with respect to θ, we have

$$y' = \frac{d}{d\theta}(3\cot(n\theta))$$

$$= 3(-\csc^2(n\theta))\frac{d}{d\theta}(n\theta) \quad \left(\text{since } \frac{d}{d\theta}(\cot\theta) = -\csc^2\theta\right)$$

$$= -3\csc^2(n\theta)(n)$$

Therefore $y' = -3n\csc^2(n\theta)$

Chapter 2 Derivatives Exercise 2.5 17E

Consider the following function:

$$f(x) = (2x-3)^4(x^2+x+1)^5$$

Find the derivative of the function by applying the product rule and chain rule.

The Product rule is as stated as below:

Suppose *f* and *g* are differentiable functions then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

The Chain Rule is stated as follows:

Suppose *f* and *g* are differentiable functions such that $F = f \circ g$ then

$$\frac{d}{dx}[F(x)] = \frac{d}{dx}[f(g(x))]\cdot\frac{d}{dx}[g(x)]$$

Differentiate *f* with respect to *x*.

$$f'(x) = \frac{d}{dx}\left[(2x-3)^4(x^2+x+1)^5\right]$$

$$= (x^2+x+1)^5\frac{d}{dx}\left[(2x-3)^4\right] + (2x-3)^4\frac{d}{dx}\left[(x^2+x+1)^5\right]$$

$$= (x^2+x+1)^5\left(4(2x-3)^3\right)\frac{d}{dx}(2x-3)$$

$$+(2x-3)^4\left(5(x^2+x+1)^4\right)\frac{d}{dx}(x^2+x+1) \quad \text{Use } \frac{d}{dx}(x^n) = nx^{n-1}$$

$$= (x^2+x+1)^5\left(4(2x-3)^3\right)(2) + (2x-3)^4\left(5(x^2+x+1)^4\right)(2x+1)$$

$$= 8(2x-3)^3(x^2+x+1)^5 + 5(2x+1)(2x-3)^4(x^2+x+1)^4$$

Continuation to the above steps

$$=(2x-3)^3(x^2+x+1)^4\left[8(x^2+x+1)+5(2x-3)(2x+1)\right]$$

Taking out the common factor

$$=(2x-3)^3(x^2+x+1)^4\left[8x^2+8x+8+5(4x^2+2x-6x-3)\right] \quad \text{Expanding}$$

$$=(2x-3)^3(x^2+x+1)^4\left[8x^2+8x+8+5(4x^2-4x-3)\right]$$

$$=(2x-3)^3(x^2+x+1)^4(8x^2+8x+8+20x^2-20x-15)$$

$$=(2x-3)^3(x^2+x+1)^4(28x^2-12x-7)$$

Hence, the derivative of the function *f* is $\boxed{f'(x)=(2x-3)^3(x^2+x+1)^4(28x^2-12x-7)}$.

Chapter 2 Derivatives Exercise 2.5 18E

To find the derivative of the function,

$$g(x)=(x^2+1)^3(x^2+2)^6$$

Differentiate both sides with respect to x,

$$\frac{d}{dx}[g(x)]=\frac{d}{dx}\left[(x^2+1)^3(x^2+2)^6\right]$$

Use **product rule** of differentiation,

$$\frac{d}{dx}[f(x)h(x)]=f(x)\frac{d}{dx}[h(x)]+h(x)\frac{d}{dx}[f(x)]$$

So,

$$g'(x)=(x^2+1)^3\frac{d}{dx}\left[(x^2+2)^6\right]+(x^2+2)^6\frac{d}{dx}\left[(x^2+1)^3\right]$$

Also, use the **power rule** of differentiation,

$$\frac{d}{dx}[f(x)]^n=n[f(x)]^{n-1}\cdot f'(x)$$

So,

$$g'(x)=(x^2+1)^3\left\{6(x^2+2)^5\frac{d}{dx}(x^2+2)\right\}+(x^2+2)^6\left\{3(x^2+1)^2\frac{d}{dx}(x^2+1)\right\}$$

$$=(x^2+1)^3\left\{6(x^2+2)^5(2x+0)\right\}+(x^2+2)^6\left\{3(x^2+1)^2(2x+0)\right\}$$

$$=(x^2+1)^3\left\{6(x^2+2)^5(2x)\right\}+(x^2+2)^6\left\{3(x^2+1)^2(2x)\right\}$$

$$=(x^2+1)^2(x^2+2)^5\left[12x(x^2+1)+(x^2+2)(6x)\right]$$

Continuation to the above

$$g'(x)=(x^2+1)^2(x^2+2)^5\left[12x(x^2+1)+(x^2+2)(6x)\right]$$

$$=(x^2+1)^2(x^2+2)^5\left[12x^3+12x+6x^3+12x\right]$$

$$=(x^2+1)^2(x^2+2)^5\left[18x^3+24x\right]$$

$$=(x^2+1)^2(x^2+2)^5 6x\left[3x^2+4\right]$$

$$=6x(x^2+1)^2(x^2+2)^5\left[3x^2+4\right]$$

Hence,

$$g'(x)=\boxed{6x(x^2+1)^2(x^2+2)^5\left[3x^2+4\right]}$$

Chapter 2 Derivatives Exercise 2.5 19E

To find the derivative of the function,

$$h(t)=(t+1)^{2/3}(2t^2-1)^3$$

Differentiate both sides with respect to t,

$$\frac{d}{dt}[h(t)]=\frac{d}{dt}\left[(t+1)^{2/3}(2t^2-1)^3\right]$$

Use **product rule** of differentiation,

$$\frac{d}{dt}[f(t)g(t)]=f(t)\frac{d}{dt}[g(t)]+g(t)\frac{d}{dt}[f(t)]$$

So,

$$h'(t)=(t+1)^{2/3}\frac{d}{dt}\left[(2t^2-1)^3\right]+(2t^2-1)^3\frac{d}{dt}\left[(t+1)^{2/3}\right]$$

Also, use the **power rule** of differentiation,

$$\frac{d}{dt}[f(t)]^n=n[f(t)]^{n-1}\cdot f'(t)$$

So,

$$\begin{aligned}h'(t)&=(t+1)^{\frac{2}{3}}\left\{3(2t^2-1)^2\frac{d}{dt}(2t^2-1)\right\}+(2t^2-1)^3\left\{\frac{2}{3}(t+1)^{\frac{2}{3}-1}\frac{d}{dt}(t+1)\right\}\\&=(t+1)^{\frac{2}{3}}\left\{3(2t^2-1)^2(4t-0)\right\}+(2t^2-1)^3\left\{\frac{2}{3}(t+1)^{\frac{-1}{3}}(1+0)\right\}\\&=(t+1)^{\frac{2}{3}}3(2t^2-1)^2(4t)+(2t^2-1)^3\frac{2}{3}(t+1)^{\frac{-1}{3}}\\&=\frac{2}{3}(t+1)^{\frac{-1}{3}}(2t^2-1)^2\left[\frac{3}{2}\cdot(t+1)\cdot 3\cdot(4t)+(2t^2-1)\right]\end{aligned}$$

Continuation to the above

$$\begin{aligned}h'(t)&=\frac{2}{3}(t+1)^{\frac{-1}{3}}(2t^2-1)^2\left[\frac{3}{2}\cdot(t+1)\cdot 3\cdot(4t)+(2t^2-1)\right]\\&=\frac{2}{3}(t+1)^{\frac{-1}{3}}(2t^2-1)^2\left[3(t+1)\cdot 3\cdot(2t)+(2t^2-1)\right]\\&=\frac{2}{3}(t+1)^{\frac{-1}{3}}(2t^2-1)^2\left[18t^2+18t+(2t^2-1)\right]\\&=\frac{2}{3}(t+1)^{\frac{-1}{3}}(2t^2-1)^2\left[20t^2+18t-1\right]\end{aligned}$$

Hence,

$$h'(t)=\boxed{\frac{2}{3}(t+1)^{\frac{-1}{3}}(2t^2-1)^2\left[20t^2+18t-1\right]}$$

Chapter 2 Derivatives Exercise 2.5 20E

Let $F(t)=(3t-1)^4(2t+1)^{-3}$

By the product rule

$$F'(t)=(3t-1)^4\frac{d}{dt}(2t+1)^{-3}+(2t+1)^{-3}\frac{d}{dt}(3t-1)^4$$

$$=(3t-1)^4(-3)(2t+1)^{-4}\frac{d}{dt}(2t+1)+(2t+1)^{-3}4(3t-1)^3\frac{d}{dt}(3t-1)$$

$$=(3t-1)^4(-3)(2t+1)^{-4}(2)+(2t+1)^{-3}4(3t-1)^3(3)$$

$$=(3t-1)^3(2t+1)^{-3}\left[-6(3t-1)(2t+1)^{-1}+12\right]$$

$$=(3t-1)^3(2t+1)^{-3}\left[-6(3t-1)(2t+1)^{-1}+12\right]$$

$$\boxed{\therefore F'(t)=(3t-1)^3(2t+1)^{-3}\left[-6(3t-1)(2t+1)^{-1}+12\right]}$$

Chapter 2 Derivatives Exercise 2.5 21E

Consider the function $y=\left(\frac{x^2+1}{x^2-1}\right)^3$

The objective is to find the derivative of $y=\left(\frac{x^2+1}{x^2-1}\right)^3$.

Here first use chain rule and then use quotient rule

The power rule combined with the chain rule If n is any real number and $u=g(x)$ is differential, then

$$\frac{d}{dx}(g(x))^n = n(g(x))^{n-1}\cdot\frac{d}{dx}g(x)$$

Quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{d}{dx}u-u\frac{d}{dx}v}{v^2}$

Differentiating y,

$$\begin{aligned}
y' &= \frac{d}{dx}\left[\left(\frac{x^2+1}{x^2-1}\right)^3\right] \\
&= 3\left(\frac{x^2+1}{x^2-1}\right)^{3-1}\cdot\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) \\
&= 3\left(\frac{x^2+1}{x^2-1}\right)^2\cdot\left[\frac{(x^2-1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}\right]
\end{aligned}$$

Continuation of the above

$$\begin{aligned}
&= 3\left(\frac{x^2+1}{x^2-1}\right)^2\cdot\left[\frac{(x^2-1)\left\{\frac{d}{dx}(x^2)+\frac{d}{dx}(1)\right\}-(x^2+1)\left\{\frac{d}{dx}(x^2)-\frac{d}{dx}(1)\right\}}{(x^2-1)^2}\right] \\
&= 3\left(\frac{x^2+1}{x^2-1}\right)^2\left[\frac{(x^2-1)\{2x^{2-1}+0\}-(x^2+1)\{2x^{2-1}-0\}}{(x^2-1)^2}\right] \\
&= 3\left(\frac{x^2+1}{x^2-1}\right)^2\left[\frac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2}\right] \\
&= 3\left(\frac{x^2+1}{x^2-1}\right)^2\left[\frac{2x\{x^2-1-x^2-1\}}{(x^2-1)^2}\right] \\
&= 3\left(\frac{x^2+1}{x^2-1}\right)^2\left[\frac{2x\{-2\}}{(x^2-1)^2}\right] \\
&= \boxed{-12x\frac{(x^2+1)^2}{(x^2-1)^4}}
\end{aligned}$$

Chapter 2 Derivatives Exercise 2.5 22E

Let $f(s)=\sqrt{\dfrac{s^2+1}{s^2+4}}=(s^2+1)^{\frac{1}{2}}(s^2+4)^{\frac{-1}{2}}$

By the product rule

$$f'(s)=(s^2+1)^{\frac{1}{2}}\frac{d}{ds}(s^2+4)^{\frac{-1}{2}}+(s^2+4)^{\frac{-1}{2}}\frac{d}{ds}(s^2+1)^{\frac{1}{2}}$$

$$=(s^2+1)^{\frac{1}{2}}-\frac{1}{2}(s^2+4)^{\frac{-3}{2}}\frac{d}{ds}(s^2+4)+(s^2+4)^{\frac{-1}{2}}\frac{1}{2}(s^2+1)^{\frac{-1}{2}}\frac{d}{ds}(s^2+1)$$

$$=(s^2+1)^{\frac{1}{2}}-\frac{1}{2}(s^2+4)^{\frac{-3}{2}}(2s)+(s^2+4)^{\frac{-1}{2}}\frac{1}{2}(s^2+1)^{\frac{-1}{2}}(2s)$$

$$=-s(s^2+1)^{\frac{1}{2}}(s^2+4)^{\frac{-3}{2}}+s(s^2+4)^{\frac{-1}{2}}(s^2+1)^{\frac{-1}{2}}$$

$$\therefore f'(s)=-s(s^2+1)^{\frac{1}{2}}(s^2+4)^{\frac{-3}{2}}+s(s^2+4)^{\frac{-1}{2}}(s^2+1)^{\frac{-1}{2}}$$

Chapter 2 Derivatives Exercise 2.5 23E

Here $y=\sin(x\cos x)$

Then,

$$\frac{dy}{dx}=\cos(x\cos x).(x\cos x)' \quad [\text{chain rule}]$$

$$=\cos(x\cos x).\left[x'\cos x+x(\cos x)'\right][\text{product rule}]$$

$$=\cos(x\cos x).[\cos x-x\sin x]$$

Chapter 2 Derivatives Exercise 2.5 24E

Consider the function $f(x)=\dfrac{x}{\sqrt{7-3x}}$

Find the derivative of the function:

Use the quotient rule.

$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right]=\frac{v(x)\frac{d}{dx}u(x)-u(x)\frac{d}{dx}v(x)}{[v(x)]^2}$$

Here $u(x)=x$ and $v(x)=(7-3x)^{\frac{1}{2}}$

The derivative of $f(x)$ is calculated as follows:

$$f'(x)=\frac{(7-3x)^{\frac{1}{2}}\frac{d}{dx}x-x\frac{d}{dx}(7-3x)^{\frac{1}{2}}}{\left[(7-3x)^{\frac{1}{2}}\right]^2}$$

$$=\frac{(7-3x)^{\frac{1}{2}}\cdot 1-x\frac{1}{2}(7-3x)^{\frac{-1}{2}}\frac{d}{dx}(7-3x)}{(7-3x)} \quad \text{Chain rule}$$

$$=\frac{(7-3x)^{\frac{1}{2}}-\frac{1}{2}x(7-3x)^{-\frac{1}{2}}(-3)}{(7-3x)}$$

$$=\frac{(7-3x)^{\frac{1}{2}}+\frac{3}{2}x(7-3x)^{-\frac{1}{2}}}{(7-3x)}$$

$$=\frac{\left[(7-3x)^{\frac{1}{2}}+\frac{3}{2}x(7-3x)^{-\frac{1}{2}}\right](7-3x)^{\frac{1}{2}}}{(7-3x)(7-3x)^{\frac{1}{2}}}$$

Proceed as follows:

$$= \frac{(7-3x)+\frac{3}{2}x}{(7-3x)^{\frac{3}{2}}}$$

$$= \frac{7-\frac{3}{2}x}{(7-3x)^{\frac{3}{2}}}$$

$$= \frac{14-3x}{2(7-3x)^{\frac{3}{2}}}$$

Therefore, $f'(x) = \boxed{\frac{14-3x}{2(7-3x)^{\frac{3}{2}}}}$

Chapter 2 Derivatives Exercise 2.5 25E

Consider the function $F(z) = \sqrt{\frac{z-1}{z+1}}$

Find the derivative of the function:

Use the quotient rule.

$$\frac{d}{dz}\left[\frac{u(z)}{v(z)}\right] = \frac{v(z)\frac{d}{dz}u(z) - u(z)\frac{d}{dz}v(z)}{[v(z)]^2}$$

Here $u(z) = \sqrt{z-1}$ and $v(z) = \sqrt{z+1}$

The derivative of $F(z)$ is calculated as follows:

$$F'(z) = \frac{\sqrt{z+1}\frac{d}{dz}\left(\sqrt{z-1}\right) - \sqrt{z-1}\frac{d}{dz}\left(\sqrt{z+1}\right)}{\left[\sqrt{z+1}\right]^2}$$

$$= \frac{\sqrt{z+1}\left(\frac{1}{2\sqrt{z-1}}\right) - \sqrt{z-1}\left(\frac{1}{2\sqrt{z+1}}\right)}{z+1}$$

$$= \frac{z+1-z+1}{2(z+1)\sqrt{z^2-1}}$$

$$= \boxed{\frac{1}{(z+1)\sqrt{z^2-1}}}$$

Chapter 2 Derivatives Exercise 2.5 26E

Here $G(y) = \frac{(y-1)^4}{(y^2+2y)^5}$

Then $G'(y) = \frac{\left[(y-1)^4\right]'(y^2+2y)^5 - (y-1)^4\left[(y^2+2y)^5\right]'}{\left[(y^2+2y)^5\right]^2}$ [Quotient rule]

$$= \frac{4(y-1)^3(y^2+2y)^5 - (y-1)^4 5(y^2+2y)^4(2y+2)}{(y^2+2y)^{10}} \quad [\text{chain rule}]$$

$$= \frac{2(y-1)^3(y^2+2y)^4\left[2y^2+4y-5(y^2-1)\right]}{(y^2+2y)^{10}}$$

$$= \frac{2(y-1)^3(-3y^2+4y+5)}{(y^2+2y)^6}$$

Chapter 2 Derivatives Exercise 2.5 27E

Consider the function $y=\frac{r}{\sqrt{r^2+1}}$.

Rewrite the function as follows:

$$y=\frac{r}{\sqrt{r^2+1}}$$
$$y=\frac{r}{\left(r^2+1\right)^{\frac{1}{2}}}$$
$$y=r\left(r^2+1\right)^{-\frac{1}{2}}$$

Find the derivative y' for the given function.

Use Product rule and Power rule combined with the Chain rule to find the derivative of the function.

Product rule:

If f and g are both differentiable, then

$$\frac{d}{dr}\left[f(r)g(r)\right]=f(r)\frac{d}{dr}\left[g(r)\right]+g(r)\frac{d}{dr}\left[f(r)\right].$$

Power rule combined with the Chain rule:

If n is any real number and $u=s(r)$ is differentiable, then

$$\frac{d}{dr}\left(u^n\right)=nu^{n-1}\frac{du}{dr}.$$

Differentiate y with respect to r.

Here, $f(r)=r$ and $g(r)=\left(r^2+1\right)^{-\frac{1}{2}}$.

$$y'=\frac{dy}{dr}$$
$$=\frac{d}{dr}\left[r\left(r^2+1\right)^{-\frac{1}{2}}\right]$$
$$=r\frac{d}{dr}\left[\left(r^2+1\right)^{-\frac{1}{2}}\right]+\left(r^2+1\right)^{-\frac{1}{2}}\frac{dr}{dr} \quad \text{[Use Product Rule.]}$$
$$=r\frac{d}{dr}\left[\left(r^2+1\right)^{-\frac{1}{2}}\right]+\left(r^2+1\right)^{-\frac{1}{2}}(1)$$
$$=\left[r\left(-\frac{1}{2}\right)\left(r^2+1\right)^{-\frac{1}{2}-1}\frac{d}{dr}\left(r^2+1\right)\right]+\left(r^2+1\right)^{-\frac{1}{2}}$$

$$\left[\begin{array}{l}\text{Use Power rule combined with the Chain rule}\\ \text{Here, } u=r^2+1 \text{ and } n=-\frac{1}{2}\end{array}\right]$$

$$=\left[\left(-\frac{r}{2}\right)\left(r^2+1\right)^{-\frac{3}{2}}(2r+0)\right]+\left(r^2+1\right)^{-\frac{1}{2}}$$

$$=\left(-r^2\right)\left(r^2+1\right)^{-\frac{3}{2}}+\left(r^2+1\right)^{-\frac{1}{2}}$$

$$=\left(r^2+1\right)^{-\frac{1}{2}}\left[1-r^2\left(r^2+1\right)^{-1}\right]$$

$$=\left(r^2+1\right)^{-\frac{1}{2}}\left[1-\frac{r^2}{r^2+1}\right]$$

$$=\left(r^2+1\right)^{-\frac{1}{2}}\left[\frac{r^2+1-r^2}{r^2+1}\right]$$

$$=\left(r^2+1\right)^{-\frac{1}{2}}\left[\frac{1}{r^2+1}\right]$$

$$=\left(r^2+1\right)^{-\frac{1}{2}-1}$$

$$=\left(r^2+1\right)^{-\frac{3}{2}}$$

Therefore, the derivative of the given function is $y'=\boxed{\left(r^2+1\right)^{-\frac{3}{2}}}$.

Chapter 2 Derivatives Exercise 2.5 28E

Here $y=\dfrac{\cos \pi x}{\sin \pi x+\cos \pi x}$

Then, $\dfrac{dy}{dx}=\dfrac{(\cos \pi x)'[\sin \pi x+\cos \pi x]-\cos \pi x[\sin \pi x+\cos \pi x]'}{(\sin \pi x+\cos \pi x)^2}$

$$=\frac{-\pi \sin \pi x(\sin \pi x+\cos \pi x)-\cos \pi x(\pi \cos \pi x-\pi \sin \pi x)}{(\sin \pi x+\cos \pi x)^2}$$

$$=\frac{-\pi\left[\sin^2 \pi x+\sin \pi x \cos \pi x+\cos^2 \pi x-\cos \pi x \sin \pi x\right]}{(\sin \pi x+\cos \pi x)^2}$$

$$=\frac{-\pi}{(\sin \pi x+\cos \pi x)^2}$$

Chapter 2 Derivatives Exercise 2.5 29E

Evaluate the derivative of the function y as follows:

$y=\sin\sqrt{1+x^2}$ — Original function

$\dfrac{dy}{dx}=\dfrac{d}{dx}\left(\sin\sqrt{1+x^2}\right)$ — Differentiate with respect to x on both sides

$=\cos\sqrt{1+x^2}\times\dfrac{d\left(\sqrt{1+x^2}\right)}{dx}$ — Use the chain rule that is $\dfrac{d}{dx}\left(\sin\left(f(x)\right)\right)=\cos\left(f(x)\right)\dfrac{df(x)}{dx}$

$=\cos\sqrt{1+x^2}\times\dfrac{d}{dx}\left(\left(1+x^2\right)^{\frac{1}{2}}\right)$ — Rewrite the square root as power $\frac{1}{2}$. That is $\sqrt{1+x^2}=\left(1+x^2\right)^{\frac{1}{2}}$

Continue the above step.

$$= \cos\sqrt{1+x^2} \times \frac{1}{2} \times \left(1+x^2\right)^{\frac{1}{2}-1} \times \frac{d}{dx}\left(1+x^2\right) \quad \left\{ \begin{array}{l} \text{Again Use the chain rule that is} \\ \frac{d}{dx}\left(f(x)\right)^n = n \cdot \left(f(x)\right)^{n-1} \frac{df(x)}{dx} \end{array} \right.$$

$$= \frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}} \cos\sqrt{1+x^2} \times \frac{d}{dx}\left(1+x^2\right) \quad \text{Simplify}$$

$$= \frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}} \cos\sqrt{1+x^2} \times \left(\frac{d}{dx}(1) + \frac{d}{dx}\left(x^2\right)\right) \quad \left\{ \begin{array}{l} \text{Use the addition rule that is} \\ \frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx} \end{array} \right.$$

$$= \frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}} \cos\sqrt{1+x^2} \times (0+2x) \quad \left\{ \begin{array}{l} \text{Use the formula} \\ \frac{d}{dx}\left(x^n\right) = n \cdot x^{n-1} \end{array} \right.$$

$$= \frac{2x}{2}\left(1+x^2\right)^{-\frac{1}{2}} \cos\sqrt{1+x^2} \quad \text{Simplify}$$

$$= \frac{x}{\sqrt{1+x^2}} \cos\sqrt{1+x^2} \quad \text{Simplify and rewrite } \left(1+x^2\right)^{-\frac{1}{2}} \text{ as } \frac{1}{\sqrt{1+x^2}}$$

Therefore the derivative of the function $y = \sin\sqrt{1+x^2}$ is $\boxed{\frac{x}{\sqrt{1+x^2}} \cos\sqrt{1+x^2}}$.

Chapter 2 Derivatives Exercise 2.5 30E

Let $F(v) = \left(\frac{v}{v^3+1}\right)^6$

Then $F'(v) = 6\left(\frac{v}{v^3+1}\right)^5 \frac{d}{dv}\left(\frac{v}{v^3+1}\right)$

$$= 6\left(\frac{v}{v^3+1}\right)^5 \frac{\left(v^3+1\right)(1) - v\left(3v^2\right)}{\left(v^3+1\right)^2}$$

$$= 6\left(\frac{v}{v^3+1}\right)^5 \frac{v^3+1-3v^3}{\left(v^3+1\right)^2}$$

$$= 6\left(\frac{v}{v^3+1}\right)^5 \frac{-2v^3+1}{\left(v^3+1\right)^2}$$

$$\boxed{\therefore F'(v) = 6\left(\frac{v}{v^3+1}\right)^5 \frac{1-2v^3}{\left(v^3+1\right)^2}}$$

Chapter 2 Derivatives Exercise 2.5 31E

Consider the equation, $y = \sin(\tan 2x)$.

The objective is to differentiate the function.

Use chain rule.

Let $u = \tan 2x$. Then $y = \sin u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{dx}(\sin u) \cdot \frac{d}{dx}(\tan 2x)$$

$$= \cos u \cdot \sec^2 2x \cdot \frac{d}{dx} 2x$$

$$= \cos(\tan 2x) \cdot \sec^2 2x \cdot 2$$

$$= 2\cos(\tan 2x)\sec^2 2x$$

Therefore, the result is $\frac{dy}{dx} = \boxed{2\cos(\tan 2x) \cdot \sec^2 2x}$

Chapter 2 Derivatives Exercise 2.5 32E

To find the derivative of the function,

$$y = \sec^2(m\theta)$$

Differentiate both sides with respect to θ,

$$\frac{d}{d\theta}(y) = \frac{d}{d\theta}\left[\sec^2(m\theta)\right]$$
$$= \frac{d}{d\theta}\left[\{\sec(m\theta)\}^2\right]$$

Use the **power rule** of differentiation,

$$\frac{d}{d\theta}\left[f(\theta)\right]^n = n\left[f(\theta)\right]^{n-1} \cdot f'(\theta)$$

So,

$$y' = 2\left[\sec(m\theta)\right]^{2-1} \frac{d}{d\theta}(\sec(m\theta))$$

Use the **chain rule** of differentiation,

$$\frac{d}{d\theta}\left[f(g(\theta))\right] = f'(g(\theta)) \cdot g'(\theta)$$

to differentiate $\sec(m\theta)$

So,

$$y' = 2\left[\sec(m\theta)\right]^{2-1} \cdot \sec(m\theta)\tan(m\theta)\frac{d}{d\theta}(m\theta)$$
$$= 2\sec(m\theta) \cdot \sec(m\theta)\tan(m\theta) \cdot m\frac{d}{d\theta}(\theta)$$
$$= 2\sec(m\theta) \cdot \sec(m\theta)\tan(m\theta) \cdot m \cdot 1$$
$$= 2m\sec^2(m\theta)\tan(m\theta)$$

Hence,

$$y' = \boxed{2m\sec^2(m\theta)\tan(m\theta)}$$

Chapter 2 Derivatives Exercise 2.5 33E

Consider the following function:

$$y = \sec^2 x + \tan^2 x$$

The objective is to find the derivative of the function by using the Chain rule.

The Chain Rule of differentiation is stated as follows:

Suppose *f* and *g* are differentiable functions such that $F = f \circ g$ then

$$\frac{d}{dx}\left[F(x)\right] = \frac{d}{dx}\left[f(g(x))\right] \cdot \frac{d}{dx}\left[g(x)\right]$$

Differentiate *y* with respect to *x*.

$$y' = \frac{d}{dx}\left(\sec^2 x + \tan^2 x\right)$$
$$= \frac{d}{dx}\left(\sec^2 x\right) + \frac{d}{dx}\left(\tan^2 x\right) \qquad \text{Use sum rule}$$
$$= 2\sec x\frac{d}{dx}(\sec x) + 2\tan x\frac{d}{dx}(\tan x) \qquad \text{Use chain rule}$$
$$= 2\sec x(\sec x\tan x) + 2\tan x\left(\sec^2 x\right)$$
$$= 2\sec^2 x\tan x + 2\sec^2 x\tan x$$
$$= 4\sec^2 x\tan x$$

Hence, the derivative of the function *y* is $\boxed{y' = 4\sec^2 x\tan x}$.

Chapter 2 Derivatives Exercise 2.5 34E

Consider the following function:

$$y = x\sin\frac{1}{x}$$

Find the derivative of the function by using the Chain rule.

The Product rule of differentiation is stated as follows:

Suppose *f* and *g* are differentiable functions then

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

The Chain Rule of differentiation is stated as follows:

Suppose *f* and *g* are differentiable functions such that $F = f \circ g$ then

$$\frac{d}{dx}\left[F(x)\right] = \frac{d}{dx}\left[f(g(x))\right]\cdot\frac{d}{dx}\left[g(x)\right]$$

Differentiate *y* with respect to *x*

$$\begin{aligned}
y' &= \frac{d}{dx}\left(x\sin\frac{1}{x}\right) \\
&= x\frac{d}{dx}\left(\sin\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)\frac{d}{dx}(x) \quad \text{Use the Product rule} \\
&= x\frac{d}{dx}\left(\sin\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)(1) \\
&= x\frac{d}{dx}\left(\sin\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)
\end{aligned}$$

Therefore, the derivative of *y* is $y' = x\frac{d}{dx}\left(\sin\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right) \quad \cdots\cdots (1)$

Use chain rule find the derivative of $\frac{d}{dx}\left(\sin\frac{1}{x}\right)$.

$$\begin{aligned}
\frac{d}{dx}\left(\sin\frac{1}{x}\right) &= \cos\left(\frac{1}{x}\right)\frac{d}{dx}\left(\frac{1}{x}\right) \quad \text{Use } \frac{d}{dx}(\sin x) = \cos x \\
&= \cos\left(\frac{1}{x}\right)\frac{d}{dx}\left(x^{-1}\right) \\
&= \cos\left(\frac{1}{x}\right)\left(-1\cdot x^{-1-1}\right) \quad \text{Use } \frac{d}{dx}x^n = nx^{n-1} \\
&= -\cos\left(\frac{1}{x}\right)\left(x^{-2}\right) \\
&= -\frac{\cos\left(\frac{1}{x}\right)}{x^2}
\end{aligned}$$

Thus, the derivative of $\sin\left(\frac{1}{x}\right)$ is $\frac{d}{dx}\left(\sin\frac{1}{x}\right) = -\frac{\cos\left(\frac{1}{x}\right)}{x^2}$.

Substitute the value of $\frac{d}{dx}\left(\sin\frac{1}{x}\right)$ in (1) to obtain the following:

$$\begin{aligned}
y' &= x\frac{d}{dx}\left(\sin\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right) \\
&= x\left[-\frac{\cos\left(\frac{1}{x}\right)}{x^2}\right] + \sin\left(\frac{1}{x}\right) \\
&= -\frac{1}{x}\cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)
\end{aligned}$$

Therefore , the derivative of *y* is $y' = \boxed{-\frac{1}{x}\cos\frac{1}{x} + \sin\frac{1}{x}}$.

Chapter 2 Derivatives Exercise 2.5 35E

Given function $y = \left(\dfrac{1-\cos 2x}{1+\cos 2x}\right)^4$

Let $u = \dfrac{1-\cos 2x}{1+\cos 2x}$ and so $y = u^4$ then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

consider $\dfrac{du}{dx} = \dfrac{d}{dx}\left(\dfrac{1-\cos 2x}{1+\cos 2x}\right)$

$$= \frac{(1+\cos 2x)\frac{d}{dx}(1-\cos 2x) - (1-\cos 2x)\frac{d}{dx}(1+\cos 2x)}{(1+\cos 2x)^2}$$

$$= \frac{(1+\cos 2x)(-(-\sin 2x))\frac{d}{dx}(2x) - (1-\cos 2x)(-\sin 2x)\frac{d}{dx}(2x)}{(1+\cos 2x)^2}$$

$$= \frac{(1+\cos 2x)(\sin 2x)(2) - (1-\cos 2x)(-\sin 2x)(2)}{(1+\cos 2x)^2}$$

$$= \frac{2\sin 2x + 2\sin 2x\cos 2x + 2\sin 2x - 2\sin 2x\cos 2x}{(1+\cos 2x)^2}$$

$$= \frac{4\sin 2x}{(1+\cos 2x)^2}$$

also $\dfrac{dy}{du} = \dfrac{d}{du}(u^4)$

$= 4u^{4-1}$

$= 4u^3$

put $\dfrac{dy}{du}$ and $\dfrac{du}{dx}$ in $\dfrac{dy}{dx}$, we have

$$\frac{dy}{dx} = 4u^3\left[\frac{4\sin 2x}{(1+\cos 2x)^2}\right]$$

$$= 4\left(\frac{1-\cos 2x}{1+\cos 2x}\right)^3\left[\frac{4\sin 2x}{(1+\cos 2x)^2}\right] \quad \text{(substitute } u \text{ from above)}$$

Therefore $\dfrac{dy}{dx} = \dfrac{16\sin 2x(1-\cos 2x)^3}{(1+\cos 2x)^5}$

Chapter 2 Derivatives Exercise 2.5 36E

Consider the function $f(t) = \sqrt{\left(\dfrac{t}{t^2+4}\right)}$.

Rewrite the function as $f(t) = \left(\dfrac{t}{t^2+4}\right)^{\frac{1}{2}}$.

Differentiate the function with respect to t. Solve as follows:

$$f'(t) = \frac{d}{dt}\left(\frac{t}{t^2+4}\right)^{\frac{1}{2}}$$

$$f'(t) = \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{\frac{1}{2}-1}\frac{d}{dt}\left(\frac{t}{t^2+4}\right)$$ Since $\dfrac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}\cdot g'(x)$

$$f'(t) = \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-\frac{1}{2}}\frac{d}{dt}\left(\frac{t}{t^2+4}\right)$$

Let $u(t)=t, v(t)=t^2+4$.

$$f'(t)=\frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-\frac{1}{2}}\left[\frac{(t^2+4)\frac{d}{dt}(t)-(t)\frac{d}{dt}(t^2+4)}{(t^2+4)^2}\right]$$

Since $\frac{d}{dt}\left(\frac{u(t)}{v(t)}\right)=\frac{v\frac{d}{dt}(u(t))-u\frac{d}{dt}(v(t))}{v(t)^2}$

$$f'(t)=\frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-\frac{1}{2}}\left[\frac{(t^2+4)\cdot 1-(t)(2t+0)}{(t^2+4)^2}\right]$$

$$=\frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-\frac{1}{2}}\left[\frac{t^2+4-2t^2}{(t^2+4)^2}\right]$$

$$=\frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-\frac{1}{2}}\left[\frac{4-t^2}{(t^2+4)^2}\right]$$

Solve further as follows:

$$=\frac{1}{2}\left(\frac{(t)^{-\frac{1}{2}}}{(t^2+4)^{-\frac{1}{2}}}\right)\left[\frac{4-t^2}{(t^2+4)^2}\right]$$

$$=\frac{1}{2}\left(\frac{(t)^{-\frac{1}{2}}(4-t^2)}{(t^2+4)^{-\frac{1}{2}}(t^2+4)^2}\right)$$

$$=\frac{1}{2}\left(\frac{(t)^{-\frac{1}{2}}(4-t^2)}{(t^2+4)^{-\frac{1}{2}+2}}\right)$$ Since $a^m\cdot a^n=a^{m+n}$

$$=\frac{1}{2}\left(\frac{(t)^{-\frac{1}{2}}(4-t^2)}{(t^2+4)^{\frac{3}{2}}}\right)$$

Hence, the final answer is as follows:

$$f'(t)=\boxed{\frac{1}{2}\left(\frac{(t)^{-\frac{1}{2}}(4-t^2)}{(t^2+4)^{\frac{3}{2}}}\right)}.$$

Chapter 2 Derivatives Exercise 2.5 37E

Here $y=\cot^2(\sin\theta)$

Then $\frac{dy}{dx}=2\cot(\sin\theta)\times[\cot(\sin\theta)]'$ [*chain rule*]

$$=2\cot(\sin\theta)\times[-\csc^2(\sin\theta)]\times(\sin\theta)' \text{ [chain rule]}$$

$$=-2\cot(\sin\theta)\csc^2(\sin\theta)\cos\theta$$

Chapter 2 Derivatives Exercise 2.5 38E

Consider the function $y=\left(ax+\sqrt{x^2+b^2}\right)^{-2}$

Need to find differentiate the function.

Differentiate with respect to x

$$y'=\frac{d}{dx}\left(ax+\sqrt{x^2+b^2}\right)^{-2}$$

$$=-2\left(ax+\sqrt{x^2+b^2}\right)^{-2-1}\frac{d}{dx}\left(ax+\sqrt{x^2+b^2}\right)$$

Since $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

$$=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\frac{d}{dx}\left(ax+\sqrt{x^2+b^2}\right)$$

Continuation to the above steps,

$$y'=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\left[\frac{d}{dx}(ax)+\frac{d}{dx}\left(\sqrt{x^2+b^2}\right)\right]$$

Since $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$

$$=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\left[a\frac{d}{dx}(x)+\frac{d}{dx}\left(\sqrt{x^2+b^2}\right)\right]$$

Since $\frac{d}{dx}(cf(x))=c\frac{d}{dx}(f(x))$

$$=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\left[a+\frac{d}{dx}\left(\sqrt{x^2+b^2}\right)\right]$$ Since $\frac{d}{dx}(x)=1$

$$=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\left[a+\frac{d}{dx}\left(x^2+b^2\right)^{\frac{1}{2}}\right]$$ Rewrite the derivative

Continuation to the above steps,

$$y'=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\left[a+\frac{1}{2}\left(x^2+b^2\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(x^2+b^2\right)\right]$$

Since $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

$$=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\left[a+\frac{1}{2}\left(x^2+b^2\right)^{-\frac{1}{2}}(2x+0)\right]$$

$$=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\left[a+\frac{1}{2}\left(x^2+b^2\right)^{-\frac{1}{2}}(2x)\right]$$

$$=-2\left(ax+\sqrt{x^2+b^2}\right)^{-3}\left[a+x\left(x^2+b^2\right)^{-\frac{1}{2}}\right]$$ Cancel like terms

$$y'=-\frac{2\left(a+\frac{x}{\sqrt{x^2+b^2}}\right)}{\left(ax+\sqrt{x^2+b^2}\right)^3}$$

Therefore,

$$\boxed{y'=-\frac{2\left(a+\frac{x}{\sqrt{x^2+b^2}}\right)}{\left(ax+\sqrt{x^2+b^2}\right)^3}}.$$

Chapter 2 Derivatives Exercise 2.5 39E

Consider the function $y=\left[x^2+(1-3x)^5\right]^3$

Need to find differentiate the function.

Differentiate with respect to x

$$y'=\frac{d}{dx}\left[x^2+(1-3x)^5\right]^3$$

$$y'=3\left[x^2+(1-3x)^5\right]^{3-1}\frac{d}{dx}\left[x^2+(1-3x)^5\right]$$

Since $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

$$=3\left[x^2+(1-3x)^5\right]^2\frac{d}{dx}\left[x^2+(1-3x)^5\right]$$

Continuation to the above steps,

$$y'=3\left[x^2+(1-3x)^5\right]^2\left[\frac{d}{dx}\left[x^2\right]+\frac{d}{dx}\left[(1-3x)^5\right]\right]$$

Since $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$

$$y'=3\left[x^2+(1-3x)^5\right]^2\left[2x+\frac{d}{dx}\left[(1-3x)^5\right]\right]$$

Since $\frac{d}{dx}(x^n)=nx^{n-1}$

Continuation to the above steps,

$$y'=3\left[x^2+(1-3x)^5\right]^2\left[2x+5(1-3x)^{5-1}\frac{d}{dx}(1-3x)\right]$$

Since $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

$$=3\left[x^2+(1-3x)^5\right]^2\left[2x+5(1-3x)^4(0-3)\right]$$

$$y'=3\left[x^2+(1-3x)^5\right]^2\left[2x-15(1-3x)^4\right]$$

Therefore,

$$y'=\boxed{3\left[x^2+(1-3x)^5\right]^2\left[2x-15(1-3x)^4\right]}.$$

Chapter 2 Derivatives Exercise 2.5 40E

Here $y=\sin(\sin(\sin x))$

Then using chain rule we have,

$$\frac{dy}{dx}=\cos(\sin(\sin x)).[\sin(\sin x)]'$$

$$=\cos(\sin(\sin x)).\cos(\sin x).(\sin x)'$$

$$=\cos(\sin(\sin x)).\cos(\sin x).\cos x$$

Chapter 2 Derivatives Exercise 2.5 41E

Now applying the "THE POWER RULE COMBINED WITH THE CHAIN RULE"

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$

Here $y = \sqrt{x + \sqrt{x}}$

$$y = \left(x + \sqrt{x}\right)^{\frac{1}{2}}$$

Then using chain rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + \sqrt{x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(x + \sqrt{x}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(x + \sqrt{x}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x + \sqrt{x}}} \times \frac{d}{dx}\left(x + \sqrt{x}\right)$$

$$= \frac{1}{2\sqrt{x + \sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x + \sqrt{x}}}$$

Chapter 2 Derivatives Exercise 2.5 42E

Consider the function,

$$y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

The objective is to find the derivative of the given function.

Rewrite the function as follows:

$$y = \left(x + \sqrt{x + \sqrt{x}}\right)^{\frac{1}{2}}$$

Differentiate y with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(x + \sqrt{x + \sqrt{x}}\right)^{\frac{1}{2}}\right]$$

Use Chain Rule to simplify the differentiation as follows:

Chain Rule:

$$\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

Apply the Chain Rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(x + \sqrt{x + \sqrt{x}}\right)^{\frac{1}{2}}\right]$$

$$= \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left[x + \sqrt{x + \sqrt{x}}\right] \qquad \text{Use } \frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$= \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-\frac{1}{2}} \frac{d}{dx}\left[x + \sqrt{x + \sqrt{x}}\right]$$

$$= \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \frac{d}{dx}\left[x + \sqrt{x + \sqrt{x}}\right] \qquad \text{......(1)}$$

Find the value of the $\frac{d}{dx}\left[x+\sqrt{x+\sqrt{x}}\right]$ above result as follows:

$$\frac{d}{dx}\left[x+\sqrt{x+\sqrt{x}}\right]=\frac{d}{dx}[x]+\frac{d}{dx}\left[\sqrt{x+\sqrt{x}}\right]$$
$$=1+\frac{d}{dx}\left[\left(x+\sqrt{x}\right)^{\frac{1}{2}}\right]$$
$$=1+\frac{1}{2}\left(x+\sqrt{x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left[x+\sqrt{x}\right] \quad \text{Apply chain rule}$$
$$=1+\frac{1}{2\sqrt{x+\sqrt{x}}}\frac{d}{dx}\left[x+\sqrt{x}\right]$$

Simplify the above result as follows:

$$\frac{d}{dx}\left[x+\sqrt{x+\sqrt{x}}\right]=1+\frac{1}{2\sqrt{x+\sqrt{x}}}\frac{d}{dx}\left[x+\sqrt{x}\right]$$
$$=1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left[\frac{d}{dx}[x]+\frac{d}{dx}\left[\sqrt{x}\right]\right]$$
$$=1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left[\frac{d}{dx}[x]+\frac{d}{dx}\left[x^{\frac{1}{2}}\right]\right]$$
$$=1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2}x^{\frac{1}{2}-1}\right)$$
$$=1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)$$

Substitute the value of $\frac{d}{dx}\left[x+\sqrt{x+\sqrt{x}}\right]=1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)$ in equation (1).

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\frac{d}{dx}\left[x+\sqrt{x+\sqrt{x}}\right]$$
$$=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right]$$
$$=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right)\right]$$
$$=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}\right]$$

Simplify the above result,

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}\right]$$
$$=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[\frac{4\sqrt{x}\sqrt{x+\sqrt{x}}+2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}\right]$$
$$=\frac{4\sqrt{x}\sqrt{x+\sqrt{x}}+2\sqrt{x}+1}{8\sqrt{x+\sqrt{x+\sqrt{x}}}\sqrt{x}\sqrt{x+\sqrt{x}}}$$

Therefore, the derivative of the given function is,

$$\boxed{\frac{dy}{dx}=\frac{4\sqrt{x}\sqrt{x+\sqrt{x}}+2\sqrt{x}+1}{8\sqrt{x}\sqrt{x+\sqrt{x+\sqrt{x}}}\sqrt{x+\sqrt{x}}}}.$$

Chapter 2 Derivatives Exercise 2.5 43E

Consider the function $g(x)=[2r\sin rx+n]^p$

Differentiate the function with respect to x as follows:

$$g'(x)=\frac{d}{dx}[2r\sin rx+n]^p$$

$$g'(x)=p[2r\sin rx+n]^{p-1}\frac{d}{dx}[2r\sin rx+n]$$

Since $\frac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

Solve further as follows:

$$g'(x)=p[2r\sin rx+n]^{p-1}\frac{d}{dx}[2r\sin rx]+\frac{d}{dx}[n]$$

Since $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$

$$g'(x)=p[2r\sin rx+n]^{p-1}\frac{d}{dx}[2r\sin rx]+0$$

Since $\frac{d}{dx}[k]=0$

Solve further as follows:

$$g'(x)=p[2r\sin rx+n]^{p-1}\,2\frac{d}{dx}[r\sin rx]$$

Since $\frac{d}{dx}(cf(x))=c\frac{d}{dx}f(x)$

Let $f(x)=r, g(x)=\sin rx$

$$g'(x)=2p[2r\sin rx+n]^{p-1}\left[r\frac{d}{dx}(\sin rx)+(\sin rx)\frac{d}{dx}(r)\right]$$

Since $\frac{d}{dx}(f(x)\cdot g(x))=f(x)\frac{d}{dx}(g(x))+g(x)\frac{d}{dx}(f(x))$

Solve further as follows:

$$g'(x)=p[2r\sin rx+n]^{p-1}\,2[r(r\cos rx)+(\sin rx)\cdot 0]$$

Since $\frac{d}{dx}(\sin rx)=r\cos rx, \frac{d}{dx}(k)=0$

$$=p[2r\sin rx+n]^{p-1}\,2[r(r\cos rx)]$$

$$g'(x)=p[2r\sin rx+n]^{p-1}[2r^2\cos rx]$$

Therefore, the result is as follows:

$$g'(x)=\boxed{p[2r\sin rx+n]^{p-1}[2r^2\cos rx]}.$$

Chapter 2 Derivatives Exercise 2.5 44E

Given $y=\cos^4(\sin^3 x)$

We know $\frac{d}{dx}(f(x))^n=n(f(x))^{n-1}f'(x)$

$$\frac{d}{dx}(\cos x)=-\sin x\,1\frac{d}{dx}(\sin x)=\cos x$$

The derivative of the given function y is

$$\frac{dy}{dx}=\frac{d}{dx}\left[\cos^4\left(\sin^3 x\right)\right]$$

$$=4\cos^3\left(\sin^3 x\right)\frac{d}{dx}\left(\cos\left(\sin^3 x\right)\right)$$

$$=4\cos^3\left(\sin^3 x\right)\left(-\sin\left(\sin^3 x\right)\right)\frac{d}{dx}\left(\sin^3 x\right)$$

$$=-4\cos^3\left(\sin^3 x\right)\sin\left(\sin^3 x\right)3\sin^2 x\frac{d}{dx}\left(\sin x\right)$$

$$=-4\cos^3\left(\sin^3 x\right)\sin\left(\sin^3 x\right)3\sin^2 x\left(\cos x\right)$$

$$=-12\cos^3\left(\sin^3 x\right)\sin\left(\sin^3 x\right)\sin^2 x\cos x$$

$$\therefore \frac{dy}{dx}=-12\cos^3\left(\sin^3 x\right)\sin\left(\sin^3 x\right)\sin^2 x\cos x$$

Chapter 2 Derivatives Exercise 2.5 45E

$$y=\cos(\sqrt{\sin(\tan(\pi x))})$$

$$y'=-\sin\left(\sqrt{\sin(\tan(\pi x))}\right)\cdot\frac{d}{dx}\sqrt{\sin(\tan(\pi x))}\left[\text{CHAINRULE}\right]$$

$$=-\sin\left(\sqrt{\sin(\tan(\pi x))}\right)\cdot\frac{1}{2}(\sin(\tan(\pi x))^{\frac{-1}{2}}\cdot\frac{d}{dx}\sin\left(\tan\left(\pi x\right)\right)$$

$$=-\sin\left(\sqrt{\sin(\tan(\pi x))}\right)\cdot\frac{1}{2}(\sin(\tan(\pi x))^{\frac{-1}{2}}\cdot\cos\left(\tan\left(\pi x\right)\right)\cdot\frac{d}{dx}\tan\left(\pi x\right)$$

$$=-\sin\left(\sqrt{\sin(\tan(\pi x))}\right)\cdot\frac{1}{2}(\sin(\tan(\pi x)))^{\frac{-1}{2}}\cdot\cos\left(\tan\left(\pi x\right)\right)\cdot\pi(\sec(\pi x))^2$$

Chapter 2 Derivatives Exercise 2.5 46E

Given $y=\left[x+\left(x+\sin^2 x\right)^3\right]^4$

We know $\frac{d}{dx}\left(f(x)\right)^n=n\left(f(x)\right)^{n-1}f'(x)$

The derivative of the given function y is

$$\frac{d}{dx}(y)=\frac{d}{dx}\left[x+\left(x+\sin^2 x\right)^3\right]^4$$

$$=4\left[x+\left(x+\sin^2 x\right)^3\right]\frac{d}{dx}\left(x+\left(x+\sin^2 x\right)^3\right)$$

$$=4\left[x+\left(x+\sin^2\right)^3\right]^3\left[1+3\left(x+\sin^2 x\right)^2\frac{d}{dx}\left(x+\sin^2 x\right)\right]$$

$$=4\left[x+x\left(+\sin^2 x\right)^3\right]^3\left[1+3\left(x+\sin^2 x\right)^2\left(1+2\sin x\cos x\right)\right]$$

$$=\boxed{4\left(x+\left(x+\sin^2 x\right)^3\right)^3\left(1+3\left(x+\sin^2\right)^2\left(1+\sin 2x\right)\right)}$$

Chapter 2 Derivatives Exercise 2.5 47E

Consider the function $y = \cos(x^2)$

To find the first derivative, using chain rule

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$

Here the outer function *f* is the cosine function and the inner function is the squaring function $g(x) = x^2$, so the chain rule gives

$\frac{dy}{dx} = \frac{d}{dx}\cos(x^2)$ Since $y = \cos(x^2)$

$= \frac{d}{dx}\cos(x^2)\frac{d}{dx}(x^2)$ Apply Chain rule

$= -\sin(x^2)\frac{d}{dx}(x^2)$ Derivative of outer function

$= -\sin(x^2)2x$ Derivative of inner function

$= -2x\sin(x^2)$ Thus the first derivative is $\boxed{\frac{dy}{dx} = -2x\sin(x^2)}$

Next find the second derivative:

$\frac{d^2y}{dx^2} = \frac{d}{dx}\cdot\frac{dy}{dx}$

$= \frac{d}{dx}(-2x\sin(x^2))$ Since $\frac{dy}{dx} = -2x\sin(x^2)$

$= -2\frac{d}{dx}(x\sin(x^2))$ Remove out -2 from parentheses

By the product rule if *f* and *g* are differentiable functions then

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

Let $f(x) = x$ and $g(x) = \sin(x^2)$

Then continuation to the above steps,

$\frac{d^2y}{dx^2} = -2\left[x\cdot\frac{d}{dx}\sin(x^2) + \sin(x^2)\frac{d}{dx}(x)\right]$ Apply product rule

$= -2\left[x\cos(x^2)\frac{d}{dx}(x^2) + \sin(x^2)\frac{d}{dx}x\right]$ Apply chain rule for $\frac{d}{dx}\sin(x^2)$

$= -2\left[x\cos(x^2)(2x) + \sin(x^2)(1)\right]$ Since $\frac{d}{dx}(x^2) = 2x$ and $\frac{d}{dx}(x) = 1$

$= -2\left[2x^2\cos(x^2) + \sin(x^2)\right]$ Simplify

Hence the second derivative for $y = \cos(x^2)$ is

$\boxed{\frac{d^2y}{dx^2} = -2\left[2x^2\cos(x^2) + \sin(x^2)\right]}$.

Chapter 2 Derivatives Exercise 2.5 48E

Let $y = \cos^2 x$

The first derivative $y' = \frac{d}{dx}\cos^2 x$

$= \frac{d}{dx}[\cos x]^2$

$= 2\cos x\frac{d}{dx}\cos x$

$= 2\cos x(-\sin x)$

$= -\sin(2x)$

The second derivative $y'' = \frac{d}{dx}(-\sin(2x))$

$$= \frac{-d}{dx}\sin(2x)$$

$$= -\cos(2x)\frac{d}{dx}(2x)$$

$$= -\cos(2x)2$$

$$= -2\cos(2x)$$

$$\therefore y' = -\sin(2x)$$
$$y'' = -2\cos(2x)$$

Chapter 2 Derivatives Exercise 2.5 49E

Consider the function:

$$H(t) = \tan 3t$$

Apply the chain rule to find the first derivative of the function.

$$H'(t) = \frac{d}{dt}(\tan 3t)$$

$$= \sec^2(3t)\frac{d}{dt}(3t)$$

$$= 3\sec^2(3t)$$

Now, again apply the chain rule to find the second derivative of the function.

$$H''(t) = \frac{d}{dt}\left(3\sec^2(3t)\right)$$

$$= 3\frac{d}{dt}\left(\sec^2(3t)\right)$$

$$= 3\times 2\sec(3t)\frac{d}{dt}\left(\sec(3t)\right)$$

$$= 6\sec(3t)\sec(3t)\tan(3t)\frac{d}{dt}(3t)$$

$$= 6\sec^2(3t)\tan(3t)\times 3$$

$$= 18\sec^2(3t)\tan(3t)$$

Hence,

$$H'(t) = 3\sec^2(3t)$$
$$H''(t) = 18\sec^2(3t)\tan(3t)$$

Let's find the first and second derivative for the function

$$y = \frac{4x}{\sqrt{x+1}}$$

First Derivative

$$\frac{d}{dx}\left(\frac{4x}{\sqrt{x+1}}\right)$$

The derivative of a constant times a function is the constant times the derivative of the function so we are able to take out the 4 from the derivative.

$$= 4\frac{d}{dx}\left(\frac{x}{\sqrt{x+1}}\right)$$

Use product rule

$$\frac{d(uv)}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$

u is x.

v is $\frac{1}{\sqrt{x+1}}$

$$= 4\left(\frac{1}{\sqrt{x+1}}\frac{d}{dx}(x) + x\frac{d}{dx}\left(\frac{1}{\sqrt{x+1}}\right)\right)$$

The derivative of xn is n*x(n-1).

$$= 4\left(x\frac{d}{dx}\left(\frac{1}{\sqrt{x+1}}\right) + \frac{1}{\sqrt{x+1}}\right)$$

Use the chain rule

$$\frac{d\,u^n}{d\,x} = n\,u^{n-1}\,\frac{d\,u}{d\,x}$$

u is x+1.

v is (-1/2).

$$= 4\left(\frac{1}{\sqrt{x+1}} - \frac{x}{2\,(x+1)^{3/2}}\;\frac{d}{d\,x}(\mathbf{x+1})\right)$$

The derivative of a sum is the sum of the derivatives.

$$= 4\left(\frac{1}{\sqrt{x+1}} - \frac{x\left(\frac{d}{d\,x}(\mathbf{1})+\frac{d}{d\,x}(\mathbf{x})\right)}{2\,(x+1)^{3/2}}\right)$$

The derivative of 1 is 0.

$$= 4\left(\frac{1}{\sqrt{x+1}} - \frac{x}{2\,(x+1)^{3/2}}\;\frac{d}{d\,x}(\mathbf{x})\right)$$

The derivative of xn is n*x(n-1).

$$= 4\left(\frac{1}{\sqrt{x+1}} - \frac{x}{2\,(x+1)^{3/2}}\right)$$

Simplify, assuming all variables are positive.

$$= \frac{2(x+2)}{(x+1)^{3/2}}$$

The answer for first derivative for function is

$$= \frac{2(x+2)}{(x+1)^{3/2}}$$

Now lets find the second derivative of the function

$$\frac{d^2}{dx^2}\left(\frac{4x}{\sqrt{x+1}}\right)$$

The second derivative is the derivative of the derivative.

$$= \frac{d}{dx}\left(\frac{d}{dx}\left(\frac{4x}{\sqrt{x+1}}\right)\right)$$

Use the first derivative from above

$$= \frac{d}{dx}\left(\frac{2(x+2)}{(x+1)^{3/2}}\right)$$

The derivative of a constant times a function is the constant times the derivative of the function, so the 2 is taken out of the derivative

$$= \mathbf{2}\,\frac{d}{dx}\left(\frac{x+2}{(x+1)^{3/2}}\right)$$

Use Product Rule

$$\frac{d(u\,v)}{dx} = \frac{du}{dx}\mathbf{v} + \boldsymbol{u}\frac{dv}{dx}$$

u is $\frac{1}{(x+1)^{3/2}}$.

v is x+2.

$$= \mathbf{2}\left((\mathbf{x}+\mathbf{2})\,\frac{d}{dx}\left(\frac{1}{(x+1)^{3/2}}\right) + \frac{1}{(x+1)^{3/2}}\,\frac{d}{dx}(\mathbf{x}+\mathbf{2})\right)$$

The derivative of a sum is the sum of the derivatives.

$$= \mathbf{2}\left(\frac{\frac{d}{dx}(\mathbf{2})+\frac{d}{dx}(\mathbf{x})}{(x+1)^{3/2}} + (\mathbf{x}+\mathbf{2})\,\frac{d}{dx}\left(\frac{1}{(x+1)^{3/2}}\right)\right)$$

The derivative of 2 is 0.

$$= \mathbf{2}\left(\frac{1}{(x+1)^{3/2}}\,\frac{d}{dx}(\mathbf{x}) + (\mathbf{x}+\mathbf{2})\,\frac{d}{dx}\left(\frac{1}{(x+1)^{3/2}}\right)\right)$$

The derivative of xn is n*x(n-1).

$$= \mathbf{2}\left((\mathbf{x}+\mathbf{2})\,\frac{d}{dx}\left(\frac{1}{(x+1)^{3/2}}\right) + \frac{1}{(x+1)^{3/2}}\right)$$

Use chain rule

$$\frac{d\,u^n}{d\,x} = n\,u^{n-1}\,\frac{d\,u}{d\,x}$$

u is x+1.

n is (-3/2).

$$= 2\left(\frac{1}{(x+1)^{3/2}} - \frac{3(x+2)}{2(x+1)^{5/2}}\,\frac{d}{d\,x}(x+1)\right)$$

The derivative of a sum is the sum of the derivatives.

$$= 2\left(\frac{1}{(x+1)^{3/2}} - \frac{3(x+2)\left(\frac{d}{d\,x}(1)+\frac{d}{d\,x}(x)\right)}{2(x+1)^{5/2}}\right)$$

The derivative of 1 is 0.

$$= 2\left(\frac{1}{(x+1)^{3/2}} - \frac{3(x+2)}{2(x+1)^{5/2}}\,\frac{d}{d\,x}(x)\right)$$

The derivative of xn is n*x(n-1).

$$= 2\left(\frac{1}{(x+1)^{3/2}} - \frac{3(x+2)}{2(x+1)^{5/2}}\right)$$

Simplify, assuming all variables are positive.

$$= \frac{-x-4}{(x+1)^{5/2}}$$

The answer for the second derivative of the function is

$$= \frac{-x-4}{(x+1)^{5/2}}$$

Chapter 2 Derivatives Exercise 2.5 51E

The equation of the curve is $y=(1+2x)^{10}$

Differentiating with respect to x, by using chain rule, we get

$$\frac{dy}{dx}=10(1+2x)^9\frac{d}{dx}(1+2x)$$
$$=10(1+2x)^9(0+2)$$
$$=20(1+2x)^9$$

The slope of the tangent line to the curve at (0, 1) is

$$\left(\frac{dy}{dx}\right)_{x=0}=20(1+0)^9=20$$

Then the equation of the tangent line at (0, 1) is

$$(y-1)=20(x-0)$$

Or, $\boxed{y=20x+1}$

Chapter 2 Derivatives Exercise 2.5 52E

Consider the curve:

$$y=\sqrt{1+x^3}$$

In order to find the tangent line equation at the point $(2,3)$ for the curve, first find the derivative of the function $y=\sqrt{1+x^3}$.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{1+x^3}\right)$$

Let $u=1+x^3$, then $\sqrt{u}=\sqrt{1+x^3}$.

Use the formula,

$$\frac{d\left(\sqrt{u}\right)}{dx}=\frac{1}{2\sqrt{u}}\frac{d(u)}{dx}.$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{1+x^3}}\cdot\frac{d}{dx}\left(1+x^3\right)$$

To find the derivative of $\left(1+x^3\right)$, use the Power Rule.

$$\frac{d\left(x^n\right)}{dx}=nx^{n-1},\text{ where } n \text{ is any real number.}$$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{1+x^3}}3x^2$$
$$=\frac{3}{2}\frac{x^2}{\sqrt{1+x^3}}$$

The slope m of the tangent line at $(2,3)$ is,

$$\left.\frac{dy}{dx}\right|_{x=2}=\frac{3}{2}\cdot\frac{2^2}{\sqrt{1+2^3}}$$
$$=\frac{3}{2}\cdot\frac{4}{\sqrt{1+8}}$$
$$=\frac{3}{2}\cdot\frac{4}{3}$$
$$=2$$

Therefore slope $m=2$.

Use the point-slope form to write an equation of the tangent line at (x_1, y_1) with the slope m is,

$$y - y_1 = m(x - x_1).$$

To find the tangent line equation for the curve $y = \sqrt{1+x^3}$ at the point $(2,3)$, substitute 2 for x_1, 3 for y_1 and 2 for m in point-slope form.

$$y - 3 = 2(x-2)$$
$$y - 3 = 2x - 4$$
$$y = 2x - 4 + 3$$
$$= 2x - 1$$

Therefore the equation to the tangent line for the curve $y = \sqrt{1+x^3}$ at $(2,3)$ is,

$$\boxed{y = 2x - 1}.$$

Chapter 2 Derivatives Exercise 2.5 53E

Here $y = \sin(\sin x)$

$$\frac{d}{dx}(y) = \frac{d}{dx}(\sin(\sin x))$$

$$\frac{dy}{dx} = \cos(\sin x)\frac{d}{dx}(\sin x) \quad \left[\text{by chain rule } \& \ \frac{d}{dx}(\sin x) = \cos x\right]$$

$$\Rightarrow \frac{dy}{dx} = \cos(\sin x).\cos x \quad \left[\text{by chain rule } \& \ \frac{d}{dx}(\sin x) = \cos x\right]$$

The slope of the tangent at $(\pi, 0)$ is

$$m = \left.\frac{dy}{dx}\right|_{x=\pi}$$
$$= \cos(\sin \pi).\cos \pi$$
$$= \cos 0.(-1)$$
$$= -1$$

The equation of the tangent at $(\pi, 0)$ is

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 0 = -1(x - \pi)$$
$$\Rightarrow y + x = \pi$$

$$\boxed{y = \pi - x}$$

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Here $y = \sin x + \sin^2 x$

Then $\frac{dy}{dx} = Cosx + 2SinxCosx = Cosx + Sin2x$

The slope of the tangent at (0, 0) is

$$m = \left.\frac{dy}{dx}\right|_{x=0}$$
$$= \cos 0 + \sin 2(0)$$
$$= \cos 0$$
$$= 1$$

The equation of the tangent at (0, 0) is

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 0 = 1(x - 0)$$
$$\Rightarrow y = x$$

(A)

The equation of the curve is

$$y = \tan\left(\frac{\pi x^2}{4}\right)$$

Now the slope of the tangent at any point x is

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(\frac{\pi x^2}{4}\right)$$

Let $g(x) = \frac{\pi x^2}{4}$

$$= \frac{d}{dx}\tan(g(x))$$

$$= \sec^2(g(x)).\frac{d}{dx}.g(x)$$

$$= \sec^2\left(\frac{\pi x^2}{4}\right).\frac{d}{dx}\left[\frac{\pi}{4}x^2\right]$$

$$= \sec^2\left(\frac{\pi x^2}{4}\right).\frac{\pi}{4}.2x$$

$$y' = \frac{\pi}{2}x.\sec^2\left(\frac{\pi x^2}{4}\right)$$

Then the slope of the tangent at (1, 1) is

$$y' = \frac{\pi}{2}x.\sec^2\left(\frac{\pi}{4}\right)$$

$$y' = \frac{\pi}{2}.\left(\sqrt{2}\right)^2 \qquad \left[\sec\frac{\pi}{4} = \sqrt{2}\right]$$

$$\boxed{y' = \pi}$$

Then the equation of the tangent line at the points is

$$(y-1) = \pi(x-1)$$

$$\Rightarrow y - 1 = \pi x - \pi$$

$$\Rightarrow \boxed{y = \pi x - \pi + 1}$$

(B) The graph of $y = \tan\left(\frac{\pi x^2}{4}\right)$ and tangent line is drawn in figure 1.

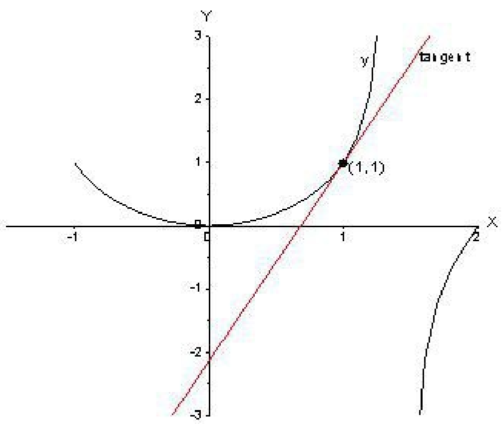

Fig1

$y = \frac{|x|}{\sqrt{2-x^2}}$ is defined as

$$y = \begin{cases} \frac{x}{\sqrt{2-x^2}} & x \ge 0 \\ \frac{-x}{\sqrt{2-x^2}} & x < 0 \end{cases}$$

(A) We want the equation of the tangent at (1, 1) so here $x \ge 0$ then we will consider y for $x \ge 0$

That is $y = \frac{x}{\sqrt{2-x^2}}$

Then the slope of the tangent at (1, 2)

$$\frac{dy}{dx} = \frac{d}{dx}\frac{x}{\sqrt{2-x^2}}$$

By Quotient rule we have

$$\frac{dy}{dx} = \frac{\left(\sqrt{2-x^2}\right)\frac{d}{dx}(x) - x\frac{d}{dx}\left(\sqrt{2-x^2}\right)}{\left(\sqrt{2-x^2}\right)^2} \qquad \text{Let } \sqrt{2-x^2} = g(x)$$

$$= \frac{\sqrt{2-x^2} - x\frac{d}{dx}g(x)}{(2-x^2)}$$

Now $\frac{d}{dx}g(x) = \frac{d}{dx}\sqrt{(2-x^2)}$ Let $(2-x^2) = h(x) \Rightarrow h'(x) = -2x$

$$= \frac{d}{dx}\sqrt{h(x)}$$

$$= \frac{1}{2}[h(x)]^{-1/2}.\frac{d}{dx}f(x) = \frac{1}{2}\frac{1}{\sqrt{2-x^2}}.-2x = \frac{-x}{\sqrt{2-x^2}}$$

Now we have $\frac{d}{dx}g(x) = \frac{-x}{\sqrt{2-x^2}}$

Then $$\frac{dy}{dx} = \frac{\left(\sqrt{2-x^2}\right) - x.\left[\frac{-x}{\sqrt{2-x^2}}\right]}{(2-x^2)}$$

$$\frac{dy}{dx} = \frac{\sqrt{2-x^2} + \frac{x^2}{\sqrt{2-x^2}}}{(2-x^2)}$$

$$= \frac{\frac{2-x^2+x^2}{\sqrt{2-x^2}}}{(2-x^2)}$$

$$\boxed{\frac{dy}{dx} = \frac{2}{(2-x^2)\sqrt{2-x^2}}}$$

Slope at (1, 1) is

$$\frac{dy}{dx}_{x=1} = \frac{2}{(2-1)\sqrt{2-1}} = 2$$

Then the equation of tangent at (1, 1)

$$(y-1) = 2.(x-1)$$
$$\Rightarrow y-1 = 2x-2$$
$$\Rightarrow y = 2x-2+1$$
$$\Rightarrow \boxed{y = 2x-1}$$

(B)

The graph of $y = \frac{x}{\sqrt{2-x^2}}$ and the tangent line is shown in figure 1 $\left[where\ x \geq 0 \right]$

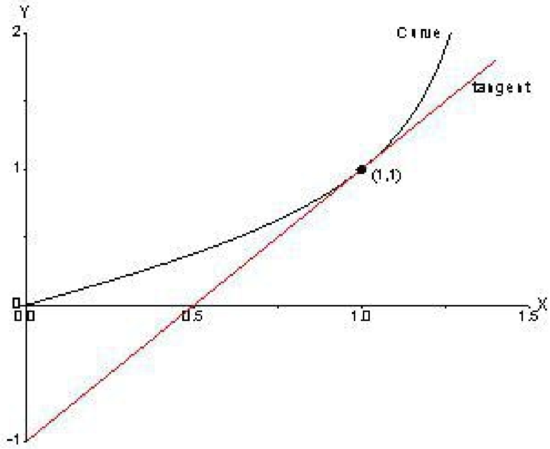

Fig.1

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We are given that a function

$$f(x) = x\sqrt{2-x^2}$$

We have to find $f'(x)$

In this problem we must use the Product Rule before using the Chian Rule

$$f'(x) = \frac{d}{dx}\left[x\sqrt{2-x^2}\right]$$

$$= \sqrt{2-x^2} \cdot \frac{d}{dx}[x] + x \cdot \frac{d}{dx}\left[\sqrt{2-x^2}\right]$$

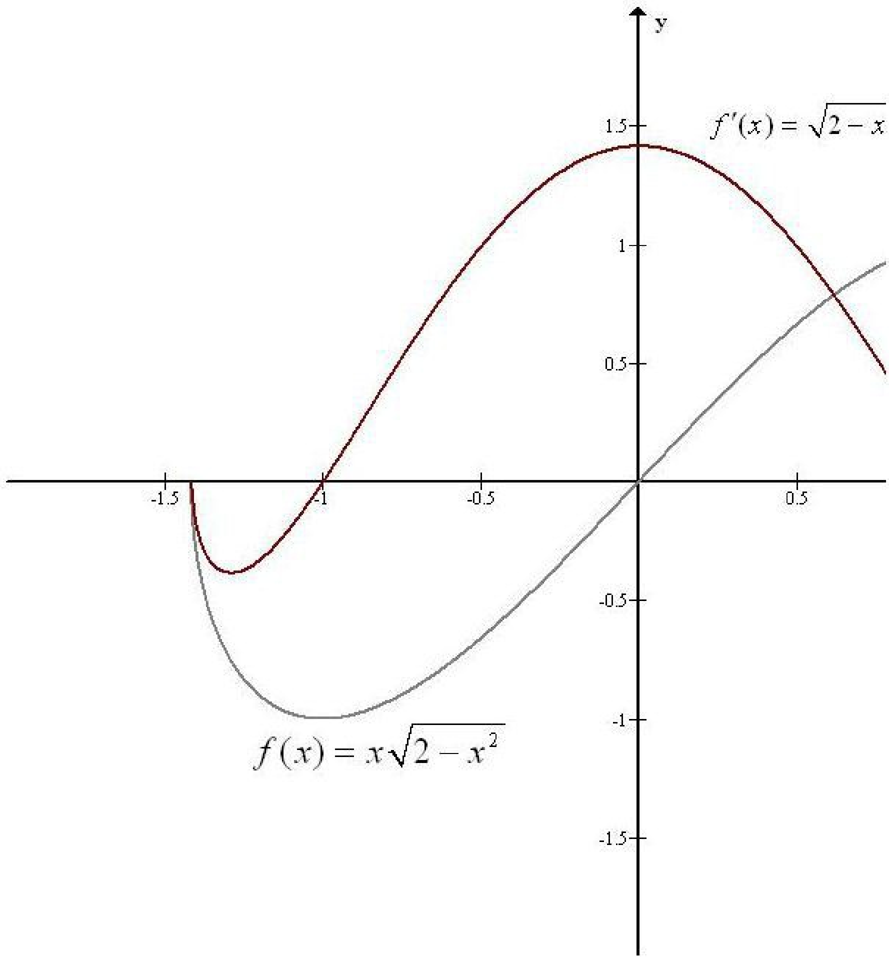
$$= \sqrt{2-x^2} + \frac{x}{2\sqrt{2-x^2}} \cdot \frac{d}{dx}\left(2-x^2\right)$$

$$= \sqrt{2-x^2} + \frac{x}{2\sqrt{2-x^2}}\left(-2x\right)$$

$$= \sqrt{2-x^2} + \frac{x(-x)}{2\sqrt{2-x^2}}$$

$$= \sqrt{2-x^2} - \frac{x^2}{\sqrt{2-x^2}}$$

$$\Rightarrow f'(x) = \sqrt{2-x^2} - \frac{x^2}{\sqrt{2-x^2}}$$



The graphs of the functions $f(x)$ and $f'(x)$ are shown in figure above. Notice that when y increases rapidly and $y' = 0$ when y has a horizontal tangent.

So, our answer appears tobe reasonable y

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(A)

The graph of $f(x) = \sin(x + \sin 2x)$ is drawn in figure 1

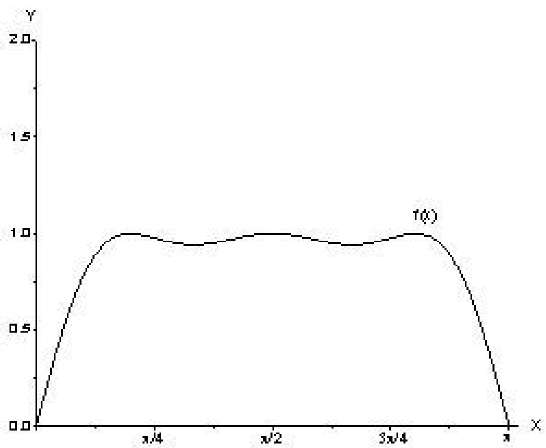


fig 1

Now we see that this graph has five horizontal tangents at different points so here the derivative of f(x) will be zero. Where f(x) is increasing, f'(x) will be positive and where f(x) is decreasing, f'(x) will be negative. Now we sketch the graph of f'(x) in figure 2.

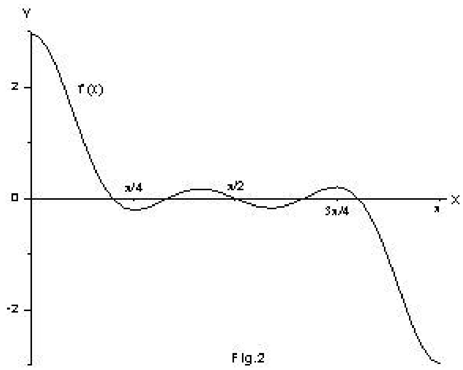


Fig.2

(B)

$$f(x) = \sin(x + \sin 2x)$$

Let $y = x + \sin 2x$

Then $f(x) = \sin y$

Differentiate $f(x)$ with respect to x

$$f'(x) = \cos y . \frac{dy}{dx} \qquad \left[\frac{d}{dx}\sin x = \cos x\right]$$

Now $y = x + \sin 2x$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}\sin 2x$$

$$= 1 + \frac{d}{dx}\sin 2x$$

Let $V = 2x$ *then* $\frac{dV}{dx} = 2$

And $\frac{dy}{dx} = 1 + \frac{d}{dx}(\sin V)$

$$= 1 + \frac{d}{dx}(\sin V) . \frac{dV}{dx}$$

$$= 1 + \cos V . (2)$$

$$= 1 + 2\cos V$$

$$\Rightarrow \frac{dy}{dx} = 1 + 2\cos 2x$$

We have

$$f'(x) = \cos y . \frac{dy}{dx}$$

$$\Rightarrow \boxed{f'(x) = \cos(x + \sin 2x).(1 + 2\cos 2x)}$$

Now we graph the function $f(x)$ and $f'(x)$ on the same screen and compare with our rough sketch which is almost same.

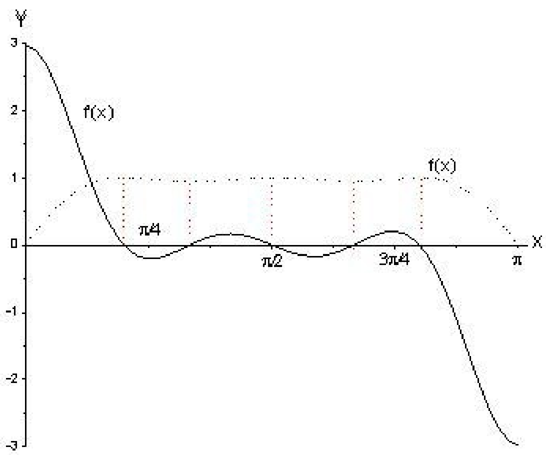


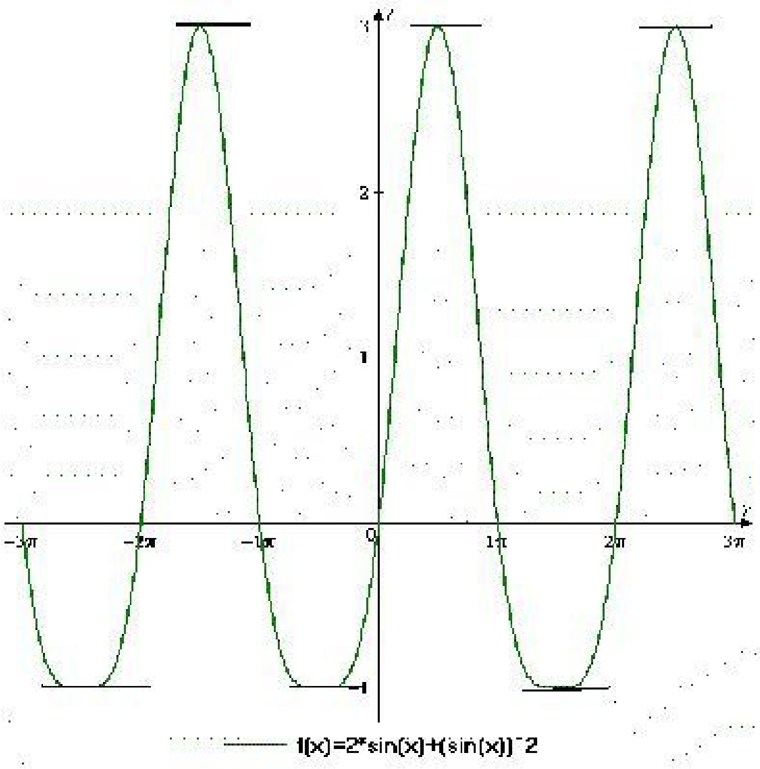
Fig.3

Chapter 2 Derivatives Exercise 2.5 59E

The function is given as $f(x) = 2\sin x + \sin^2 x$

Differentiating with respect to x

$$f'(x) = \frac{d}{dx}\left(2\sin x + \sin^2 x\right)$$
$$= 2\frac{d}{dx}(\sin x) + \frac{d}{dx}\left(\sin^2 x\right)$$
$$= 2(\cos x) + (2\sin x)\left[\frac{d}{dx}(\sin x)\right] \qquad \text{[Chain rule]}$$
$$= 2\cos x + 2\sin x\cos x$$
$$= 2\cos x(1 + \sin x)$$



Tangent line will be horizontal when

$$f'(x) = 0$$

$\Rightarrow \quad 2\cos x(1 + \sin x) = 0$

$\Rightarrow \quad \cos x = 0 \quad \text{or} \quad \sin x = -1$

$\Rightarrow \quad x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots \quad \text{or} \quad x = -\frac{\pi}{2}, -\frac{5\pi}{2}, \ldots, \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$

$\Rightarrow \quad x = \frac{\pi}{2} + 2n\pi \quad \text{or} \quad x = \frac{3\pi}{2} + 2n\pi \qquad$ (Here n is an integer)

Now we have $f(\pi/2) = 2\sin(\pi/2) + \sin^2(\pi/2) = 3$

And $\qquad f(3\pi/2) = 2\sin(3\pi/2) + \sin^2(3\pi/2) = -1$

Hence the points are $\boxed{\left(\frac{\pi}{2} + 2n\pi,\ 3\right) \text{ and } \left(\frac{3\pi}{2} + 2n\pi,\ -1\right)}$ at which the function has horizontal tangent lines

Chapter 2 Derivatives Exercise 2.5 60E

Consider the function:

$y = \sin 2x - 2\sin x$

The objective is to determine the points at which the tangent line to $f(x)$ is horizontal.

Differentiate the above function with respect to *x*.

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(\sin 2x - 2\sin x)$$

$$= \frac{d}{dx}(\sin 2x) - \frac{d}{dx}(2\sin x) \qquad \text{Use } \frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$= \frac{d}{dx}(\sin 2x) - 2\frac{d}{dx}(\sin x) \qquad \text{Use } \frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

$$= 2\cos 2x - 2(\cos x) \qquad \text{Use } \frac{d}{dx}(\sin x) = \cos x$$

$$= 2(\cos 2x - \cos x) \qquad \text{Simplify}$$

$$= 2(2\cos^2 x - 1) - 2\cos x$$

$$= 4\cos^2 x - 2 - 2\cos x$$

The tangent line is horizontal when the derivative is zero.

$$\frac{dy}{dx} = 0$$

$$4\cos^2 x - 2\cos x - 2 = 0$$

Add and subtract $-2\cos x$ for factorizing, we get

$$4\cos^2 x - 2\cos x - 2\cos x - 2 = -2\cos x$$

$$4\cos^2 x - 4\cos x - 2 + 2\cos x = 0$$

$$4\cos x(\cos x - 1) + 2(\cos x - 1) = 0$$

$$(\cos x - 1)(4\cos x + 2) = 0$$

$$(\cos x - 1)2(2\cos x + 1) = 0$$

$$(\cos x - 1)(2\cos x + 1) = 0$$

$$\cos x - 1 = 0 \quad \text{or} \quad 2\cos x + 1 = 0$$

$$\cos x = 1 \quad \text{or} \quad \cos x = -1/2.$$

For the equation $\cos x = 1$, we get

$$x = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots$$

The general solution is $x = 2n\pi$, where *n* is an integer

For the equation $\cos x = -1/2$, we get

$$x = \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}, \pm\frac{8\pi}{3}, \pm\frac{10\pi}{3}, \ldots$$

The general solution is $x = (2n+1)\pi \pm \frac{\pi}{3}$, where n is an integer

Hence the $x-$coordinates are $\boxed{x = 2n\pi \quad \text{or} \quad x = (2n+1)\pi \pm \frac{\pi}{3}}$ at which the tangent lines are horizontal.

Chapter 2 Derivatives Exercise 2.5 61E

Consider the function

$$F(x) = f(g(x))$$

where $f(-2) = 8, f'(-2) = 4, f'(5) = 3, g(5) = -2, \text{and } g'(5) = 6$

To find $F'(5)$:

The **chain rule** states that if *f* and *g* are both differentiable functions and $F(x) = f \circ g$ is the composite function defined by $F(x) = f(g(x))$ then *F* is differentiable and

$$F'(x) = f'(g(x)) \cdot g'(x)$$

So

$$\begin{aligned} F'(5) &= f'(g(5)) \cdot g'(5) \\ &= f'(-2) \cdot 6 && [g(5) = -2, g'(5) = 6] \\ &= 4 \cdot 6 && [f'(-2) = 4] \\ &= 24 \end{aligned}$$

Therefore

$$F'(5) = \boxed{24}$$

Chapter 2 Derivatives Exercise 2.5 62E

Consider the function

$$h(x) = \sqrt{4 + 3f(x)}$$

where $f(1) = 7 \text{ and } f'(1) = 4$

The power rule combined with the Chain rule:

If $y = [g(x)]^n$, then $y' = n[g(x)]^{n-1} \cdot g'(x)$

To find $h'(1)$:

Use chain rule, so that

$$\begin{aligned} h'(x) &= \left(\sqrt{4 + 3f(x)}\right)' \\ &= \left[(4 + 3f(x))^{1/2}\right]' \\ &= \frac{1}{2}(4 + 3f(x))^{-1/2} \cdot (4 + 3f(x))' \\ &= \frac{1}{2}(4 + 3f(x))^{-1/2}(0 + 3f'(x)) \end{aligned}$$

$$= \frac{1}{2}(4 + 3f(x))^{-1/2}(3f'(x))$$

$$= \frac{3f'(x)}{2\sqrt{4 + 3f(x)}}$$

Substitute 1 for x into $h'(x)$, to get

$$h'(1)=\frac{3f'(1)}{2\sqrt{4+3f(1)}}$$

$$=\frac{3\cdot4}{2\sqrt{4+3\cdot7}} \qquad (f'(1)=4,\ f(1)=7)$$

$$=\frac{12}{2\sqrt{4+21}}$$

$$=\frac{12}{2\sqrt{25}}$$

$$=\frac{6}{5}$$

Therefore $h'(1)=\boxed{\frac{6}{5}}$

Chapter 2 Derivatives Exercise 2.5 63E

(A)

$$h(x)=f(g(x))$$
$$\Rightarrow h'(x)=f'(g(x)).g'(x)$$
$$\Rightarrow h'(1)=f'(g(1)).g'(1)$$
$$=f'(2).6$$
$$=5(6)$$
$$=30$$

(B)

$$H(x)=g(f(x))$$
$$\Rightarrow H'(x)=g'(f(x)).f'(x)$$
$$\Rightarrow H'(1)=g'(f(1)).f'(1)$$
$$=g'(3).4$$
$$=9(4)$$
$$=36$$

Chapter 2 Derivatives Exercise 2.5 64E

(A)

$$F(x)=f(f(x))$$
$$\Rightarrow F'(x)=f'(f(x)).f'(x) \quad \text{chain rule}$$
$$\Rightarrow F'(2)=f'(f(2)).f'(2)$$
$$=f'(1).5$$
$$=4(5)$$
$$=20$$

(B)

$$G(x)=g(g(x))$$
$$\Rightarrow G'(x)=g'(g(x)).g'(x) \quad \text{chain rule}$$
$$\Rightarrow G'(3)=g'(g(3)).g'(3)$$
$$=g'(2).9$$
$$=7\times9$$
$$=\boxed{63}$$

Chapter 2 Derivatives Exercise 2.5 65E

We have

$$u(x)=f(g(x)),\ v(x)=g(f(x)),\text{ and } w(x)=g(g(x))$$

$f(1)=2$ and $g(1)=3$

$f'(1)=2=$ Slope of f at $x=1$

$g'(1)=-3=$ Slope of f at $x=1$

$f'(3)=-\frac{1}{4}=$ Slope of f at $x=3$

$f'(2)$ does not exist because f has a corner at $x=2$.

$g'(2)$ does not exist because g has a corner at $x=2$.

$g'(3)=\frac{2}{3}$ is the slope of g at $x=3$.

Now we have all the value of $f(x), g(x), f'(x)$, and $g'(x)$ for $x=1, 2, 3$.

(A) Given $u(x)=f(g(x))$

Differentiate by the chain rule with respect to x, we get

$$u'(x)=f'(g(x)).g'(x)$$

Put $x=1$

$$u'(1)=f'(g(1)).g'(1)$$

$$\Rightarrow u'(1)=f'(3).g'(1) \qquad [g(1)=3]$$

$$\Rightarrow u'(1)=-\frac{1}{4}.(-3)$$

$$\boxed{u'(1)=\frac{3}{4}}$$

(B) We have $V(x)=g(f(x))$

By the chain rule we differentiate $V(x)$ with respect to x.

$$V'(x)=g'(f(x)).f'(x)$$

Put $x=1$

$$\Rightarrow V'(1)=g'(f(1)).f'(1)$$

$$\Rightarrow V'(1)=g'(2).f'(1)$$

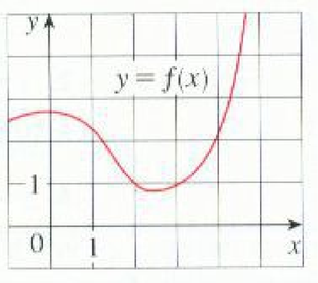
$\boxed{V'(1)\text{ does not exist}}$ because $g'(2)$ does not exist.

(C) We have $w(x)=g(g(x))$

By chain rule we differentiate $w(x)$ with respect to x

$$\Rightarrow w'(x)=g'(g(x)).g'(x)$$

Chapter 2 Derivatives Exercise 2.5 66E



(A) We have $h(x)=f(f(x))$

Differentiating with respect to x

$h'(x)=f'(f(x))f'(x) \qquad [\text{chain rule}]$

Then $h'(2)=f'(f(2)).f'(2)$

From the graph we have

Now we have calculate

$$F'(x)=\lim_{h\to 0}\frac{F(x+h)-F(x)}{h}$$
$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$
let $h=1$
$$f'(2)=\frac{f(3)-f(2)}{1}=\frac{0-1}{1}\approx -1,$$
$$f(2)\approx 1$$
$$f'(2)\approx -1, f(2)\approx 1$$

And

$$F'(x)=\lim_{h\to 0}\frac{F(x+h)-F(x)}{h}$$
$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$
let $h=1$
$$f'(1)=\frac{f(2)-f(1)}{1}=\frac{1-2}{1}\approx -1$$
$$f'(1)\approx -1$$

And then $h'(2)\approx f'(1).f'(2)$

Or $h'(2)\approx(-1).(-1)$

Or $\boxed{h'(2)\approx 1}$

(B) We have $g(x)=f(x^2)$

Then $g'(x)=f'(x^2).\frac{d}{dx}(x^2) \qquad \text{Chain rule}$

$=2xf'(x^2)$

Now put x = 2

$\Rightarrow \quad g'(2)=2\cdot 2f'(4)$

$=4f'(4)$

From the graph $f'(4)\approx 1.5$

Therefore,

$g'(2)\approx 4(1.5)$

$\Rightarrow \quad \boxed{g'(2)\approx 6}$

Chapter 2 Derivatives Exercise 2.5 68E

(A)

Since $F(x)=f(x^{\alpha})$, where $\alpha\in\mathbb{R}$

Then $F'(x)=f'(x^{\alpha}).(x^{\alpha})'$

$=\alpha x^{\alpha-1}.f'(x^{\alpha})$

(B)

Since $G(x)=[f(x)]^{\alpha}$

$$G'(x)=\alpha[f(x)]^{\alpha-1}\times f'(x)$$

Chapter 2 Derivatives Exercise 2.5 69E

THE CHAIN RULE: If g is differentiable at x and f is differentiable at g(x),
Then the composite function $F=f\circ g$ defined by $F(x)=f(g(x))$ is
Differentiable at x and F' is given by the product $F'(x)=f'(g(x))\cdot g'(x)$

Since $r(x)=f(g(h(x)))$,

We have by chain rule,

$$r'(x)=f'(g(h(x)))\times g'(h(x))\times h'(x)$$

$$\Rightarrow r'(1)=f'(g(h(1)))\times g'(h(1))\times h'(1)$$

$$=f'(g(2))\times g'(2)\times 4$$

$$=f'(3)\times 5\times 4$$

$$=6\cdot 5\cdot 4$$

$$=120$$

Chapter 2 Derivatives Exercise 2.5 70E

Suppose that *g* is a twice differentiable function and $f(x)=xg(x^2)$.

Chain rule:

If f and g are both differentiable and $F=f\circ g$ is the composite function define by $F(x)=f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x)=f'(g(x))\cdot g'(x).$$

The product rule:

If *f* and *g* are both differentiable, then

$$(fg)'=fg'+gf'$$

The sum rule:

If *f* and *g* are both differentiable, then

$$(f+g)'=f'+g'$$

To find f'' in terms of $g, g', \text{and } g''$:

To find the first derivative of $f(x)$ we use the product rule.

$$f'(x)=[xg(x^2)]'$$

$$=x[g(x^2)]'+g(x^2)(x)'$$

$$=x[g(x^2)]'+g(x^2) \qquad ((x)'=1)$$

To find the derivative of $g(x^2)$ we use the chain rule.

$$x[g(x^2)]'+g(x^2)=x\cdot g'(x^2)\cdot(x^2)'+g(x^2)$$

$$=xg'(x^2)\cdot 2x+g(x^2) \qquad ((x^n)'=nx^{n-1})$$

$$=2x^2g'(x^2)+g(x^2)$$

Therefore, $f'(x)=2x^2g'(x^2)+g(x^2)$

Now we find the second derivative by finding the derivative of $f'(x)$.

$$f''(x)=[f'(x)]'$$
$$=[2x^2g'(x^2)+g(x^2)]' \quad \text{By using the sum rule}$$
$$=[2x^2g'(x^2)]'+[g(x^2)]'$$
$$=[2x^2g'(x^2)]'+g'(x^2)\cdot(2x) \quad \text{By using the chain rule}$$

The derivative of the first part is found with the product rule:

$$[2x^2g'(x^2)]'+2xg'(x^2)=2x^2[g'(x^2)]'+g'(x^2)(2x^2)'+2xg'(x^2)$$
$$=2x^2[g'(x^2)]'+g'(x^2)\cdot 4x+2xg'(x^2) \quad \left((x^n)'=nx^{n-1}\right)$$
$$=2x^2[g'(x^2)]'+6xg'(x^2)$$

To continue with the derivative, we use the chain rule to differentiate the function $F(x)=g'(x^2)$. So,

$$2x^2[g'(x^2)]'+6xg'(x^2)=2x^2\cdot g''(x^2)\cdot(x^2)'+6xg'(x^2)$$
$$=2x^2g''(x^2)\cdot(2x)+6xg'(x^2) \quad \left((x^n)'=nx^{n-1}\right)$$
$$=4x^3g''(x^2)+6xg'(x^2)$$

Therefore

$$f''(x)=\boxed{4x^3g''(x^2)+6xg'(x^2)}$$

Chapter 2 Derivatives Exercise 2.5 71E

We are given that

$$F(x)=f(3f(4f(x)))$$

We first find

$$F'(x)$$

We must use the chain rule.

$$F'(x)=\frac{d}{dx}\left[f(3f(4f(x)))\right]$$
$$=f'(3f(4f(x)))\cdot\frac{d}{dx}(3f(4f(x)))$$
$$=f'(3f(4f(x)))\cdot 3f'(4f(x))\cdot\frac{d}{dx}(4f(x))$$
$$=f'(3f(4f(x)))\cdot 3f'(4f(x))\cdot 4f'(x)$$
$$=12f'(x)\cdot f'(4f(x))\cdot f'(3f(4f(x)))$$
$$\Rightarrow F'(x)=12f'(x)\cdot f'(4f(x))\cdot f'(3f(4f(x)))$$

Now $F'(x)$ at $x=0$ is $F'(0)$

$$F'(0)=12f'(0)\cdot f'(4f(0))\cdot f'(3f(4f(0)))$$

$$=12(2)\cdot f'(4(0))\cdot f'(3f(4(0)))$$

$$=24\cdot f'(0)\cdot f'(3f(0))$$

$$=24\cdot 2\cdot f'(3(0))$$

$$=48\cdot f'(0)$$

$$=48\cdot 2 \quad =96$$

$$\Rightarrow F'(0)=96$$

The solution is $F'(0)=96$

Chapter 2 Derivatives Exercise 2.5 72E

We are given that

$$F(x)=f(\mathrm{xf}(\mathrm{xf}(x)))$$

We have to find

$$F'(1)$$

We first find $F'(x)$

We must use the chain rule before using the product rule:

$$F'(x)=\frac{d}{\mathrm{dx}}\left[f(\mathrm{xf}(\mathrm{xf}(x)))\right]$$

$$=f'(\mathrm{xf}(\mathrm{xf}(x)))\cdot\frac{d}{\mathrm{dx}}(\mathrm{xf}(\mathrm{xf}(x)))$$

$$=f'(\mathrm{xf}(\mathrm{xf}(x)))\cdot\left[f(\mathrm{xf}(x))+\mathrm{xf}'(\mathrm{xf}(x))\cdot\frac{d}{\mathrm{dx}}[\mathrm{xf}(x)]\right]$$

$$=f'(\mathrm{xf}(\mathrm{xf}(x)))\cdot\left[f(\mathrm{xf}(x))+xf'\ (\mathrm{xf}(x))(f(x)+f'(x)\cdot x)\right]$$

$$\Rightarrow F'(x)=f'(\mathrm{xf}(\mathrm{xf}(x)))\cdot\left[f(\mathrm{xf}(x))+xf'\ (\mathrm{xf}(x))(f(x)+f'(x)\cdot x)\right]$$

Now $F'(x)$ at $x=1$ is $F'(1)$

$$F'(1) = f'(1f(1f(1))) \left[f(1f(1)) + 1f'(1f(1))(f(1) + f'(1)\cdot 1)\right]$$

$$= f'(f(f(1))) \left[f(f(1)) + f'(f(1))\cdot(f(1) + f'(1))\right]$$

$$= f'(f(2))\left[f(2) + f'(2)\cdot(2+4)\right]$$

$$= f'(3)\left[3 + 6(5)\right]$$

$$= 6[3 + 30]$$

$$= 6[33] \;\; = 198$$

Hence the solution is $F'(1) = 198$

Chapter 2 Derivatives Exercise 2.5 73E

Consider the following derivative

$$D^{103}\cos 2x$$

Need to find first few derivatives.

Let $D = \cos 2x$

Differentiate with respect to x

$$D' = \frac{d}{dx}(\cos 2x)$$

$D' = -2\sin 2x$ Since $\frac{d}{dx}(\cos ax) = -a\sin ax$

Again differentiate with respect to x

$$D'' = \frac{d}{dx}(-2\sin 2x)$$

$D'' = -2\frac{d}{dx}(\sin 2x)$ Since $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$

$D'' = -2(2\cos 2x)$ Since $\frac{d}{dx}(\sin ax) = a\cos ax$

$$D'' = (-2^2)\cos 2x$$

Again differentiate with respect to x

$$D^3 = \frac{d}{dx}\left[-(2^2)\cos 2x\right]$$

$D^3 = (-2^2)\frac{d}{dx}[\cos 2x]$ Since $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$

$D^3 = -(2^2)(-2\sin 2x)$ Since $\frac{d}{dx}(\cos ax) = -a\sin ax$

$$D^3 = (2^3)(\sin 2x)$$

Again differentiate with respect to x

$$D^4 = \frac{d}{dx}(2^3\sin 2x)$$

$D^4 = 2^3\frac{d}{dx}(\sin 2x)$ Since $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$

$D^4 = 2^3(2\cos 2x)$ Since $\frac{d}{dx}(\sin ax) = a\cos ax$

$$D^4 = (2^4)\cos 2x$$

In view of the above differentiations, it can be followed that 1st, 2nd, 5th, 6th, 9th, 10th, and so on derivatives have – sign while 3rd, 4th, 7th, and 8th, and so on have positive sign.

Continue this pattern up to hundred times differentiation, it can be followed that 101st, 102nd derivatives have – sign while 103rd derivative has positive sign. ...... (1)

Further, the above derivatives give that 1st, 3rd are sine functions while 2nd and 4th are the cosine functions.

In other words, odd derivatives result in sine function while even derivative is cosine.

While 103 is an odd number, the resulting derivative is the sine function. ...... (2)

Generalize the observations (1) and (2) as

$$D^n(\cos ax) = \begin{cases} -a^{4n+1}\sin(ax) & (i) \\ -a^{4n+2}\cos(ax) & (ii) \\ a^{4n+3}\sin(ax) & (iii) \\ a^{4n+4}\cos(ax) & (iv) \end{cases}$$

One application of this observation is

$D^{103}\cos 2x = 2^{103}\sin 2x$ By (*iii*) of the definition of the derivative above

Therefore,

$$D^{103}\cos 2x = \boxed{2^{103}\sin 2x}.$$

Chapter 2 Derivatives Exercise 2.5 74E

Consider the following function:

$$f(x) = x\sin\pi x$$

Find $D^{35}x\sin\pi x.$

Find the few derivatives and observing the pattern that occurs.

$$D^1(x\sin\pi x) = \sin\pi x + \pi x\cos\pi x$$

$$\begin{aligned} D^2(x\sin\pi x) &= (\sin\pi x + \pi x\cos\pi x)' \\ &= \pi\cos\pi x + \pi\cos\pi x - \pi^2 x\sin\pi x \\ &= 2\pi\cos\pi x - \pi^2 x\sin\pi x \end{aligned}$$

$$\begin{aligned} D^3(x\sin\pi x) &= (2\pi\cos\pi x - \pi^2 x\sin\pi x)' \\ &= -2\pi^2\sin\pi x - \pi^2\sin\pi x - \pi^3 x\cos\pi x \\ &= -3\pi^2\sin\pi x - \pi^3 x\cos\pi x \end{aligned}$$

Simplify further as shown below:

$$D^{2k}x\sin\pi x = (-1)^{k+1}2k\pi^{2k-1}\cos\pi x + (-1)^{k+1}x\pi^{2k}\sin\pi x$$

$$D^{2k+1}x\sin\pi x = (-1)^{k+1}(2k+1)\pi^{2k}\sin\pi x + (-1)^k x\pi^{2k+1}\cos\pi x$$

$$D^{35}x\sin\pi x = (-1)^{18}35\pi^{34}\sin\pi x + (-1)^{17}x\pi^{35}\cos\pi x$$

Therefore, $D^{35}x\sin\pi x = \boxed{(-1)^{18}35\pi^{34}\sin\pi x + (-1)^{17}x\pi^{35}\cos\pi x}.$

Chapter 2 Derivatives Exercise 2.5 75E

The velocity of the particle at t seconds is given by

$v(t) = \dfrac{dS}{dt}$, where $S = S(t)$ is the displacement after t seconds

Since $S(t) = 10 + \dfrac{1}{4}\sin(10\pi t)$

$$V(t) = S'(t) = \frac{1}{4}\cos(10\pi t)\times 10\pi$$

$$\Rightarrow V(t) = \frac{5\pi}{2}\cos(10\pi t) \qquad \text{Cm/sec}$$

Chapter 2 Derivatives Exercise 2.5 76E

(A)

$$S = A\cos(\omega t + \delta)$$

Velocity at time t is

$$V(t) = \frac{dS}{dt} = \frac{d}{dt}\left(A\cos(\omega t+\delta)\right)$$

$$\Rightarrow \boxed{V(t) = -A\omega\sin(\omega t+\delta)}$$

$$\Rightarrow t = \frac{-\delta}{\omega}, \frac{\pi-\delta}{\omega}, \frac{2\pi-\delta}{\omega}, \frac{3\pi-\delta}{\omega}$$

But time t can not be negative

So we take $t = \frac{\pi-\delta}{\omega}, \frac{2\pi-\delta}{\omega}, ---$

$$\Rightarrow \boxed{t = \frac{n\pi-\delta}{\omega}}$$ Where n = 1, 2, 3 ---

Chapter 2 Derivatives Exercise 2.5 77E

(A)

The rate of change of the brightness $B(t)$ after t days is given by

$$\frac{dB}{dt} = \left[4 + 0.35\sin\left(\frac{2\pi t}{5.4}\right)\right]'$$

$$= 0.35\cos\left(\frac{2\pi t}{5.4}\right)\times\frac{2\pi}{5.4}$$

$$= \frac{7}{54}\pi\cos\left(\frac{2\pi t}{5.4}\right)$$

(B) Now we find the rate of change of the brightness $B(t)$ after 1 day , that is

$$\left.\frac{dB}{dt}\right|_{t=1} = \frac{7}{54}\pi\cos\left(\frac{2\pi}{5.4}\right) = 0.16$$

Chapter 2 Derivatives Exercise 2.5 78E

The rate of increasing of the day light is the derivative of $L(t)$ with respect to t, i.e.,

$$L'(t) = \frac{d}{dt}\left[12 + 2.8\sin\left[\frac{2\pi}{365}(t-80)\right]\right]$$

Now let $y = \frac{2\pi}{365}(t-80)$

Then $\frac{dy}{dt} = \frac{2\pi}{365}$

Then we have

$$L'(t) = \frac{d}{dt}(12) + 2.8\frac{d}{dt}\sin y$$

$$= 0 + 2.8.\frac{d}{dy}.\sin y.\frac{dy}{dx}$$

$$= 2.8\cos y.\left(\frac{2\pi}{365}\right)$$

$$L'(t) = 2.8\times\frac{2\pi}{365}.\cos\left(\frac{2\pi}{365}(t-80)\right)$$

$$\Rightarrow \boxed{L'(t) = \frac{5.6\pi}{365}.\cos\left(\frac{2\pi}{365}(t-80)\right)\text{ hours/day}}$$

March 21 is the 80^{th} day of the year.
Then rate of increasing of day light on March 21 is the rate of increasing of day light on the 80^{th} day of the year [February = 28 days].
That is

$$L'(80) = \frac{5.6\pi}{365}\cos\left[\frac{2\pi}{365}(80-80)\right] = \frac{5.6\pi}{365}.\cos(0)$$

$$\boxed{L'(80) = \frac{5.6\pi}{365} \text{hours}/_{\text{day}}} \approx 0.0482 \quad \text{hours/day}$$

May 21 is 141^{st} day of the year. [February =28]

Then rate of increasing of day light on May 21 is the rate of increasing of day light on the 141^{st} day of the year.
That is,

$$L'(141) = \frac{5.6\pi}{365}\cos\left[\frac{2\pi}{365}(141-80)\right]$$

$$\approx \frac{5.6\pi}{365}\cos\left(\frac{\pi}{3}\right)$$

$$\approx \frac{5.6\pi}{365}\cdot\frac{1}{2}$$

$$\boxed{L'(141) \approx \frac{2.8\pi}{365} \text{hours}/_{\text{days}}} \approx 0.02398 \quad \text{hours/day}$$

which is half of $L'(80)$.

Chapter 2 Derivatives Exercise 2.5 79E

Velocity as a function of position: $v = v(s)$, and position as a function of time: $s = s(t)$. Then we can write velocity as a composition of functions: $v = v(s(t))$.

The derivative of the composition with respect to time by using the chain rule:

$$\frac{d}{dt}(v) = \frac{d}{dt}\left[v(s(t))\right]$$
$$= \frac{dv}{ds}\cdot\frac{ds}{dt}$$

Acceleration is the derivative of velocity with respect to time: $a(t) = \frac{dv}{dt}$.

Also, velocity is the derivative of position with respect to time: $v(t) = \frac{ds}{dt}$.

Hence by substitution:

$$\frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt}$$
$$a(t) = \frac{dv}{ds}\cdot v(t)$$
$$a(t) = v(t)\frac{dv}{ds}$$

The derivative $\frac{dv}{dt}$ is the rate of change of velocity with respect to time (acceleration).

However, the derivative $\frac{dv}{ds}$ is the rate of change of velocity with respect to displacement.

For example,

If we envision a roller coaster on a track, $\frac{dv}{dt}$ indicates how the velocity changes at a particular instant while the cars are moving.

However, $\frac{dv}{ds}$ indicates how the velocity changes when the cars are at a specific location on the track.

So, $\frac{dv}{dt}$ is the rate of change of velocity with respect to time $\frac{dv}{ds}$ is the rate of change of velocity with respect to displacement.

Chapter 2 Derivatives Exercise 2.5 80E

Air is being pumped into a spherical weather balloon. At any time t, the volume of the balloon is $V(t)$ and its radius is $r(t)$.

(a)

The derivative $\frac{dV}{dr}$ is the rate of change of volume with respect to the size of the radius.

However, the derivative $\frac{dV}{dt}$ is the rate of change of volume with respect to time.

(b)

The balloon is spherical; the volume is measured by the formula:

$$V(r)=\frac{4}{3}\pi r^3$$

Hence volume is a function of the radius.

We can calculate $\frac{dV}{dr}$ from the volume formula for a sphere:

$$\begin{aligned}\frac{d}{dr}(V)&=\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\\ \frac{dV}{dr}&=\frac{4}{3}\pi\frac{d}{dr}\left(r^3\right)\\ &=\frac{4}{3}\pi\left(3r^2\right)\\ &=4\pi r^2\end{aligned}$$

If the radius is changing over time, then we find the derivative of the volume, $V(r(t))$, with respect to time by using the chain rule:

$$\begin{aligned}\frac{d}{dt}(V)&=\frac{d}{dt}\left[V(r(t))\right]\\ \frac{dV}{dt}&=\frac{dV}{dr}\cdot\frac{dr}{dt}\end{aligned}$$

$$\frac{dV}{dt}=4\pi r^2\cdot\frac{dr}{dt} \quad \text{Substitute.}$$

Therefore,

$$\frac{dV}{dt}=\boxed{4\pi r^2\frac{dr}{dt}}.$$

Chapter 2 Derivatives Exercise 2.5 81E

Consider the following function

$$g(t)=\left(\frac{t-2}{2t+1}\right)^9$$

Find $g'(t)$:

$$g'(t)=9\left(\frac{t-2}{2t+1}\right)^8\frac{d}{dt}\left(\frac{t-2}{2t+1}\right) \quad \text{Use Power rule, chain rule}$$

$$=9\left(\frac{t-2}{2t+1}\right)^8\frac{(2t+1)\cdot 1-2(t-2)}{(2t+1)^2} \quad \text{Use quotient rule}$$

$$=9\left(\frac{t-2}{2t+1}\right)^8\left(\frac{2t+1-2t+4}{(2t+1)^2}\right)$$

$$=45\frac{(t-2)^8}{(2t+1)^{10}}$$

Therefore,

$$g'(t)=45\frac{(t-2)^8}{(2t+1)^{10}} \quad \text{...... (1)}$$

Find the derivative by using CAS:

By computer algebra system maple, to find the derivative:

The input command is

Diff(((t-2)/(2*t+1))^9,t)

The output command is

$$> diff\left(\left(\frac{(t-2)}{(2\cdot t+1)}\right)^9, t\right)$$

$$\frac{9(t-2)^8}{(2t+1)^9} - \frac{18(t-2)^9}{(2t+1)^{10}}$$

Now simplify this derivative

The input command is

Simplify(Diff(((t-2)/(2*t+1))^9,t))

The output command is

$$> simplify\left(diff\left(\left(\frac{(t-2)}{(2\cdot t+1)}\right)^9, t\right)\right);$$

$$\frac{45(t-2)^8}{(2t+1)^{10}}$$

Therefore,

$$g'(t) = 45\frac{(t-2)^8}{(2t+1)^{10}} \quad \ldots\ldots (2)$$

Hence, (1) and (2) are same.

b)

Consider the following function

$$y = (2x+1)^5 (x^3-x+1)^4$$

Find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = (2x+1)^5 \frac{d}{dx}(x^3-x+1)^4 + (x^3-x+1)^4 \frac{d}{dx}(2x+1)^5 \qquad \text{Product rule}$$

$$= (2x+1)^5\, 4(x^3-x+1)^3 \frac{d}{dx}(x^3-x+1) + (x^3-x+1)^4\, 5(2x+1)^4 \frac{d}{dx}(2x+1)$$

Power rule and chain rule

$$= 4(2x+1)^5(x^3-x+1)^3(3x^2-1) + 5(x^3-x+1)^4(2x+1)^4\cdot 2$$

$$= 2(2x+1)^4(x^3-x+1)^3\left[2(2x+1)(3x^2-1) + 5(x^3-x+1)\right]$$

$$= 2(2x+1)^4(x^3-x+1)^3\left[12x^3-4x+6x^2-2+5x^3-5x+5\right]$$

$$= 2(2x+1)^4(x^3-x+1)^3\left[17x^3+6x^2-9x+3\right]$$

Therefore,

$$\frac{dy}{dx} = \boxed{2(2x+1)^4(x^3-x+1)^3\left[17x^3+6x^2-9x+3\right]} \quad \ldots\ldots (3)$$

Find the derivative by using CAS:

By computer algebra system maple, to find the derivative:

The input command is

Diff((2*x+1)^5*(x^3-x+1)^4,x)

The output command is

> $diff\left((2\cdot x+1)^5\cdot(x^3-x+1)^4, x\right);$

$$10\,(2x+1)^4\,(x^3-x+1)^4+4\,(2x+1)^5\,(x^3-x+1)^3\,(3x^2-1)$$

Observe that each term has common factor

Now use factor command in Maple

The input command is

Factor(Diff((2*x+1)^5*(x^3-x+1)^4,x))

The output command is

> $factor\left(diff\left((2\cdot x+1)^5\cdot(x^3-x+1)^4, x\right)\right);$

$$2\,(17x^3+6x^2-9x+3)\,(x^3-x+1)^3\,(2x+1)^4$$

Therefore,

$$\frac{dy}{dx}=2\left(17x^3+6x^2-9x+3\right)(2x+1)^4\left(x^3-x+1\right)^3 \;\ldots\ldots\; (4)$$

Observe that the equations (3) and (4) are same. Factor form is the best for locating horizontal tangents.

Chapter 2 Derivatives Exercise 2.5 82E

Consider the function $f(x)=\sqrt{\frac{x^4-x+1}{x^4+x+1}}$

(a) Use a CAS to differentiate the function

$$f'(x)=\frac{1}{2}\frac{4x^3-1}{\sqrt{x^4-x+1}\sqrt{x^4+x+1}}-\frac{1}{2}\frac{\sqrt{x^4-x+1}\left(4x^3+1\right)}{\left(x^4+x+1\right)^{3/2}}$$

Simplify collects terms together symbolically, so

$$\boxed{f'(x)=\frac{3x^4-1}{\sqrt{x^4-x+1}\left(x^4+x+1\right)^{3/2}}}$$

(b) Required to find horizontal tangents of the graph f.

To find the horizontal tangents $f'(x)$ must be equal to zero.

$$\frac{3x^4-1}{\sqrt{x^4-x+1}\left(x^4+x+1\right)^{3/2}}=0$$

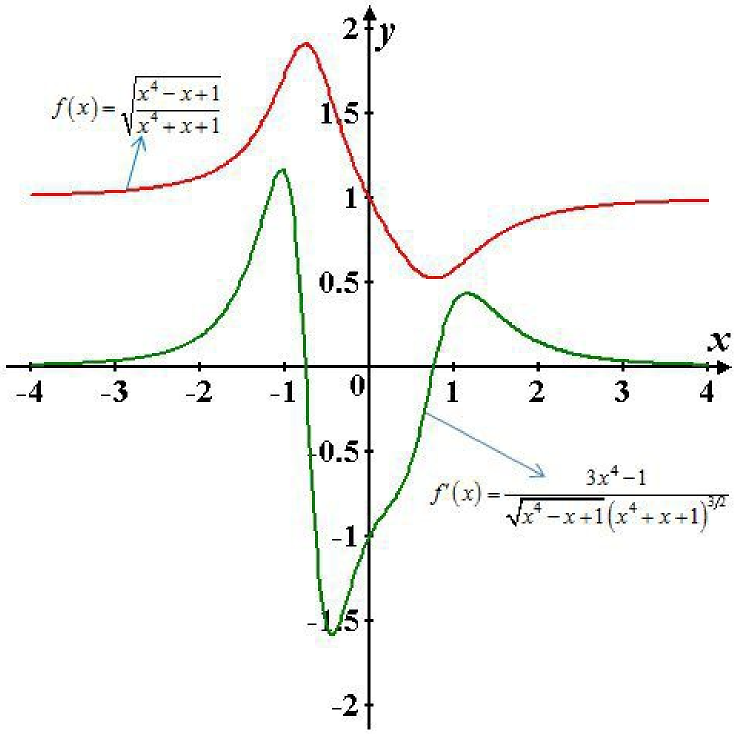
$$3x^4-1=0$$

$$x^4=\frac{1}{3}$$

$$x=\pm\frac{1}{\sqrt[4]{3}}$$

Therefore, horizontal tangents at $\boxed{x=\pm\frac{1}{\sqrt[4]{3}}}$

(c) Graph of f and f'



Graphical results are consistent with analytical results.

Chapter 2 Derivatives Exercise 2.5 83E

(A)

Let $f(x)$ be an even functions, then

$$f(-x)=f(x)$$

Differentiating both sides we have

$$f'(-x)\times(-x)'=f'(x)$$
$$\Rightarrow -f'(-x)=f'(x)$$
$$\Rightarrow f'(-x)=-f'(x)$$
$$\Rightarrow f'(x) \text{ is an odd function}$$

(B)

Let $g(x)$ be an odd function, then

$$g(-x)=-g(x)$$
$$g'(-x)\times(-x)'=-g'(x)$$
$$\Rightarrow -g'(-x)=-g'(x)$$
$$\Rightarrow g'(-x)=g'(x)$$
$$\Rightarrow g'(x) \text{ is an even function.}$$

Chapter 2 Derivatives Exercise 2.5 84E

Let $y=f(x)\left[g(x)\right]^{-1}$

Then $\frac{dy}{dx}=f'(x)\left[g(x)\right]^{-1}+f(x)\left(\left[g(x)\right]^{-1}\right)'$ (product rule)

$$=\frac{f'(x)}{g(x)}+f(x).(-1)\left(g(x)\right)^{-2}.g'(x) \quad \text{(chain rule)}$$

$$=\frac{f'(x)}{g(x)}-\frac{f(x)g'(x)}{\left[g(x)\right]^2}$$

$$\Rightarrow \left[\frac{f(x)}{g(x)}\right]'=\frac{f'(x)g(x)-f(x)g'(x)}{\left[g(x)\right]^2}$$

This is the quotient rule.

(A)

We have to get $\frac{d}{dx}\left(\sin^n x\cos nx\right)$

Let $u=\sin x$ *and* $v=\cos nx$ *and* $f(x)=\sin^n x\cos nx$

Then

$$\frac{d}{dx}f(x)=\frac{d}{dx}\left(u^n.v\right)$$

By the product rule, we have

$$\frac{d}{dx}\left(u^n.v\right)=u^n\frac{d}{dx}(v)+v.\frac{d}{dx}\left(u^n\right) \quad \text{--- (1)}$$

Now

$$\frac{d}{dx}\left(u^n\right)=\frac{d}{du}\left(u^n\right).\frac{du}{dx}$$

$$\Rightarrow\frac{du^n}{dx}=nu^{n-1}.\cos x$$

$$\Rightarrow\frac{du^n}{dx}=n\sin^{n-1}.\cos x \quad \text{--- (2)}$$

And

$$\frac{d}{dx}(v)=\frac{d}{dx}.\cos nx$$

Let $y=nx\,then\,\frac{dy}{dx}=n$

Then $\frac{dv}{dx}=\frac{d}{dy}\cos y.\frac{dy}{dx}$

$=-n\sin y$

$$\Rightarrow\frac{dv}{dx}=-n\sin nx \quad \text{--- (3)}$$

From equations (2) and (3) put the values of $\frac{du^n}{dx}\,and\,\frac{dv}{dx}$ in equation (1)

We have

$$\frac{d}{dx}\left(u^n.v\right)=\sin^n x.(-n\sin nx)+\cos nx.\left(n\sin^{n-1}x\cos x\right)$$

$$=n\sin^{n-1}x\cos x.\cos nx-n\sin^n x.\sin nx$$

$$=n\sin^{n-1}x\left[\cos x.\cos nx-\sin x.\sin nx\right]$$

Use the identity

$$\cos(A+B)=\cos A.\cos B-\sin A\sin B$$

$$\frac{d}{dx}\left(u^n.v\right)=n\sin^{n-1}x\left[\cos(x+nx)\right]$$

Thus we have

$$\boxed{\frac{d}{dx}\left(\sin^n x.\cos nx\right)=n\sin^{n-1}x\cos\left[(n+1)x\right]}\ \text{Proved}$$

(B) We have to get the derivative $y=\cos^n x.\cos nx$

Let $u=\cos x\,and\,v=\cos nx$

Thus $y=u^n.v$

Then $\frac{dy}{dx}=\frac{d}{dx}\left(u^n.v\right)$

By the product rule

$$\frac{dy}{dx}=u^n.\frac{dv}{dx}+v.\frac{du^n}{dx} \quad \text{--- (1)}$$

Now $\frac{d}{dx}\left(u^n\right)=\frac{d}{du}\left(u^n\right)\frac{du}{dx}$

$$=nu^{n-1}(-\sin x)$$

$$\Rightarrow\frac{du^n}{dx}=-n\cos^{n-1}x.\sin x \quad \text{--- (2)}$$

And

$$\frac{dv}{dx}=\frac{d}{dx}\cos nx$$

Let $y=nx$ and $\frac{dy}{dx}=n$

Then

$$\frac{dv}{dx}=\frac{d}{dy}\cos y.\frac{dy}{dx}$$
$$=-n\sin y$$

$$\frac{dv}{dx}=-n\sin nx \qquad \text{--- (3)}$$

From equation (2) and (3), put the values of $\frac{du^n}{dx}$ *and* $\frac{dv}{dx}$ in equation (1)

We have,

$$\frac{dy}{dx}=\cos^n x.(-n\sin nx)+\cos nx(-n\cos^{n-1}x.\sin x)$$
$$=-n\cos^n x.\sin nx-n\cos nx.\cos^{n-1}x.\sin x$$
$$=-n\cos^{n-1}x[\cos x.\sin nx+\sin x.\cos nx]$$

Use the identity

$$\sin(A+B)=\sin A\cos B+\cos A.\sin B$$

$$\frac{dy}{dx}=-n\cos^{n-1}x[\sin(nx+x)]$$

Thus we have

$$\boxed{\frac{dy}{dx}=(\cos^n x.\cos nx)=-n\cos^{n-1}x.\sin[(n+1)x]}$$

Chapter 2 Derivatives Exercise 2.5 86E

The rate of change of y^5 with respect to x is

$$\frac{dy^5}{dx}=\frac{d}{dy}(y^5).\frac{dy}{dx}$$
$$=5y^4\frac{dy}{dx} \qquad \left[\frac{d}{dx}x^n=nx^{n-1}\right]$$

Now according to the given condition,

$$5y^4\frac{dy}{dx}=80\times(\text{The rate of change of } y \text{ with respect to } x)$$

$$5y^4\frac{dy}{dx}=80\frac{dy}{dx}$$

By comparing we have

$$5y^4=80$$
$$\Rightarrow y^4=16$$
$$\Rightarrow y=\sqrt[4]{16}$$
$$\Rightarrow \boxed{|y|=2}\Rightarrow y=2 \qquad \text{Since } y>0 \text{ for all } x$$

So we have for y = 2, the rate of change of y^5 with respect to x eighty times the rate of change of y with respect to x.

Chapter 2 Derivatives Exercise 2.5 87E

We have $\frac{d}{d\theta}\sin\theta=\cos\theta$ when θ is measured in radian.

When θ is measured in degrees then

We have

$$\sin\theta^\circ=\sin\frac{\pi}{180}.\theta$$

Then

$$\frac{d}{d\theta}\sin\theta^\circ=\frac{d}{d\theta}\sin\left(\frac{\pi}{180}.\theta\right)$$

Let

$$y=\frac{\pi}{180}.\theta \; then \; \frac{dy}{d\theta}=\frac{\pi}{180}$$

Thus we have

$$\frac{d}{d\theta}\sin\theta^{\circ}=\frac{d}{d\theta}\sin y$$

By using chain rule

$$\frac{d}{d\theta}\sin\theta^{\circ}=\frac{d}{dy}\sin y.\frac{dy}{d\theta}$$

$$=\cos.\left(\frac{\pi}{180}\theta\right).\frac{\pi}{180}$$

$$==\frac{\pi}{180}\cos.\left(\frac{\pi}{180}\theta\right).$$

But $\cos\left(\frac{\pi}{180}\theta\right)=\cos\theta^{\circ}$

Then we have

$$\boxed{\frac{d}{d\theta}\sin\theta^{\circ}=\frac{\pi}{180}\cos\theta^{\circ}}$$ Proved

Chapter 2 Derivatives Exercise 2.5 88E

(A) We write $|x|=\sqrt{x^2}$

Then $\frac{d}{dx}\sqrt{x^2}=\frac{2x}{2\sqrt{x^2}}$ [Chain rule]

$$=\frac{x}{\sqrt{x^2}}$$

$$=\frac{x}{|x|}$$

Thus $\boxed{\frac{d}{dx}|x|=\frac{x}{|x|}}$

(B) We have $f(x)=|\sin x|$

We can write $f(x)=|\sin x|=\sqrt{\sin^2 x}$

Then $f'(x)=\frac{d}{dx}|\sin x|=\frac{d}{dx}\sqrt{\sin^2 x}$

$$=\frac{1}{2\sqrt{\sin^2 x}}(2\sin x\cos x)$$

$$=\frac{\sin x\cos x}{\sqrt{\sin^2 x}}$$

Thus $\boxed{f'(x)=\frac{\sin x\cos x}{|\sin x|}}$

Since $f'(x)$ is not defined when $\sin x=0 \Rightarrow x=n\pi$

So $f(x)=|\sin x|$ is not differentiable when $x=n\pi$

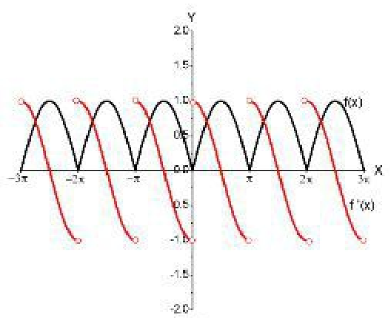


Fig.1

Chapter 2 Derivatives Exercise 2.5 89E

Here

$y = f(u)$ and $u = g(x)$

Here f and g are twice differentiable functions.

We first find $\frac{dy}{du}$ and $\frac{du}{dx}$

$$\frac{dy}{du} = f'(u); \ \frac{du}{dx} = g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \text{ .... (1)}$$

By the differentiation of the equation (1) of the both sides with respect to x, we get

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left[\frac{dy}{du} \cdot \frac{du}{dx}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{du} \cdot \frac{du}{dx}\right]$$

By using the Product Rule and Chain Rule, we get

$$\frac{d^2y}{dx^2} = \frac{du}{dx} \cdot \frac{d}{dx}\left(\frac{dy}{du}\right) + \frac{dy}{du} \cdot \frac{d}{dx}\left(\frac{du}{dx}\right)$$

$$= \frac{du}{dx} \cdot \frac{d^2y}{du^2} \cdot \frac{du}{dx} + \frac{dy}{du} \cdot \frac{d^2u}{dx^2}$$

$$= \left(\frac{du}{dx}\right)^2 \frac{d^2y}{du^2} + \frac{dy}{du} \cdot \frac{d^2u}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d^2y}{du^2} \cdot \left(\frac{du}{dx}\right)^2 + \frac{dy}{du} \cdot \frac{d^2u}{dx^2}$$

Proved

Chapter 2 Derivatives Exercise 2.5 90E

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$$= \frac{du}{dx}\cdot\frac{d^2y}{du^2}\cdot\frac{du}{dx} + \frac{dy}{du}\cdot\frac{d^2u}{dx^2}$$

$$= \left(\frac{du}{dx}\right)^2\frac{d^2y}{du^2} + \frac{dy}{du}\cdot\frac{d^2u}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d^2y}{du^2}\cdot\left(\frac{du}{dx}\right)^2 + \frac{dy}{du}\cdot\frac{d^2u}{dx^2}$$

Now,

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}\left[\frac{d^2y}{du^2}\left(\frac{du}{dx}\right)^2 + \frac{dy}{du}\cdot\frac{d^2u}{dx^2}\right]$$

$$= \frac{d}{dx}\left(\frac{d^2y}{du^2}\cdot\left(\frac{du}{dx}\right)^2\right) + \frac{d}{dx}\left(\frac{dy}{du}\cdot\frac{d^2u}{dx^2}\right)$$

$$= \frac{d^2y}{du^2}\cdot\frac{d}{dx}\left(\frac{du}{dx}\right)^2 + \left(\frac{du}{dx}\right)^2\cdot\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$$

$$+ \frac{dy}{du}\cdot\frac{d}{dx}\left(\frac{d^2u}{dx^2}\right) + \frac{d^2u}{dx^2}\cdot\frac{d}{dx}\left(\frac{dy}{du}\right)$$

$$= \frac{d^2y}{du^2}\cdot 2\frac{du}{dx}\cdot\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^2\cdot\frac{d^3y}{du^3}\;\frac{du}{dx}$$

$$+ \frac{dy}{du}\cdot\frac{d^3u}{dx^3} + \frac{d^2u}{dx^2}\cdot\frac{d^2y}{du^2}\cdot\frac{du}{dx}$$

$$= 3\frac{d^2y}{du^2}\cdot\frac{d^2u}{dx^2}\cdot\frac{du}{dx} + \left(\frac{du}{dx}\right)^3\cdot\frac{d^3y}{dx^3} + \frac{dy}{du}\cdot\frac{d^3u}{dx^3}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{d^3y}{dx^3}\left(\frac{du}{dx}\right)^3 + 3\frac{d^2y}{du^2}\cdot\frac{d^2u}{dx^2}\cdot\frac{du}{dx} + \frac{d^3u}{dx^3}\cdot\frac{dy}{du}$$

This is the formula for $\frac{d^3y}{dx^3}$