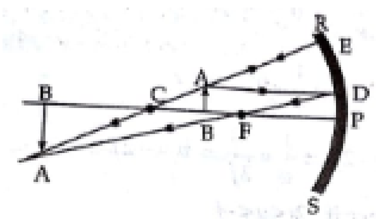
Long Answer Type Question

Q.1. (i) Draw a ray diagram to show image formation when the concave mirror produces a real, inverted and magnified image of the object.

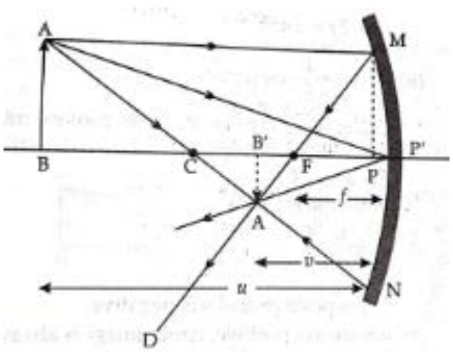
(ii) Obtain the mirror formula and write the expression for the linear magnification.

(iii) Explain two advantages of a reflecting telescope over a refracting telescope.

Ans. (i) When object is between C and F, image is beyond C. It is real, inverted and magnified image of the object.



(ii)



In right-angled triangles, A’B’F and MPF are similar. (For paraxial rays, P and P’ coincides each other and MP’ are same as MP. Hence MP can be considered to be a straight line perpendicular to CP.)

Therefore, right angle triangles Δ A’B’F’ ~ ΔMPF

Hence

$$\frac{B'A'}{PM} = \frac{B'F'}{FP}$$

or $\frac{B'A'}{BA} = \frac{B'F'}{BF} [\because PM = AB]$ **...(i)**

The right-angled triangles A’B’P’ and ABP are also similar as both are right angle triangles and ∠APB = ∠A’PB’ (angle of incidence = angle of reflection)

So, $\frac{B'A'}{BA} = \frac{B'F}{BP}$ **...(ii)**

Comparing eq(i) and (ii) we get,

$\frac{B'F}{FP} = \frac{B'F-FP}{FP} = \frac{B'P}{BP}$

Now putting the values of BP = -u, FP = -f and B'P = -v (applying sign convention also)

$\frac{-v+f}{-f} = \frac{-v}{-u}$

or $\frac{v-f}{f} = \frac{v}{u}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

This relation is known as mirror equation

Linear magnification m = $-\frac{v}{u}$ or $\frac{h'}{h}$

Where v and u are image and object distances respectively from the mirror. h' is image size and h is object's size.

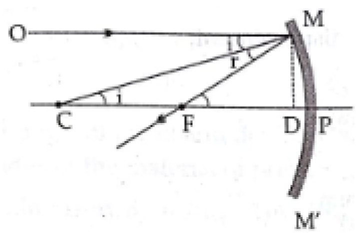
(iii) (a) There are no chromatic and spherical aberrations in reflecting telescope.

(b) It is economical and light as compare to refracting telescope.

Note for students: part (c) is the question of Topic – 3.

Q.2. (i) Prove that for a concave mirror, the radius of curvature is twice the focal length for small aperture as compare to its radius of curvature.

Ans. (i)



Let us consider MM' as small aperture concave mirror. the line through centre of curvature C is perpendicular to the mirror. Applying law of reflections

∠OMC = ∠CMF (angle of incidence = angle of reflection)

∠MCF = ∠OMC (alternate angle)

Hence, ∠MCF = ∠CMF

So, CF = MF

Since aperture of mirror is very small, M is very near P and we can put

CF = FP

PC = 2PF

C = 2f

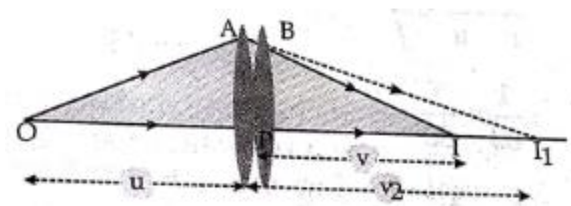
or f = $\frac{C}{2}$

Hence proved.

Q.3. (i) Draw a ray diagram to show the image formation by a combination of two thin convex lenses in contact. Obtain the expression for the power of this combination in terms of the focal lengths of the lenses.

(ii) A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when the angle of incidence is $\frac{3}{4}$th of the angle of prism. calculate the speed of light in the prism.

Ans. (i)



Two thin lenses, of focal length f_1 and f_2 are kept in contact. Let O be the position of object Let u be the object distance. The distance of the image (which is at I_1) for the first lens is v_1.

This image serves as object for the second lens. let the final image be at I. We than have.

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u}$$

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1}$$

Adding, we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\therefore P = P_1 + P_2$$

(ii) At minimum deviation

$$r = \frac{A}{2} = 30^o$$

We are given that

$$i = \frac{3}{4}A = 45o$$

$\therefore \mu = \frac{\sin 45^o}{\sin 30^o} = \sqrt{2}$

$\therefore$ Speed of light in the prism = $\frac{c}{\sqrt{2}}$

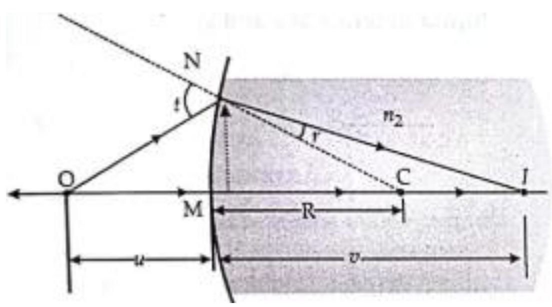
(≅ 2.1 × 108 ms-1)

[Award ½ marks if the student writes the formula: $\mu = \frac{\sin(A+\delta_m)/2}{\sin(A/2)}$ but does not do any calculations.]

Q.4. (i) A point object is placed on the principal axis of a convex spherical surface of radius of curvature R, which separated the two media of refractive indices $\mathbf{n_1}$ and $\mathbf{n_2}$ ($\mathbf{n_2} > \mathbf{n_1}$). Draw the ray diagram and deduce the relation between the object distance (u), image distance (v) and the radius of curvature (R) for refraction to take place at the convex spherical surface from rarer to denser medium.

(ii) A converging lens has a focal length of 20 cm in air. It is made of material of refractive index 1.6. If it is immersed in a liquid of refractive index 1.3, find its new focal length.

Ans. (i)



For small angles

tan ∠NOM = $\frac{\text{MN}}{\text{OM}}$: tan ∠NCM = $\frac{\text{MN}}{\text{NC}}$ and tan ∠NIM = $\frac{\text{MN}}{\text{MI}}$

For Δ NOC, i is exterior angle, therefore

i = ∠NOM + ∠NCM = $\frac{\text{MN}}{\text{CM}} + \frac{\text{MN}}{\text{NC}}$

Similarly, r = $\frac{\text{MN}}{\text{MC}} - \frac{\text{MN}}{\text{MI}}$

For small angles Snells law can be written as $n_1 i = n_2 r$

$\therefore \frac{n_1}{OM} - \frac{n_2}{MI} = \frac{n_2 - n_1}{R}$

$\therefore$ OM = -u, MI = +v MC = +R (using sign conversion)

$\therefore \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

(ii) Lens marker formula is

$\frac{1}{f_o} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\therefore \frac{1}{20} = (1.6 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\therefore \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{20 \times 0.6} = \frac{1}{12}$

Let f be the focal length of the lens in water

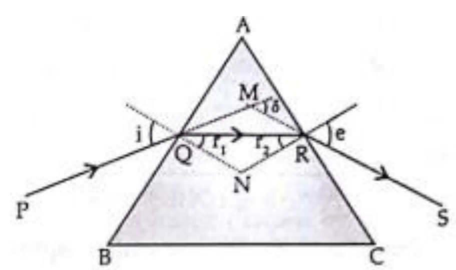
$\therefore \frac{1}{f\prime} = \left(\frac{1.6-1.3}{1.3}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{0.3}{12 \times 1.3}$

or $f' = \frac{120 \times 1.3}{3} = 52\ cm$

Q.5. (i) Draw the ray diagram showing refraction of light through a glass prism and hence obtain the relation between the refractive index μ of the prism, angle of prism and angle of minimum deviation.

(ii) Determine the value of the angle of incidence for a ray of light travelling from a medium of refractive index μ_1 = $\sqrt{2}$ into the medium of refractive index μ_2 = 1, so that it just grazes along the surface of separation.

Ans. (i)



From fig ∠A + ∠QNR = 180o **(1)**

From triangle ∠QNR $r_1 + r_2 + \angle QNR = 180^o$ **(2)**

Hence from eq (1) and (2)

∴ ∠A = $r_1 + r_2$

The angle of Deviation

δ = $(i - r_1)$ + (e $- r_2$) = i + ρ – A

At minimum deviation i = e and r_1 and r_2

∴ r = $\frac{A}{2}$

And i = $\frac{A+\delta_m}{2}$

Hence refractive index

μ = $\frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2}$

(ii) From Snell's law μ_1 sin i = μ_2 sin r

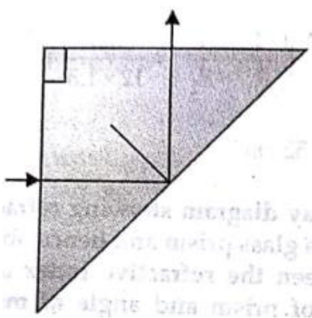
Given $\mu_1 = \sqrt{2},\ \mu_2 = 1$ and $r = 90^o$ (just grazing)

∴ $\sqrt{2}\sin i = 1 \times \sin 90^o$ ⇨ $\sin i = \frac{1}{\sqrt{2}}$ or $i = 45^o$

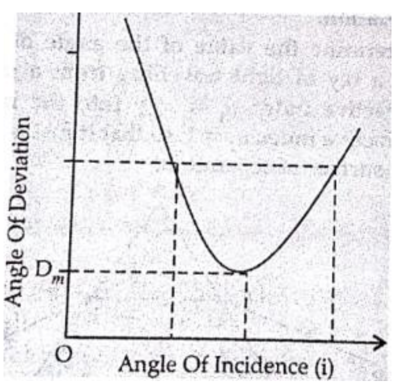
Q.6. (i) Plot a graph to show variation of the angle of deviation as a function of angle of incidence for light passing through a prism. derive an expression for refractive index of the prism in terms of angle of minimum deviation and angle of prism.

(ii) What is dispersion of light? What is its cause?

(iii) A ray of light incident normally on one face of right isosceles prism is totally reflected as shown in fig. What must be the minimum value of refractive index of glass? Give relevant calculations.



Ans. (i)



From figure δ = $D_m, i = e$ which implies $r_1 = r_2$

$2r = A, or\ r = A/2$

Using δ = $i + e - A$

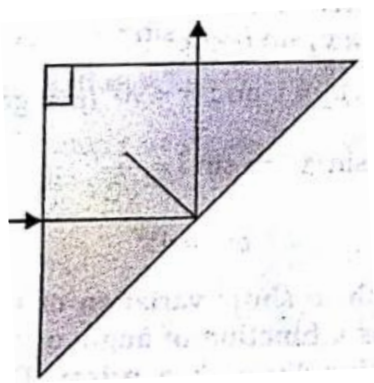
$D_m = 2i - A$

$i = \frac{A+D_m}{2}$

$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+Dm}{2}\right)}{\sin(A/2)}$

(ii) The phenomenon of splitting of white light into its constituent colours.

Cause: Refractive index of the material is different for different colours According to the equation, δ ≇ (μ – 1) A, where A is the angle of prism, different colours will deviate through different amount.



For total internal reflection, $\angle i \geq \angle i_c$ (critical angle)

⇨ 45o ≥ $\angle i_c, i.e., \angle i_c \leq 45^o$

$\sin i_c \leq \sin 45^o$

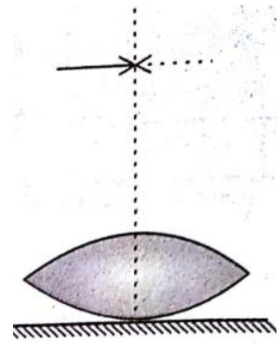
≤ $\frac{1}{\sqrt{2}}$

$\frac{1}{\sin i_c} \geq \sqrt{2}$

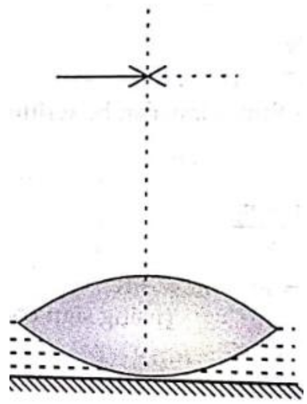
⇨ $\mu \geq \sqrt{2}$

Hence, the minimum value of refractive index must be $\sqrt{2}$.

Q.7. A symmetric biconvex lens of radius of curvature and made of glass of refractive index 1.5 is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its top on the principal axis of the lens is moved along the axis until its real inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x. on removing the liquid layer and repeating the experiment the distance is found to be y. Obtain the expression for the refractive index of the liquid in terms of x and y.



Ans. When a liquid is placed on top of plane mirror and convex lens is placed over it, then this whole system would become a combination of convex lens of glass and planoconcave lens of liquid. This is shown in the figure.



Let focal length of convex lens = f_1

focal length of planoconcave liquid lens = f_2

combined focal length = F

in both case image coincides with needle; hence ray is normal to plane mirror. So, needle position is focal lengths of convex lens and combined system respectively. According to the question,

$f_1 = y$ unit

F = x unit

We also know that for combination of two lenses

$$\frac{1}{\mathrm{F}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{\mathrm{F}} - \frac{1}{f_1}$$

$$\frac{1}{f_2} = \frac{1}{x} - \frac{1}{y}$$

$f_2 = \frac{xy}{x-y}$

for glass lens, let $\mathrm{R}_1 = \mathrm{R}, \mathrm{R}_2 = -\mathrm{R}$. From lens maker formula

$\frac{1}{f} = (n-1)\left(\frac{1}{R} - \left(-\frac{1}{R}\right)\right)$

$\frac{1}{y} = (1.5-1)\left(\frac{1}{R} + \frac{1}{R}\right)$

$\frac{1}{y} = \frac{1}{R}$

R = y unit

For liquid planoconcave lens

$R_1 = -R, R_2 = \infty$. From lens maker formula

$\frac{1}{f_2} = (n_l - 1)\left(-\frac{1}{R} - \frac{1}{\infty}\right)$

$\frac{y-x}{yx} = -(n_1 - 1) \times \frac{1}{y}$ (∵ R = y)

$1 - n_l = \frac{y-x}{x}$

$n_1 = 1 - \frac{y-x}{x}$

$n_1 = \frac{x-y+x}{x}$

$n_1 = \frac{2x-y}{x}$

Q.8. A telescope objective of focal length 1.0 m forms a real image of the moon 0.92 cm in diameter. Calculate the diameter of the moon, taking its mean distance from the moon to be 38 × 104 km. if the telescope uses an eyepiece of focal length 5.0 cm, what would be the distance between the two lenses for (a) the final image to be formed at infinity, (b) the final image (virtual) at 25 cm from the eye?

Ans. Let d be the diameter of the moon. Now, the angle (α) subtended by the moon at the objective of the telescope is

α = $\frac{\text{diameter of the moon}}{\text{distance of moon from earth}}$

= $\frac{d}{38\times10^4 \text{ km}}$

= $\frac{d}{3.8\times10^8 m}$

The angle subtended at the objective by the image of the moon formed by the objective in its focal plane α = $\frac{\text{diameter of moon's image}}{\text{focal length of the objective}}$

= $\frac{0.92\text{cm}}{100cm}$

= 0.0092 rad.

Thus, $\frac{d}{3.8\times10^8\text{m}} = 0.0092$rad

or $d =$ (3.8 × 108 m) × 0.0092 = 3.5 × 106 m.

(a) Distance between the two lenses when the final image is formed at infinity is

$f_o + f_e = 100$cm + 5.0 cm = 105cm

(b) When the telescope is not in normal adjustment, the distance between the objective and the eye-piece is $f_o + u_e$, where u_e is the distance of the image (formed by the objective) from the eye piece.

Let us use the lens formula for the eye-piece:

$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$.

Here, $v_e = -25cm$ and $f_e = +5.0$cm.

∴ $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5.0} = -\frac{6}{25}$

or $u_e = -\frac{25}{6} = -4.17$ cm,

∴ Distance between the objective and the eye-piece is $|f_o| + |u_e| = 100\text{cm} + 4.17\text{cm} = 104.17\text{cm}$.

Q.9. A compound microscope consists of an objective of focal length 2cm eye-piece of focal length 5cm. When an object in kept 2.4cm from an objective, final image formed is virtual and 25 cm form an eye-piece. Determine magnifying power of this compound microscope in this set up, i.e., in normal use.

Ans. Here $f_o = 2\text{cm},\ f_e = 5\text{cm},\ u_o = 2.4\text{cm}, v_e = D = 25\text{cm}$.

In normal use, magnifying power

M = $-\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

For objective, $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

$$\frac{1}{2} = \frac{1}{v_o} - \frac{1}{(-2.4)}$$

$$\frac{1}{v_e} = \frac{1}{2} - \frac{1}{2.4} = \frac{6-5}{12}$$

$$v_e = 12\text{cm}$$

So, M = $-\frac{12}{24}\left(1 + \frac{25}{5}\right)$

= –5 × 6 = –30

Q.10. Find the maximum magnifying power of a microscope having a 25D lens as the objective, a 5 D lens as the eye-piece and the separation 30cm between the two lenses. The least distance for clear vision is 25cm.

Ans. Maximum magnifying power will be in the case when final image formed at the least distance of distinct vision.

M = $-\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

Separation between the lenses in this case $|v_o| + |u_o| = 30\text{cm}$

$v_e = D = 25\text{cm}$

Use $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$ here $f_e = \frac{1}{p} = \frac{1}{5}$m = 20cm

$\frac{1}{20} = \frac{1}{-25} - \frac{1}{u_e}$

or $\frac{-1}{u_e} = \frac{1}{20} + \frac{1}{25} = \frac{5+4}{100}$

or $u_e = -100/9\text{cm}$

but $|v_o| + |u_e| = 30\text{cm}$

So $|v_o| = 30 - |u_e| = 30 - \frac{100}{9} = \frac{170}{9}$cm

Now, for objective

$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$ here $f_o = \frac{1}{p} = \frac{1}{25}m$ = 4 cm

$\frac{1}{4} = \frac{1}{170/9} - \frac{1}{u_e}$

$\frac{1}{u_o} = -\frac{9}{170} - \frac{1}{4} = \frac{360-170}{4\times170}$

or $u_o = -\frac{4\times170}{134} = \frac{340}{67}cm$

So, magnifying power

M = $-\frac{\frac{170}{9}}{\frac{340}{67}}\left(1 + \frac{25}{20}\right) = -8.4.$

Q.11. A compound microscope has a magnification of 30. The focal length of its eye-piece is 5cm. Assuming the final image to be formed at least distance of distinct vision (25 cm), calculate the magnification produced by the objective.

Ans. $M = m_o \times m_e$

or $M = m_o\left(1 + \frac{D}{f_e}\right)$

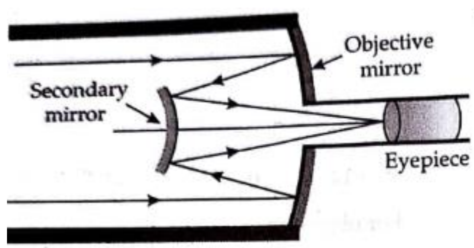
$-30 = m_o\left(1 + \frac{25}{5}\right)$

$m_o = -5.$

Q.12. (i) Draw a labelled ray diagram of an astronomical telescope to show the image formation of a distant object. Write the main consideration required in selection the objective and eyepiece lenses in order to have large magnifying power and high resolution of the telescope.

(ii) A compound microscope has an objective of focal length 1.25 cm and eyepiece of focal length 5 cm. a small object is kept as 2.5 cm from the objective. If the final image formed is at infinity, find the distance between the objective and the eyepiece.

Ans. (i)



For a large magnifying power, f_o should be large and f_e should be small.

For a higher resolution, the diameter of the objective should be large.

Note for students: You may draw refractive or reflective telescope.

(ii) $\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$

$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o}$

$\frac{1}{v_o} = \frac{1}{1.25} - \frac{1}{2.5}$

$\frac{1}{v_o} = \frac{1}{2.5} \Rightarrow v_o = 2.5\ cm$

If the final image is at infinity it means image from objective lens would be at focal length of eye piece.

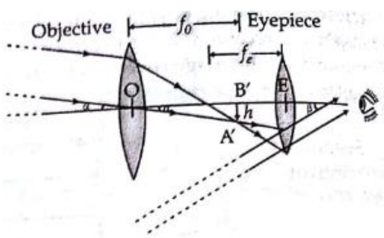
Additionally, this is 2.5 cm away from objective lens.

Hence tube length in this case = 2.5 cm + 5.0 cm = 7.5 cm

Q.13. (i) Draw a labelled ray diagram showing the image formation of a distant object by a refracting telescope. Deduce the expression for its magnifying power when the final image is formed at infinity.

(ii) The sum of focal lengths of the two lenses of the two lenses of a refracting telescope is 105 cm. the focal length of one lens is 20 times that of the other. Determine the total magnification of the telescope when the final image is formed at infinity.

Ans. (i)



Magnification = $\frac{angle\ subtended\ by\ image\ at\ observer's\ eye}{angle\ subtended\ by\ object\ at\ observer's\ eye} = \frac{\beta}{\alpha}$

For small angle β = tan β = (as eye is very near to eyepiece)

Angle α = tan α = $\frac{-A'B'}{fe}$

m = $\frac{\frac{-A'B'}{-f_e}}{\frac{-A'B'}{f_o}} = -\frac{f_o}{f_e}$

(ii) Given,

$f_o + f_e = 105$ cm

and $f_o = 20f_e$

$20f_e + f_e = 105$

Since $f_o = 20f_e$

= 20 × = 100 cm

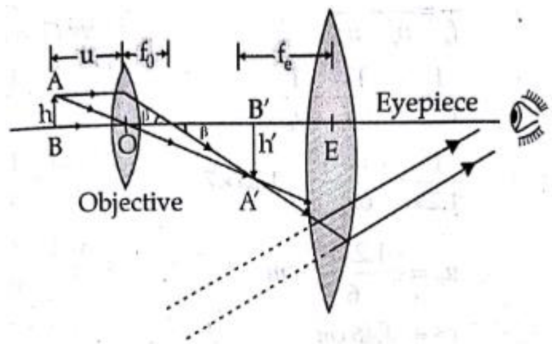
∴ magnification, $m = \frac{f_o}{f_e} = \frac{100}{5} = 20$

Q.14. (i) Draw a ray diagram showing the image formation by a compound microscope. Obtain expression for total magnification when the image is formed at infinity.

(ii) How does the resolving power of a compound microscope get affected, when

(a) Focal length of the objective is decreased.
(b) The wavelength of light is increased? Give reasons to justify your answer.

Ans. (i)



Magnification by objective lens = $\frac{\tan \beta}{\tan \alpha}$

tan β = $\frac{h\prime}{L} = \frac{h}{f_o}$

$\frac{h\prime}{L} = \frac{h}{f_o} =$ (where L = The distance L, i.e., the distance between the second focal point of the eyepiece is called the tube length of the compound microscope.)

Eyepiece will act as simple microscope; hence we may use the formula of magnification by simple microscope for normal adjustment.

$\mathrm{m}_e = \frac{D}{f_e}$

Total magnification, m = $\mathrm{m}_o \times \mathrm{m}_e$

= $\frac{L}{f_o} \times \frac{D}{f_e}$

(ii) $d_{min} = \frac{1.22 f\lambda}{D}$

(a) From the equation, it is clear that resolving power increases when the focal length of the objective is decreased. This is because the minimum separation, d_{min} decrease when f is decreased.

(b) Resolving power decreases when the wavelength of light is increased. This is because the minimum separation, d_{min} increase when λ is increased.

Q.15. (i) Define magnifying power of a telescope.

(ii) Write its expression. A small telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. if this telescope is used to view a 100 m high tower 3 × 105 cm away, find the height of the final image when it is formed 25 cm away from the eye piece.

Ans. (i) Magnifying power is the ratio of the angle subtended at the eye by the image to the angle subtended at the unaided eye by the object. $m = \frac{\beta}{\alpha} = \frac{f_o}{f_e}$

(ii) Expression

or $m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

[Award 1 mark if the student writes expression with –ve sign]

Using the lens equation for an objective lens,

$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

⇨ $\frac{1}{150} = \frac{1}{v_o} - \frac{1}{3\times10^5}$

⇨ $\frac{1}{v_o} = \frac{1}{150} - \frac{1}{3\times10^5} = \frac{2000-1}{3\times10^5}$

⇨ $v_o = \frac{3\times10^5}{1999} cm$

≈ 150 cm

Hence, magnification due to the objective lens

$m_o = \frac{v_o}{u_o} = \frac{150\times10^{-2}m}{3000\, m}$

$m_o \approx \frac{10^{-2}}{20}$ = 0.05 × 10-2

Using lens formula for eyepiece,

$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

⇨ $\frac{1}{5} = \frac{1}{-25} - \frac{1}{u_e}$

⇨ $\frac{1}{u_e} = \frac{1}{-25} - \frac{1}{5} = \frac{-1-5}{25}$

⇨ $u_e = \frac{-25}{6} cm$

∴Magnification due to eyepiece $m_e = \frac{-25}{-\frac{25}{6}} = 6$

Hence, total magnification ⇨ $m = m_e \times m_o$

$m = 6 \times 5 \times 10^{-4} = 30 \times 10^{-4}$

Hence, size of the final image

= 30 × $10^{-4} \times 100\, m$

= 30 cm

Q.16. How is the working of a telescope different from that of a microscope? The focal lengths of the objective and eyepiece of a microscope are 1.25 cm and 5 cm respectively. Find the position of the objective lens in order to obtain an angular magnification of 30 in normal adjustment.

Ans. Working difference:

(i) The objective of a telescope forms the image of a very far off object at, or within, the focus of its eyepiece. The microscope does the same for a small object kept just beyond the focus of its objective.

(ii) The final image formed by a telescope is magnified relative to its size as seen by the unaided eye, while the final image formed by a microscope is magnified relative to its absolute size.

(iii) The objective of a telescope has large focal length and large aperture, while the corresponding for a microscope have very small values.

Given: $f_o = 1.25cm, f_e = 5cm$

Angular magnification $m = 30$

Now, $m = m_e \times m_o$

In normal adjustment, the angular magnification of an eyepiece

$m_e = \frac{d}{f_e} = \frac{25}{5} = 5$

Hence, $m_0 = 6$

But $m_0 = \frac{v_o}{u_o} \Rightarrow -6 = \frac{v_o}{u_o}$

$v_o = -6u_o$

Applying lens equation to the objective lens:

$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

$\frac{1}{1.25} = \frac{1}{-6u_o} - \frac{1}{u_o}$

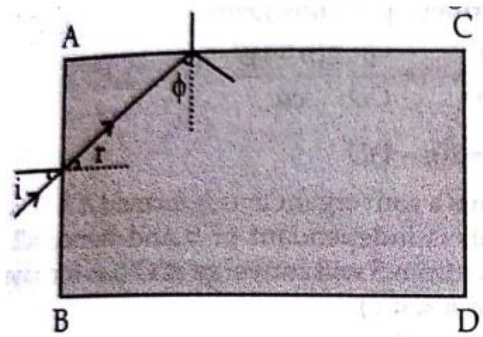
$\frac{1}{1.25} = \frac{-1-6}{6u_o} = -1.25 \times 7$

$u_o = \frac{-1.25\times7}{6} cm$

$u_o = -1.46cm$

Q.17. Show that for a material with refractive index μ ≥ $\sqrt{2}$, light incident at any angle shall be guided along a length perpendicular to the incident face.

Ans. Any ray entering at an angle i shall be guided along AC if the angle the ray makes with the face AC(ϕ) is greater than the critical angle.



⇨ sin $r \geq \frac{1}{\mu}$

⇨ $\cos r \; \geq \frac{1}{\mu}$

Or, 1 – cos2 $r \leq 1 - \frac{1}{\mu^2}$

i.e. sin2 $r \leq 1 - \frac{1}{\mu^2}$

Since sin $i = \mu \sin r$

$$\frac{1}{\mu^2} sin^2 i \leq 1 - \frac{1}{\mu^2}$$

Or, sin2$i \leq \mu^2 - 1$

The smallest angle ϕ shall be when $i = \frac{\pi}{2}$. If that is greater than the critical angle then all other angle of incidence shall be more than the critical angle.

Thus 1 ≤ $\mu^2 - 1$

Or, $\mu^2 \geq 2$

⇨ μ ≥ $\sqrt{2}$

Q.18. If light passes near a massive object, the gravitational interaction causes a bending of the ray, this can be thought of as happening due to a change in the effective refractive index of the medium given by

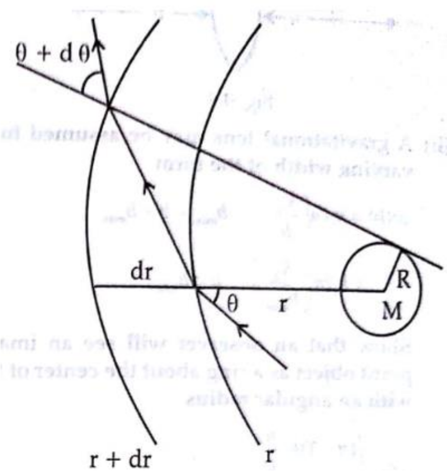
$\boldsymbol{n(r) = 1 + 2GM/rc^2}$

where $r$is the distance of the point of consideration from the centre of the mass of the massive body? G is the universal gravitational constant. M is the mass of the body and c is the speed of light in vacuum. Considering a spherical object find the deviation of the ray from the original path as it grazes the object.

Ans. Consider two planes at r and r + dr. Let the light be incident at an angle θ at the plane at r and leave $r + dr$ at an angle θ + dθ

Then from Snell's law

$n(r)\sin\theta = n(r+dr)\sin(\theta+d\theta)$



⇨ $n(r)\sin\theta; \left(n(r)+\frac{dn}{dr}dr\right)(\sin\theta\cos d\theta+\cos\theta\sin d\theta)\ ;\ \left(n(r)+\frac{dn}{dr}dr\right)(\sin\theta+\cos\theta\, d\theta)$

Neglecting produces of differentials

$n(r)+\sin\theta\ ; n(r)\sin\theta+\frac{dn}{dr}dr\sin\theta+n(r)\cos\theta\, d\theta$

⇨ $-\frac{dn}{dr}\tan\theta = n(r)\frac{d\theta}{dr}$

⇨ $\frac{2GM}{r^2c^2}\tan\theta = \left(1+\frac{2GM}{rc^2}\right)\frac{d\theta}{dr} \approx \frac{d\theta}{dr}$

$\therefore \int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{\tan\theta dr}{r^2}$

Now $r^2 = x^2 + R^2$ and tan θ = $\frac{R}{x}$

$2rdr = 2xdx$

$\therefore \int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{R}{x}\frac{xdx}{(x^2+R^2)^{\frac{3}{2}}}$

Put $x = R\tan\phi$

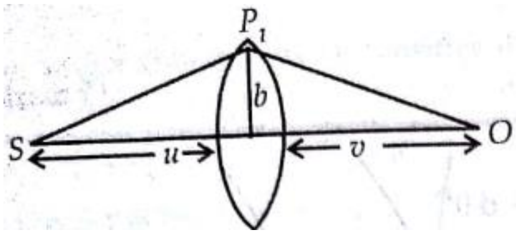
$dx = R\sec^2\phi d\phi$

$\therefore \theta_0 = \frac{2GMR}{c^2}\int_{-\pi/2}^{\pi/2}\frac{Rsec^2\phi d\phi}{R^2sec^3\phi}$

= $\frac{2GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\cos\phi d\phi = \frac{4GM}{Rc^2}$

Q.19. (i) Consider a thin lens placed between a source (S) and an observer (O) (Fig.). Let the thickness of the lens vary as $w(b) = w_o - \frac{b^2}{\alpha}$, where b is the verticle distance from the pole. w_0 is a constant. Using Fermat's principle i.e., the time of transit for ray between

the source and observer is an extremum, find the condition that all paraxial rays starting from the source will converge at a point O on the axis. Find the focal length.



(ii) A gravitational lens may be assumed to have a varying width of the form

$$w(b) = k_1 ln\left(\frac{k_2}{b}\right) \quad b_{min} < b < b_{max}$$

$$= k_1 ln\left(\frac{k_2}{b_{min}}\right) \; b < b_{min}$$

Show that an observer will see an image of a point object as a ring about the centre of the lens with an angular radius $\beta = \sqrt{\frac{(n-1)k_1\frac{u}{v}}{u+v}}$

Ans. **(i)** The time required to travel from S to P_1 is $t_1 = \frac{SP_1}{c} = \frac{\sqrt{u^2+b^2}}{c}; \frac{u}{c}\left(1+\frac{1}{2}\frac{b^2}{u^2}\right)$ assuming b < < u_o

The time required to travel from P_1 to O is

$$t_2 = \frac{P_1O}{c} = \frac{\sqrt{v^2+b^2}}{c}; \frac{v}{c}\left(1+\frac{1}{2}\frac{b^2}{v^2}\right)$$

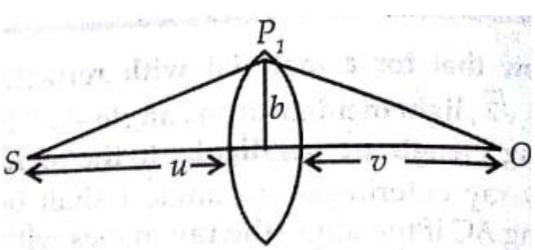
The time required to travel through the lens is $t_1 = \frac{(n-1)w(b)}{c}$ where n is the refractive index.

Thus, the total time is

$$t = \frac{1}{c}\left[u+v+\frac{1}{2}b^2\left(\frac{1}{u}+\frac{1}{v}\right)+(n-1)w(b)\right].$$

Put $\frac{1}{D} = \frac{1}{u}+\frac{1}{v}$

Then $t = \frac{1}{c}\left(u+v+\frac{1}{2}\frac{b^2}{D}+(n-1)\left(w_0+\frac{b^2}{\alpha}\right)\right)$



Fermet’s principles gives

$\frac{dt}{db}$ =∩= $\frac{b}{CD}\frac{2(n-1)b}{c\alpha}$

$\alpha = 2(n-1)D$

Thus, a convergent lens is formed if α = 2(n – 1) D. This is independent of b and hence all paraxial rays from S will converge at O (i.e. for rays b < < n and b < < v).

Since $\frac{1}{D} = \frac{1}{u} + \frac{1}{v}$, the focal length is D.

(ii) In this case

$t = \frac{1}{c}\left(u + v + \frac{1}{2}\frac{b^2}{D} + (n-1)k_1 ln\left(\frac{k_2}{b}\right)\right)$

$\frac{dt}{db} = 0 = \frac{b}{D} - (n-1)\frac{k_1}{b}$

⇨ $b^2 = (n-1)k_1 D$

$\therefore b = \sqrt{(n-1)k_1 D}$

Thus, all rays passing at a height b shall contribute to the image. The ray paths make an angle

$\beta; \frac{b}{v} = \frac{\sqrt{(n-1)k_1 D}}{v^2} = \sqrt{\frac{(n-1)k_1 uv}{v^2(u+v)}} = \sqrt{\frac{(n-1)k_1 u}{(u+v)v}}$.