HOTS (Higher Order Thinking Skills)

Que 1. In Fig. 7.58, $\Delta FEC \cong \Delta GDB$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \sim \Delta ABC$.

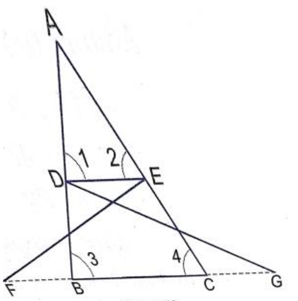

Fig. 7.58

Sol. Since $\Delta FEC \cong \Delta GDB$

⇒ EC = BD ….(i)

It is given that

$\angle 1 = \angle 2$

⇒ AE = AD [Sides opposite to equal angles are equal] ….(ii)

From (i) and (ii), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

⇒ DE||BC [By the converse of basic proportionality theorem]

⇒ $\angle 1 = \angle 3$ *and* $\angle 2 = \angle 4$ [Corresponding angles]

Thus, in $\Delta' s\ ADE$ *and* ABC, we have

$\angle A = \angle A$ [Common]

$\angle 1 = \angle 3$

$\angle 2 = \angle 4$ [Proved above]

So, by AAA criterion of similarity, we have

$\Delta ADE \sim \Delta ABC$

Que 2. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Sol. Given: In ΔABC and $\Delta PQR, AD$ and PM are their medians respectively

Such that $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ …(i)

To prove: $\Delta ABC \sim \Delta PQR$

Construction: Produce AD to E such that AD = DE and produce PM to N such that PM = MN.

Join BE, CE, QN, RN.

Proof: Quadrilateral ABEC and PQNR are ||^{gm} because their diagonals bisect each other

at D and M respectively.

$\Rightarrow \quad BE = AC \quad and \quad QN = PR$

$\Rightarrow \quad \frac{BE}{QN} = \frac{AC}{PR} \quad \Rightarrow \quad \frac{BE}{QN} = \frac{AB}{PQ}$ [From (i)]

$i.e., \quad \frac{AB}{PQ} = \frac{BE}{QN}$ …(ii)

From (i) $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AE}{PN}$ …..(iii)

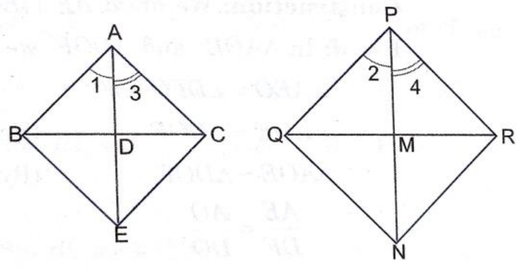

Fig. 7.59

From (ii) and (iii)

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Rightarrow \quad \Delta ABE \sim \Delta PQN$ (SSS similarity criterion)

$\Rightarrow \quad \angle 1 = \angle 2$ ……(iv)

Similarly, we can prove

$\Delta ACE \sim \Delta PRN \quad \Rightarrow \quad \angle 3 = \angle 4$ …….(v)

Adding (iv) and (v), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4 \quad \Rightarrow \quad \angle A = \angle P$

And $\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

$\therefore \quad \Delta ABC \sim \Delta PQR$ (By SAS criterion of similarity)

Que 3. In Fig. 7.60, P is the mid-point of BC and Q is the mid-point of AP. If BQ when produced meets AC at R, prove that RA = $\frac{1}{3}CA$.

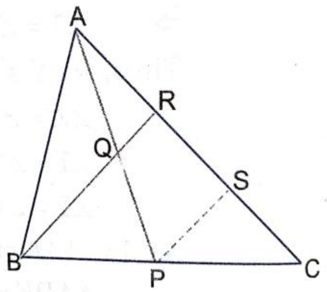

Fig. 7.60

Sol. Given: In $\Delta ABC,$ P is the mid-point of BC, Q is the mid-point of AP such that BQ produced meets AC at R.

To prove: RA = $\frac{1}{3}CA$.

Construction: Draw PS||BR, meeting AC at S.
Proof: In $\Delta BCR, P$ is the mid-point of BC
and PS||BR.
∴ S is the mid-point of CR.
⇒ CS = SR ….(i)
In $\Delta APS, Q$ is the mid-point of AP and QR||PS.
∴ R is the mid-point of AS.
⇒ AR = RS ….(ii)
From (i) and (ii), we get
AR = RS = SC

$$\Rightarrow \quad AC = AR + RS + SC = 3\,AR \quad \Rightarrow \quad AR = \frac{1}{3}AC = \frac{1}{3}CA$$

Que 4. In Fig. 7.61, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\mathbf{ar(\Delta ABC)}}{\mathbf{ar(\Delta DBC)}} = \frac{\mathbf{AO}}{\mathbf{DO}}$.

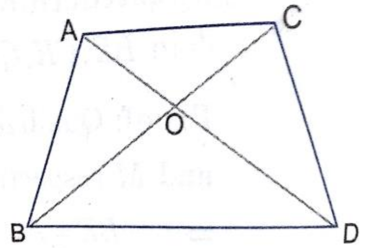

Fig. 7.61

Sol. Given: Two triangles ΔABC and ΔDBCwhich stand on the same base but on opposite sides of BC.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction: We draw AE ⊥ BC and DF ⊥ BC.
Proof: In ΔAOE and $\Delta DOF,$ we have

$\angle AEO = \angle DFO = 90°$

$\angle AOE = \angle DOF$ (Vertically opposite angles)

∴ $\Delta AOE \sim \Delta DOF$ (By AA criterion of similarity)

⇒ $\frac{AE}{DF} = \frac{AO}{DO}$

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF}$

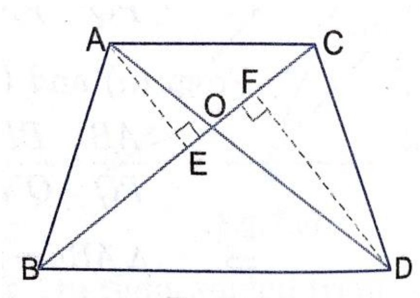

Fig. 7.62

⇒ $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AE}{DE}$ …(ii)

From (i) and (ii), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

Que 5. Two poles of height a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

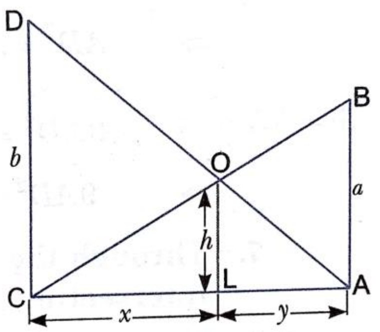

Fig. 7.63

Sol. Let AB and CD be two poles of height a and b metres respectively such that the poles are p metres apart i.e., AC = p metres. Suppose the lines AD and BC meet at O such that

OL = h metres.

Let CL = x and LA = y. Then, x + y = p.

In ΔABC and ΔLOC, we have

$\angle CAB = \angle CLO$ [Each equal to 90 °]

$\angle C = \angle C$ [Common]

$\therefore \quad \Delta ABC \sim \Delta LOC$ [By AA criterion of similarity]

$\Rightarrow \quad \frac{CA}{CL} = \frac{AB}{LO} \quad \Rightarrow \quad \frac{P}{x} = \frac{a}{h}$

$\Rightarrow \quad x = \frac{ph}{a}$ ....(i)

In $\Delta ALO \ and \ \Delta ACD,$ we have

$\angle ALO = \angle ACD$ [Each equal to 90°]

$\angle A = \angle A$ [Common]

$\therefore \quad \Delta ALO \sim \Delta ACD$ [By AA criterion of similarity]

$\Rightarrow \quad \frac{AL}{AC} = \frac{OL}{DC} \quad \Rightarrow \quad \frac{y}{p} = \frac{h}{b}$

$\Rightarrow \quad y = \frac{ph}{b}$ .....(ii)

From (i) and (ii), we have

$x + y = \frac{ph}{a} + \frac{ph}{b} \quad \Rightarrow \quad p = ph\left(\frac{1}{a} + \frac{1}{b}\right)$ [$\because \ x + y = p$]

$\Rightarrow \quad 1 = h\left(\frac{a+b}{ab}\right) \quad \Rightarrow \quad h = \frac{ab}{a+b} \ metres.$

Hence, the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ metres.