Chapter-09
Z-transform

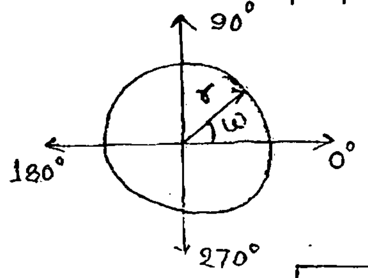
* Discrete time fourier transform (DTFT) exists for E & P signals where as Z-TF also exist for NENP sig. (upto certain only).
* The replacement $z = e^{j\omega}$ is used for z-transform to DTFT conversion only for absolutely summable signal.

$$x(n) \rightleftharpoons X(z)$$

where; z = complex variable

$$= re^{j\omega}$$

damping factor ⟵ r ; $e^{j\omega}$ ⟶ oscillation factor.



$Z \longrightarrow 2\pi$

$X(z) \longrightarrow 2\pi$

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}}$$

Que:→ $x(n) = a^n u(n) \rightleftharpoons X(z) = ?$

Sol^n→

$$X(z) = \sum_{n=-\infty}^{\infty} a^n z^{-n}$$

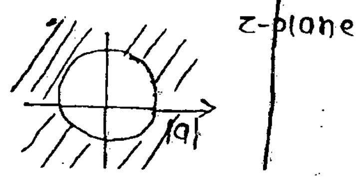
$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= (az^{-1})^0 + (az^{-1}) + (az^{-1})^2 + \ldots\ldots$$

$$X(z) = \frac{1}{1-az^{-1}} \quad ; \quad |az^{-1}| < 1$$

$$a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}} \quad ; \quad \text{ROC: } |z| > a$$



Que.→ $x(n) = -a^n u(-n-1)$ cal. ZTF & ROC.

Sol^n→

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{-\infty}^{\infty} -a^n u(-n-1) z^{-n}$$

$$= \sum_{-\infty}^{-1} (-a^n) z^{-n}$$

$$= -\sum_{-\infty}^{-1} (az^{-1})^n$$

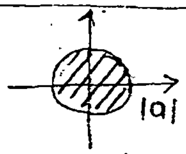
$$X(z) = -\left[(az^{-1})^{-1} + (az^{-1})^{-2} + \cdots\cdots\right]$$

$$= -\left[a^{-1}z + (a^{-1}z)^2 + \cdots\cdots\right]$$

$$= \frac{-a^{-1}z}{1-a^{-1}z} \; ; \; |a^{-1}z| < 1$$

$$X(z) = \frac{1}{1-az^{-1}} \; ; \; |z| < |a|$$

$$\boxed{-a^n u(-n-1) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \; |z| < |a|}$$



$$-a^n u(-n-1) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \; |z| < |a|$$

$$a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \; |z| > |a|$$

(a=1) (a=1)

$$u(n) \rightleftharpoons \frac{1}{1-z^{-1}} \; ; \; |z| > 1$$

$$-u(-n-1) \rightleftharpoons \frac{1}{1-z^{-1}} \; ; \; |z| < 1$$

Properties of z-transform →

1.) linearity :- $a_1 x_1(n) + a_2 x_2(n) \rightleftharpoons a_1 X_1(z) + a_2 X_2(z)$

2.) Time-reversal :- $x(-n) \rightleftharpoons X(z^{-1})$

3.) Conjugation :- $x^*(n) \rightleftharpoons X^*(z^*)$

4.) Time-shifting :- $x(n-n_0) \rightleftharpoons X(z).z^{-n_0}$

$$\downarrow (n_0 = 1)$$

$$x(n-1) \rightleftharpoons z^{-1}X(z)$$

~~5.) Sca~~ling of z :- $a^n x(n) \rightleftharpoons X(a^{-1}z)$

6.) Convolution in time :-

$$x_1(n) * x_2(n) \rightleftharpoons X_1(z) . X_2(z)$$

7.) Multiplication in time :-

$$x_1(n) . x_2(n) \rightleftharpoons \frac{1}{2\pi j}\left[X_1(z) * X_2(z)\right]$$

8.) Successive diff./difference in time →

$$\frac{dx(n)}{dn} = \frac{x(n) - x(n-1)}{n-(n-1)}$$

$$= x(n) - x(n-1) \rightleftharpoons X(z) - z^{-1}X(z)$$

$$x(n) - x(n-1) \rightleftharpoons (1-z^{-1})X(z)$$

9.) Accumulation/Integration in time →

$$\sum_{k=-\infty}^{n} x(k) \rightleftharpoons \frac{X(z)}{1-z^{-1}}$$

10.) Differentiation in Freq. →

$$n.x(n) \rightleftharpoons -z\frac{dX(z)}{dz}$$

(11) Initial value theorem →

$$x(t)\Big|_{t=0} = \lim_{s\to\infty} sX(s)$$

Cond^n.- Applicable only for causal type signal i.e.

(12) Final value theorem →

$$x(t)\Big|_{t=\infty} = \lim_{s\to 0}[sX(s)]$$

$$x(n)\Big|_{n=\infty} = \lim_{z\to 1}[(1-z^{-1})X(z)]$$

Condn :- (i) ~~Applicable only~~ For causal signals. i.e.

$$x(n)=0,\ n<0$$

(ii) poles of term $[(1-z^{-1})X(z)]$ should lie inside unit circle in z-plane.

DATE-30/10/14

Region of Convergance (ROC) → It is defined as the range of complex variable z in z-plane for which z-transform of signal is convergant or Finite.

Properties of ROC →

(1) ROC does not include any pole.

(2) For right sided signal ROC will be outside circule in z-plane.

(3) For left sided signal ROC will be inside circule in z-plane.

RS LS
↑ ↑
ROC I
↓ ↓
out side inside

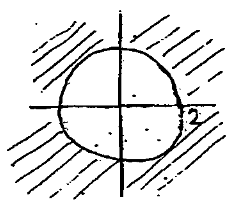
(4) For both sided signal ROC is a ring in z-plane.

(5) For stability, ROC includes unit circle in z-plane.

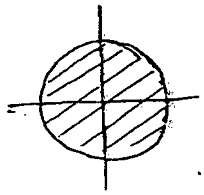
(6) For finite duration sig. ROC is intire z-plane excluding possibly z=0 &/or $\pm\infty$.

Que.→ Check stability of sys. & comment about extension of h(n).

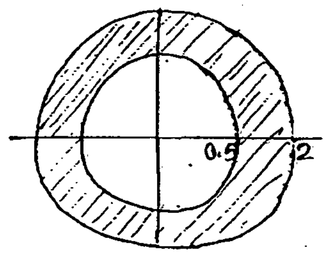
(i) ROC: |z|>2



(ii) ROC: |z|<2



(iii) ROC: 0.5<|z|<2



Soln → h(n) = RS+US

h(n) = LS+S

h(n) = BS+S

Que.→ $x(n) = \{2, \underset{\uparrow}{5}, 3, 7, 8\}$ $X(z) = ?$, ROC = ?

Sol^n→

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{-1}^{3} x(n) z^{-n}$$

$$= x(-1)z^{1} + x(0)z^{0} + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}$$

$$= 2z + 5 + 3z^{-1} + 7z^{-2} + 8z^{-3}$$

ROC → Entire z-plane excluding

$z = 0, +\infty, -\infty$ (excluded) [Because they give ∞ sol^n in $X(z)$]

Que.→

$$x(n) \rightleftharpoons X(z) \longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y(n)$$

$$h(n) = 2\delta(n-3)$$

$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$. Find $y(4)$

(a) −6 (b) 0 (c) 2 (d) −4

Sol^n→

$$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$$

$$x(n) = \{1, 0, 1, -2, 2, 0, 0, 0, -3\}$$

$\delta(n) \rightleftharpoons 1$

$2\delta(n) \rightleftharpoons 2$

$2\delta(n-3) \rightleftharpoons 2z^{-3}$

$$y(n) = x(n) * h(n)$$

$$Y(z) = X(z) \cdot H(z)$$

$$= 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})$$

$$Y(z) = 2z + 2z^{-1} - 4z^{-2} + 4z^{-3} - 6z^{-7}$$

$$y(n) = \{2, \underset{\uparrow}{0}, 2, -4, 4, 0, 0, 0, -6\}$$

$$y(4) = 0$$

$$\boxed{y(4) = 0}$$

Que.→ $X(z) = 1 - 3z^{-1} \rightleftharpoons x(n) \neq$ i/p

$Y(z) = 1 + 2z^{-2} \rightleftharpoons y(n) \neq$ o/p

An LTI sys has impulse responce $h(n)$ defined as $h(n) = x(n-1) * y(n)$.
The o/p of sys for i/p $\delta(n-1)$ ~~has ZT~~.

$\underline{\text{sol}^n \rightarrow}$ $\quad i/p \rightarrow \delta(n-1) = z^{-1}$

$$p(n) = P(z) \longrightarrow \boxed{\begin{matrix}\text{LTI}\\ \text{system}\end{matrix}} \longrightarrow q(n) = Q(z)$$

$P(z) = z^{-1}$ $\qquad h(n) \rightleftharpoons H(z)$

$$h(n) = x(n-1) * y(n)$$

$$H(z) = z^{-1} X(z) \cdot Y(z)$$

$$q(n) = p(n) * h(n)$$

$$Q(z) = P(z) \cdot H(z)$$

$$= (z^{-1}) [z^{-1} X(z) \cdot Y(z)]$$

$$Q(z) = z^{-2} X(z) \cdot Y(z)$$

$$= z^{-2} (1 - 3z^{-1})(1 + 2z^{-2})$$

$$= z^{-2} (1 - 3z^{-1} + 2z^{-2} - 6z^{-3})$$

$$Q(z) = z^{-2} - 3z^{-3} + 2z^{-4} - 6z^{-5}$$

$$q(n) = \delta(n-2) - 3\delta(n-3) + 2\delta(n-4) - 6\delta(n-5)$$

ans (b)

$\underline{2^{nd} \text{ method} \rightarrow}$

$$q(n) = p(n) * h(n)$$

$$= \delta(n-1) * h(n)$$

$$= h(n-1)$$

$$h(n) = x(n-1) * y(n)$$

$$X(z) = 1 - 3z^{-1} \rightleftharpoons x(n) = (\underset{\uparrow}{1}, -3)$$

$$x(n-1) = (\underset{\uparrow}{0}, 1, -3)$$

$$Y(z) = 1 + 2z^{-2} \rightleftharpoons y(n) = (\underset{\uparrow}{1}, 0, 2)$$

$$h(n) = (\underset{\uparrow}{0}, 1, 2) * (\underset{\uparrow}{1}, 0, 2)$$

Que.→ $x(n) = \left(-\frac{1}{2}\right)^{n} u(-n+1) + 3^{n} u(n)$

$X(z) = ?$

Soln→

$\cancel{|z| < \left|\frac{-1}{2}\right|}$ $\quad$ $\cancel{|z| > |3|}$

$\cancel{|z| < (1/2)}$ $\quad$ $\cancel{|z| > 3}$

$\cancel{\left(\frac{1}{2}\right) < |z| < 3}$

$(-2)^{n} u(-n-3)$ (LS)

$|z| < |-2|$

$|z| < 2$

$x(n) = \left(-\frac{1}{2}\right)^{n} u(-n+1) + 3^{n} u(n)$

$= \underbrace{(-2)^{-n} u(-n+1)}_{|z|<2} + \underbrace{3^{n} u(n)}_{|z|>3}$

ZT of the $x(n)$ will not exist because no common ROC

Que.→ $x(n) = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^{n} u(n)$ $\quad$ ROC=?

Soln→ $x(n) = \left(\frac{1}{3}\right)^{n} - \left(\frac{1}{2}\right)^{n} u(n)$

$$\left(\frac{1}{3}\right)^{n} = \begin{cases} \left(\frac{1}{3}\right)^{-n} ; & n<0 \\ \left(\frac{1}{3}\right)^{n} ; & n \geq 0 \end{cases}$$

$$= \begin{cases} 3^{n} ; & n<0 \\ (1/3)^{n} ; & n \geq 0 \end{cases}$$

$= 3^{n} u(-n-1) + \left(\frac{1}{3}\right)^{n} u(n)$

$= \quad |z| < 3 \qquad |z| > 1/3$

$x(n) = \left(\frac{1}{3}\right)^{n} - \left(\frac{1}{2}\right)^{n} u(n)$

$= \underbrace{3^{n} u(-n-1)}_{|z|<3} + \underbrace{\left(\frac{1}{3}\right)^{n} u(n)}_{|z|>1/3} - \underbrace{\left(\frac{1}{2}\right)^{n} u(n)}_{|z|>1/2}$

$|z| < 3 \qquad |z| > 1/3$

$\frac{1}{3} < |z| < 3$

$$\boxed{ROC = \frac{1}{3} < |z| < 3}$$

Que.→ $x(n) = 2^{r(n)}$ ZT=?

Soln→ $x(n) = 2^{r(n)}$

$$\cancel{X(z) = \sum_{-\infty}^{\infty} 2^{r(n)} z^{-n}}$$

$$x(n) = 2^{r(n)}$$

$$= 2^{n u(n)}$$

$$= \begin{cases} 1 & ; n<0 \\ 2^n & ; n\geq 0 \end{cases}$$

$$= u(-n-1) + 2^n u(n)$$

$$|z|<1 \qquad |z|>2$$

ZT will not exists because no common ROC.

Que.→ $x(n) = \cos\omega_0 n \cdot u(n)$ $X(z)=?$, ROC=?

Soln→ $x(n) = \cos\omega_0 n\, u(n)$

$$a^n u(n) \rightleftharpoons \frac{z}{z-a}, |z|>|a|$$

$a = e^{j\omega_0}$

$$e^{j\omega_0 n} u(n) \longrightarrow \frac{z}{z-e^{j\omega_0}} \quad |z|>$$

$a = e^{-j\omega_0}$

$$e^{-j\omega_0 n} u(n) \longrightarrow \frac{z}{z-e^{-j\omega_0}} \quad |z|>1$$

$$\cos\omega_0 n = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}$$

$$x(n) = \frac{1}{2} e^{j\omega_0 n} u(n) + \frac{1}{2} e^{-j\omega_0 n} u(n)$$

$$X(z) = \frac{1}{2}\left[\frac{z}{z-e^{j\omega_0}} + \frac{z}{z-e^{-j\omega_0}}\right]; \ |z|>1$$

$$= \frac{1}{2}\left[\frac{z(z-e^{-j\omega_0}) + z(z-e^{j\omega_0})}{z^2 - z(e^{j\omega_0}+e^{-j\omega_0})+1}\right], \ |z|>1$$

$$= \frac{1}{2}\left[\frac{2z^2 - z(e^{j\omega_0}+e^{-j\omega_0})}{z^2 - 2z\cos\omega_0 + 1}\right], \ |z|>1$$

$$= \frac{1}{2}\left[\frac{2z^2 - 2z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}\right], \ |z|>1$$

$$= \frac{z^2 - z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1} \qquad |z|>1$$

Important signals →

$x(n)$	$X(z)$	ROC
$\delta(n)$	1	entire z-plane
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$n\cdot a^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\cos\omega_0 n$	$\frac{z^2 z\cos\omega_0}{z^2-2z\cos\omega_0+1}$	$\lvert z\rvert > 1$
$\sin\omega_0 n$	$\frac{z\sin\omega_0}{z^2-2z\cos\omega_0+1}$	$\lvert z\rvert > 1$

Que.→ $x(n) \rightleftharpoons X(z) = \frac{0.5}{1-2z^{-1}}$

It is given that ROC of $X(z)$ includes unit circle is

The value of $x(0)$ is :-

(a) 0.5 (b) 0 (c) 0.25 (d) 0.5

Soln→

$$X(z) = \frac{0.5}{1-2z^{-1}}$$

Pole:- $1-2z^{-1}=0$

$z=2$

ROC:- $|z|<2$ (Given)

So inverse will be left sided, $x(n)$

$x(n) = -0.5(2)^n u(-n-1)$

$u(-n-1) \to (-\infty \text{ to } -1)$

Que→ Find inverse ZT of

$$X(z) = \frac{z}{(z-1)(z-2)^2} \quad \text{if}$$

(i) $|z|>2$ (ii) $|z|<1$ (iii) $1<|z|<2$

Sol^n→

$$X(z) = \frac{z}{(z-1)(z-2)^2}$$

$$X(z) = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2}$$

$$\frac{X(z)}{z} = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2}$$

$$X(z) = \frac{Az}{(z-1)} + \frac{Bz}{(z-2)} + \frac{Cz}{(z-2)} \times \frac{z^{-2}}{z^{-2}}$$

$$X(z) = \frac{A}{1-z^{-1}} + \frac{B}{1-2z^{-1}} + \frac{C}{2}\left[\frac{2z^{-1}}{(1-2z^{-1})^2}\right]$$

$A=1,\ B=-1,\ C=1$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{z}{1-2z^{-1}} + \frac{1}{2}\left[\frac{2z^{-1}}{(1-2z^{-1})^2}\right]$$

$$\boxed{\begin{array}{l} f(n) = a^n u(n) \rightarrow F(z) = \dfrac{z}{z-a} \\ f(n-1) = a^{n-1}u(n-1) \\ z^{-1}F(z) = \dfrac{1}{(z-a)} \end{array}}$$

Poles→ $z = 1, 2$

(i) ROC: $|z|>2$

* $x(n)$ will be RS.
* $x(n) = u(n) - 2^n u(n) + \frac{1}{2} n 2^n u(n)$

(ii) ROC: $|z|<1$

* $x(n)$ will be LS.
* $x(n) = -u(-n-1) - [-2^n u(-n-1)] + \frac{1}{2}[-n 2^n u(-n-1)]$

(iii) ROC: $1<|z|<2$

* $x(n)$ will be both sided.
* $x(n) = u(n) - [-2^n u(-u-1)] + \frac{1}{2}[-n 2^n u(-n-1)]$

Que.→ $x(n) = 4^n u(n) \rightleftharpoons X(z)$

$y(n) \rightleftharpoons Y(z) = X^2(z)$ Find $y(n)$

(a) $4^n u(n)$ (b) $(n+1)4^n u(n+1)$ (c) $(n+1)4^n u(n)$ (d) $n \cdot 4^n u(n+1)$

Soln→

$$X(z) = \frac{1}{1-4z^{-1}}$$

$$X^2(z) = \left(\frac{1}{1-4z^{-1}}\right)^2 = \frac{1}{(1-4z^{-1})^2} = \frac{1}{1+16z^{-2}-8z^{-1}}$$

$$Y(z) = X^2(z) = \frac{1}{(1-4z^{-1})^2}$$

$$Y(z) = \frac{z^2}{(z-4)^2} = \frac{z(z-4)+4z}{(z-4)^2}$$

$$= \frac{z}{(z-4)} + \frac{4z}{(z-4)^2} \times \frac{z^{-2}}{z^{-2}}$$

$$Y(z) = \frac{1}{1-4z^{-1}} + \frac{4z^{-1}}{(1-4z^{-1})^2}$$

$$y(n) = 4^n u(n) + n 4^n u(n)$$

$$\boxed{y(n) = (n+1)\, 4^n u(n)}$$

ans: (b) & (c)

$$(n+1)4^n u(n+1) = (0, \underset{\uparrow}{1}, 8 \ldots\ldots)$$

$$(n+1)4^n u(n) = (\underset{\uparrow}{1}, 8 \ldots\ldots)$$

Que.→ $X(z) = \log(1+az^{-1})$; $|z|>|a|$ Find $x(n) = ?$

Soln→ diff in Freq:

$$x(n) \rightleftharpoons X(z)$$

$$n x(n) \rightleftharpoons -z\frac{dX(z)}{dz} = -z\left[\frac{1}{1+az^{-1}}(-az^{-2})\right]$$

$$n \cdot x(n) = \frac{az^{-1}}{1+az^{-1}} ; \quad |z|>|a|$$

$$n x(n) = 1 - \frac{1}{1+az^{-1}} \quad |z|>|a|$$

$$n x(n) = \delta(n) - (-a)^n u(n)$$

Que→ $X(z) = \dfrac{1+z^{-1}}{1+\frac{1}{3}z^{-1}} \rightleftharpoons x(n)$

(a) Assuming ROC to be $|z| < 1/3$. determine $x(0), x(-1), x(-2)$

(b) Assuming ROC to be $|z| > 1/3$ determine $x(0), x(1)$ & $x(2)$

Soln→

$$X(z) = \frac{1+z^{-1}}{1+\left(\frac{1}{3}\right)z^{-1}}$$

(a) $|z| < \frac{1}{3}$; $x(n)$ will be LS sig.

Arrange numerator & denominator polynomials in asending powers of 'z'.

$$X(z) = \frac{z^{-1}+z^{0}}{\left(\frac{1}{3}\right)z^{-1}+z^{0}}$$

LED (LA)

$$\tfrac{1}{3}z^{-1}+1 \,\big)\, z^{-1}+1 \,\big(\, 3-6z+18z^{2}+\cdots$$

$$\begin{array}{l} \cancel{z^{-1}}+3 \\ \hline -2 \\ -\cancel{2}-6z \\ \hline 6z \\ \cancel{6z}+18z^{2} \\ \hline -18z^{2} \\ \vdots \end{array}$$

$$X(z) = 3-6z+18z^{2}+\cdots\cdots$$

$$= x(0)+x(-1)z+x(-2)z^{2}+\cdots$$

$$x(0)=3, \quad x(-1)=-6, \quad x(-2)=18$$

(b) $|z| > \frac{1}{3}$; $x(n)$ will be RS sig.

Arrange nume. & deno. poly. in decensing powers of 'z'.

$$X(z) = \frac{1+z^{-1}}{1+\left(\frac{1}{3}\right)z^{-1}}$$

$$1+\tfrac{1}{3}z^{-1} \,\big)\, 1+z^{-1} \,\big(\, 1+\tfrac{2}{3}z^{-1}-\tfrac{2}{9}z^{-2}+\cdots$$

$$\begin{array}{l} \cancel{1}+\frac{1}{3}z^{-1} \\ \hline \frac{2}{3}z^{-1} \\ \frac{2}{3}z^{-1}+\frac{2}{9}z^{-2} \\ \hline -\frac{2}{9}z^{-2} \end{array}$$

$$X(z) = 1 + \frac{2}{3}z^{-1} - \frac{2}{9}z^{-2} + \cdots\cdots$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + \cdots\cdots$$

$$x(0) = 1,\ x(1) = \frac{2}{3},\ x(2) = \frac{-2}{9}$$

Que.→ $x(n) \rightleftharpoons X(z) = \dfrac{z}{2z^2 - 3z + 1}$, $|z| < \frac{1}{2}$. Find $x(-2)$

(a) 0 (b) 1 (c) 2 (d) 3

Solⁿ→

$$X(z) = \frac{z}{1 - 3z + 2z^2}$$

$$1 - 3z + 2z^2 \,)\, z \,(\, z + 3z^2 + 7z^3 + \cdots\cdots$$

$$\underline{z - 3z^2 + 2z^3}$$

$$3z^2 - 2z^3$$

$$\underline{3z^2 - 9z^3}$$

$$7z^3$$

$$\underline{7z^3 - 21z^4 + 14z^5}$$

$$\boxed{x(-2) = 3}$$

Que.→ $x(n) \rightleftharpoons X(z) = \dfrac{z + z^{-3}}{z + z^{-1}}$; $x(n)$ series has

(a) Alternate -1s (c) Alternate 1s

(b) Alternate 0s (d) Alternate 2s.

solⁿ→

$$X(z) = \frac{z + z^{-3}}{z + z^{-1}}$$

Here Roc is not given, so devide in the given form.

$$z + z^{-1} \,)\, z + z^{-3} \,(\, 1 - z^{-2} + 2z^{-4} - 2z^{-6} + 2z^{-8} - 2z^{-10}$$

$$\underline{z + z^{-1}}$$

$$-z^{-1} + z^{-3}$$

$$\underline{-z^{-1} - z^{-3}}$$

$$2z^{-3}$$

$$\underline{2z^{-3} + 2z^{-5}}$$

$$-2z^{-5}$$

$$\underline{-2z^{-5} - 2z^{-7}}$$

$$\vdots$$

$$X(z) = 1 - z^{-2} + 2z^{-4} + 2z^{-6} + 2z^{-8} + \cdots\cdots$$

$$x(n) = (1, 0, -1, 0, 2, 0, -2, 0, 2 \cdots)$$

ans. (b)

*** Causal system →**

(1) $h(n) = 0,\ n < 0$

(2) $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{n=0}^{\infty} h(n) z^{-n}$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + \cdots = \frac{N(z)}{D(z)}$$

Note → For causal sys, expansion of TF does not include +ve powers of z.

(3) $\lim_{z\to\infty} H(z) = h(0) = 0$ (or) finite.

Note → * For causal sys., order of numerator can't exceed order of denominator

* For causal sys. ROC will be outside circle in z-plane.

* For stability of ~~d/t~~ discrete time causal sys., poles of TF should lie inside unit circle in z-plane.

*** Anticausal system →**

* For this sys. ROC will be inside circle in z-plane.

* For stability of this sys, poles of TF should lie outside unit circle in z-plane.

Que → A causal LTI sys. is described by the difference eqn

$$2y(n) = \alpha y(n-2) - 2x(n) + \beta x(n-1)$$

The sys. is stable only if

(a) $|\alpha| = 2,\ |\beta| < 2$

(b) $|\alpha| > 2,\ |\beta| > 2$

(c) $|\alpha| < 2$, for any value of 'β'

(d) $|\beta| < 2$, for any value of 'α'

Sol$^n \rightarrow$ $\quad 2Y(z) = aY(z)\cdot z^{-2} - 2X(z) + \beta X(z) z^{-1}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-2+\beta z^{-1}}{2-az^{-2}}$$

Poles:- $\quad 2-az^{-2}=0$

$$z = \sqrt{\frac{a}{2}}$$

For stability of causal sys.

$$|\text{pole}| < 1$$

$$\sqrt{\frac{a}{2}} < 1$$

$$\boxed{|a| < 2}$$

Que.$\rightarrow$ $x(n) \rightleftharpoons X(z) = \dfrac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$

Find $x(\infty) = ?$ $\quad$ (a) $1/4$ (b) 0 (c) 1 (d) ∞

Sol$^n \rightarrow$

$$x(n) \rightleftharpoons X(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$$

(i) $\lim_{z\to\infty} X(z) = 0$

Cond^n for causality is satisfied.

(ii) $(1-z^{-1})X(z) = \dfrac{z^{-1}(1-z^{-4})}{4(1-z^{-1})}$

$$= \frac{z^{-1}(1+z^{-2})(1-z^{-2})}{4(1-z^{-1})}$$

$$= \frac{z^{-1}(1+z^{-2})(1+z^{-1})\cancel{(1-z^{-1})}}{4\cancel{(1-z^{-1})}}$$

$$= \frac{z^{-1}(1+z^{-2})(1+z^{-1})}{4}$$

Pole:- $z=0<1$

Both the cond^n are satisfied. so we can use final value theorem.

Final value theorem →

$$x(\infty) = \lim_{z\to 1}\left[(1-z^{-1})X(z)\right]$$

$$= \lim_{z\to 1}\left[\frac{z^{-1}(1+z^{-2})(1+z^{-1})}{4}\right]$$

$$= \frac{2\times 2}{4}$$

$$= 1$$

Que → A stable & causal sys. is described by the diff. eqn

$$y(n) + \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = -2x(n) + \frac{5}{4}x(n-1)$$

h(n) of the sys. is

(a.) $\left(\frac{1}{4}\right)^n u(n) + 3\left(\frac{-1}{2}\right)^n u(n)$ (b.) $\left(\frac{1}{4}\right)^n u(n) - 3\left(\frac{-1}{2}\right)^n u(n)$

(c.) $\left(\frac{1}{4}\right)^n u(n) - 3u(n)$ (d.) $\left(\frac{1}{4}\right)^{n-1} u(n-1) - 3u(n-1)$

Soln →

$$Y(z) + \frac{1}{4}Y(z)z^{-1} - \frac{1}{8}Y(z)z^{-2} = -2X(z) + \frac{5}{4}X(z)z^{-1}$$

$$Y(z)\left[1 + \frac{1}{4}z^{-1} - \frac{z^{-2}}{8}\right] = X(z)\left[-2 + \frac{5}{4}z^{-1}\right]$$

$$H(z) = \frac{-2 + \frac{5}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{z^{-2}}{8}}$$

from option

(a.) $|z| > \frac{1}{4}$ $|z| > \frac{1}{2}$ common ROC $|z| > \frac{1}{2}$ (stable)

(b.) common ROC $|z| > \frac{1}{2}$ (stable)

(c.) common ROC $|z| > 1$ (US)

(d.) common ROC $|z| > 1$ (US)

Here sys. is causal so follow the Initial value theorem

$$h(0) = \lim_{z\to\infty} H(z) = -2$$

(a). $h(0) = 1 + 3 = 4$ (b.) $h(0) = 1 - 3 = -2$

* Jury Test →

(1.) To check stability of contineous time causal LTI sys. Routh-hurwitz criteria is used.

(2.) Jury test is used to check stability of discrete time causal LTI sys.

$$H(z) = \frac{k_m z^m + k_{m-1} z^{m-1} + \cdots + k_0}{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0} = \frac{N(z)}{D(z)}$$

For causality:- $n \geq m$

c/s eqn :- $D(z) = 0$

necessary condn For stability →

(1.) $D(1) > 0$

(2.) $D(-1) \begin{cases} > 0 \;;\; n = \text{even} \\ < 0 \;;\; n = \text{odd} \end{cases}$

Jury table → no. of rows $= 2n-3$

	z^0	z^1	z^2	-----	z^n
1.	a_0	a_1	a_2	-----	a_n
2.	a_n	a_{n-1}	a_{n-2}	-----	a_0
3.	b_0	b_1	b_2	------	b_{n-1}
4.	b_{n-1}	b_{n-2}	b_{n-3}	----	b_0
5.	c_0	c_1	c_2	-----	c_{n-2}

$$b_i = \begin{vmatrix} a_0 & a_{n-i} \\ a_n & a_i \end{vmatrix} = a_0 a_i - a_n a_{n-i}$$

$$b_0 = a_0 a_0 - a_n a_{n-0} = a_0^2 - a_n^2$$

$$b_1 = a_0 a_1 - a_n \cdot a_{n-1}$$

$$c_0 = b_0^2 - b_{n-1}^2$$

$$c_1 = b_0 b_1 - b_{n-1} \cdot b_{n-2}$$

Sufficient cond^n →

(1) $|a_n| > |a_0|$

(2) $|b_{n-1}| < |b_0|$

(3) $|c_{n-2}| < |c_0|$

(4) $|d_{n-3}| < |d_0|$

⋮

Que → $H(z) = \dfrac{2z^3 + 2z^2 + 3z + 1}{2z^4 + 3z^3 + z^2 - 1}$ check stability of sys.

Sol^n → $D(z) = 2z^4 + 3z^3 + z^2 - 1$

Order → $n = 4$, (even)

necessity cond^n:-

(i) $D(1) > 0$ ✓

(ii) $D(-1) > 0$, n = even ✗

Que → check stability of sys. having

$D(z) = 5z^3 + 2z^2 + 4z + 1$

Sol^n → $D(z) = 5z^3 + 2z^2 + 4z + 1$

Order – odd ($n = 3$)

Cond^n → (i) $D(1) > 0$

(ii) $D(-1) < 0$, n = odd

$\downarrow$

-6

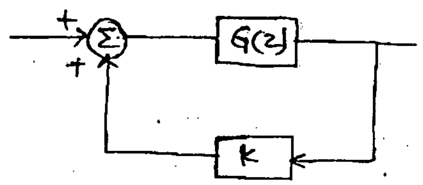
Jury table → No. of rows $2n - 3 = 2 \times 3 - 3 = 3$

		z^0	z^1	z^2	z^3	
a_0	1.	1	4	2	5	a_n
	2.	5	2	4	1	
b_0	3.	−24	−6	−18	0	b_{n-1}
	4	$\cancel{0}$	$\cancel{-18}$	$\cancel{-6}$	$\cancel{-24}$	

Sufficient cond^n:-

(1) $|a_n| > |a_0|$ ✓ (2) $|b_{n-1}| < |b_0|$ ✓

Que.→



$g(n) \rightleftharpoons G(z)$

$g(n) = (\underset{\uparrow}{0}, 1, 1)$

The sys is stable for range of ~~value~~ value of 'k'

(a) $(-1, 1/2)$ (b) $(-1, 1)$ (c) $(-1/2, 1)$ (d) $(-\frac{1}{2}, 2)$

Sol^n→

$$G(z) = z^{-1} + z^{-2}$$

$$H(z) = \frac{G(z)}{1 - kG(z)}$$

$$= \frac{z^{-1} + z^{-2}}{1 - k(z^{-1} + z^{-2})} \times \frac{z^2}{z^2}$$

$$= \frac{z+1}{z^2 - k(z+1)}$$

$$D(z) = z^{2} - kz + 1 \xrightarrow{\text{even}}$$

* $D(1) > 0$; $1 - 2k > 0 \Rightarrow k < \frac{1}{2}$ ----(i)

* $D(-1) > 0$; $1 > 0$

* $|a_n| > |a_0|$

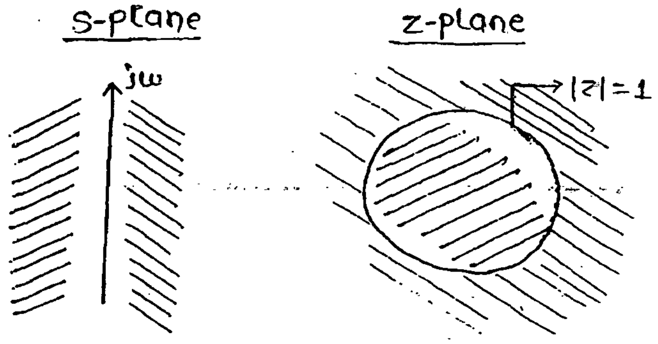
$1 > |k|$

$(-1 < k < 1)$ -----(ii)

from (i) & (ii)

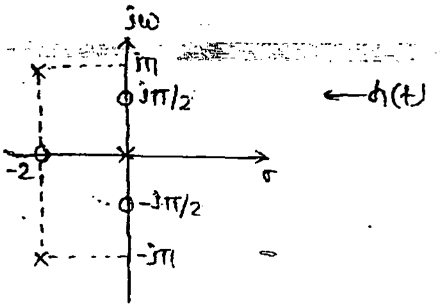
$$\cancel{-\frac{1}{2}} \quad \boxed{-1 < k < \frac{1}{2}}$$

Mapping between s-plane & z-plane →



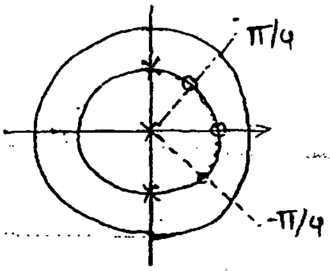
Pole mapping – $\boxed{z=e^{sT}}$ T – sampling time interval

Que →

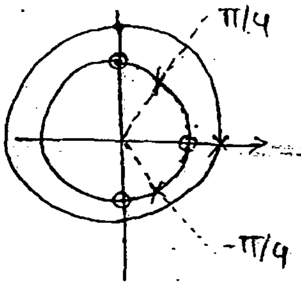


The impulse response of h(t) is sampled at 2kHz to get h(n). which one of the following represents equivalent pole-zero plot of H(z) in z-plane? (The concentric circles are $|z|=1$, $|z|=\frac{1}{e}$)

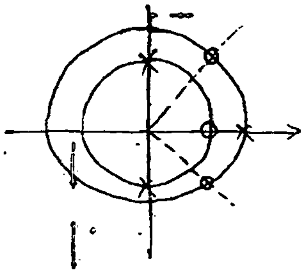
(a.)



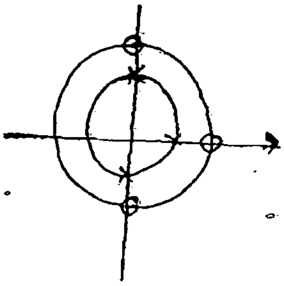
(b.)



(c.)



(d.)



Sol^n → **s-plane**

$$s = 0,\ -2+j\pi,\ -2-j\pi$$

z-plane

$$z = e^{sT} = e^{s/T} = e^{s/2}$$

$$= 1,\ e^{-1+j\pi/2},\ e^{-1-j\pi/2}$$

$$= 1e^{0j},\ \frac{1}{e}e^{j\pi/2},\ \frac{1}{e}e^{-j\pi/2}$$

*** DTFT →**

$$x(n) = X(e^{j\omega})$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\Big\downarrow z = e^{j\omega}$$

$$\boxed{X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}}$$

$$\Big\downarrow (\omega = 0)$$

$$\boxed{X(e^{j0}) = \sum_{n=-\infty}^{\infty} x(n)}$$

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega.$$

$$\Big\downarrow (n = 0)$$

$$x(0) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})\, d\omega$$

$$\boxed{2\pi x(0) = \int_{2\pi} X(e^{j\omega})\, d\omega}$$

16/18

$$Y(e^{j0}) = \sum_{-\infty}^{\infty} y(n)$$

$$= \sum_{0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= 1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \cdots$$

$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

31/20

$$h(n) = \frac{1}{2}\left[\delta(n) + \delta(n-2)\right] \qquad |H(e^{j\omega})| = ?, \quad \omega = \Omega$$

$$\left.\begin{aligned} H(z) &= \frac{1}{2}\left(1 + z^{-2}\right) \\ H(e^{j\omega}) &= \frac{1}{2}\left(1 + e^{-j2\omega}\right) \end{aligned}\right) z = e^{j\omega}$$

$$H(e^{j\omega}) = e^{-j\omega}\left[\frac{e^{j\omega} + e^{-j\omega}}{2}\right]$$

$$|H(e^{j\omega})| = |\cancel{e^{-j\omega}}|^{1} \, |\cos\omega|$$

$$|H(e^{j\omega})| = |\cos\omega|$$

45/31

$$x(n) = \sin(\omega_0 n + \phi) \longrightarrow \boxed{\begin{matrix}\text{LTI} \\ \text{sys.}\end{matrix}}_{H(e^{j\omega})} \longrightarrow y(n) = Ax(n-n_0)$$

$$\angle H(e^{j\omega_0}) = ?$$

$$y(n) = Ax(n-n_0)$$

$$Y(z) = AX(z)\cdot z^{-n_0}$$

$$H(z) = Az^{-n_0}$$

$$\downarrow (z = e^{j\omega})$$

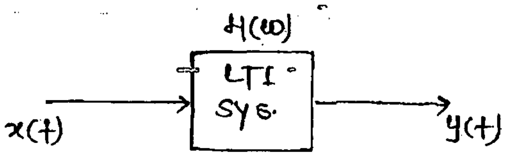
$$H(e^{j\omega}) = A\,e^{-j\omega n_0} \quad \downarrow \omega = \omega_0$$

$$H(e^{j\omega_0}) = Ae^{-j\omega_0 n_0} \cdot e^{j2\pi k} \longrightarrow k = \text{an int.}$$

$$= Ae^{j(2\pi k - n_0\omega_0)}$$

$$\angle H(e^{j\omega_0}) = 2\pi k - n_0\omega_0$$

**



$$x(t) = A_0 \sin(\omega_0 t + \phi)$$

$$y(t) = A_0 \times |H(\omega_0)| \times \sin\left\{[(\omega_0 t + \phi)] + \angle H(\omega_0)\right\}$$

21/19

$$h(t) = e^{-2t}u(t)$$

$$x(t) = 2\cos(2t) \; ; \omega_0 = 2$$

$$y(t) = ?$$

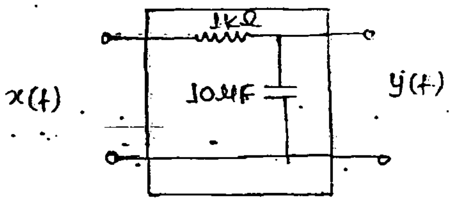
$$H(\omega) = \frac{1}{j\omega + 2}$$

$$H(\omega_0) = \frac{1}{2+2j} \qquad |H(\omega_0)| = \frac{1}{2\sqrt{2}} \;, \angle H(\omega_0) = -\pi/4$$

$$y(t) = 2 \times \frac{1}{2\sqrt{2}} \times \cos(2t - \pi/4)$$

$$= e^{-0.5}\cos(2t - 0.25\pi)$$

11/22



$$x(t) = 3 + 4\sin 100t$$

$$\omega_0 = 100$$

$$H(s) = \frac{1}{1 + sCR}$$

$$\big\downarrow (s = j\omega)$$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$\big\downarrow (\omega = \omega_0)$$

$$H(\omega_0) = \frac{1}{1 + j\omega_0 RC} \qquad \left\{ \begin{array}{l} \omega_0 RC = 100 \times 10^3 \times 10 \times 10^{-6} \\ \quad = 1 \end{array} \right.$$

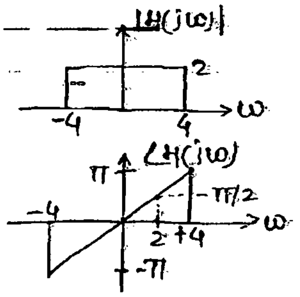
$$= \frac{1}{1+j}$$

$$|H(\omega_0)| = \frac{1}{\sqrt{2}} \;, \angle H(\omega_0) = -\frac{\pi}{4}$$

$$y(t) = 3 + 4 \times \frac{1}{\sqrt{2}} \times \sin[100t + (-\pi/4)]$$

$$= 3 + \frac{4}{\sqrt{2}} \sin(100t - \pi/4)$$

Que.→



$x(t) = 2\sin 2t$

$y(t) = ?$

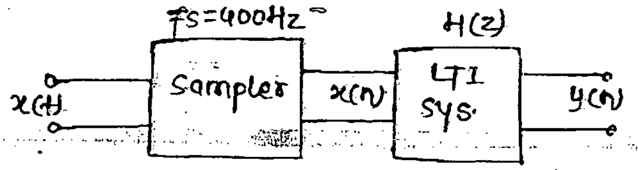
Sol^n→

$\omega_0 = 2 \quad x(t) = 2\sin 2t$

$y(t) = 2\times 2\times \sin(2t + \pi/2)$

$y(t) = 4\cos 2t$

Que.→



$x(t) = 2 + 5\sin(100\pi t)$

$H(z) = \frac{1}{N}\left[\frac{1-z^{-N}}{1-z^{-1}}\right]$ where N = no. of samples per cycle

The o/p. y(n) of sys. under steady state is

(a) 0 (b) 1 (c) 2 (d) 5

Sol^n→ $y(\infty) = ?$; N = no. of samples per cycle = sampling freq.

$= 400$

$x(t) = 2 + 5\sin(100\pi t)$

$\downarrow t = nT_s = \frac{n}{f_s} = \frac{n}{400}$

$x(n) = 2 + 5\sin\left(\cancel{100}\pi \times \frac{n}{\cancel{400}}\right)$

$= 2 + 5\sin\left(\frac{n\pi}{4}\right) \quad ; \omega_0 = \frac{\pi}{4}$

$(z = e^{j\omega})$ $H(\omega) = \frac{1}{N}\left[\frac{1-e^{-j\omega N}}{1-e^{-j\omega}}\right]$ $\Big\downarrow (\omega = \omega_0 = \frac{\pi}{4})$

$H(\omega_0) = \frac{1}{N}\left[\frac{1-e^{-j\frac{\pi}{4}N}}{1-e^{-j\pi/4}}\right]^{N=400} \neq 1$ $\quad (\because e^{-j100\pi} = 1)$

$H(\omega_0) = 0$

$y(n) = 2 +$ o/p due to ~~sinusoidal~~ part $\to 0$

$= 2$

$\boxed{y(\infty) = 2}$

Que.$\to$ $x(n) \rightleftharpoons X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$

$x(n)$ is orthogonal to the signal.

(a.) $y_1(n) = Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$

(b.) $y_2(n) = Y_2(z) = \sum_{n=0}^{\infty} (5^n \cdot n)\, z^{-(2n+1)}$

(c.) $y_3(n) = Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$

(d.) $y_4(n) = Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

Sol^n$\to$

$$\sum_{n=-\infty}^{\infty} x_1(n) \cdot x_2(n) = 0$$

(orthogonal)

$y_2(n)$ is available for odd instants of 'n'

ans.(b)

$x_1(n)\, y_2(n) = 0$ so $\sum_{-\infty}^{\infty} x(n)\, y_2(n) = 0$

so; they are orthogonal.

$\frac{3}{4}$ $3y^2(t) + 2y^2(t) + y(t) = x^2(t) + x(t)$

all values are present (static)

NL (sq. term)

$\frac{10}{4}$ anticipator – anticausal sys.

$\frac{11}{4}$ $h(n) = \delta(n+2) - \delta(n-2)$ (OR) $H(z) = z^2 - z^{-2}$

$y(n) = x(n+2) - x(n-2)$ (a.) $H(z) =$

$(n = n-2)$

$y(n-2) = x(n) - x(n-4)$

$\frac{14}{5}$ $y(n) = x(n) * h(n)$

$= \sum x(k) \cdot h(n-k)$

$= \sum a^k u(k) \cdot b^{n-k} u(n-k)$

16/5

$$x(n) = \begin{cases} 0, & n<-2 \text{ or } n>4 \\ 1 & \text{otherwise.} \end{cases}$$

$\downarrow$

$(n-2)$

$x(n-2) = 0 \quad n-2<-2 \quad (-n-2)>4.$

$n>0 \quad n<-6$

22/6

even part of $u(t) = \dfrac{u(t)+u(-t)}{2} = \dfrac{1}{2}$

Odd part of $u(t) = \dfrac{u(t)-u(-t)}{2}$ $\qquad \because u(t)+u(-t)=1$

$u(-t) = 1-u(t)$

$= \dfrac{u(t)-1+u(t)}{2}$

$= \dfrac{2u(t)-1}{2}$ $\qquad \because u(t) = \dfrac{1+sgn(t)}{2}$

$= \dfrac{sgn(t)}{2}$ $\qquad sgn(t) = 2u(t)-1$

$= \dfrac{x(t)}{2}$

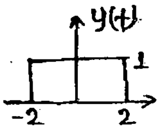
Ans. $\left(\dfrac{1}{2}, \dfrac{x(t)}{2}\right)$

27/7

$x(t) = \delta(t+2) - \delta(t-2)$

$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

$= u(t+2) - u(t-2)$



$E_y = \int_{-\infty}^{\infty} |y(t)|^2 dt$

$= \int_{-2}^{2} 1^2 dt = 4$

28/7 scaling property of convoⁿ

$(t^2+t) \quad (t)$

$x_1(t) * x_2(t) = y(t)$

$x_1(at) * x_2(at) = \frac{1}{|a|} y(at)$

$\downarrow a=3$

$x_1(3t) * x_2(3t) = \frac{1}{3} y(3t)$

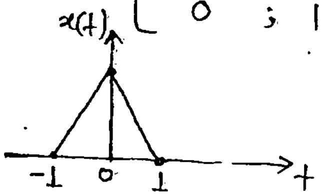
$(t=t+3)$

$(9t^2+3t) \quad (3t)$

32/8

$$x(t) = \begin{cases} 1-|t| & |t|\le 1 \\ 0 & |t|\ge 1 \end{cases} \quad (-1\le t\le 1)$$

$$= \begin{cases} 1+t \; ; & -1\le t\le 0 \\ 1-t \; ; & 0\le t\le 1 \\ 0 \; ; & |t|>1 \end{cases}$$



Chapter-02

1/9

$y(n+2) - 5y(n+1) + 6y(n) = x(n)$

$H(z) = \frac{1}{z^2-5z+6}$

$H(z) = \frac{1}{(z-3)(z-2)}$

Poles:- 3, 2

4/9

$y(t) + \int_{0^-}^{\infty} y(\tau)x(t-\tau)d\tau = \delta(t) + x(t)$

$y(t) + y(t) * x(t) = \delta(t) + x(t)$

$\downarrow (LT)$

$Y(s) + Y(s)\cdot X(s) = 1 + X(s)$

$Y(s)[1+X(s)] = 1 + X(s)$

$h(n) = (1, \frac{1}{2}, \frac{1}{4})$

$x(n) = (1, 0, 1)$

$y(n) = x(n) * h(n)$

Tabular method

15/10

$x(t) = u(t) \longrightarrow y(t) = 0.5(1-e^{-2t})u(t)$

$X(s) = \frac{1}{s}$; $Y(s) = 0.5\left(\frac{1}{s} - \frac{1}{s+2}\right)$

$= \frac{1}{s(s+2)}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$

$h(t) = e^{-2t}u(t)$

17/10

(i) $y(t) = t\,x(t) \longrightarrow L$

(ii) $y(t) = t\,x^2(t) \longrightarrow NL$

(iii) $y(t) = x(2t) \longrightarrow L$

18/11

$y(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau)d\tau$

$\downarrow y(t)=h(t) \qquad \downarrow x(t)=\delta(t)$

$h(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} \delta(\tau)d\tau$

$h(t) = \frac{1}{T}\left[u(\tau)\right]_{t-T/2}^{t+T/2}$

$= \frac{1}{T}\left[u(t+T/2) - u(t-T/2)\right]$

$1/T \; h(t)$

19/11

$h(t) = \delta(t) + \delta(t-1) \rightleftharpoons H(s) = 1 + e^{-s}$

$y(t) = u(t) + u(t-1) \rightleftharpoons Y(s) = \frac{1}{s} + \frac{1}{s}e^{-s}$

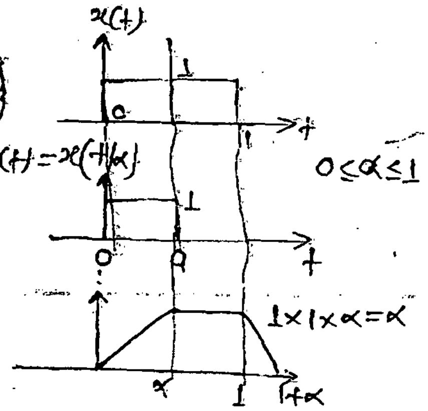
$= \frac{1}{s}(1+e^{-s})$

$H(s) = \frac{Y(s)}{X(s)}$

$X(s) = \frac{1}{s}$

$x(t) = u(t)$

20/11



$0 \le \alpha \le 1$

21/12

LPF; $f_c = 100\,Hz$

$v(t) = 30\sqrt{2}\sin(1256t)$

$f_i = \frac{1256}{2\pi} \approx 200\,Hz$

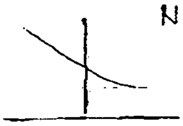
Chapter-03

9/14

$x_1(t) \to e^{j20t}$ (P)

$x_2(t) \to e^{(-2+j)t} = e^{-2t} e^{jt}$

$x_2(t)$ ↓ NP, e^{-2t} ↓ N, e^{jt} ↓ P



13/14

$e^{-|t|}$ ↓ NP

$\sin 25t$ ↓ P

13/15

$v(t) = (10\sin 2\pi t)\ 100t$

↓ NP, ↓ P, ↓ NP

Chapter-04

4/16

C+P → CTFS; D+NP

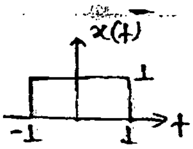
CTFT C+NP

D+P DTFS

DTFT D+NP

Dirchelts cond^n NP

20/19



$X(\omega) = 2Sa(\omega)$

$X(\omega) = 0,\ \omega = ?$

$2Sa(\omega) = 0$

$Sa(\omega) = 0$

$\frac{\sin\omega}{\omega} = 0$

$\sin\omega = 0$

$\omega = n\pi;\ n \neq 0$

8/17

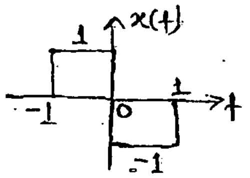
$y(n) - \frac{1}{2}y(n-1) = x(n) = k\delta(n)$

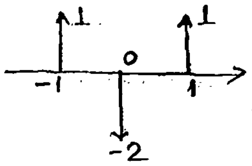
$Y(z) - \frac{1}{2}Y(z)z^{-1} = X(z) = k$

$Y(z) = \frac{k}{1-\frac{1}{2}z^{-1}}$

$y(n) = k\left(\frac{1}{2}\right)^n u(n)$

(24/19)



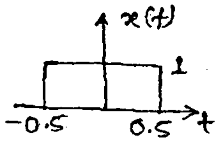


$$\frac{dx(t)}{dt} = \delta(t+1) + \delta(t-1) - 2\delta(t)$$

$$j\omega X(\omega) = (e^{j\omega} + e^{-j\omega}) - 2$$

$$X(\omega) = \frac{2\cos\omega - 2}{j\omega}$$

(25/19)



$\rightleftharpoons X(\omega) = Sa\left(\frac{\omega}{2}\right)$

$$h(t) = e^{j\omega_0 t} \rightleftharpoons H(\omega) = 2\pi\delta(\omega - \omega_0)$$

$y(t) = 0,\ \omega_0 = ?$

$Y(\omega) = 0$

$X(\omega)\cdot H(\omega) = 0$

$$Sa\left(\frac{\omega}{2}\right)\cdot 2\pi\delta(\omega-\omega_0) = 0$$

$$Sa\left(\frac{\omega_0}{2}\right) 2\pi\delta(\omega-\omega_0) = 0$$

$$Sa\left(\frac{\omega_0}{2}\right) = 0$$

$$\frac{\sin\left(\frac{\omega_0}{2}\right)}{\left(\frac{\omega_0}{2}\right)} = 0$$

$$\sin\left(\frac{\omega_0}{2}\right) = 0$$

$$\frac{\omega_0}{2} = n\pi,\ n \neq 0$$

$$\omega_0 = 2n\pi,\ n \neq 0$$

Chapter-05

(2/21)

$$x(t) = u(t) \rightleftharpoons y(t) = t^2 e^{-2t} u(t)$$

$$t^2 u(t) \rightleftharpoons \frac{2}{s^3}$$

$$e^{-2t} t^2 u(t) \rightleftharpoons \frac{2}{(s+2)^3}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{2}{(s+2)^3}}{1/s} = \frac{2s}{(s+2)^3}$$

(9/21)

$$\frac{d^2y(t)}{dt^2} = x(t-2)u(t-2) + \frac{d^2x(t)}{dt^2}$$

$H(s) \longrightarrow$ LTI

(12/22)

$$f(t) \rightleftharpoons F(s) = \frac{s+2}{s^2+1}$$

$$g(t) \rightleftharpoons G(s) = \frac{s^2+1}{(s+3)(s+2)}$$

$$h(t) = \int_0^t f(\tau)\cdot g(t-\tau)d\tau \rightleftharpoons H(s) = ?$$

$$= \int_{-\infty}^{\infty} f(\tau)\cdot g(t-\tau)d\tau$$

$$h(t) = f(t) * g(t)$$

Let assump $\rightarrow f(t)$ & $g(t) \rightarrow$ causal

$$H(s) = F(s)\,G(s) = \frac{1}{s+3}$$

(23/24)

$x(t) = \sin\omega t$

$y(\infty) = 0,\ \omega = ?$

$y(t) = 0,\ \omega = ?$

$Y(s) = 0$

$H(s)\cdot X(s) = 0$

$(s = j\omega)$ $H(s) = 0$

$H(\omega) = 0,\ \omega = ?$

26/24

$f(t) \rightleftharpoons F_1(s)$

$f(t-\tau) \rightleftharpoons F_2(s) = F_1(s)\, e^{-s\tau}$

$g(t) \rightleftharpoons G(s) = \dfrac{F_2(s)\cdot F_1(s)}{|F(s)|^2}$

?

$G(s) = \dfrac{F_2(s)\cdot F_1^*(s)}{F_1(s)\cdot F_1^*(s)} = e^{-s\tau}$

A

$g(t) = \delta(t-\tau)$

29/24

$y(t)=0,\ \omega = ?$

$Y(s)=0$

$\frac{1}{s} - \frac{3}{s+1} + \frac{3}{s+2} = 0$

$(s+1)(s+2) - 3s(s+2) + 3s(s+1) = 0$

$s^2 + 3s + 2 - 3s^2 - 6s + 3s^2 + 3s = 0$

$s^2 + 2 = 0$

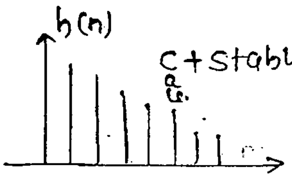
$(j\omega)^2 + 2 = 0$

$-\omega^2 + 2 = 0$

$\omega = \sqrt{2}$ rad/s.

$\boxed{\text{Chapter-06}}$

1/25



C+stable

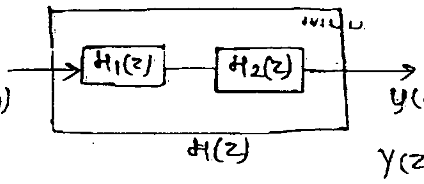
$|\text{Poles}| < 1$

7/26

$a^n u(n) \rightleftharpoons \frac{z}{z-a}$, $|z|>|a|$

$-a^n u(-n-1) \rightleftharpoons \frac{z}{z-a}$ $|z|<|a|$

8/25



$y(n) = x(n-2)$

$Y(z) = X(z) z^{-2}$

$H(z) = z^{-2}$

$H(z) = H_1(z)\cdot H_2(z)$

$H_2(z) = \dfrac{H(z)}{H_1(z)}$

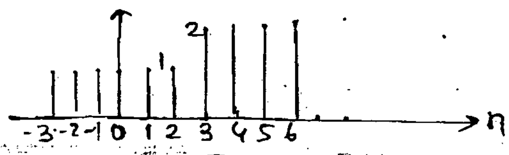
$H_2(z) = \dfrac{z^{-2}(z-0.8)}{z-0.5}$

$= \dfrac{z^{-1} - 0.8z^{-2}}{z-0.5} \times \dfrac{z^{-1}}{z^{-1}}$

$= \dfrac{z^{-2} - 0.8z^{-3}}{1-0.5z^{-1}}$

15/27

$h(n) = u(n+3) + u(n-3) - 2u(n-7)$



35/30

$x(n)$ $y(n)$ LTI sys.

$u(n) \rightarrow s(n) = $ step responce

$\downarrow d/dn$

$\delta(n) \longrightarrow h(n) = \frac{ds(n)}{dn}$

$= s(n) - s(n-1)$

$h(n) = s(n) - s(n-1)$

$s(n) = (\underset{\uparrow}{1}, 1/2, 1/4, 1/8 \ldots)$

$s(n-1) = (\underset{\uparrow}{0}, 1, 1/2, 1/4 \ldots)$

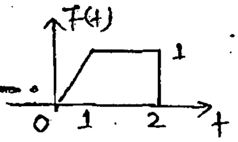
$h(n) = (1, -1/2, -1/4, -1/8 \ldots)$

56/32

$G(s) \rightarrow$ Poles $\Rightarrow s = 0.5$

$z = e^{sT} = 1, e^{-sT}$

$\frac{1}{1})$



$$f(t) = r(t) - r(t-1) - u(t-2)$$
$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}$$
$$= \frac{1}{s^2}\left(1 - e^{-s} - s e^{-2s}\right)$$

$\frac{1}{1})$

$$\frac{d^2y}{dt^2} = x(t-2) + \frac{d^2x}{dt^2}$$
$$s^2 Y(s) = e^{-2s} X(s) + s^2 X(s)$$
$$\frac{Y(s)}{X(s)} = \frac{s^2 + e^{-2s}}{s^2} = 1 + \frac{e^{-2s}}{s^2}$$

)

$$F(s) = \frac{27s+97}{s(s+33)} \rightleftharpoons f(t)$$
$$\frac{27s+97}{s(s+33)} = \frac{A}{s} + \frac{B}{s+33} \qquad s=0,\ A = \frac{97}{33}$$
$$s=-33,\ B = \frac{-27\times 33+97}{-33\times 33} = \frac{794}{1089}$$
$$f(t) = \frac{97}{33}u(t) + \frac{794}{1089}e^{-33t}u(t)$$
$$f(0^+) = \frac{97}{33}u(0^+) + \frac{794}{1089}e^{-33(0^+)} = \frac{97}{33}$$

2)

$$x_1(t) = e^{k1t}u(t) \qquad x_2(t) = e^{-k2t}u(t)$$