

CHAPTER 15

Investment, Time, and Capital Markets

In Chapter 14 we saw that in competitive markets, firms compare the marginal revenue product of each factor to its cost to decide how much to purchase each month. The decisions of all firms determine the market demand for each factor, and the market price is the one that equates the quantity demanded with the quantity supplied. For factor inputs such as labor and raw materials, this picture is reasonably complete, but not so for capital. The reason is that capital is *durable*—it can last and contribute to production for years after it is purchased.

Firms sometimes rent capital much the way they hire workers. For example, a firm might rent office space for a monthly fee, just as it hires a worker for a monthly wage. But more often, capital expenditures involve the purchases of factories and equipment that are expected to last for years. This introduces the element of *time*. When a firm decides whether to build a factory or purchase machines, it must compare the outlays it would have to make *now* with the additional profit the new capital will generate *in the future*. To make this comparison, the firm must address the following question: *How much are future profits worth today?* This problem does not arise when hiring labor or purchasing raw materials. To make those choices, the firm need only compare its *current* expenditure on the factor, e.g., the wage or the price of steel, with the factor's *current* marginal revenue product.

In this chapter we will learn how to calculate the current value of future flows of money. This is the basis for our study of the firm's investment decisions. Most of these decisions involve comparing an outlay today with profits that will be received in the future; we will see how firms can make this comparison and determine whether the outlay is warranted. Often, the future profits resulting from a capital investment may be higher or lower than anticipated. We will see how firms can take this kind of uncertainty into account.

We will also examine other intertemporal decisions that firms sometimes face. For example, producing a depletable resource, such as coal or oil, now means that less will be available to produce in the future. How should a producer take this into account? And how long should a timber company let the trees on its land grow before harvesting them for lumber?

The answers to these investment and production decisions depend in part on the *interest rate* that one pays or receives when borrowing or lending money. We will discuss what determines interest rates, and why interest rates on government bonds, corporate bonds, and savings accounts differ.

15.1 Stocks Versus Flows

Before proceeding, we must be clear about how to measure capital and other factor inputs that firms purchase. Capital is measured as a *stock*, i.e., a quantity of plant and equipment that the firm owns. For example, if a firm owns an electric motor factory worth \$10 million, we say that it has a *capital stock* worth \$10 million. Inputs of labor and raw materials, on the other hand, are measured as *flows*, as is the output of the firm. For example, this same firm might use 20,000 man-hours of labor and 50,000 pounds of copper *per month* to produce 8,000 electric motors *per month*. (The choice of monthly units is arbitrary; we could just as well have expressed these quantities in weekly or annual terms, for example, 240,000 man-hours of labor, per year, 600,000 pounds of copper per year, and 96,000 motors per year.)

Let's look at this producer of electric motors in more detail. Both variable cost and the rate of output are flows. Suppose the wage rate is \$15 per hour, and the price of copper is 80 cents per pound. Then variable cost is $(20,000)(\$15) + (50,000)(\$0.80) = \$340,000$ *per month*. Average variable cost, on the other hand, is a cost *per unit*: $(\$340,000 \text{ per month}) / (8,000 \text{ units per month}) = \42.50 per unit.

Suppose the firm sells its electric motors for \$52.50 each. Then its average profit is $\$52.50 - \$42.50 = \$10.00$ per unit, and its total profit is \$80,000 *per month*. (Note that this is also a flow.) To make and sell these motors, however, the firm needs capital—the factory that it built for \$10 million. *Thus, the firm's \$10 million capital stock allows it to earn a flow of profit of \$80,000 per month.*

Was the \$10 million investment in this factory a sound decision? To answer this question, we need to translate the \$80,000 per month profit flow into a number that we can compare with the factory's \$10 million cost. Suppose the factory is expected to last for 20 years. Then, simply put, the problem is: What is the value today of \$80,000 per month for the next 20 years? If that value is greater than \$10 million, the investment was a good one.

A profit of \$80,000 per month for 20 years comes to $(\$80,000)(20)(12) = \19.2 million. That would make the factory seem like an excellent investment. But

is \$80,000 five years-or 20 years-from now worth \$80,000 today? No, because money today can be invested in a bank account, a bond, or other interest-bearing assets to yield more money in the future. As a result, \$19.2 million received over the next 20 years is worth *less* than \$19.2 million today.

15.2 Present Discounted Value

We will return to the \$10 million electric motor factory in Section 15.4, but first we must address a basic problem: *How much is \$1 paid in the future worth today?* The answer depends on the *interest rate*, the rate at which one can borrow or lend money.

Suppose the interest rate is R . (Don't worry about which interest rate this actually is; later, we'll discuss how to choose among the various types of interest rates.) Then \$1 today can be invested to yield $(1 + R)$ dollars a year from now. Therefore, $1 + R$ dollars is *the future value* of \$1 today. Now, what is the value today, i.e., the *present discounted value* (PDV), of \$1 paid one year from now? The answer is easy, once we see that $1 + R$ dollars one year from now is worth $(1 + R)/(1 + R) = \$1$ today. Therefore, *\$1 a year from now is worth $\$1/(1 + R)$ today.* This is the amount of money that will yield \$1 after one year if invested at the rate R .

What is the value today of \$1 paid *two* years from now? If \$1 were invested today at the interest rate R , it would be worth $1 + R$ dollars after one year, and $(1 + R)(1 + R) = (1 + R)^2$ dollars at the end of two years. Since $(1 + R)^2$ dollars two years from now is worth \$1 today, \$1 two years from now is worth

TABLE 15.1 PDV of \$1 Paid in the Future

Interest Rate	1 Year	2 Years	5 Years	10 Years	20 Years	30 Years
0.01	\$0.990	\$0.980	\$0.951	\$0.905	\$0.820	\$0.742
0.02	0.980	0.961	0.906	0.820	0.673	0.552
0.03	0.971	0.943	0.863	0.744	0.554	0.412
0.04	0.962	0.925	0.822	0.676	0.456	0.308
0.05	0.952	0.907	0.784	0.614	0.377	0.231
0.06	0.943	0.890	0.747	0.558	0.312	0.174
0.07	0.935	0.873	0.713	0.508	0.258	0.131
0.08	0.926	0.857	0.681	0.463	0.215	0.099
0.09	0.917	0.842	0.650	0.422	0.178	0.075
0.10	0.909	0.826	0.621	0.386	0.149	0.057
0.15	0.870	0.756	0.497	0.247	0.061	0.015
F.20	0.833	0.694	0.402	0.162	0.026	0.004

$\$1/(1 + R)^2$ today. Similarly, \$1 paid three years from now is worth $\$1/(1 + R)^3$ today, and \$1 paid n years from now is worth $\$1/(1 + R)^n$ today.¹ We can summarize this as follows:

$\begin{aligned} \text{PDV of \$1 paid after 1 year} &= \frac{\$1}{(1 + R)} \\ \text{PDV of \$1 paid after 2 years} &= \frac{\$1}{(1 + R)^2} \\ \text{PDV of \$1 paid after 3 years} &= \frac{\$1}{(1 + R)^3} \\ &\vdots \\ \text{PDV of \$1 paid after } n \text{ years} &= \frac{\$1}{(1 + R)^n} \end{aligned}$

Table 15.1 shows, for different interest rates, the present value of \$1 paid after 1, 2, 5, 10, 20, and 30 years. Note that for interest rates above 6 or 7 percent, \$1 paid 20 or 30 years from now is worth very little today; but this is not the case for low interest rates. For example, if R is 3 percent, the PDV of \$1 paid 20 years from now is about 55 cents. Put another way, if 55 cents were invested now at the rate of 3 percent, it would yield about \$1 after 20 years.

Valuing Payment Streams

We can now determine the present value of a stream of payments over time. For example, consider the two payment streams in Table 15.2. Stream A comes to \$200: \$100 paid now and \$100 a year from now. Stream B comes to \$220: \$20 paid now, \$100 a year from now, and \$100 two years from now. Which of these two payment streams would you prefer to receive? The answer depends on the interest rate.

	Today	1 Year	2 Years
Payment Stream A:	\$100	\$100	0
Payment Stream B:	\$ 20	\$100	\$100

¹ We are assuming that the annual rate of interest R is constant from year to year. Suppose the annual interest rate were expected to change, so that R_1 is the rate in year 1, R_2 is the rate in year 2, and so forth. After two years, \$1 invested today would be worth $(1 + R_1)(1 + R_2)$, so that the PDV of \$1 received two years from now is $\$1/(1 + R_1)(1 + R_2)$. Similarly, the PDV of \$1 received n years from now is $\$1/(1 + R_1)(1 + R_2)(1 + R_3) \cdots (1 + R_n)$.

TABLE 15.3 PDV of Payment Streams

	$R = .05$	$R = .10$	$R = .15$	$R = .20$
PDV of Stream A:	\$195.24	\$190.90	\$186.96	\$183.33
PDV of Stream B:	205.94	193.54	182.57	172.77

To calculate the present discounted value of these two streams, we compute and add the present values of each year's payment:

$$\text{PDV of Stream A} = \frac{\$100}{(1 + R)}$$

$$\text{PDV of Stream B} = \frac{\$100}{(1 + R)} + \frac{\$100}{(1 + R)^2}$$

Table 15.3 shows the present values of the two streams for interest rates of 5, 10, 15, and 20 percent. As the table shows, which stream is preferred depends on the interest rate. For interest rates of 10 percent or less, Stream B is worth more; for interest rates of 15 percent or more, Stream A is worth more. The reason is that less is paid out in Stream A, but it is paid out sooner.

EXAMPLE 15.1 THE VALUE OF LOST EARNINGS

In legal cases involving accidents, victims or their heirs (if the victim is killed) sue the injuring party (or an insurance company) to recover damages. In addition to compensating for pain and suffering, those damages include the future income that the injured or deceased person would have earned had the accident not occurred. To see how the present value of these lost earnings can be calculated, let's examine an actual 1986 accident case. (The names and some of the data have been changed to preserve anonymity.)

Harold Jennings died in an automobile accident on January 1, 1986, at the age of 53. His family sued the driver of the other car for negligence. A major part of the damages they asked to be awarded was the present value of the earnings that Mr. Jennings would have received from his job as an airline pilot had he not been killed. The calculation of present value was typical of cases like this.

Had he worked in 1986, Mr. Jennings' salary would have been \$85,000, and the normal age of retirement for an airline pilot is age 60. To calculate the present value of Mr. Jennings' lost earnings, we need to take several things into account. First, Mr. Jennings' salary would probably have increased over the years. Second, we cannot be sure that he would have lived to retirement had the accident not occurred; he might have died from some other cause. The PDV of his lost earnings until retirement at the end of 1993 is therefore

$$\begin{aligned} \text{PDV} = & W_0 + \frac{W_0(1+g)(1-m_1)}{(1+R)} + \frac{W_0(1+g)^2(1-m_2)}{(1+R)^2} \\ & + \dots + \frac{W_0(1+g)^7(1-m_7)}{(1+R)^7} \end{aligned}$$

where W_0 is his salary in 1986, g is the annual percentage rate at which his salary is likely to have grown (so that $W_0(1+g)$ would be his salary in 1987, $W_0(1+g)^2$ would be his salary in 1988, etc.), and m_1, m_2, \dots, m_7 are *mortality rates*, i.e., the probabilities that he would have died from some other cause by 1987, 1988, . . . , 1993.

To calculate this PDV, we need to know the mortality rates m_1, \dots, m_7 , the expected rate of growth of Mr. Jennings' salary g , and the interest rate R . Mortality data are available from insurance tables that provide death rates for men of similar age and race.² As a value for g , we can use 8 percent, the average rate of growth of wages for airline pilots over the past decade. Finally, for the interest rate we can use the rate on government bonds, which in 1986 was about 9 percent. (We will say more about how one chooses the correct interest rate to discount future cash flows in Sections 15.4 and 15.5.) Table 15.4 shows the details of the present value calculation.

By summing the last column we obtain a PDV of \$650,252. If Mr. Jennings's family were successful in proving that the defendant was at fault, and there were no other damage issues involved in the case, they could recover this amount as compensation.³

TABLE 15.4 Calculating Lost Wages

Year	$W_0(1+g)^t$	$(1-m_t)$	$1/(1+R)^t$	$W_0(1+g)^t(1-m_t)/(1+R)^t$
1986	\$ 85,000	.991	1.000	\$84,235
1987	91,800	.990	.917	83,339
1988	99,144	.989	.842	82,561
1989	107,076	.988	.772	81,671
1990	115,642	.987	.708	80,810
1991	124,893	.986	.650	80,043
1992	134,884	.985	.596	79,185
1993	145,675	.984	.547	78,408

² See, for example, the *Statistical Abstract of the United States*, 1992, Table 107.

³ Actually, this sum should be reduced by the amount of Mr. Jennings' wages that would have been spent on his own consumption, and therefore would not have benefitted his wife or children.

15.3 *The Value of a Bond*

A bond is a contract in which a borrower agrees to pay the bondholder (the lender) a stream of money. For example, a corporate bond (a bond issued by a corporation) might make "coupon" payments of \$100 per year for the next ten years, and then a principal payment of \$1000 at the end of ten years.⁴ How much would you pay for such a bond? To find out how much the bond is worth, we simply compute the present value of the payment stream:

$$PDV = \frac{\$100}{(1 + R)} + \frac{\$100}{(1 + R)^2} + \cdots + \frac{\$100}{(1 + R)^{10}} + \frac{\$1000}{(1 + R)^{10}} \quad (15.1)$$

Again, the present value depends on the interest rate. Figure 15.1 shows the value of the bond—the present value of its payment stream—for interest rates up to 20 percent. Note that the higher the interest rate, the lower the value of

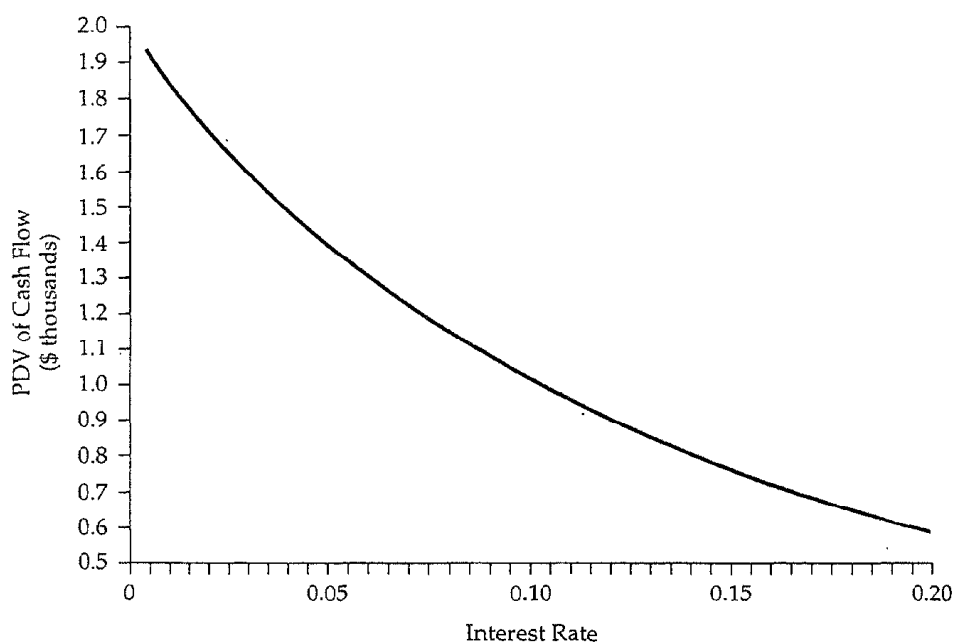


FIGURE 15.1 Present Value of the Cash Flow from a Bond. Because most of the bond's payments occur in the future, the present discounted value declines as the interest rate increases. For example, when the interest rate is 5 percent, the PDV of a 10-year bond paying \$100 per year on a principal of \$1000 is \$1386.

In the United States, the coupon payments on most corporate bonds are made in semiannual installments. To keep the arithmetic simple, we will assume that the payments are made annually.

the bond. At an interest rate of 5 percent, the bond is worth about \$1386, but at an interest rate of 15 percent, its value is only \$749.

Perpetuities

A *perpetuity* is a bond that pays out a fixed amount of money each year, *forever*. How much is a perpetuity that pays \$100 per year worth? The present value of the payment stream is given by the infinite summation:

$$\text{PDV} = \frac{\$100}{(1 + R)} + \frac{\$100}{(1 + R)^2} + \frac{\$100}{(1 + R)^3} + \frac{\$100}{(1 + R)^4} + \dots$$

Fortunately, it isn't necessary to calculate and add up all these terms to find the value of this perpetuity; the summation can be expressed in terms of a simple formula:⁵

$$\text{PDV} = \$100/R \quad (15.2)$$

So if the interest rate is 5 percent, the perpetuity is worth $\$100/(.05) = \2000 , but if the interest rate is 20 percent, the perpetuity is worth only \$500.

The Effective Yield on a Bond

Many corporate and most government bonds are traded in the *bond market*. The value of a traded bond can be determined directly by looking at its market price, since this is what buyers and sellers agree that the bond is worth.⁶ Thus, we usually know the value of a bond, but to compare the bond with other investment opportunities, we would like to determine the interest rate consistent with that value.

Equations (15.1) and (15.2) show how the values of two different bonds depend on the interest rate used to discount future payments. These equations can be "turned around" to relate the interest rate to the bond's value. This is particularly easy to do for the perpetuity. Suppose the market price—and hence the value—of the perpetuity is P . Then from equation (15.2), $P = \$100/R$, and $R = \$100/P$. So if the price of the perpetuity is \$1000, we know that the interest rate is $R = \$100/\$1000 = 0.10$, or 10 percent. This interest rate is called the *effective yield*, or *rate of return*. It is the percentage return that one receives by investing in the perpetuity.

For the ten-year coupon bond in equation (15.1), calculating the effective yield is a bit more complicated. If the price of the bond is P , we write equation (15.1) as

⁵ Let x be the PDV of \$1 per year in perpetuity, so $x = 1/(1 + R) + 1/(1 + R)^2 + \dots$. Then $x(1 + R) = 1 + 1/(1 + R) + 1/(1 + R)^2 + \dots$, so $x(1 + R) = 1 + x$, $xR = 1$, and $x = 1/R$.

⁶ The prices of actively traded corporate and U.S. government bonds are shown daily in newspapers, such as the *Wall Street Journal* and the *New York Times*.

$$P = \frac{\$100}{(1+R)} + \frac{\$100}{(1+R)^2} + \frac{\$100}{(1+R)^3} + \cdots + \frac{\$100}{(1+R)^{10}} + \frac{\$1000}{(1+R)^{10}}$$

Given the price P , this equation must be solved for R . Although no simple formula expresses R in terms of P in this case, there are methods (sometimes available on hand-held calculators) for calculating R numerically. Figure 15.2, which plots the same curve that is in Figure 15.1, shows how R depends on P for this bond. Note that if the price of the bond is \$1000, the effective yield is 10 percent. If the price rises to \$1300, the figure shows that the effective yield drops to about 6 percent. If the price falls to \$700, the effective yield rises to over 16 percent.

Yields can differ considerably among different bonds. Corporate bonds generally yield more than government bonds, and as Example 15.2 shows, the bonds of some corporations yield much more than the bonds of others. One of the most important reasons for this is that different bonds carry different degrees of risk. The U.S. government is less likely to *default* (fail to make interest or principal payments) on its bonds than is a private corporation. And

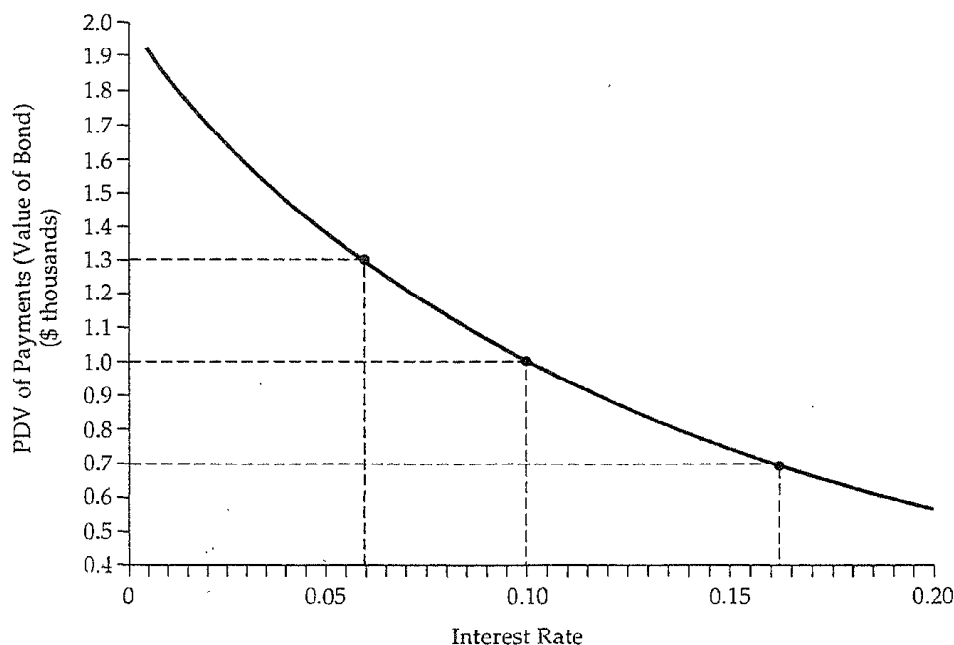


FIGURE 15.2 Effective Yield on a Bond. The effective yield is the interest rate that equates the present value of the bond's payment stream with the bond's market price. The figure shows the present value of the payment stream as a function of the interest rate, so the effective yield can be found by drawing a horizontal line at the level of the bond's price. For example, if the price of this bond were \$1000, its effective yield would be about 10 percent. If the price were \$1300, the effective yield would be about 6 percent, and if the price were \$700, it would be 16.2 percent.

some corporations are financially stronger and therefore less likely to default on their bonds than others. As we saw in Chapter 5, the more risky an investment is, the greater the return that an investor demands. As a result, riskier bonds have higher yields.

EXAMPLE 15.2 THE YIELDS ON CORPORATE BONDS

To see how corporate bond yields are calculated, and how they can differ from one corporation to another, *let's* examine the yields for two coupon bonds—one issued by IBM and the other by the Chrysler Corporation. Each has a *face value* of \$100, which means that when the bond matures, the holder receives a principal payment of that amount. Each bond makes a "coupon" (i.e., interest) payment every six months.

We calculate the bond yields using the closing prices on September 21, 1990. The following information on the bonds appeared on the bond page of the newspapers on September 22:

For IBM:

IBM 9¾04 9.4 206 100⅝ 99⅞ 99⅞ -⅝

For Chrysler:

Chrysler 13s97 13.6 258 96 94¾ 95¼ -¾

What do these numbers mean? For IBM, 9¾ refers to the coupon payments *over one year*. This bond pays \$4.687 every six months, for a total of \$9.375 per year. The number 04 means that the bond matures in 2004 (at which time the holder will receive \$100 in principal). The next number, 9.4, is the annual coupon divided by the bond's closing price (i.e., $9,375/99.875$). The number 206 refers to the number of these IBM bonds traded that day. The next three numbers, 100⅝, 99⅞, and 99⅞, are the high, low, and closing prices for the bond. (Some newspapers only report the closing price.) Finally, the -⅝ means that the closing price was ⅝ point lower than the preceding day's close.⁷

What is the yield on this bond? For simplicity, we'll assume that the coupon payments are made annually, instead of every six months. (The error that this introduces is very small.) Because the bond matures in 2004, payments will be made for $2004 - 1990 = 14$ years. The yield is then given by the following equation:

$$99.875 = \frac{9.375}{(1+R)} + \frac{9.375}{(1+R)^2} + \frac{9.375}{(1+R)^3} + \cdots + \frac{9.375}{(1+R)^{14}} + \frac{100}{(1+R)^{14}}$$

This equation must be solved for R . You can check (by substituting and seeing whether the equation is satisfied) that the solution is $R^* = 9.4$ percent.

⁷ These bonds actually have a face value of \$1000, not \$100. The prices and coupon payments are listed as though the face value were \$100 to save space. To get the actual prices and payments, just multiply the numbers that appear in the newspaper by ten.

The yield on the Chrysler bond is found in the same way. This bond makes coupon payments of \$13 per year, matures in the year 1997, and had a closing price of 95¼. Since the bond has seven years to mature, the equation for its yield is

$$95.25 = \frac{13}{(1+R)} + \frac{13}{(1+R)^2} + \frac{13}{(1+R)^3} + \cdots + \frac{13}{(1+R)^7} + \frac{100}{(1+R)^7}$$

The solution to this equation is $R^* = 14.1$ percent.

Why was the yield on the Chrysler Corporation bond so much higher than that on the IBM bond? Because it was much riskier. In 1990, Chrysler's sales had been falling, it was losing money, and its prospects for recovery were in doubt. Given Chrysler's uncertain financial situation, investors required a higher return before they would hold its bonds.

15.4 *The Net Present Value Criterion for Capital Investment Decisions*

One of the most common and important decisions that firms make is to invest in new capital. Millions of dollars may be invested in a factory or machines that will last and affect the firm's profits for many years. The future cash flows that the investment will generate are often uncertain. And once the factory has been built, the firm usually cannot disassemble and resell it to recoup its investment—it becomes a sunk cost.

How should a firm decide whether a particular capital investment is worthwhile? It should calculate the present value of the future cash flows that it expects to receive from the investment, and compare it with the cost of the investment. This is the *Net Present Value (NPV) criterion*:

NPV Criterion: Invest if the present value of the expected future cash flows from an investment is larger than the cost of the investment.

Suppose a capital investment costs C and is expected to generate profits over the next ten years of amounts $\pi_1, \pi_2, \dots, \pi_{10}$. Then we write the net present value as

$$NPV = -C + \frac{\pi_1}{(1+R)} + \frac{\pi_2}{(1+R)^2} + \cdots + \frac{\pi_{10}}{(1+R)^{10}} \quad (15.3)$$

where R is the *discount rate* that we use to discount the future stream of profits. (R might be a market interest rate, or it might be some other rate; we will discuss how to choose it shortly.) Equation (15.3) describes the net benefit to

the firm from the investment. The firm should make the investment only if that net benefit is positive, i.e., *only if* $NPV > 0$.

What discount rate should the firm use? The answer depends on the alternative ways that the firm could use its money. For example, instead of this investment, the firm might invest in another piece of capital that generates a different stream of profits. Or it might invest in a bond that yields a different return. As a result, we can think of R as the *firm's opportunity cost of capital*. Had the firm not invested in this project, it could have earned a return by investing in something else. *The correct value for R is therefore the return that the firm could earn on a "similar" investment.*

By "similar" investment, we mean one with the same *risk*. As we saw in Chapter 5, the more risky an investment, the greater the return one expects to receive from it. Therefore, the opportunity cost of investing in this project is the return that one could earn from another project or asset of similar riskiness.

We'll see how to evaluate the riskiness of an investment in the next section. For now, let's assume that this project has *no risk* (i.e., the firm is sure that the future profit flows will be π_1 , π_2 , etc.). Then the opportunity cost of the investment is the *risk-free* return, e.g., the return one could earn on a government bond. If the project is expected to last for ten years, the firm could use the annual interest rate on a ten-year government bond to compute the NPV of the project, as in equation (15.3).⁸ If the NPV is zero, the benefit from the investment would just equal the opportunity cost, so the firm should be indifferent between investing and not investing. If the NPV is greater than zero,⁹ the benefit exceeds the opportunity cost, so the investment should be made.

The Electric Motor Factory

In Section 15.1, we discussed a decision to invest \$10 million in a factory to produce electric motors. This factory would enable the firm to use labor and copper to produce 8000 motors per month for 20 years, at a cost of \$42.50 each. The motors could be sold for \$52.50 each, for a profit of \$10 per unit, or \$80,000 per month. We will assume that after 20 years the factory will be obsolete, but it can be sold for scrap for \$1 million. Is this a good investment? To find out, we must calculate its net present value.

We will assume for now that the \$42.50 production cost and the \$52.50 price at which the motors can be sold are certain, so that the firm is sure it will receive \$80,000 per month, or \$960,000 per year, in profit. We also assume that the \$1 million scrap value of the factory is certain. The firm should therefore use a risk-free interest rate to discount future profits. Writing the cash flows in millions of dollars, the NPV is

⁸This is an approximation. To be precise, the firm should use the rate on a one-year bond to discount π_1 , the rate on a two-year bond to discount π_2 , etc.

⁹This NPV rule is incorrect when the investment is irreversible, subject to uncertainty, and can be delayed. For a treatment of irreversible investment, see Avinash Dixit and Robert Pindyck, *Investment Under Uncertainty* (Princeton, NJ: Princeton University Press, 1994).

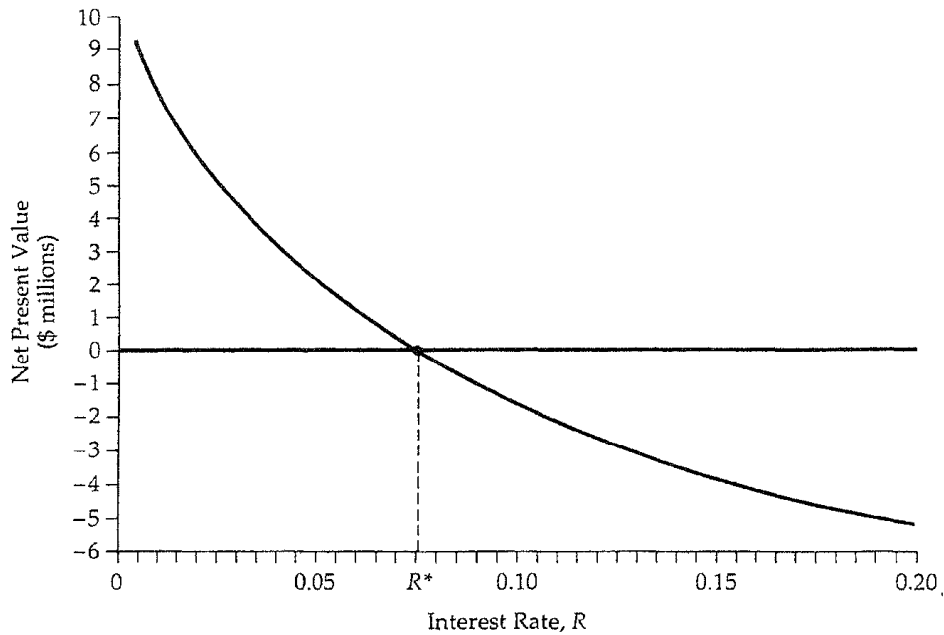


FIGURE 15.3 Net Present Value of a Factory. The NPV of a factory is the present discounted value of all the cash flows involved in building and operating it. Here it is the PDV of the flow of future profits less the current cost of construction. The NPV declines as the interest rate increases. At the interest rate R^* , the NPV is zero.

$$\begin{aligned} \text{NPV} = & -10 + \frac{.96}{(1+R)} + \frac{.96}{(1+R)^2} + \frac{.96}{(1+R)^3} \\ & + \cdots + \frac{.96}{(1+R)^{20}} + \frac{1}{(1+R)^{20}} \end{aligned} \quad (15.4)$$

Figure 15.3 shows the NPV as a function of the discount rate R . Note that at the rate R^* , which is about 7.5 percent, the NPV is equal to zero.¹⁰ For discount rates below 7.5 percent, the NPV is positive, so the firm should invest in the factory. For discount rates above 7.5 percent, the NPV is negative, and the firm should not invest.

Real Versus Nominal Discount Rates

In the example above, we assumed that future cash flows are certain, so that the discount rate R should be a risk-free interest rate, such as the rate on U.S.

¹¹ The rate R^* is sometimes referred to as the *internal rate of return* on the investment.

government bonds. Suppose that rate happened to be 9 percent. Does that mean the NPV is negative, and the firm should not invest?

To answer this question, we must distinguish between real and nominal discount rates, and between real and nominal cash flows. *Let's* begin with the cash flows. In Chapter 1 we discussed real versus nominal prices, and we explained that the real price is *net of inflation*, whereas the nominal price includes inflation. In our example, we assumed that the electric motors coming out of the factory could be sold for \$52.50 each over the next 20 years. We said nothing, however, about the effect of inflation. Is the \$52.50 a real price, net of inflation, or does it include inflation? As we will see, the answer to this question can be critical.

Let's assume that the \$52.50 price-and the \$42.50 production cost-are in *real* terms. (This means that if we expect a 5 percent annual rate of inflation, the nominal price of the motors will increase from \$52.50 in the first year to $(1.05)(52.50) = \$55.13$ in the second year, to $(1.05)(55.13) = \$57.88$ in the third year, and so on.) Therefore, our profit of \$960,000 per year is also in real terms.

Now let's turn to the discount rate. *If the cash flows are in real terms, the discount rate must also be in real terms.* The reason is that the discount rate is the opportunity cost of the investment. If inflation is not included in the cash flows, it should not be included in the opportunity cost either.

In our example, the discount rate should therefore be the *real* interest rate on government bonds. The nominal interest rate (9 percent) is the rate that we see in the newspapers; it includes inflation. *The real interest rate is the nominal rate minus the expected rate of inflation.*¹¹ If we expect inflation to be 5 percent per year on average, the real interest rate would be $9 - 5 = 4$ percent. This is the discount rate that should be used to calculate the NPV of the investment in the electric motor factory. Note from Figure 15.3 that at this rate the NPV is clearly positive, so the investment should be undertaken.

When using the NPV rule to evaluate investments, the numbers in the calculations may be in real or in nominal terms, as long as they are consistent. If cash flows are in real terms, the discount rate should also be in real terms. If a nominal discount rate is used, the effect of future inflation must also be included in the cash flows.

Negative Future Cash Flows

Factories and other production facilities can take several years to build and equip. Then, the cost of the investment will also be spread out over several years, instead of occurring only at the outset. In addition, some investments are expected to result in *losses*, rather than profits, for the first few years. (For example, demand may be low until consumers learn about the product, or costs may start high and fall only when managers and workers have moved

¹¹ People can have different views about future inflation, and may therefore have different estimates of the real interest rate.

down the learning curve.) Negative future cash flows create no problem for the NPV rule; they are simply discounted just like positive cash flows.

For example, suppose that our electric motor factory will take a year to build: \$5 million is spent right away, and another \$5 million is spent next year. Also, suppose the factory is expected to *lose* \$1 million in its first year of operation and \$0.5 million in its second year. Afterwards, it will earn \$0.96 million a year until year 20, when it will be scrapped for \$1 million, as before. (All these cash flows are in real terms.) Now the net present value is

$$\begin{aligned} \text{NPV} = & -5 - \frac{5}{(1+R)} - \frac{1}{(1+R)^2} - \frac{.5}{(1+R)^3} + \frac{.96}{(1+R)^4} + \frac{.96}{(1+R)^5} \\ & + \cdots + \frac{.96}{(1+R)^{20}} + \frac{1}{(1+R)^{20}} \end{aligned} \quad (15.5)$$

Suppose the real interest rate is 4 percent. Should the firm build this factory? You can confirm that the NPV is positive, so this project is a good investment.

15.5 Adjustments for Risk

We have seen that a risk-free interest rate is an appropriate discount rate for future cash flows that are certain. For most projects, however, future cash flows are far from certain. For example, for our electric motor factory, we would expect uncertainty over future copper prices, over the future demand and hence the price of motors, and even over future wage rates. Thus, the firm cannot know what its profits from the factory will be over the next 20 years. Its best estimate of profits might be \$960,000 per year, but actual profits may turn out to be higher or lower than this. How should the firm take this uncertainty into account when calculating the net present value of the project?

A common practice is to increase the discount rate by adding a *risk premium* to the risk-free rate. The idea is that the owners of the firm are risk averse, which makes future cash flows that are risky worth less than those that are certain. Increasing the discount rate takes this into account by reducing the present value of those future cash flows. But how large should the risk premium be? We discuss this below.

Diversifiable Versus Nondiversifiable Risk

Adding a risk premium to the discount rate must be done with care. If the firm's managers are operating in the stockholders' interests, they must dis-

tinguish between two kinds of risk—*diversifiable* and *nondiversifiable risk*.¹² Diversifiable risk can be eliminated by investing in many projects or by holding the stocks of many companies. Nondiversifiable risk cannot be eliminated in this way. *Only nondiversifiable risk affects the opportunity cost of capital, and should enter into the risk premium,*

To understand this, recall from Chapter 5 that diversifying can eliminate many risks. For example, I cannot know whether the result of a coin flip will be heads or tails. But I can be reasonably sure that of a thousand coin flips, roughly half will be heads. Similarly, an insurance company that sells me life insurance cannot know how long I will live. But by selling life insurance to thousands of people, it can be reasonably sure about the fraction of those people who will die each year.

Much the same is true about capital investment decisions. Although the profit flow from a single investment may be very risky, if the firm invests in dozens of projects (as most large firms do), its overall risk will be much less. Furthermore, even if the company invests in only one project, the stockholders can easily diversify by holding the stocks of a dozen or more different companies, or by holding a mutual fund that invests in many stocks. So the stockholders, i.e., the owners of the firm, can eliminate diversifiable risk.

Because investors can eliminate diversifiable risk, they cannot expect to earn a return higher than the risk-free rate by bearing it. (No one will pay you for bearing a risk that there is no need to bear.) And indeed, assets that have only diversifiable risk tend on average to earn a return close to the risk-free rate. Now, remember that the discount rate for a project is the opportunity cost of investing in that project, rather than in some other project or asset with similar risk characteristics. Therefore, if the project's only risk is diversifiable, the opportunity cost is the risk-free rate, *and no risk premium should be added to the discount rate.*

What about nondiversifiable risk? First, let's be clear about how it can arise. For a life insurance company, the possibility of a major war poses nondiversifiable risk. A war may increase mortality rates sharply, and the company could not expect that an "average" number of its customers would die each year, no matter how many customers it had. As a result, most insurance policies, whether for life, health, or property, do not cover losses resulting from acts of war.

For capital investments, nondiversifiable risk arises because firms' profits tend to depend on the overall economy. When economic growth is strong, corporate profits tend to be higher. (For our electric motor factory, the demand for motors is likely to be strong, so profits increase.) On the other hand, profits tend to fall in a recession. Because future economic growth is uncertain, diversification cannot eliminate all risk. Investors should (and indeed can) earn a higher return by bearing this risk.

¹²Diversifiable risk is also called *nonsystematic* risk and nondiversifiable risk is called *systematic* risk. Adding a simple risk premium to the discount rate may not always be the correct way of dealing with risk—See, for example, Richard Brealey and Stewart Myers, *Principles of Corporate Finance* (New York: McGraw-Hill, 1991).

To the extent that a project has nondiversifiable risk, the opportunity cost of investing in that project is higher than the risk-free rate, and a risk premium must be included in the discount rate. Let's see how the size of that risk premium can be determined.

The Capital Asset Pricing Model

The *Capital Asset Pricing Model* (CAPM) measures the risk premium for a capital investment by comparing the expected return on that investment with the expected return on the entire stock market. To understand the model, suppose, first, that you invest in the entire stock market (say, through a mutual fund). Then your investment would be completely diversified, and you would bear no diversifiable risk. You would, however, bear nondiversifiable risk because the stock market tends to move with the overall economy. (The stock market reflects expected future profits, which depend in part on the economy.) As a result, the expected return on the stock market is higher than the risk-free rate. Denoting the expected return on the stock market by r_m and the risk-free rate by r_f , the risk premium on the market is $r_m - r_f$. This is the additional expected return one can expect to earn by bearing the nondiversifiable risk associated with the stock market.

Now consider the nondiversifiable risk associated with one asset, such as a company's stock. We can measure that risk in terms of the extent to which the return on the asset tends to be *correlated* with (move in the same direction as) the return on the stock market as a whole. For example, one company's stock might have almost no correlation with the market as a whole. On average, the price of that stock would move independently of changes in the market, so it would have little or no nondiversifiable risk. The return on that stock should therefore be about the same as the risk-free rate. Another stock, however, might be highly correlated with the market. Its price changes might even amplify changes in the market as a whole. That stock would have substantial nondiversifiable risk, perhaps more than the stock market as a whole, in which case its return on average will exceed the market return r_m .

The CAPM summarizes this relationship between expected returns and the risk premium by the following equation:

$$r_i - r_f = \beta(r_m - r_f) \quad (15.6)$$

where r_i is the expected return on an asset. The equation says that the risk premium on the asset (its expected return less the risk-free rate) is proportional to the risk premium on the market. The constant of proportionality, β , is called the *asset beta*. It measures how sensitive the asset's return is to market movements and therefore the asset's nondiversifiable risk. If a 1 percent rise in the market tends to result in a 2 percent rise in price of the asset, the beta is 2. If a 1 percent rise in the market tends to result in a 1 percent rise in the price of the asset, the beta is 1. And if a 1 percent rise in the market tends

to result in no change in the price of the asset, the beta is zero. As equation (15.6) shows, the larger beta is, the greater is the expected return on the asset because the greater is the asset's nondiversifiable risk.

Given beta, we can determine the correct discount rate to use in computing an asset's net present value. That discount rate is the expected return on the asset or on another asset with the same risk. It is therefore the risk-free rate plus a risk premium to reflect nondiversifiable risk:

$$\text{Discount rate} = r_f + \beta(r_m - r_f) \quad (15.7)$$

Over the past 60 years, the risk premium on the stock market, $(r_m - r_f)$, has been about 8 percent on average. If the real risk-free rate were 4 percent and beta were 0.6, the correct discount rate would thus be $0.04 + 0.6(0.08) = 0.09$, or 9 percent.

If the asset is a stock, its beta can usually be estimated statistically.¹³ When the asset is a new factory, however, determining its beta is more difficult. Many firms therefore use a company cost of capital as a (nominal) discount rate. The company cost of capital is a weighted average of the expected return on the company's stock (which depends on the beta of the stock) and the interest rate it pays for debt. This approach is correct as long as the capital investment in question is typical for the company as a whole. It can be misleading, however, if the capital investment has much more or much less nondiversifiable risk than the company as a whole. In that case it may be better to make a reasoned guess as to how much the revenues from the investment are likely to depend on the overall economy.

EXAMPLE 15.3 CAPITAL INVESTMENT IN THE DISPOSABLE DIAPER INDUSTRY

In Example 13.6, we discussed the disposable diaper industry, which has been dominated by Procter & Gamble, with about a 60 percent market share, and Kimberly-Clark, with another 30 percent. We explained that their confirming R&D (research and development) expenditures have given these firms a cost advantage that deters entry. Now we'll examine the capital investment decision of a potential entrant.

Suppose you are considering entering this industry. To take advantage of scale economies, both in production and in advertising and distribution, you would need to build three plants at a cost of \$60 million each, which, when operating at capacity, would produce 2.5 billion diapers per year. These would be sold at wholesale for about 16 cents per diaper, yielding revenues of about \$400 million per year. You can expect your variable production costs to be about \$290 million per year, for a net revenue of \$110 million per year.

¹³One can estimate beta by running a linear regression of the return on the stock against the excess return on the market, $r_m - r_f$. One would find, for example, that the beta for Digital Equipment is about 1.4, the beta for Eastman Kodak is about 0.8, and the beta for General Motors is about 0.5.

You will have other expenses, however. Using the experience of P&G and Kimberly-Clark as a guide, you can expect to spend about \$60 million in R&D before start-up to design an efficient manufacturing process, and another \$20 million in R&D during each year of production to maintain and improve that process. Finally, once you are operating at full capacity, you can expect to spend another \$50 million per year for a sales force, advertising, and marketing, for a net operating profit of \$40 million per year. The plants will last for 15 years and will then be obsolete.¹⁴

Is the investment a good idea? To find out, let's calculate its net present value. Table 15.5 shows the relevant numbers. We assume that production begins at 33 percent of capacity in 1993, takes two years to reach full capacity, and continues through the year 2008. Given the net cash flows, the NPV is calculated as

$$\begin{aligned} \text{NPV} = & -120 - \frac{93.4}{(1+R)} - \frac{56.6}{(1+R)^2} + \frac{40}{(1+R)^3} \\ & + \frac{40}{(1+R)^4} + \cdots + \frac{40}{(1+R)^{15}} \end{aligned}$$

The table shows the NPV for discount rates of 5, 10, and 15 percent.

Note that the NPV is positive for a discount rate of 5 percent, but it is negative for discount rates of 10 or 15 percent. What is the correct discount rate?

TABLE 15.5 Data for NPV Calculation (\$ Millions)

	Pre-1993	1993	1994	1995	2008
Sales		133.3	266.7	400.0	400.0
LESS					
Variable cost		96.7	193.3	290.0	290.0
Ongoing R&D		20.0	20.0	20.0	20.0
Sales force, ads, and promotion		50.0	50.0	50.0	50.0
Operating profit		-33.4	3.4	40.0	40.0
LESS					
Construction cost	60.0	60.0	60.0		
Initial R&D	60.0				
NETCASHFLOW	-120.0	-93.4	-56.6	40.0	40.0
Discount Rate:		0.05	0.10	0.15	
NPV:		80.5	-16.9	-75.1	

¹⁴These numbers are from Michael E. Porter, "The Disposable Diaper Industry," Harvard Business School Case 9-380-175, July 1981, and articles from the *New York Times* and *Wall Street Journal*.

First, we have ignored inflation, so the discount rate should be in *real* terms. Second, the cash flows are risky—we don't know how efficient our plants will be, how effective our advertising and promotion will be, or even what the future demand for disposable diapers will be. Some of this risk is nondiversifiable. To calculate the risk premium, we will use a beta of 1, which is typical for a producer of consumer products of this sort. Using 4 percent for the real risk-free interest rate and 8 percent for the risk premium on the stock market, our discount rate should be

$$R = 0.04 + 1(0.08) = 0.12$$

At this discount rate, the NPV is clearly negative, so the investment does not make sense. We will not enter the industry; P&G and Kimberly-Clark can breathe a sigh of relief. You should not be surprised, however, that these firms can make money in this market while we cannot. Their experience, years of earlier R&D, and brand name recognition give them a competitive advantage that a new entrant would find hard to overcome. (For example, they would not need to spend \$60 million on R&D before building new plants.)

15.6 *Investment Decisions by Consumers*

We have seen how firms value future cash flows and thereby decide whether to invest in long-lived capital. Consumers face similar decisions when they purchase a durable good, such as a car or major appliance. Unlike the decision to purchase food, entertainment, or clothing, buying a durable good involves comparing a flow of *future* benefits with the current purchase cost.

Suppose you are deciding whether to buy a new car. If you keep the car for six or seven years, most of the benefits (and costs of operation) will occur in the future. You must therefore compare the future flow of net benefits from owning the car (the benefit of having transportation less the cost of insurance, maintenance, and gasoline to operate the car) with the purchase price. Likewise, when deciding whether to buy a new air conditioner, you must compare its price with the present value of the flow of net benefits (the benefit of a cool room less the cost of electricity to operate the unit).

These problems are analogous to the problem of a firm, which must compare a future flow of profits with the current cost of plant and equipment when making a capital investment decision. We can therefore analyze these problems just as we analyzed the firm's investment problem. Let's do this for a consumer's decision to buy a car.

The main benefit from owning a car is the flow of transportation services it provides. The value of those services differs from consumer to consumer. Let's assume our consumer values the service at S dollars per year. Let's also as"

sume that the total operating expense (insurance, maintenance, and gasoline) is E dollars per year, that the car costs \$10,000, and that after six years its resale value will be \$2000. The decision to buy the car can then be framed in net present value terms:

$$\begin{aligned} \text{NPV} = & -10,000 + (S - E) + \frac{(S - E)}{(1 + R)} + \frac{(S - E)}{(1 + R)^2} \\ & + \cdots + \frac{(S - E)}{(1 + R)^6} + \frac{2000}{(1 + R)^6} \end{aligned} \quad (15.8)$$

What discount rate R should the consumer use? The consumer should apply the same principle that the firm does—the discount rate is the opportunity cost of money. If the consumer already has \$10,000 and does not need a loan, the correct discount rate is the return that could be carried by investing the money in another asset, say, a savings account or a government bond. On the other hand, if the consumer is in debt, the discount rate would be the borrowing rate that he or she is already paying. This rate is likely to be much higher than the interest rate on a bond or savings account, so the NPV of the investment will be smaller.

EXAMPLE 15.4 CHOOSING AN AIR CONDITIONER

Buying a new air conditioner involves making a trade-off. Some air conditioners cost less but are less efficient—they consume a lot of electricity relative to their cooling power. Other air conditioners cost more but are also more efficient. Should you buy an inefficient air conditioner that costs less now but will cost more in the future to operate, or an efficient one that costs more now but will cost less to operate?

Let's assume you are comparing air conditioners of equivalent cooling power, so that they yield the same flow of benefits. We can then compare the present discounted values of their costs. Assuming an eight-year lifetime and no resale, the PDV of the costs of buying and operating air conditioner i is

$$\text{PDV} = C_i + OC_i + \frac{OC_i}{(1 + R)} + \frac{OC_i}{(1 + R)^2} + \cdots + \frac{OC_i}{(1 + R)^8}$$

where C_i is the purchase price of air conditioner i and OC_i is its average annual operating cost.

Which air conditioner is best depends on your discount rate. If you have little free cash and must borrow, you should use a high discount rate. This would make the present value of the future operating costs smaller, so you would probably choose a less expensive but relatively inefficient unit. If you have plenty of free cash, so that your opportunity cost of money (and hence your discount rate) is low, you would probably buy the more expensive unit.

An econometric study of household purchases of air conditioners shows that consumers tend to trade off capital costs and expected future operating costs

in just this way, although the discount rates that people use are high—about 20 percent for the population as a whole.¹⁵ (American consumers seem to behave myopically by overdiscounting future savings.) The study also shows that consumers' discount rates vary inversely with their incomes. (For example; people whose 1978 annual income was between \$25,000 and \$35,000 used discount rates of about 9 percent, while those with incomes under \$10,000 used discount rates of 39 percent or more.) We would expect this because higher-income people are likely to have more free cash available and therefore have a lower opportunity cost of money.

***15.7** *Intertemporal Production Decisions—Depletable Resources*

Firms' production decisions often have *intertemporal* aspects—production today affects sales or costs in the future. The learning curve, which we discussed in Chapter 7, is an example of this. By producing today, the firm gains experience that lowers its future costs. In this case production today is partly an investment in future cost reduction, and the value of this must be taken into account when comparing costs and benefits. Another example is the production of a depletable resource. When the owner of an oil well pumps oil today, less oil is available for future production. This must be taken into account when deciding how much to produce.

Production decisions in cases like these involve comparisons between costs and benefits today with costs and benefits in the future. We can make those comparisons using the concept of present discounted value. We'll look in detail at the case of a depletable resource, although the same principles apply to other intertemporal production decisions.

The Production Decision of an Individual Resource Producer

Suppose your rich uncle gave you an oil well. The well contains 1000 barrels of oil that can be produced at a constant average and marginal cost of \$10 per barrel. Should you produce all the oil today, or should you save it for the future?¹⁶

¹⁵ See Jerry A. Hausman, "Individual Discount Rates and the Purchase and Utilization of Energy-Using Durables," *Bell Journal of Economics* 10 (Spring 1979): 33-54.

¹⁶ For most real oil wells, marginal and average cost are not constant, and it would be extremely costly to extract all the oil in a short time. We will ignore this complication.

You might think that the answer depends on the profit you can earn if you remove the oil from the ground. After all, why not remove the oil if its price is greater than the cost of extraction? However, this ignores the opportunity cost of using up the oil today so that it is not available for the future.

The correct answer, then, depends not on the current profit level, but on how fast you expect the price of oil to rise. Oil in the ground is like money in the bank, and you should keep it in the ground only if it earns a return at least as high as the market interest rate. So if you expect the price of oil to remain constant or rise very slowly, you would be better off extracting and selling all of it now and investing the proceeds. But if you expect the price of oil to rise rapidly, you should leave it in the ground.

How fast must the price rise for you to keep the oil in the ground? The value of each barrel of oil in your well is equal to the price of oil, less the \$10 cost of extracting it. (This is the profit you can obtain, by extracting and selling each barrel.) This value must rise at least as fast as the rate of interest for you to keep the oil. Your production decision rule is therefore: *Keep all your oil if you expect its price less its extraction cost to rise faster than the rate of interest. Extract and sell all of it if you expect price less cost to rise at less than the rate of interest.* And what if you expect price less cost to rise at exactly the rate of interest? Then you would be indifferent between extracting the oil and leaving it in the ground. Letting P_t be the price of oil this year, P_{t+i} be the price next year, and c the cost of extraction, we can write this production rule as follows:

If $(P_{t+i} - c) > (1 + R)(P_t - c)$, keep the oil in the ground.

If $(P_{t+i} - c) < (1 + R)(P_t - c)$, sell all the oil now.

If $(P_{t+i} - c) = (1 + R)(P_t - c)$, makes no difference.

Given our expectation about the growth rate of oil prices, we can use this rule to determine production. But how fast should we expect the market price of oil to rise?

The Behavior of Market Price

Suppose there were no OPEC cartel, and the oil market consisted of many competitive producers with oil wells like our own. We could then determine how fast oil prices are likely to rise by considering the production decisions of other producers. If other producers want to earn the highest possible return, they will follow the production rule we stated above. This means that *price less marginal cost must rise at exactly the rate of interest*.¹⁷ To see why, suppose price less cost were to rise faster than the rate of interest. Then no one would sell any oil. Inevitably, this would drive the current price of oil up. If, on the other hand, price less cost were to rise at a rate less than the rate of in-

¹⁷This result is called the Hotelling rule because it was first demonstrated by Harold Hotelling in "The Economics of Exhaustible Resources," *Journal of Political Economy* 39 (April 1931): 137-175.

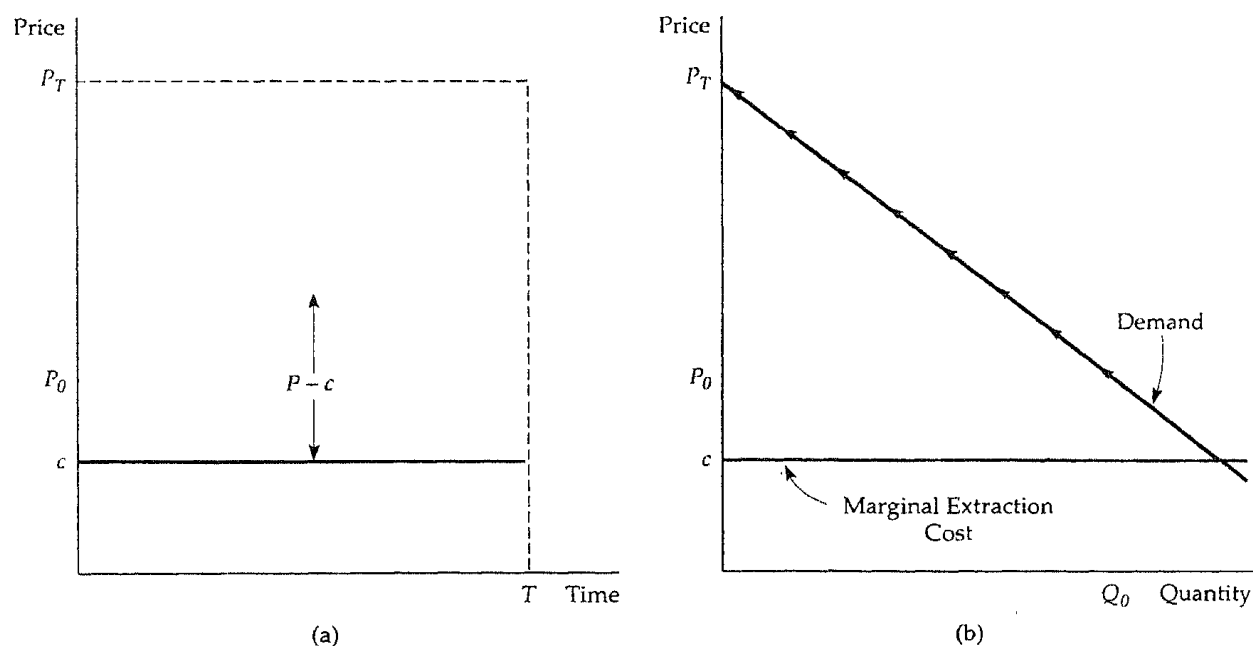


FIGURE 15.4 Price of an Exhaustible Resource. In (a), the price is shown rising over time. Units of a resource in the ground must earn a return commensurate with that on other assets. Therefore, in a competitive market, price less marginal production cost will rise at the rate of interest. Part (b) shows movement up the demand curve as price rises.

terest, everyone would try to sell all of their oil immediately, which would drive the current price down.

Figure 15.4 illustrates how the market price must rise. The marginal cost of extraction is c , and the price and total quantity produced are initially P_0 and Q_0 . Figure 15.4a shows the net price, $P - c$, rising at the rate of interest. Figure 15.4b shows that as price rises, the quantity demanded falls. This continues until time T , when all the oil has been used up, and the price P_T is such that demand is just zero.

User Cost

We saw in Chapter 8 that a competitive firm always produces up to the point where price is equal to marginal cost. However, in a competitive market for an exhaustible resource, price *exceeds* marginal cost (and the difference between price and marginal cost rises over time). Does this conflict with what we learned in Chapter 8?

No, once we recognize that the *total* marginal cost of producing an exhaustible resource is greater than the marginal cost of extracting it from the

ground. There is an additional opportunity cost because producing and selling a unit today makes it unavailable for production and sale in the future. We call this opportunity cost the *user cost* of production. In Figure 15.4, user cost is the difference between price and marginal production cost. It rises over time because as the resource remaining in the ground becomes scarcer, the opportunity cost of depleting another unit becomes higher.

Resource Production by a Monopolist

What if the resource is produced by a *monopolist* rather than a competitive industry? Should price less marginal cost still rise at the rate of interest?

Suppose a monopolist is deciding between keeping an incremental unit of a resource in the ground, or producing and selling it. The value of that unit is the *marginal revenue* less the marginal cost. The unit should be left in the ground if its value is expected to *rise faster* than the rate of interest; it should be produced and sold if its value is expected to rise at *less* than the rate of interest. Since the monopolist controls total output, it will produce so that marginal revenue less marginal cost, i.e., the value of an incremental unit of resource, rises at exactly the rate of interest:

$$(MR_{t+1} - c) - (1+R)(MR_t - c)$$

Note that this rule also holds for a competitive firm, but for a competitive firm, marginal revenue equals the market price p .

For a monopolist facing a downward-sloping demand curve, price is greater than marginal revenue. Hence, if marginal revenue, less marginal cost rises at the rate of interest, *price* less marginal cost will rise at less than the rate of interest. We thus have the interesting result that a monopolist is *more conservationist* than a competitive industry. In exercising monopoly power, the monopolist starts out charging a higher price and depletes the resource more slowly.

EXAMPLE 15.5 HOW DEPLETABLE ARE DEPLETABLE RESOURCES?

Resources like oil, natural gas, coal, uranium, copper, iron, lead, zinc, nickel, and helium are all depletable—there is a finite amount of each in the earth's crust, so that ultimately the production and consumption of each will cease. Nonetheless, some resources are more depletable than others.

For oil, natural gas, and helium, known and potentially discoverable in-ground reserves are equal to only 50 to 100 years of current consumption. For these resources, the user cost of depletion can be a significant component of the market price. Other resources, such as coal and iron, have a proved and potential reserve base equal to several hundred or even thousands of years of current consumption. For these resources, the user cost is very small.

The user cost for a resource can be estimated from geological information about existing and potentially discoverable reserves, and from knowledge of the demand curve and the rate at which that curve is likely to shift out over time in response to economic growth. Or if the market is competitive, user cost can be determined from the economic rent earned by the owners of resource-bearing lands.

Table 15.6 shows estimates of user cost as a fraction of the competitive price for crude oil, natural gas, uranium, copper, bauxite, and nickel.¹⁸ Note that only for crude oil and natural gas is user cost a substantial component of price. For the other resources, it is small and in some cases almost-negligible. Moreover, although most of these resources have experienced sharp price fluctuations, user cost had almost nothing to do with those fluctuations. For example, oil prices changed because of OPEC and political turmoil in the Persian Gulf, natural gas prices because of changes in government price controls, uranium and bauxite because of cartelization during the 1970s, and copper because of major changes in demand.

Resource depletion, then, has not been very important as a determinant of resource prices over the past few decades. Much more important have been market structure and changes in market demand. But the role of depletion should not be ignored. Over the long term it will be the ultimate determinant of resource prices.

TABLE 15.6 User Cost as a Fraction of Competitive Price

Resource	User Cost/Competitive Price
Crude oil	.4 to .5
Natural gas	.4 to .5
Uranium	.1 to .2
Copper	.2
Bauxite	.05 to .1
Nickel	.1 to .2

¹⁸ These numbers are based on Robert S. Pindyck, "Gains to Producers from the Cartelization of Exhaustible Resources," *Review of Economics and Statistics* 60 (1978): 238-251; Michael J. Mueller, "Scarcity and Ricardian Rents for Crude Oil," *Economic Inquiry* 23 (1985): 703-724; Kenneth R. Stollery, "Mineral Depletion with Cost as the Extraction Limit: A Model Applied to the Behavior of Prices in the Nickel Industry," *Journal of Environmental Economics and Management* 10 (1983): 151-165; and Robert S. Pindyck, "On Monopoly Power in Extractive Resource Markets," *Journal of Environmental Economics and Management* 14 (1987): 128-142.

15.8 How Are Interest Rates Determined?

We have seen how market interest rates are used to help make capital investment and intertemporal production decisions. But what determines how high interest rates will be and why they fluctuate over time? To answer these questions, remember that an interest rate is the price that borrowers pay lenders to use their funds. Like any market price, interest rates are determined by supply and demand—in this case the supply and demand for loanable funds.

The *supply of loanable funds* comes from households that wish to save part of their incomes in order to consume more in the future (or make bequests to their heirs). For example, some households have high incomes now but expect to earn less in the future after retirement. Saving lets them spread their consumption more evenly over time. Also, because they receive interest on the money they lend, they can consume more in the future in return for consuming less now. As a result, the higher the interest rate, the greater the incentive to save. The supply of loanable funds is therefore an upward-sloping curve, labeled S in Figure 15.5.

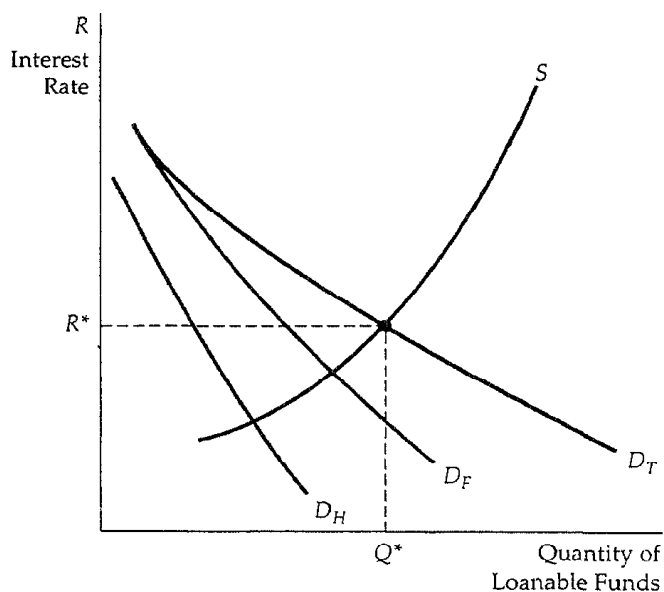


FIGURE 15.5 Supply and Demand for Loanable Funds. Market interest rates are determined by the demand and supply of loanable funds. Households supply funds to consume more in the future; the higher the interest rate, the more they supply. Households and firms both demand funds, but they demand less the higher the interest rate is. Shifts in demand or supply cause changes in interest rates.

The *demand for loanable funds* has two components. First, some households want to consume more than their current incomes, either because their incomes are low now but are expected to grow, or because they want to make a large purchase (e.g., a house) that has to be paid for out of future income. These households are willing to pay interest in return for not having to wait to consume. However, the higher the interest rate, the greater the cost of consuming rather than waiting, so the less willing these households will be to borrow. The household demand for loanable funds is therefore a declining function of the interest rate. In Figure 15.5, it is the curve labeled D_H .

The second source of demand for loanable funds is firms that want to make capital investments. Remember that firms will invest in projects with NPVs that are positive because a positive NPV means that the expected return on the project exceeds the opportunity cost of funds. That opportunity cost—the discount rate used to calculate the NPV—is the interest rate, perhaps adjusted for risk. Often firms borrow to invest because the flow of profits from an investment comes in the future, while the cost of an investment must usually be paid now. Firms' desires to invest are thus an important source of demand for loanable funds.

As we saw earlier, however, the higher the interest rate, the lower the NPV of a project. If interest rates rise, some investment projects that had positive NPVs will now have negative NPVs, and will therefore be cancelled. Overall, because firms' willingness to invest falls when interest rates rise, their demand for loanable funds also falls. The demand for loanable funds by firms is thus a downward-sloping curve; in Figure 15.5 it is labeled D_F .

The total demand for loanable funds is the sum of the household demand and the firm demand; in Figure 15.5 it is the curve D_T . This total demand curve together with the supply curve determine the equilibrium interest rate. In Figure 15.5, that rate is R^* .

Figure 15.5 can also help us understand why interest rates change. Suppose the economy goes into a recession. Firms will then expect lower sales and lower future profits from new capital investments. The NPVs of projects will fall, and firms' willingness to invest will decline, as will their demand for loanable funds. D_F , and therefore D_T , will shift to the left, and the equilibrium interest rate will fall. Or suppose the federal government spends much more money than it takes through taxes, i.e., runs large deficits, as it has done since the 1980s. It will have to borrow to finance these deficits, shifting the total demand for loanable funds D_T to the right, so that R increases. The monetary policies of the Federal Reserve are another important determinant of interest rates. The Federal Reserve can create money, shifting the supply of loanable funds to the right and reducing R .

A Variety of Interest Rates

Figure 15.5 aggregates individual demands and supplies as though there were a single market interest rate. In fact, households, firms, and the government

lend and borrow under a variety of terms and conditions. As a result, there is a wide range of "market interest rates. Here we briefly describe some of the more important interest rates that are quoted in the newspapers, and sometimes used for capital investment decisions.

Treasury Bill Rate A Treasury bill is a short-term (one year or less) bond issued by the U.S. government. It is a pure *discount bond*, i.e., it makes no coupon payments, but instead is sold at a price less than its redemption value at maturity. For example, a three-month Treasury bill might be sold for \$98. In three months it can be redeemed for \$100; it thus has an effective three-month yield of about 2 percent and an effective annual yield of about 8 percent.¹⁹ The Treasury bill rate can be viewed as a short-term, risk-free rate.

Treasury Bond Rate A Treasury bond is a longer-term bond (more than one year, and typically 10 to 30 years) issued by the U.S. government. Rates vary, depending on the maturity of the bond.

Discount Rate Commercial banks sometimes borrow for short periods from the Federal Reserve. These loans are called *discounts*, and the rate that the Federal Reserve charges on them is the discount rate.

Commercial Paper Rate Commercial paper refers to short-term (six months or less) discount bonds issued by high-quality corporate borrowers. Because commercial paper is only slightly riskier than a Treasury bill, the commercial paper rate is usually less than 1 percent higher than the Treasury bill rate.

Prime Rate This is the rate (sometimes called the *reference rate*) that large banks post as a reference point for short-term loans to their biggest corporate borrowers. As we saw in Example 12.4, this rate does not fluctuate from day to day as other rates do.

Corporate Bond Rate Newspapers and government publications report the average annual yields on long-term (typically 20-year) corporate bonds in different risk categories (e.g., high-grade bonds, medium-grade bonds, etc.). These average yields indicate how much corporations are paying for long-term debt. However, as we saw in Example 15.2, the yields on corporate bonds can vary considerably depending on the financial strength of the corporation and the time to maturity for the bond.

Summary

1. A firm's holding of capital is measured as a stock, but inputs of labor and raw materials are flows. Its stock of capital enables a firm to earn a flow of profits over time.

¹⁹To be exact, the three-month yield is $(100/98) - 1 = 0.0204$, and the annual yield is $(100/98)^4 - 1 = 0.0842$, or 8.42 percent

2. When a firm makes a capital investment, it spends money now, so that it can earn profits in the future. To decide whether the investment is worthwhile, the firm must determine the present value of future profits by discounting them.
3. The present discounted value (PDV) of \$1 paid one year from now is $\$1/(1 + R)$, where R is the interest rate. The PDV of \$1 paid n years from now is $\$1/(1 + R)^n$.
4. A bond is a contract in which a lender agrees to pay the bondholder a stream of money. The value of the bond is the PDV of that stream. The effective yield on a bond is the interest rate that equates that value with the bond's market price. Bond yields differ because of differences in riskiness and time to maturity.
5. Firms can decide whether to undertake a capital investment by applying the Net Present Value (NPV) criterion: Invest if the present value of the expected future cash flows from an investment is larger than the cost of the investment.
6. The discount rate that a firm uses to calculate the NPV for an investment should be the opportunity cost of capital, i.e., the return the firm could earn on a similar investment.
7. When calculating NPVs, if cash flows are in nominal terms (i.e., include inflation), the discount rate should also be nominal, but if cash flows are in real terms (i.e., are net of inflation), a real discount rate should be used.
8. An adjustment for risk can be made by adding a risk premium to the discount rate. However, the risk premium should reflect only nondiversifiable risk. Using the Capital Asset Pricing Model (CAPM), the risk premium is the "beta" for the project times the risk premium on the stock market as a whole. The "beta" measures the sensitivity of the projects return to movements in the market.
9. Consumers are also faced with investment decisions that require the same kind of analysis as those of firms. When deciding whether to buy a durable good like a car or a major appliance, the consumer must consider the present value of future operating costs.
10. An exhaustible resource in the ground is like money in the bank and must earn a comparable return. Therefore, if the market is competitive, price less marginal extraction cost will grow at the rate of interest. The difference between price and marginal cost is called *user cost*—it is the opportunity cost of depleting a unit of the resource.
11. Market interest rates are determined by the demand and supply of loanable funds. Households supply funds so they can consume more in the future. Households, firms, and the government demand funds. Changes in demand or supply cause changes in interest rates.

Questions for Review

1. A firm uses cloth and labor to produce shirts in a factory that it bought for \$10 million. Which of its factor inputs are measured as flows and which as stocks? How would your answer change if the firm had leased a factory instead of buying one? Is its output measured as a flow or a stock? Its profit?
2. Suppose the interest rate is 10 percent. If \$100 is invested at this rate today, how much will it be worth after one year? After two years? After five years? What is the value today of \$100 paid one year from now? Paid two years from now? Paid five years from now?

3. You are offered the choice of two payment streams: (a) \$100 paid one year from now and \$100 paid two years from now; (b) \$80 paid one year from now and \$130 paid two years from now. Which payment stream would you prefer if the interest rate is 5 percent? If it is 15 percent?
4. How does one calculate the present value of a bond? If the interest rate is 5 percent, what is the present value of a perpetuity that pays \$1000 per year forever?
5. What is the *effective yield* on a bond? How does one calculate it? Why do some corporate bonds have effective yields that are higher than others?
6. What is the Net Present Value (NPV) criterion for investment decisions? How does one calculate the NPV of an investment project? If all the cash flows for the project are certain, what discount rate should be used to calculate the NPV?
7. What is the difference between a real discount rate and a nominal discount rate? When should a real discount rate be used in an NPV calculation, and when should a nominal rate be used?
8. How is a risk premium used to account for risk in NPV calculations? What is the difference between diversifiable and nondiversifiable risk? Why should only nondiversifiable risk enter into the risk premium?
9. What is meant by the "market return" in the Capital Asset Pricing Model (CAPM)? Why is the market return greater than the risk-free interest rate? What does an β measure in the CAPM? Why should high-beta assets have a higher expected return than low-beta assets?
10. Suppose you are deciding whether to invest \$100 million in a steel mill. You know the expected cash flows for the project, but they are risky—steel prices could rise or fall in the future. How would the CAPM help you select a discount rate for an NPV calculation?
11. How does a consumer trade off current and future costs when selecting an air conditioner or other major appliance? How could this selection be aided by an NPV calculation?
12. What is meant by the "user cost" of producing an exhaustible resource? Why does price minus extraction cost rise at the rate of interest in a competitive exhaustible resource market?
13. What determines the supply of loanable funds? The demand for loanable funds? What might cause the supply or demand for loanable funds to shift, and how would that affect interest rates?

Exercises

1. Suppose the interest rate is 10 percent. What is the value of a coupon bond that pays \$80 per year for each of the next five years, and then makes a principal repayment of \$1000 in the sixth year? Repeat for an interest rate of 15 percent.
 2. A bond has two years to mature. It makes a coupon payment of \$100 after one year, and both a coupon payment of \$100 and a principal repayment of \$1000 after two years. The bond is selling for \$966. What is its effective yield?
 3. Equation (15.5) shows the net present value of an investment in an electric motor factory, where half of the \$10 million cost is paid initially and the other half after a year, and where the factory is expected to lose money during its first two years of operation. If the discount rate is 4 percent, what is the NPV? Is the investment worthwhile?
 4. The market interest rate is 10 percent and is expected to stay at that level. Consumers can borrow and lend all they want at this rate. Explain your choice in each of the following situations:
 - a. Would you prefer a \$500 gift today or a \$540 gift next year?
 - b. Would you prefer a \$100 gift now or a \$500 loan without interest for four years?
 - c. Would you prefer a \$250 rebate on an \$8000 car or one year of financing for the full price of the car at 5 percent interest?
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- d.** Suppose you win the "million dollar lottery." You will receive \$50,000 a year for the next 20 years. How much is this worth to you today?
- e.** You win the "honest million" jackpot. You can have \$1 million today or \$50,000 per year for eternity (this right can be passed on to your heirs). Which do you prefer?
- f.** Until recently, an adult child had to pay taxes on gifts of over \$10,000 from her parents, but parents could loan money to their children interest-free. Why did some people call this unfair? To whom were the rules unfair?
- 5.** Ralph is trying to decide whether go to graduate school. If he spends two years in graduate school, paying \$10,000 tuition each year, he will get a job that will pay \$50,000 per year for the rest of his working life. If he does not go to school, he will go into the work force immediately. He will then make \$20,000 per year for the next three years, \$30,000 for the following three years, and \$50,000 per year every year after that. If the interest rate is 10 percent, is graduate school a good financial investment?
- 6.** Suppose your uncle gave you an oil well like the one described in Section 15.7. (Marginal production cost is constant at \$10.) The price of oil is currently \$20 but is controlled by a cartel that accounts for a large fraction of total production. Should you produce and sell all your oil now or wait to produce? Explain your answer.
- *7.** You are planning to invest in fine wine. Each case costs \$100, and you know from experience that the value of a case of wine held for t years is $(100)t^{1/2}$. One hundred cases of wine are available for sale, and the interest rate is 10 percent.
- a.** How many cases should you buy, how long should you wait to sell them, and how much money will you receive at the time of their sale?
- b.** Suppose that at the time of purchase, someone offers you \$130 per case immediately. Should you take the offer?
- c.** How would your answers change if the interest rate were only 5 percent?
- 8.** Reexamine the capital investment decision in the disposable diaper industry (Example 15.3) from the point of view of an incumbent firm. If P&G or Kimberly-Clark were to expand capacity by building three new plants, they would not need to spend \$60 million on R&D before start-up. How does this affect the NPV calculations in Table 15.5? Is the investment profitable at a discount rate of 12 percent?