Heat Transfer Test I

Number of Questions 25 **Time:60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. A general heat diffusion equation is given by

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=0$$

The above equation is known as
(A) Steady, Poisson's equation.
(B) Unsteady, Poisson's equation.
(C) Steady, Fourier's equation.
(D) Unsteady, Fourier's equation.

2. If the inner and outer surface areas of the cylinder are 1000 mm^2 and 2000 mm^2 then to transfer the cylinder into an equivalent slab, the log mean area of the cylinder will be (in mm^2)
(A) 1443 (B) 1000
(C) 1243 (D) 1343

3. For a very thin cylinder with negligible thickness, the log mean area of the cylinder is
(A) 0 (B) ∞
(C) 1 (D) Undefined

4. The outer surface of the sphere is faced to stagnant water which is maintained at a constant temperature. If the sphere does not have any heat source then the temperature in the sphere will
(A) vary linearly with radius
(B) vary logarithmically with radius
(C) vary hyperbolically with radius
(D) decrease and independent of radius.

5. Match List-I with List-II and select the correct answer using the codes given below the lists:

	List – I		List – II
P.	Transient conduction	1.	Linear
Q.	Conduction through a cylindrical wall profile.	2.	Biot Number
R.	Temperature profile through a plane slab	3.	Constant
S.	Temperature profile through a sphere.	4.	Hyperbolic
		5.	Logarithmic

	P	*Q*	*R*	*S*
(A)	2	1	3	5
(B)	2	3	1	4
(C)	2	5	3	4
(D)	2	5	1	4

6. In unsteady state condition for bodies with negligible temperature gradients, the time temperature variation curve is
(A) Parabolic (B) Sinusoidal
(C) Linear (D) Exponential

7. The ratio of heat flow through walls A and B (Q_A/Q_B) having the same thickness having thermal conductivity $k_A = 4k_B$ for the temperature difference $(\Delta T)_B = 2(\Delta T_A)$ will be
(A) 0.5 (B) 2
(C) 0.25 (D) 1

8. The effectiveness of a fin will be maximum in an environment with
(A) Free convection (B) Radiation
(C) Forced convection (D) Convection and Radiation

9. Which of the following profiles has highest fin efficiency?
(A) Rectangular (B) Triangular
(C) Parabolic (D) All of the above

10. If the inner and outer surface areas of a sphere are 10 cm^2 and 12 cm^2 respectively, then the geometric mean area of the sphere (in cm^2) is
(A) 11.50 (B) 12.63
(C) 10.95 (D) 9.75

11. An insulated pipe of 100 mm diameter of emissivity 0.8 is laid in a room at 30°C. If the surface temperature is 300°C and the convective heat transfer coefficient is 12 W/m^2-K, then the heat loss per unit length of pipe (in Watts/m) will be
(A) 1018 (B) 1416
(C) 2434 (D) 1863

12. The temperature of the inner surface of a wall of length 6 m, height 5 m and thickness 0.25 m is 120°C. If the outer surface of the wall is 30°C and thermal conductivity of the wall is 0.8 W/m-K then the temperature at an interior point of the wall, 0.20 m from the inner wall will be
(A) 56°C (B) 192°C
(C) 38°C (D) 48°C

13. A hollow cylinder of 10 cm inner diameter and 20 cm outer diameter has an inner surface temperature of 400°C and an outer surface temperature of 200°C. The temperature of the point half way between the inner and outer surfaces will be
(A) 268°C (B) 301°C
(C) 258°C (D) 283°C

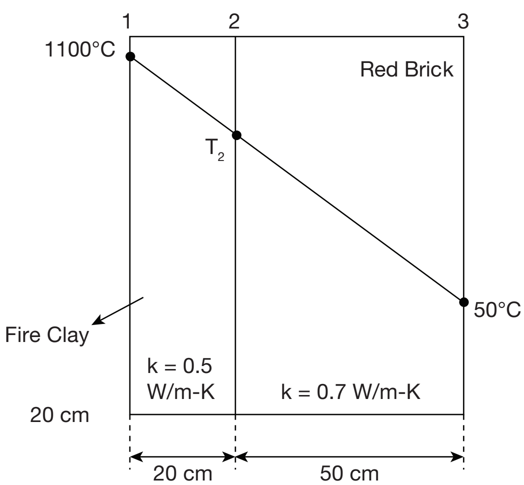
14. A hollow sphere 20 cm inner diameter and 40 cm outer diameter of a material having thermal conductivity of 60 W/m-K is filled with hot oil. Its inner and outer surface temperatures are 400°C and 100°C respectively. The temperature at a point a quarter of the way between the inner and outer surface will be

(A) 300°C (B) 280°C
(C) 245°C (D) 321°C

15. A thick furnace wall of 100 cm thickness have surface temperature of 400°C and 100°C. If the thermal conductivity (k) of the wall material is given by $k = 0.01T - (5 \times 10^{-6})T^2$; where T is temperature in °C. The heat loss per unit area (W/m^2) will be
(A) 465 (B) 565
(C) 435 (D) 645

16. A hot gas of 2000°C is flowing inside a furnace wall whose thermal conductance is 60 W/m^2-K. The inner surface of the wall is at 1000°C and the heat flow by radiation from gases to inside surface of the wall is 24 kW/m^2. If the convective heat transfer coefficient at the interior surface is 12 W/m^2-K then the surface temperature of the external surface of the wall will be
(A) 500°C (B) 600°C
(C) 400°C (D) 450°C

17. A furnace is made of fireclay and red brick. The temperatures of the surface of the furnace is shown in the figure. The heat loss through the furnace is 1200 W/m^2. It is desired to reduce the thickness of the red brick layer in this furnace to half by filling in the space between the two layers by diatomite whose thermal conductivity is given by $k = 0.1 + 0.0002T$ (W/m-K) where T is the temperature in °C. What is the thickness of the filling to ensure an identical loss of heat for the same outside and inside temperatures?



(A) 24.73 mm (B) 26.12 mm
(C) 21.23 mm (D) 30.9 mm

18. A pipe line (k = 60 W/m-K) of inner diameter 200 mm and outer diameter 210 mm is to be covered with two layers of insulation each having a thickness of 100 cm. The thermal conductivity of the first insulation material is 0.06 W/m-K and that of the second is 0.15 W/m-K. If the temperature of the inside tube surface is 300°C and that of the outside surface of the insulation is 50°C then the interface temperature between the two layers of insulation will be
(A) 110°C (B) 98°C
(C) 120°C (D) 90°C

19. A plane wall 12 cm thick generates heat at the rate of 6×10^4 W/m^3 when an electric current is passed through it. The convective heat transfer coefficient between each face of the wall and the ambient air is 60 W/m^2-K. Assuming the ambient air temperature to be 20°C and the thermal conductivity of the wall material to be 20 W/m-K, calculate the maximum temperature in the wall.
(A) 65°C (B) 85.4°C
(C) 80°C (D) 75°C

20. A long solid rod base of diameter 30 mm is placed in a furnace with a large portion of it projecting into the room air at 27°C. After steady state conditions are reached, the temperatures at two points, 200 mm apart are found to be 120°C and 80°C respectively. If the convective heat transfer coefficient between the rod surface and the surrounding air is 30 W/m^2-K, the thermal conductivity of the rod material (in W/m-k) will be
(A) 471 (B) 529
(C) 407 (D) 506

21. A fin 4 cm long and having cross-sectional area of 4 cm^2 and perimeter 10 cm is made of steel (k = 24 W/m-K). The base surface temperature is 527°C. The fin is exposed to a hot fluid at 900°C and the heat transfer coefficient between the fin and fluid is 300 W/m^2-K. Assuming the tip of the fin to be insulated, the rate of heat flow at the base of the fin (in watts) at a distance of 2 cm from the base will be
(A) 195.65 and 766.54°C (B) 210.7 and 816.1°C
(C) 173.65 and 741.67°C (D) 189.7 and 793.4°C

22. A fin of thermal conductivity of 20 W/m-K, 10 cm long and 2 cm diameter is fitted to a wall which is exposed to a boiling water convection situation where heat transfer coefficient is 4000 W/m^2-K. Which of these one is CORRECT regarding the heat dissipation when fin is used?
(A) Increase by 81.65% (B) Decrease by 65.81%
(C) Decrease by 18.35% (D) No changes

23. A large slab of 100 cm thickness and of thermal conductivity of 2 W/m-K is heated from one side and the temperature distribution across the slab as measured by thermocouples approximates to the following relation:
$T = 100 - 100x + 12x^2 + 30x^3 - 20x^4$
where T is temperature in °C and x in meters considering an area of 10 m^2, the rate of heat energy stored (in watts) will be
(A) 1320 (B) 1200
(C) 680 (D) 1120

24. The average heat transfer coefficient for flow of 200°C air over a flat plate is measured by observing the temperature time history of a 5 cm thick copper slab exposed to 200°C air. In one test run, the initial temperature of the plate was 410°C, and in 6 minutes the temperature decreased by 50°C, the heat transfer coefficient will be (in W/m^2-K)

Properties of copper: Density = 9000 kg/m^3, Specific heat = 0.38 kJ/kg-K, Thermal conductivity = 370 W/m-K

(A) 64.5 (B) 164.5
(C) 36.5 (D) 136.5

25. A sphere of 8 cm diameter has a conductivity of 60 W/m-K, which is to be insulated with an insulating material of conductivity of 0.3 W/m-K. If the convective heat transfer coefficient with the ambient atmosphere is 6 W/m^2-K, the critical thickness of insulation will be

(A) 8 cm (B) 4 cm
(C) 6 cm (D) 10 cm

Answer Keys

1. A	**2.** A	**3.** D	**4.** C	**5.** B	**6.** D	**7.** B	**8.** A	**9.** C	**10.** C
11. C	**12.** D	**13.** D	**14.** B	**15.** D	**16.** C	**17.** A	**18.** B	**19.** B	**20.** D
21. A	**22.** C	**23.** C	**24.** A	**25.** C					

Hints and Explanations

1. The temperature at any point in the material does not change with time, i.e., $\frac{\partial T}{\partial t} = 0$. Therefore steady, Poisson's equation. Choice (A)

2. $$A_m = \frac{A_o - A_i}{\ln\left(\frac{A_o}{A_i}\right)} = \frac{2000-1000}{\ln\left(\frac{2000}{1000}\right)}$$

$\Rightarrow$ $A_m = 1442.692$ mm^2
~ 1443 mm^2 Choice (A)

3. $$A_m = \frac{A_o - A_i}{\ln\left(\frac{A_o}{A_i}\right)}$$

Here $A_o = A_i$ because $d_o = d_i$ and $t_o = t_i$

$\therefore$ $A_m = \frac{0}{0}$ which is undefined Choice (D)

4. $$(T_1 - T_2) = \frac{Q(r_2 - r_1)}{4\pi k\, r_1 r_2}$$

Its is a hyperbolic decay in temperature profile.
Choice (C)

5. Choice (B)

6. Choice (D)

7. $$Q = -k\frac{dT}{dx}$$

$$\frac{Q_A}{Q_B} = \frac{k_A(\Delta T)_A}{k_B(\Delta T)_B} = \frac{4}{2} = 2$$ Choice (B)

8. Choice (A)

9. Choice (C)

10. $A_g = \sqrt{A_0\, A_i} = \sqrt{10 \times 12}$

$\Rightarrow$ $A_g = 10.95$ cm^2 Choice (C)

11. The heat loss is due to combined modes of convection and radiation.

$$Q = hA(T_S - T_\infty) + \in A\,\sigma\left(T_S^4 - T_\infty^4\right)$$

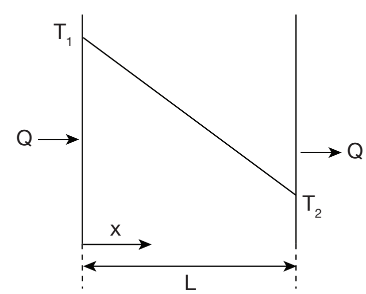
$$\Rightarrow\ Q = h\pi DL(T_S - T_\infty) + \in \pi DL\sigma\left(T_S^4 - T_\infty^4\right)$$

$$\Rightarrow\ \frac{Q}{L} = h\pi\, D\left(T_S - T_\infty\right) + \in\pi\, D\sigma\left(T_S^4 - T_\infty^4\right)$$

$$\Rightarrow \frac{Q}{L} = \left[(12\times\pi\times 0.1)(300-30)\right] + \left[0.8\times\pi\times 0.1\times 5.67\times 10^{-8}\times\left(573^4 - 303^4\right)\right]$$

$$\Rightarrow\ \frac{Q}{L} = 2433.94\ \text{Watt/m}\ \sim 2434\ \text{Watt/m}$$ Choice (C)

12. Temperature distribution is given as



$$T = \left[\frac{T_2 - T_1}{L}\right]x + T_1$$

Here, $T_1 = 120 + 273 = 393$ K
$T_2 = 30 + 273 = 303$ K
$k = 0.8$ W/m-K
$L = 0.25$ m

At $x = 0.20$ m, $T = \left[\frac{(303-303)}{0.25}\times 0.20\right] + 393$

$\Rightarrow$ $T = 321$ K or 48°C Choice (D)

13. Throughout the cylinder, heat transfer remains constant.

$$\Rightarrow \quad Q = \frac{2\pi kL(T_i - T_o)}{\ln\left(\frac{r_o}{r_i}\right)} = \text{Constant}$$

At halfway between r_i and r_o, radius

$$r^1 = \frac{20+10}{2} = 15\ cm$$

$\therefore$ Q remains constant

$$\therefore \quad \frac{T_i - T_o}{\ln\left(\frac{r_o}{r_i}\right)} = \frac{T_i - T}{\ln\left(\frac{r^1}{r_i}\right)} \Rightarrow \frac{400-200}{\ln\left(\frac{20}{10}\right)} = \frac{400-T}{\ln\left(\frac{15}{10}\right)}$$

$\Rightarrow \quad T = 283°C$ Choice (D)

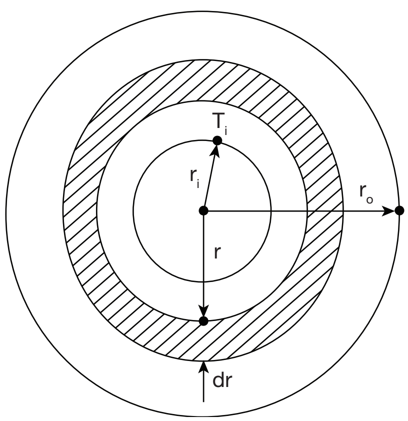
14. Heat conduction equation is

$$\frac{d}{dr}\left[r^2 \frac{dT}{dr}\right] = 0$$

Integrating $dT = C_1 \frac{dr}{r^2}$

Integrating again

$$T = -\frac{C_1}{r} + C_2$$



Boundary conditions

$T = T_i$ at $r = r_i$; $T_i = -\frac{C_1}{r_i} + C_2$ $\rightarrow (1)$

$T = T_o$ at $r = r_o$; $T_o = -\frac{C_1}{r_o} + C_2$ $\rightarrow (2)$

From equation (1) and (2) we get

$$C_1 = \frac{T_i - T_o}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)} \text{ and } C_2 = T_i + \frac{1}{r_i}\left[\frac{(T_i - T_o)}{\frac{1}{r_o} - \frac{1}{r_i}}\right]$$

$$\therefore \quad T = T_i - \frac{(T_i - T_o)}{\left[\frac{1}{r_o} - \frac{1}{r_i}\right]}\left[\frac{1}{r} - \frac{1}{r_i}\right]$$

$$\Rightarrow \quad T = \frac{r_o}{r}\left[\frac{r - r_i}{r_o - r_i}\right](T_o - T_i) + T_i$$

At $r = 10 + \frac{1}{4}(20 - 10) = 12.5$ cm

$$\therefore \quad T = \frac{0.20}{0.125}\left[\frac{0.125 - 0.10}{0.20 - 0.10}\right](100 - 400) + 400$$

$\Rightarrow \quad T = 280°C$ Choice (B)

15. $Q = -kA\frac{dT}{dx} \Rightarrow \int_0^L Q dx = -\int_{T_1}^{T_2} \text{kA dT}$

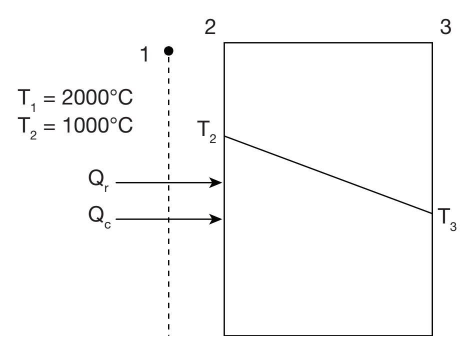
$$\therefore \quad \frac{QL}{A} = -\int_{T_1}^{T_2}\left[0.01T - 5\times10^{-6}T^2\right]dT$$

$$\Rightarrow \quad \frac{Q}{A} = +\frac{1}{L}\left[0.01\frac{T^2}{2} - 5\times10^{-6}\frac{T^3}{3}\right]_{100}^{400}$$ (Taking $L = 1\ m$)

$$\Rightarrow \frac{Q}{A} = +\frac{1}{1}\left[\frac{0.01}{2}(400^2 - 100^2) - \frac{5\times10^{-6}}{3}(400^3 - 100^3)\right]$$

$\Rightarrow \quad \frac{Q}{A} = 645\ \text{W/m}^2$ Choice (D)

16.



$$\frac{Q_r}{A} = q_r = 24\ \text{kW/m}^2$$

$$\frac{Q_C}{A} = q_c = h_1(T_1 - T_2) = 12(2000 - 1000)$$

$\Rightarrow \quad q_c = 12\ \text{kW/m}^2$

$\therefore$ Total q entering the wall = 24 + 12 = 36 kW/m²

Thermal resistance of the wall $R = \frac{1}{\text{conductance}}$

$$= \frac{1}{60}\frac{m^2 - K}{W}$$

Now, $q = \frac{(T_2 - T_3)}{R} \Rightarrow 36 = \frac{1000 - T_3}{\frac{1}{60}\times 1000}$

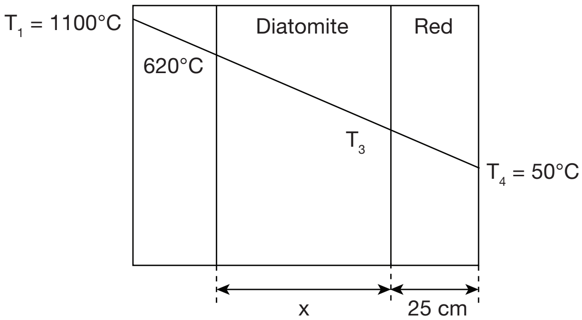
$\Rightarrow \quad T_3 = 400°C$ Choice (C)

17. $q = 1200$ W/m^2

Now $q = \dfrac{1100 - T_2}{\dfrac{0.2}{0.5}}$

$\Rightarrow \quad T_2 = 1100 - \left[1200 \times \dfrac{0.2}{0.5}\right]$

$\Rightarrow \quad T_2 = 620°C$



Now $q = \dfrac{T_3 - T_4}{\dfrac{0.25}{0.7}}$

$\Rightarrow \quad 1200 \times \dfrac{0.25}{0.7} = T_3 - 50$

$\Rightarrow \quad T_3 = 478.57°C$

Mean thermal conductivity, k_m

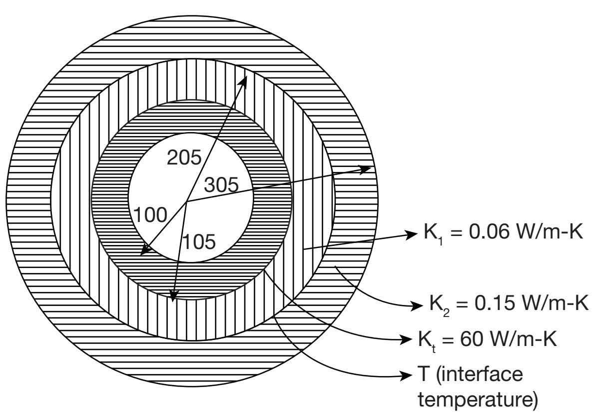
$= 0.1 + 0.0002\left[\dfrac{620 + 478.57}{2}\right]$

$\therefore \quad k_m = 0.209857$ W/m-K

$\therefore \quad q = \dfrac{T_2 - T_3}{\dfrac{x}{K_m}} \Rightarrow 1200 = \dfrac{620 - 478.57}{\dfrac{x}{0.209857}}$

$\Rightarrow \quad x = 0.0247334$ m or 24.73 mm Choice (A)

18.



$$\frac{Q}{L} = \frac{2\pi[300 - 50]}{\dfrac{\ln\left(\dfrac{105}{100}\right)}{60} + \dfrac{\ln\left(\dfrac{205}{105}\right)}{0.06} + \dfrac{\ln\left(\dfrac{305}{205}\right)}{0.15}}$$

$\Rightarrow \quad \dfrac{Q}{L} = 113.823$ W/m

Interface temperature, T

$$\frac{Q}{L} = \frac{2\pi(T - 50)}{\dfrac{\ln\left(\dfrac{305}{205}\right)}{0.15}} = 113.823$$

$\Rightarrow \quad T = 97.98°C \sim 98°C$ Choice (B)

19. $\dot{q} = 6 \times 10^4$ W/m^2, $h = 60$ W/m^2-K, $L = 12$ cm

$T_\infty = 20°C$, $k = 20$ W/m-K

T_W = Surface temperature $= T_\infty + \dfrac{\dot{q}L}{2h}$

$\Rightarrow \quad T_W = 20 + \dfrac{(6\times10^4)\times0.12}{2\times60} = 80^oC$

$T_{max} = T_W + \dfrac{\dot{q}L^2}{8k} = 80 + \dfrac{(6\times10^4)\times0.12^2}{8\times20}$

$\Rightarrow \quad T_{max} = 85.4^oC$ Choice (B)

20. For an infinitely long rod

$\dfrac{T - T_\infty}{T_o - T_\infty} = e^{-mx}$

At $x = 0$, $T = T_o = 120^oC$

At $x = 200$ mm, $T = 80^oC$

$\therefore \quad \dfrac{80 - 27}{120 - 27} = e^{-(m\times0.2)}$

or $m = 2.81154 = \sqrt{\dfrac{hP}{kA}} \Rightarrow \sqrt{\dfrac{30\times\pi\times D}{k\times\dfrac{\pi}{4}\times D^2}} = 2.81154$

$\Rightarrow \quad \sqrt{\dfrac{4\times30}{k\times D}} = 2.81154$

$\Rightarrow \quad k = \dfrac{4\times30}{2.81154^2\times0.03}$

$\Rightarrow \quad k = 506.02$ W/m-K Choice (D)

21. $\dfrac{T - T_\infty}{T_o - T_\infty} = \dfrac{\text{Cosh}[m(L-x)]}{\text{Cosh}(mL)}$

$L = 0.04$ m, $A = 4 \times 10^{-4}$ m^2, $P = 0.1$ m, $k = 24$ W/m-K, $h = 300$ W/m^2–k, $T_o = 527^oC$, $T_\infty = 900^oC$

$m = \sqrt{\dfrac{hP}{kA}} = \sqrt{\dfrac{300\times0.1}{24\times4\times10^{-4}}} = 55.9$

Heat transfer rate, $Q = \sqrt{hPkA} \times (T_o - T_\infty)$ tan $h(mL)$

$\Rightarrow \quad Q = \sqrt{300\times0.1\times24\times4\times10^{-4}} \times (-373) \times$ tan $h(55.9 \times 0.04)$

$\Rightarrow \quad Q = -195.65$ watts

Now $T - T_\infty = (T_o - T_\infty)\left[\dfrac{\text{Cos}\,h\,m(L-x)}{\text{Cos}\,h(mL)}\right]$

At $x = 0.02$ m

$$T = 900 + (-373)\frac{\text{Cos}\,h\left[55.9(0.04-0.02)\right]}{\text{Cos}(55.9\times0.04)}$$

$T = 766.54°C$ Choice (A)

22. $$\frac{Q_{with\ fin}}{Q_{without\ fin}} = \frac{\tan h(mL)}{\sqrt{\dfrac{hA}{kP}}}$$

$$= \frac{\tan\ h\left[\sqrt{\dfrac{6000\times\pi\times0.02\times4}{20\times\pi\times0.02^2}}\times0.1\right]}{\sqrt{\dfrac{6000\times\pi\times0.02^2}{4\times20\times\pi\times0.02}}}$$

∴ $Q_{\text{with fin}} = Q_{\text{without fin}} \times 0.8165$

∴ Heat dissipation will decrease by 18.35% by using the fin. Choice (C)

23. Heat entering the slab, $Q_i = -kA\left(\dfrac{dT}{dx}\right)\Bigg|_{x=0}$

⇒ $Q_i = -2 \times 10 \times (-100)|_{x=0}$

⇒ $Q_i = 2000\ W$

Heat leaving the slab, $Q_o = -kA\left(\dfrac{dT}{dx}\right)\Bigg|_{x=1\,m}$

⇒ $Q_o = -2 \times 10 \times [-100 + 24x + 90x^2 - 80x^3]_{x=1\,m}$

⇒ $Q_o = 1320$ W

∴ Rate of heat storage $= Q_i - Q_o$

$= 2000 - 1320 = 680$ W Choice (C)

24. Thickness of plate $= 2L = 0.05$ m

$$\therefore\ L_C = L = \frac{0.05}{2} = 0.025\,\text{m}$$

$$\frac{hA}{\rho cv} = \frac{h}{\rho_c L_c} = \frac{h}{9000\times0.38\times0.025} = 1.17\times10^{-5}\,h$$

$$\text{Now}\ \frac{T-T_\infty}{T_o-T_\infty} = \exp\left[-\left(\frac{hA}{\rho cv}\right)t\right]$$

$$\frac{360-200}{410-200} = \exp\left[-\left(1.17\times10^{-5}\,h\right)\times360\right]$$

$= h = 64.56$ W/m^2-K Choice (A)

25. Critical radius, $r_c = \dfrac{2k}{h_o} = \dfrac{2\times0.3}{6}$

⇒ $r_C = 0.01$ m $= 10$ cm

Critical thickness of insulation $= (10 - 4) = 6$ cm

Choice (C)