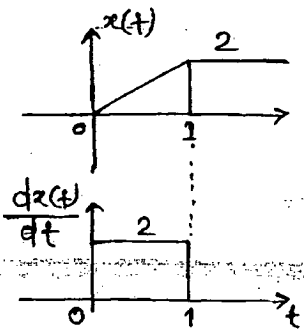
Chapter-02
Different operations of signal

(1.) Differentiation →

$$x(t) = \frac{dx(t)}{dt} = \text{slope of } x(t) \text{ wrt } 't'$$

* Graphical diff is applicable for triangular & rectangular type signal.

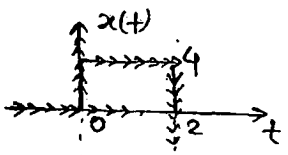
(1.)



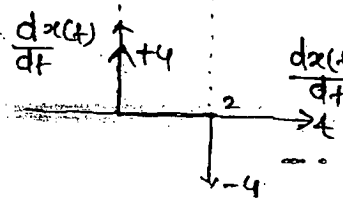
$x(t) = 2r(t) - 2r(t-1)$

$\frac{dx(t)}{dt} = 2u(t) - 2u(t-1)$

(2.)

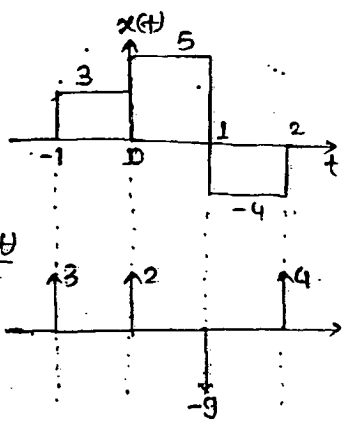


$x(t) = 4[u(t) - u(t-2)]$

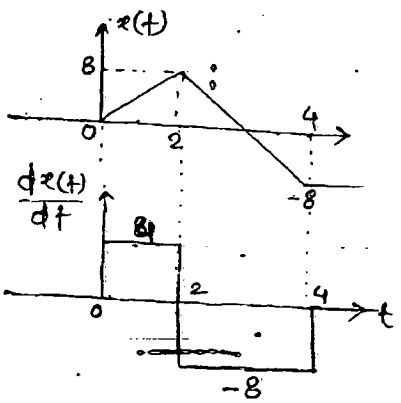


$\frac{dx(t)}{dt} = 4[\delta(t) - \delta(t-2)]$

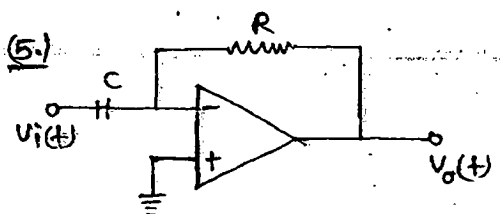
(3.)



(4.)

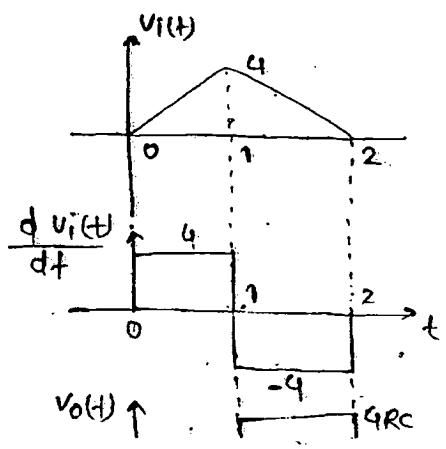


(5.)



$$V_o(t) = -RC\frac{dv_i(t)}{dt}$$

↓ amp reversal



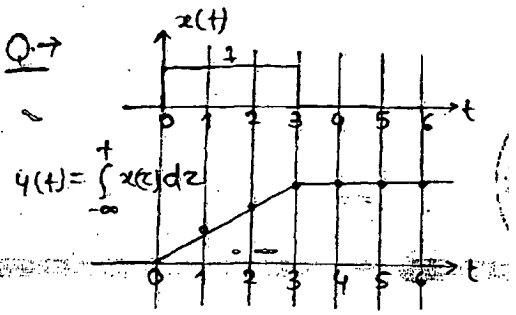
(2.) Integration →

$$x(t) = y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

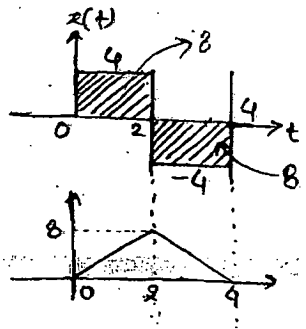
= area of signal $x(t)$ wrt 't'

* Graphical integration is applicable only for rectangular type waveform.

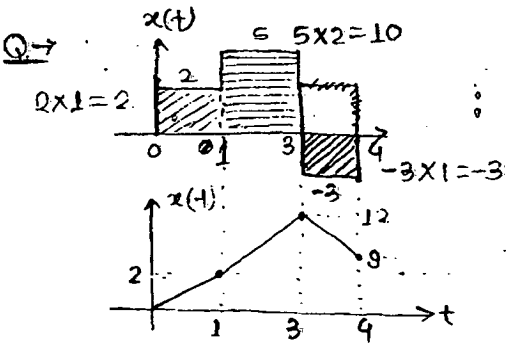
Q.→



Q.→

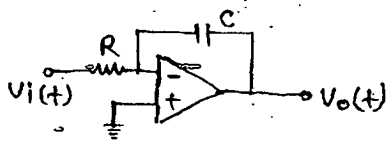


Q.→

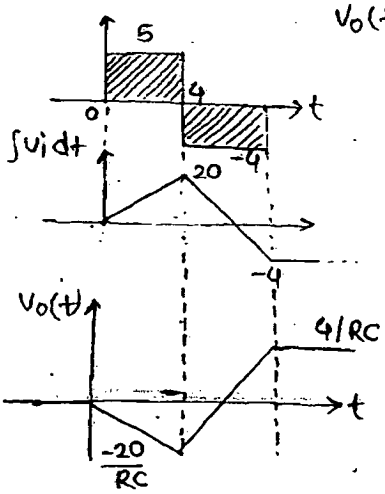


Total area = 2 + 10 − 3 = 9

Q.→



$$V_o(t) = \frac{-1}{RC}\int v_i\,dt$$

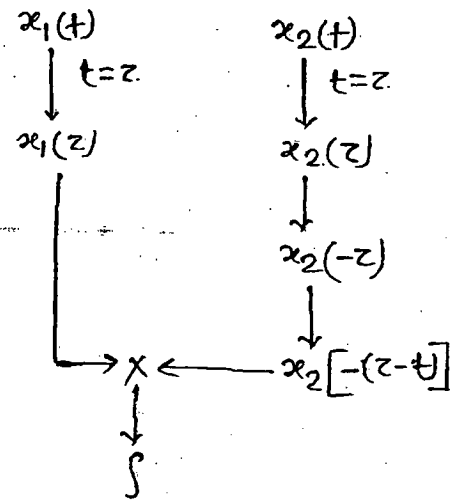


(3.) Convolution → It is a mathematical operator & it is used for calculation of responce of LTI system.

$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\cdot x_2(t-\tau)\,d\tau$$

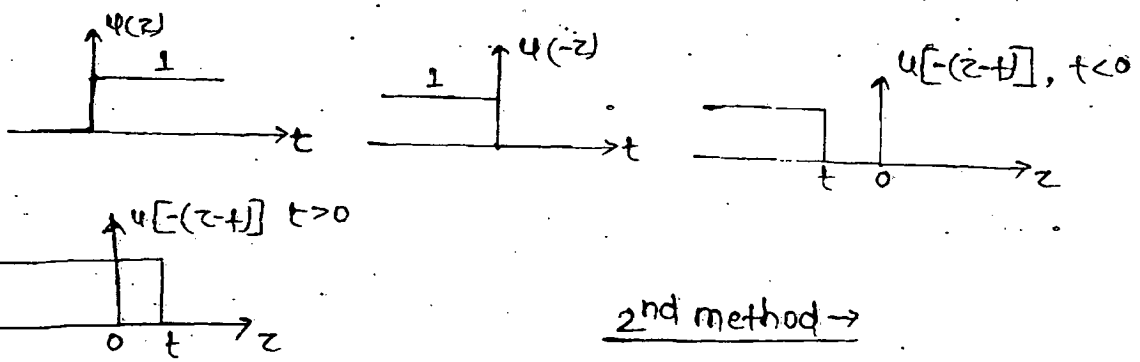
$$\boxed{x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\cdot x_2(t-\tau)\,d\tau}$$



Steps →
(1.) Folding
(2.) Shifting
(3.) Multiplication
(4.) Integration.

Q.→ $y(t) = u(t) * u(t)$

Soln→ $= \int_{-\infty}^{\infty} u(z) \cdot u(t-z)\, dz$



$$y(t) = \int_{-\infty}^{\infty} u(z) \cdot u(t-z)\, dz$$

$$= \begin{cases} 0 & ;t<0 \\ \int_0^t dt & ;t>0 \end{cases}$$

$$= \begin{cases} 0 & ,t<0 \\ t & ,t>0 \end{cases}$$

$$= r(t)$$

2nd method →

$$y(t) = x_1(t) * x_2(t)$$

$$Y(s) = X_1(s) \cdot X_2(s)$$

$$Y(s) = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2}$$

$$\boxed{y(t) = r(t)}$$

* Properties of Convolution →

(1.) Commutative →

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

$$\int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$

(2.) Associative →

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

(3.) Distributive →

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

(4.) Impulse Responce →

$$x(t) * \delta(t-t_1) = x(t-t_1)$$

$$\downarrow t_1 = 0$$

$$x(t) * \delta(t) = x(t)$$

Eg:- (1.) $u(t-1) * \delta(t+2) = u[(t+2)-1] = u(t+1)$

(5.) Derivative →

$$y(t) = x_1(t) * x_2(t)$$

$$\frac{dy(t)}{dt} = \frac{d}{dt}x_1(t) * x_2(t) \qquad x_1(t) * \frac{d}{dt}x_2(t)$$

Eg:- Find $y(t) = ?$

$$y(t) = \frac{d}{dt}[r(t) * u(t)]$$

Soln →

$$y(t) = \frac{d}{dt} r(t) * u(t) \qquad \text{(OR)} \quad y(t) = r(t) * \frac{d}{dt} u(t)$$

$$= u(t) * u(t) \qquad = r(t) * \delta(t)$$

$$= r(t) \qquad = r(t)$$

(6.) Step Responce →

$$y(t) = x(t) * u(t) = ?$$

$$y(t) = \int_{-\infty}^{t} \frac{dy(t)}{dt}\, dt = \int_{-\infty}^{t} \left[x(t) * \frac{du(t)}{dt}\right] dt$$

Eg.→ (1) $u(t) * u(t) = \int_{-\infty}^{t} u(t)\,dt = r(t)$

(2) $r(t) * u(t) = \int_{-\infty}^{t} r(t)\,dt = p(t)$

(7) Time Scaling →

If $x_1(t) * x_2(t) = y(t)$ then;

$x_1(at) * x_2(at) = \frac{1}{|a|} y(at)$ $(a \neq 0)$

(8) Area →

If $x_1(t) * x_2(t) = y(t)$ then;

$\text{Area}_{y(t)} = \text{Area}_{x_1(t)} \times \text{Area}_{x_2(t)}$

(9) Time delay →

$x_1(t) * x_2(t) = y(t)$

$x_1[(t-t_1)] * x_2(t-t_2) = y[t-(t_1+t_2)]$

Eg:- (1) $u(t-1) * u(t-2) = r(t-3)$

$= (t-3)u(t-3)$

(2) $r(t-1) * u(t+3) = p(t+2)$

$= \frac{(t+2)^2}{2} u(t+2)$

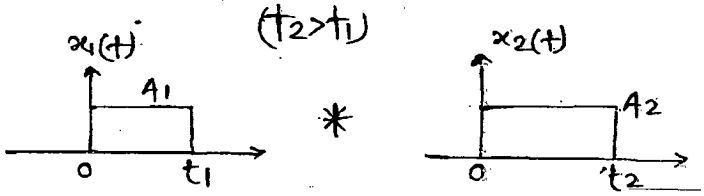
(10) Duration →

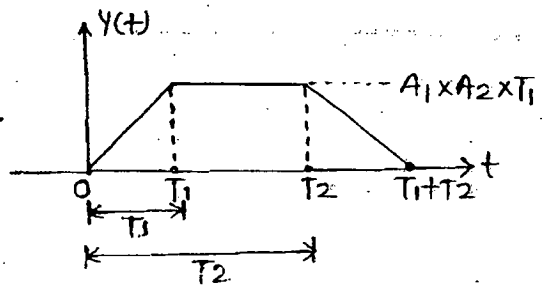
$\therefore y(t) = x_1(t) * x_2(t)$

Signal	Extension
$x_1(t)$	$t_1 \leq t \leq t_2$
$x_2(t)$	$t_3 \leq t \leq t_4$
$y(t)$	$t_1+t_3 \leq t \leq t_2+t_4$

* Convolution of 2 rectangular pulses of equal duration will be a triangle.
* Convolution of 2 rectangular pulses of unequal duration will be a Trapezoid.

Trapezoid →

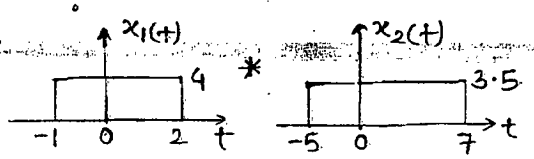




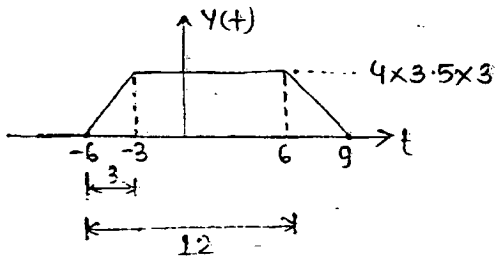
T_1 = Smaller duration

T_2 = Larger duration

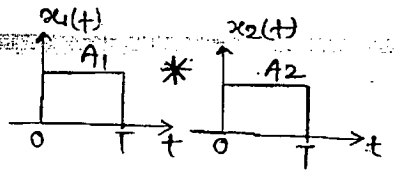
Que.→



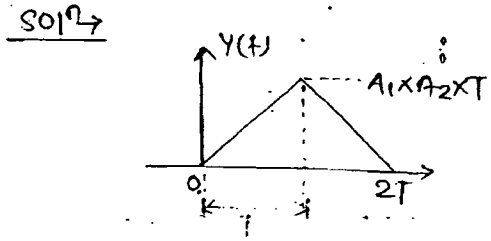
Soln→



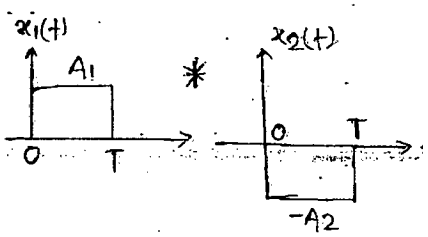
Que.→



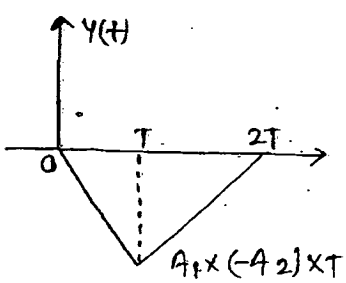
Soln→



Que.→

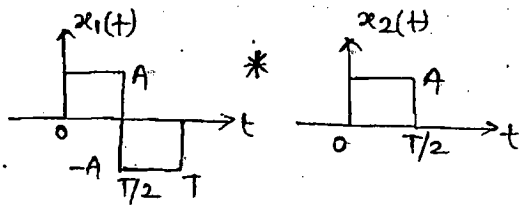


Soln→

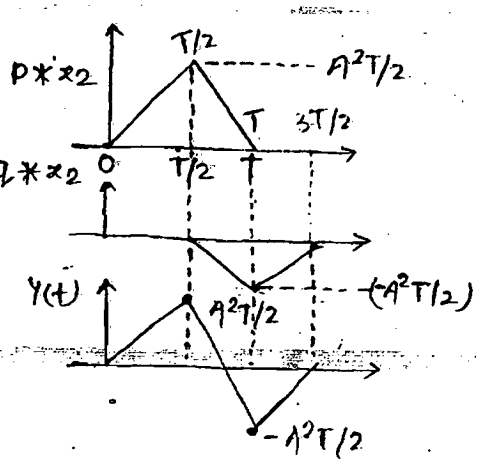


Because $(-A_2)\,\Delta$ will be -ve.

Q.→



Soln→

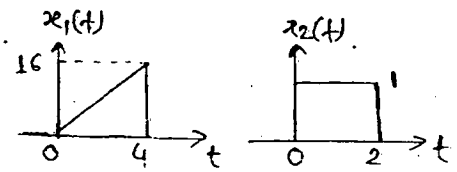


$y(t) = x_1(t) * x_2(t)$

$= [p(t) + q(t)] * x_2(t)$

$= [p(t) * x_2(t)] + [q(t) * x_2(t)]$

Q.→



$y(t) = x_1(t) * x_2(t)$

Find value of $y(2)$

(a) 4 (b) 8 (c) 16 (d) 32

Soln→

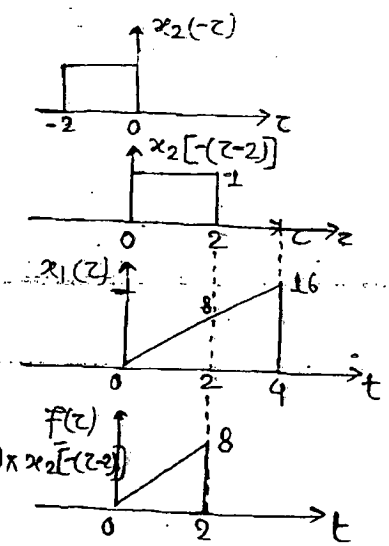
$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau$$

$$y(2) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(2-\tau)\, d\tau \quad \rightarrow x_2[-(\tau-2)]$$

$$= \int_{-\infty}^{\infty} f(\tau)\, d\tau = \text{area of } f(\tau)$$

where $f(\tau) = x_1(\tau) \cdot x_2[-(\tau-2)]$



Area $= \frac{1}{2} \times 2 \times 8 = 8$

Area = 8