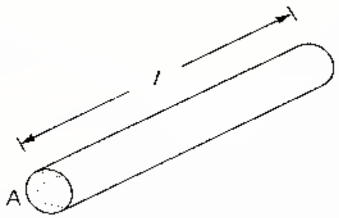
Circuit Elements and Signal Waveform

1

Electrical Network Components

Resistor

It is the property of a substance due to which it opposes the flow of current (i.e. electrons) through it.



Resistance, $R = \rho \frac{l}{A}$, Ohm (Ω)

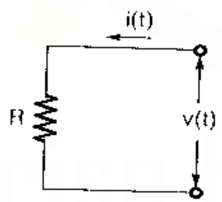
where, l = Length of conductor, metre (m)

A = Area of cross-section, m^2

ρ = Resistivity of the material, Ω-m

Note:

- Extension of wire result into increase in resistance while compression of wire result into decrease in resistance.



$$v(t) = Ri(t)$$
$$i(t) = \frac{1}{R} v(t)$$
... in time domain

$$V(s) = RI(s)$$
$$I(s) = \frac{1}{R} V(s)$$
... in s-domain

Power loss in resistor,

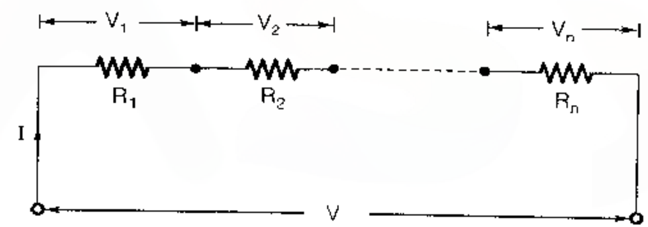
$$p = vi = i^2 R = \frac{v^2}{R}$$

Energy dissipated in resistor,

$$E_R = \int_{t_1}^{t_2} p\,dt$$

Combination of Resistors

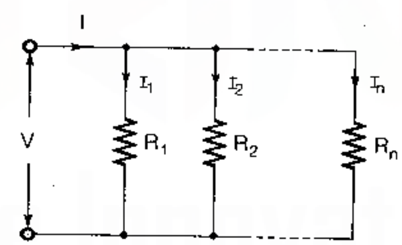
1. Resistors in series



$$V = IR_{eq}$$

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

2. Resistors in parallel

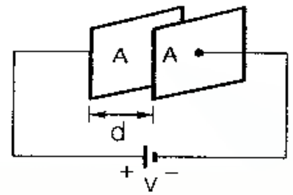


$$I = \frac{V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n}$$

Capacitor

The circuit element that stores energy in an electric field is a capacitor or capacitance.



Capacitance $\boxed{C = \frac{\epsilon_0 \epsilon_r A}{d}}$, Farad

where ϵ_0 = Permittivity of free space, F/m

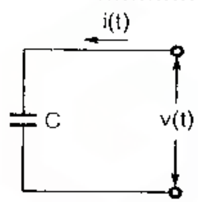
ϵ_r = Relative permittivity of the dielectric

A = Cross-sectional area of parallel plates, m^2

d = Seperation of plates, m

Note:

- When the capacitance is removed from the source, the capacitor retains the charge and the electric field until a discharge path is provided.



$$v(t) = \frac{1}{C}\int_0^t i(t)\,dt$$
$$i(t) = C\frac{dv(t)}{dt}$$
... in time domain

$$V(s) = \frac{1}{sC}I(s)$$
$$I(s) = sCV(s)$$
... in s-domain

$$Z_c(s) = \frac{1}{sC}$$
$$Y_c(s) = sC$$

Power in capacitance,

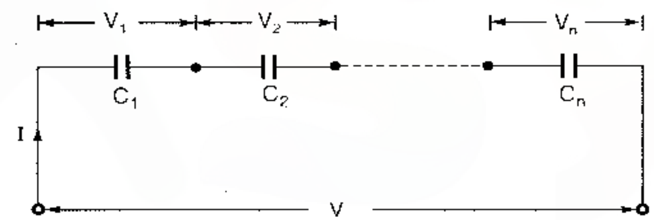
$$p = vi = Cv\frac{dv}{dt}$$

Energy stored in capacitance,

$$E_c = \frac{1}{2}Cv^2$$

Combination of Capacitors

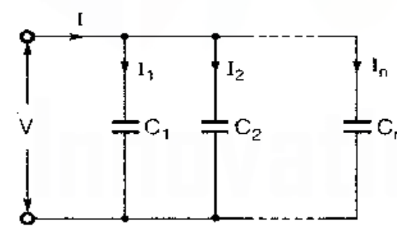
1. Capacitors in series



$$V = \frac{1}{C_{eq}}\int_0^t i\,dt$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

2. Capacitors in parallel

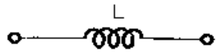


$$I = C_{eq}\frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_n$$

Inductor

The circuit element that stores energy in a magnetic field is an inductor or inductance.



Inductance $L = \dfrac{\mu_0 N^2 A}{l}$, Henry

where μ_0 = Permeability of free space, H/m

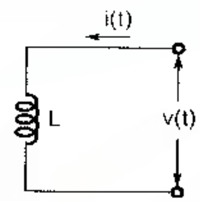
N = Total number of turns in coil

A = Cross-sectional area of coil, m^2

l = Length of coil, m

Note:

- When the inductance is removed from the source, the magnetic field will collapse i.e. no energy is stored without a connected sources.



$$v(t) = L\frac{di(t)}{dt}$$
$$i(t) = \frac{1}{L}\int_0^{\infty} v(t)\,dt$$
··· in time domain

$$V(s) = sL\, I(s)$$
$$I(s) = \frac{1}{sL}V(s)$$
··· in s-domain

$$Z_L(s) = sL$$
$$Y_L(s) = \frac{1}{sL}$$

Power in indutance,

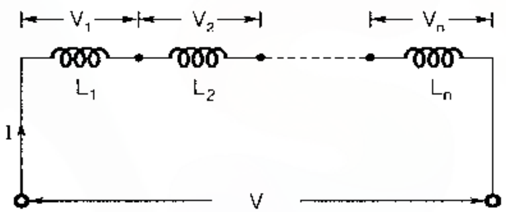
$$p = vi = Li\frac{di}{dt}$$

Energy stored in an inductance,

$$E_L = \frac{1}{2}Li^2$$

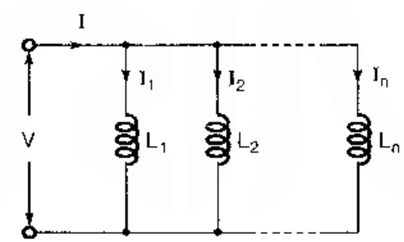
Combination of Inductor

1. Inductors in series



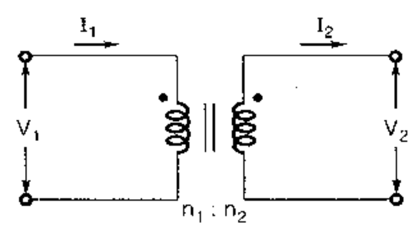
$$V = L_{eq}\frac{di}{dt} \quad ; \quad L_{eq} = L_1 + L_2 + L_3 + \ldots\ldots + L_n$$

2. Inductors in parallel



$$I = \frac{1}{L_{eq}}\int_0^t v\,dt \quad ; \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\ldots \frac{1}{L_n}$$

Ideal Transformer



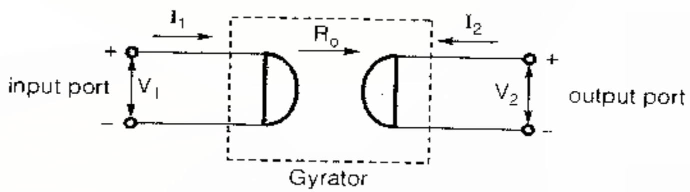
$$\boxed{\frac{V_2}{V_1} = \frac{n_2}{n_1} = \frac{I_1}{I_2}}$$

If $n_1 < n_2$, step up transformer

and $n_1 > n_2$, step down transformer

Gyrator

Gyrator shows an impedance inversion.



$$\boxed{\begin{aligned} V_1 &= +R_o I_2 \\ V_2 &= -R_o I_1 \end{aligned}}$$

where, R_o = Coefficient of gyrator and it depends upon

(i) Op-amp parameter

(ii) Externally connected R and C values

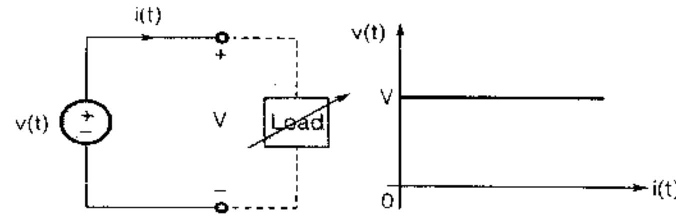
Remember:

- Linear network is one which holds the principle of superposition and principle of homogenity both.
- Element conducts in both directions is called bilateral element.
- A network containing circuit elements without any energy source is called passive network.
- A network containing energy source together with other circuit elements is called active network.

Ideal Sources

1. Ideal Voltage Source

Voltage always remains constant for any value of current passing through it.

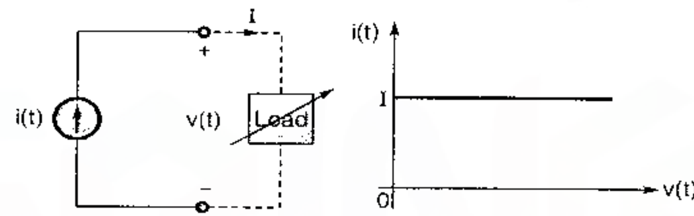


Note:

- The current through any voltage source is purely arbitrary. It will depends upon
  (a) The current source which is connect in series with it.
  (b) The load resistance which is connected in parallel with it.
- We cannot write KCL equation at a load with which any voltage source is connected. Since, the current through this voltage source is purely arbitrary.

2. Ideal current source

- Current always remains constant for any value of voltage across it



Note:

- The voltage across current source is purely arbitrary. It will depends upon
  (a) Voltage source which is connected in parallel with it.
  (b) The load resistance which is connected in series with it.
- We cannot write any KVL equation in a loop in which a current source is present. Since, the voltage across the current source is purely arbitrary.

Dependent Sources

(a)  $v(t) = Ki_1 \Rightarrow$ Current Controlled Voltage Source (CCVS)

(b) 

$v(t) = Kv_2 \Rightarrow$ Voltage Controlled Voltage Source (VCVC)

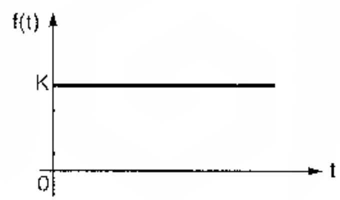
(c) 
$i(t) = Kv_1 \Rightarrow$ Voltage Controlled Current Source (VCCS)

(d) 
$i(t) = Ki_2 \Rightarrow$ Current Controlled Current Source (CCCS)

where, K is a constant.

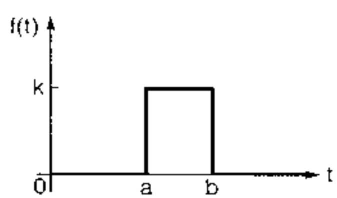
Standard Input Signals

1. Step function



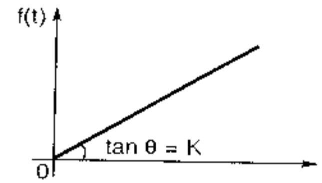
$$f(t) = \begin{cases} 0 & ; \; t \le 0 \\ K & ; \; t > 0 \end{cases}$$

2. Gate Function



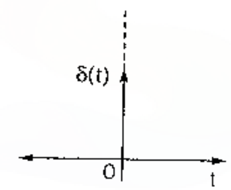
$$f(t) = K[u(t-a) - u(t-b)]$$

3. Ramp function



$$f(t) = \begin{cases} 0 & ; \; t \le 0 \\ Kt & ; \; t > 0 \end{cases}$$

4. Impulse function



$$\delta(t) = \begin{cases} 0 & ; \; t \ne 0 \\ \infty & ; \; t = 0 \end{cases}$$

- Area under the impulse function is always unity.

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- Sampling property of impulse function

$$\int_{-\infty}^{\infty} f(t) \cdot \delta(t - t_0)dt = f(t_0)$$

Miscellaneous

- Average value

$$Y_{av} = \frac{1}{T}\int_0^T y(t)dt$$

where T is the time period of periodic function y(t)

- RMS or effective value

$$Y_{rms} = \sqrt{\frac{1}{T}\int_0^T [y(t)]^2\,dt}$$

- If $y(t) = a_0 + (a_1 \cos\omega t + a_2 \cos 2\omega t + \ldots) + (b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots)$

$$Y_{rms} = \sqrt{a_0^2 + \frac{1}{2}(a_1^2 + a_2^2 + \ldots) + \frac{1}{2}(b_1^2 + b_2^2 + \ldots)}$$

- Form factor

$$FF = \frac{Y_{rms}}{Y_{av}} = \frac{\sqrt{\frac{1}{T}\int_0^T [y(t)]^2\,dt}}{\frac{1}{T}\int_0^T y(t)\,dt}$$

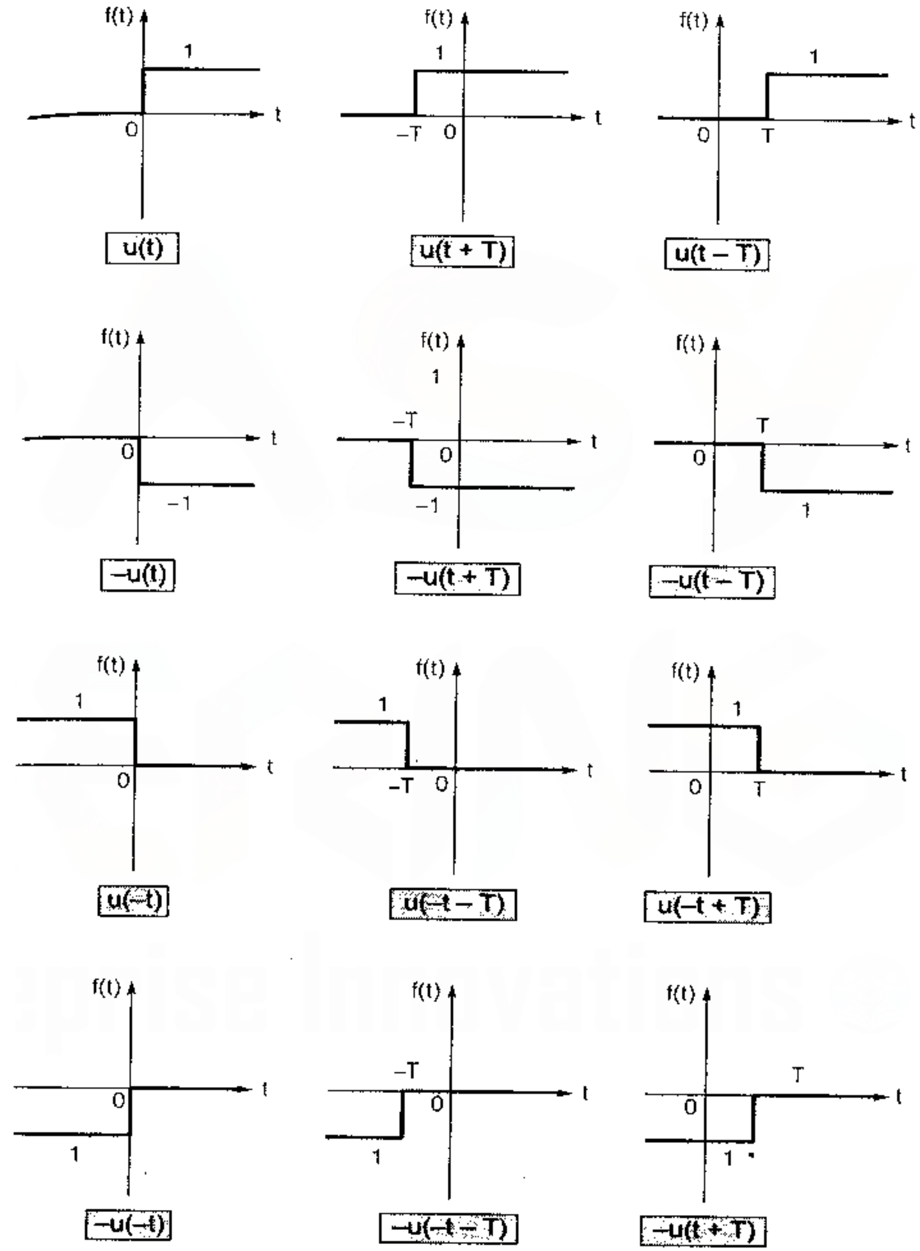
Remember: ....

- RMS value of $\sin \omega t$ and $\cos \omega t$ is $\frac{1}{\sqrt{2}}$.
- $\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$ and $\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$

  $\sinh \omega t = \frac{e^{\omega t} - e^{-\omega t}}{2}$ and $\cosh \omega t = \frac{e^{\omega t} + e^{-\omega t}}{2}$

....

Representation of Shifted Unit Step Function



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