RATIONAL EXPRESSIONS

IMPORTANT FACTS AT A GLANCE

1. An expression of the form $\frac{p(x)}{q(x)}$, where p (x) and q (x) are polynomials and q (x) $\neq$ 0, is called a ***rational expression***.

2. Every polynomial is a rational expression but a rational expression need not be a polynomial.

3. If p (x) and q (x) are polynomials such that g.c.d. {p (x), q (x)} = 1, then $\frac{p(x)}{q(x)}$, is a ***rational expression in its lowest terms.***

4. **Results on Rational expressions:**

(A) Sum (or difference) of two rational expressions is a rational expression.

(B) Product of two rational expressions is a rational expression.

(C) Addition as well as multiplication on rational expressions satisfies the commutative and associative laws.

(D) 0 is the additive identity.

(E) Additive inverse of $\frac{p(x)}{q(x)}$ is $\frac{-p(x)}{q(x)}$.

EXERCISE

1. The expression to be added to $(5x^2 - 7x + 2)$ to produce $(7x^2 - 1)$ is :-
(A) $2x^2 + 7x + 3$ (B) $2x^2 + 7x - 3$ (C) $12x^2 - 7x + 1$ (D) $2x^2 - 3$

2. What must be added to $1 - x + x^2 - 2x^3$ to produce x^3?
(A) $x^3 - x^2 + x - 1$ (B) $-1 + x + x^2 - 3x^3$ (C) $3x^3 - x^2 + x - 1$ (D) None of these

3. What must be added to the sum of $4x^2 + 3x - 7$ and $3x^2 + 6x + 5$ to get 1 ?
(A) $7x^2 + 9x - 3$ (B) $3x - 9x - 7x^2$ (C) $7x^2 + 9x - 2$ (D) None of these

4. What should be subtracted from $6x^3 - 2x^2 + 9x + 2$ so as to get $10x^3 - x^2 - 7x + 10$?
(A) $-4x^3 - x^2 + 16x - 8$ (B) $4x^3 + x^2 - 16x + 8$
(C) $16x^3 - 3x^2 + 2x + 12$ (D) None of these

5. $\left(\frac{x+1}{x^2-1} - \frac{2}{x}\right)$ expressed as a rational expression is :-
(A) $-\frac{(x+2)}{x(x-1)}$ (B) $\frac{x^2-2}{x(x^2-1)}$ (C) $\frac{1}{x(x+1)}$ (D) $\frac{2(x+1)}{(x^3-1)}$

6. The rational expression which should be subtracted from $\left(\frac{x^2+2}{x-1}\right)$ to get $\left(\frac{x^2+6x-1}{x^2-1}\right)$ is :-
(A) $\frac{x^2+x-3}{x+1}$ (B) $\frac{x^2-x-6}{x+1}$ (C) $\frac{x^2+x-6}{x-1}$ (D) $\frac{x+3}{x-1}$

7. The value of $\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\left(1+\frac{1}{x+3}\right)\left(1+\frac{1}{x+4}\right)$ is :-
(A) $1+\frac{1}{x+5}$ (B) $\frac{1}{x+5}$ (C) $\frac{x+1}{x+5}$ (D) $\frac{x+5}{x+1}$

8. $\left(\frac{x^2+8x+12}{x^2+4x-12}\right)$ in lowest terms is :-
(A) $\frac{x+2}{x-2}$ (B) $\frac{x+6}{x-2}$ (C) $\frac{x+2}{x+6}$ (D) $\frac{x+6}{x-2}$

9. The expression $\frac{(x-1)(x-2)(x^2-9x+14)}{(x-7)(x^2-3x+2)}$ simplifies to :-
(A) $(x - 1)$ (B) $(x - 2)$ (C) $(x - 7)$ (D) $\frac{1}{(x-7)}$

10. The simplified form of the rational expression $\left(\frac{x^2}{x^2-y^2}-1\right)\left(\frac{x-y}{y}+2\right)$ is ::-
(A) $\frac{x}{(x+y)}$ (B) $\frac{y}{(x+y)}$ (C) $\frac{y}{(x-y)}$ (D) $\frac{x}{(x-y)}$

11. The simplified form of $\frac{(x^2-y^2)}{(x^2+2xy+y^2)} \times \frac{(xy+y^2)}{(x^2-xy)}$ is :-
(A) $\frac{y}{x}$ (B) $\frac{x}{y}$ (C) $\frac{x}{x+y}$ (D) $\frac{y}{x-y}$

12. If $a = \frac{1+x}{2-x}$, then $\frac{1}{a+1}+\frac{2a+1}{a^2-1}$ is equal to :-

(A) $\frac{(1+x)(2+x)}{(2x-1)}$ (B) $\frac{(1-x)(2-x)}{(x-2)}$ (C) $\frac{(1+x)(2-x)}{2x-1}$ (D) $\frac{(1-x)(2-x)}{2x+1}$

13. If $x = \frac{2T}{(1+T^2)}$ and $y = \frac{1-T^2}{1+T^2}$, then $(x^2 + y^2)$ is :-

(A) 0 (B) 1 (C) T^2 (D) T^4

14. $\left(\frac{x^2-5x+6}{x^2-9x+20}\times\frac{x^2-5x+4}{x^2-3x+2}\right)$ in simplified form is :

(A) $\frac{x-2}{x-3}$ (B) $\frac{x-3}{x-5}$ (C) $\frac{x-2}{x-5}$ (D) $\frac{x-5}{x-2}$

15. Which of the following statements are correct ?

1. Every polynomial is a rational expression.
2. Every rational expression is a polynomial
3. Degree of $(x^5 - 7x^2 + 3)$ is 5.

Of these statement the correct ones are :

(A) 1 & 2 (B) 2 & 3 (C) 1 & 3 (D) 1, 2 & 3

16. Which of the following statements is correct ?

(A) The sum of two rational expressions is a rational expression.
(B) The product of two rational expressions is not always a rational expression.
(C) The difference of two rational expressions is not always a rational expression.
(D) The division of a rational expression by a non-zero rational expression need not be rational expression.

17. If $x = \frac{a}{a+b}$ and $y = \frac{b}{a-b}$, then $\frac{1}{x}+\frac{1}{y}$ is equal to :

(A) $\frac{ab}{a^2+b^2}$ (B) $\frac{ab}{a^2-b^2}$ (C) $\frac{a^2+b^2}{ab}$ (D) $\frac{a^2-b^2}{ab}$

18. $\left(\frac{\sqrt{2}x-\sqrt{8}}{(x^2-4)}\times\frac{\sqrt{8}x+\sqrt{32}}{(x^2+4)}\right)$ on simplification gives :

(A) $\frac{4}{x^2+4}$ (B) $\frac{\sqrt{2}}{x-2}$ (C) $\frac{2\sqrt{2}}{x+2}$ (D) $\frac{2}{x^2-4}$

19. $\left(\frac{x^2-7x+12}{x^2-2x-35}\times\frac{x^2-13x+42}{x^2-x-6}\times\frac{x^2+7x+10}{x^2-10x+24}\right)$ in simplified form is :-

(A) $\frac{x-1}{x-3}$ (B) $\frac{x-6}{x+7}$ (C) $\frac{x-7}{x+12}$ (D) 1

20. The value of $\frac{1-x^4}{1+x}\div\frac{1+x^2}{x}\times\frac{1}{x(1+x)}$ is :

(A) $\frac{1}{x}$ (B) $1 + x$ (C) $1 - x^2$ (D) 1

21. The value of $\left(\dfrac{\frac{x}{y}-1}{1-\frac{x}{y}}\right)$ is :

(A) 0 (B) 1 (C) –1 (D) $\dfrac{x}{y}$

22. $\dfrac{(x+y)^2-(x-y)^2}{x^2y-xy^2}$ is equal to :

(A) $\dfrac{1}{xy}$ (B) $\dfrac{1}{x-y}$ (C) $\dfrac{2}{x-y}$ (D) $\dfrac{4}{x-y}$

23. The value of $\left(\dfrac{x-3}{x^2-x-6}+\dfrac{2x-1}{2x^2+5x-3}-\dfrac{2x+5}{x^2+5x+6}\right)$ is :

(A) 1 (B) 0 (C) – 1 (D) $\left(\dfrac{x-3}{x-2}\right)$

24. The value of $\left[\dfrac{x^2-(y-z)^2}{(x+z)^2-y^2}+\dfrac{y^2-(x-z)^2}{(x+y)^2-z^2}+\dfrac{z^2-(x-y)^2}{(y+z)^2-x^2}\right]$ is :

(A) 0 (B) 1 (C) – 1 (D) None of these

25. The simplified form of $\left(\dfrac{x^3-27}{x^2+3x+9}\right)$ is :

(A) (x – 3) (B) $\dfrac{1}{(x-3)}$ (C) $\dfrac{x-3}{x+3}$ (D) $\dfrac{1}{x+3}$

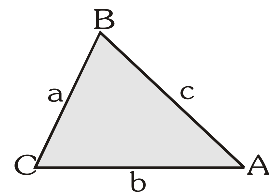
ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	B	A	A	A	D	A	B	C	A	C	B	B	D
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	A	C	A	D	D	C	D	B	B	A					

TRIANGLES

FUNDAMENTAL PROPERTIES OF TRIANGLES

1. Sum of any two sides is always greater than the third side.
2. The difference of any two sides is always less than the third side.
3. Greater angle has a greater side opposite to it and smaller angle has a smaller side opposite to it i.e., if two sides of a triangle are not congruent then the angle opposite to the greater side is greater.
4. Let a, b and c be the three sides of a ΔABC and c is the largest side. Then

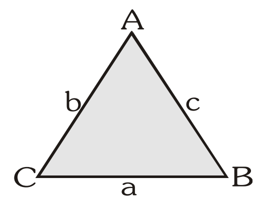


(i) if $c^2 < a^2 + b^2$, the triangle is **acute angled triangle.**

(ii) if $c^2 = a^2 + b^2$, the triangle is **right angled triangle.**

(iii) If $c^2 > a^2 + b^2$, the triangle is **obtuse angled triangle.**

5. **Sine rule :** In a ΔABC, if a, b, c be the three sides opposite to the angles A, B, C respectively, then

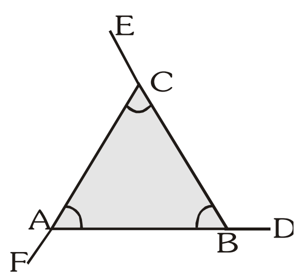


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

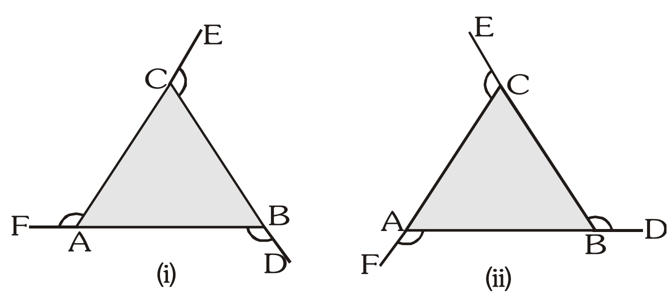
6. **Cosine rule :** In a ΔABC, if a, b, c be the sides opposite to angle A, B and C respectively, then

$$. . . . . \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

7. The sum of all the three interior angles is always 180° i.e., $\angle CAB + \angle ABC + \angle BCA = 180°$



8. The sum of three (ordered) exterior angles of a triangle is 360°

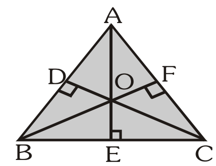
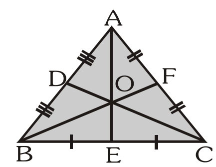
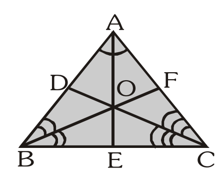
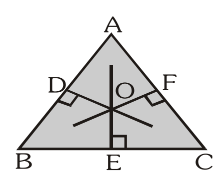
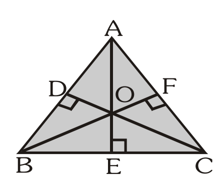


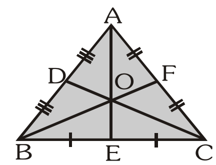
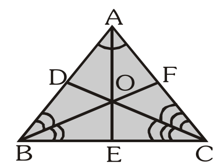
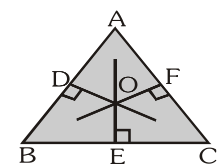
In fig. (i) : $(\angle FAC + \angle ECB + \angle DBA) = 360°$

In fig. (ii) : $(\angle FAB + \angle DBC + \angle ECA) = 360°$

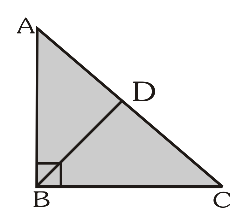
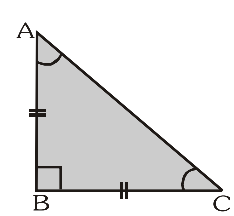
9. The sum of an interior angle and its adjacent exterior angle is 180°.
10. A triangle must has at least two acute angles.
11. In a triangle, the measure of an exterior angle equals the sum of the measures of the interior opposite angles.
12. The measure of an exterior angle of a triangle is greater than the measure of each of the opposite interior angles.

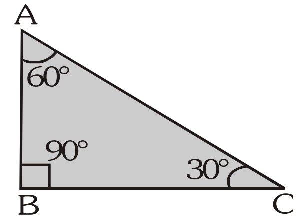
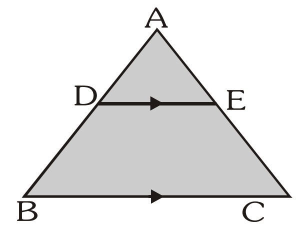
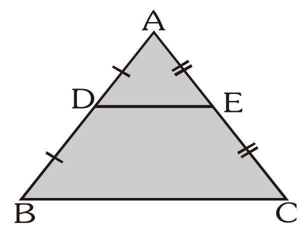
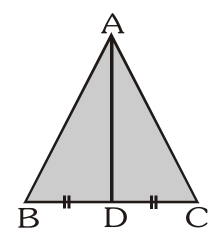
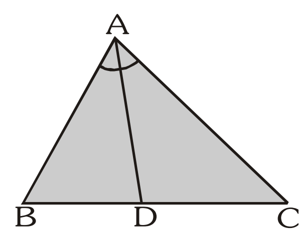
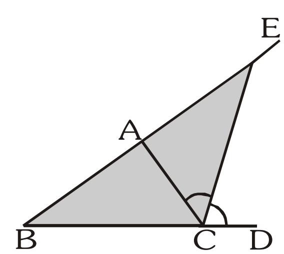
SOME IMPORTANT DEFINITIONS :

Nomenclature	Property/Definition	Diagram
Altitude (or height)	The perpendicular drawn from the opposite vertex of a side in a triangle is called an altitude of the triangle. ⇒ There are three altitudes in a triangle.	 AE, CD and BF are the altitudes
Median	The line segment joining the mid-point of a side to the vertex opposite to the side is called a median. ⇒ There are three medians in a triangle. ⇒ A median bisects the area of the triangle i.e., $ar(ABE) = ar(AEC) = \frac{1}{2} ar(\Delta ABC)$ etc.	 AE, CD and BF are the medians (BE=CE, AD=BD, AF=CF)
Angle bisector	A line segment which originates from a vertex and bisects the same angle is called an angle bisector. $\left(\angle BAE = \angle CAE = \frac{1}{2}\angle BAC\right)$ etc.	 AE, CD and BF are the angle bisectors.
Perpendicular bisector	A line segment which bisects a side perpendicularly (i.e., at right angle) is called a perpendicular bisector of a side of triangle. ⇒ All points on the perpendicular bisector of a line are equidistant from the ends of the line.	 DO, EO and FO are the perpendicular bisectors.
Orthocentre	The point of intersection of the three altitudes of the triangle is called as the orthocentre. $\angle BOC = 180° - \angle A,$ $\angle COA = 180° - \angle B$ $\angle AOB = 180° - \angle C$	 'O' is the orthocentre

Types of Triangles	Property/Definition	Diagram
Centroid	The point of intersection of the three medians of a triangle is called the centroid. A centroid divides each median in the ratio 2 : 1 (vertex : base) $\frac{AO}{OE}=\frac{CO}{OD}=\frac{BO}{OF}=\frac{2}{1}$	 'O' is centroid.
Incentre	The point of intersection of the angle bisectors of a triangle is called the incentre. Incentre O is always equidistant from all three sides i.e., the perpendicular distance between the sides and incentre is always same for all the three sides.	 'O' is the incentre.
Circumcentre	The point of intersection of the perpendicular bisectors of the sides of a triangle is called the circumcentre. OA = OB = OC = (circum radius). Circumcentre O is always equidistant from all the three vertices A, B and C	 'O' is the circumcentre.

IMPORTANT THEOREMS ON TRIANGLES

Theorem	Statement/Explanation	Diagram
Pythagoras Theorem	The square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the other two sides. i.e., $(AC)^2 = (AB)^2 + (BC)^2$ ⇒ The converse of this theorem is also true. ⇒ The numbers which satisfy this relations, are called Pythagorean triplets. e.g., (3, 4, 5) (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (12, 35, 37), (16, 63, 65), (20, 21, 29), (28, 45, 53), (33, 56, 65) **Note :** All the multiples (or submultiples) of Pythagorean triplets also satisfy the relation. e.g., (6, 8, 10), (15, 36, 39), (1.5, 2, 2.5) etc.	 $\angle B = 90°$ $\overline{AC} \rightarrow$ Hypotenuse AD = CD = BD (D is the mid point of AC)
$45° - 45° - 90°$ Triangle Theorem	If the angles of a triangle are 45°, 45° and 90°, then the hypotenuse (i.e., longest side) is $\sqrt{2}$ times of any smaller side. ⇒ Excluding hypotenuse rest two sides are equal. i.e., $AB = BC$ and $AC = \sqrt{2}\,AB = \sqrt{2}\,BC$.	 $\angle A = 45°$ $\angle B = 90°$ $\angle C = 45°$

$30° - 60° - 90°$ Triangle Theorem	If the angles of a triangle are 30°, 60° and 90°, then the sides opposite to 30° angle is half of the hypotenuse and the side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse. e.g., $AB = \frac{AC}{2}$ and $BC = \frac{\sqrt{3}}{2}AC$. $\therefore AB : BC : AC = 1 : \sqrt{3} : 2$	
Basic proportionality theorem (BPT) or Thales theorem	Any line parallel to one side of a triangle divides the other two sides proportionally. So if DE is drawn parallel to BC, it would divide sides AB and AC proportionally i.e., $\frac{AD}{DB} = \frac{AE}{EC}$ or $\frac{AD}{AB} = \frac{AE}{AC}$ $\frac{AD}{DE} = \frac{AB}{BC} = \frac{AE}{DE} = \frac{AC}{BC}$	
Mid-point theorem	If the mid-points of two adjacent sides of a triangle are joined by a line segment, then this segment is parallel to the third side. i.e., if AD = BD and AE = CE then DE||BC	
Apollonius theorem	In a triangle, sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third sides. i.e., $AB^2 + AC^2 = 2(AD^2 + BD^2)$	 BD = CD AD is the median
Interior angle bisector theorem	In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e., $\frac{BD}{CD} = \frac{AB}{AC}$ and BD ЧAC – CD ЧAB = AD^2	
Exterior angle bisector theorem	In a triangle the angle bisector of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides i.e., $\frac{BE}{AE} = \frac{BC}{AC}$	

EXERCISE

1. In a triangle ABC, if AB, BC and AC are the three sides of the triangle, then which of the statements is necessarily true?

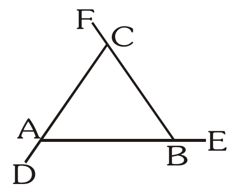
(A) AB + BC < AC (B) AB + BC > AC (C) AB + BC = AC (D) $AB^2 + BC^2 = AC^2$.

2. The sides of a triangle are 12 cm, 8 cm and 6 cm respectively, the triangle is :

(A) acute (B) obtuse (C) right (D) can't be determined

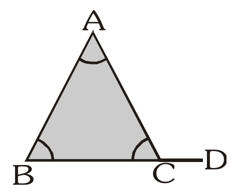
3. If the sides of a triangle are produced then the sum of the exterior angles i.e, ∠DAB + ∠EBC + ∠FCA is equal to :

(A) 180°
(B) 270°
(C) 360°
(D) 240°



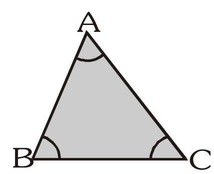
4. In the given figure BC is produced to D and ∠BAC = 40° and ∠ABC = 70°. Find the value of∠ACD:

(A) 30°
(B) 40°
(C) 70°
(D) 110°



5. In a ΔABC, ∠BAC > 90°, then ∠ABC and ∠ACB must be :

(A) acute
(B) obtuse
(C) one acute and one obtuse
(D) can't be determined

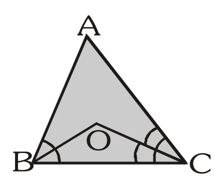


6. If the angles of a triangle are in the ratio 1 : 4 : 7, then the value of the largest angle is :

(A) 135° (B) 84° (C) 105° (D) None of these

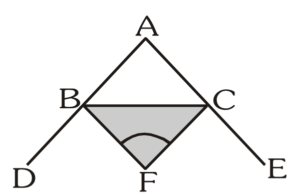
7. In the adjoining figure ∠B = 70° and ∠C = 30°. BO and CO are the angle bisectors of ∠ABC and ∠ACB. Find the value of ∠BOC :

(A) 30°
(B) 40°
(C) 120°
(D) 130°



8. In the given diagram of ΔABC, ∠B = 80°, ∠C = 30°. BF and CF are the angle bisectors of ∠CBD and ∠BCE respectively. Find the value of ∠BFC :

(A) 110°
(B) 50°
(C) 125°
(D) 55°



9. In an equilateral triangle, the incentre, circumcentre, orthocentre and centroid are:

(A) concylic (B) coincident (C) collinear (D) none of these

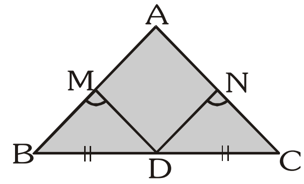
10. In the adjoining figure D is the midpoint of BC of a ΔABC. DM and DN are the perpendiculars on AB and AC respectively and DM = DN, then the ΔABC is :

(A) right angled

(B) isosceles

(C) equilateral

(D) scalene



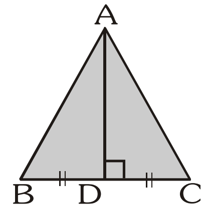
11. In the adjoining figure of ΔABC, AD is the perpendicular bisector of side BC. The triangle ABC is :

(A) right angled

(B) isosceles

(C) scalene

(D) equilateral



12. Triangle ABC is such that AB = 9 cm, BC = 6 cm, AC = 7.5 cm. Triangle DEF is similar to ΔABC, If EF = 12 cm then DE is :

(A) 6 cm (B) 16 cm (C) 18 cm (D) 15 cm

13. In ΔABC, AB = 5 cm, AC= 7 cm. If AD is the angle bisector of $\angle A$. Then BD : CD is:

(A) 25 : 49 (B) 49 : 25 (C) 6 : 1 (D) 5 : 7

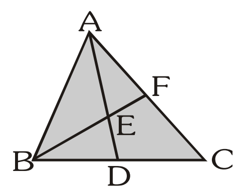
14. In a ΔABC, D is the mid-point of BC and E is mid-point of AD, BF passes through E. What is the ratio of AF : FC ?

(A) 1 : 1

(B) 1 : 2

(C) 1 : 3

(D) 2 : 3



15. In a ΔABC, AB = AC and $AD \perp BC$, then :

(A) AB < AD (B) AB > AD (C) AB = AD (D) $AB \leq AD$

16. The difference between altitude and base of a right angled triangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle is ?

(A) 24 cm (B) 31 cm (C) 34 cm (D) can't be determined

17. If AB, BC and AC be the three sides of a triangle ABC, which one of the following is true?

(A) AB – BC = AC (B) (AB – BC) > AC (C) (AB – BA) < AC (D) $AB^2 - BC^2 = AC^2$

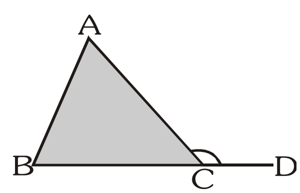
18. In the triangle ABC, side BC is produced to D. $\angle ACD = 100°$ if BC = AC, then $\angle ABC$ is :

(A) 40°

(B) 50°

(C) 80°

(D) can't be determined



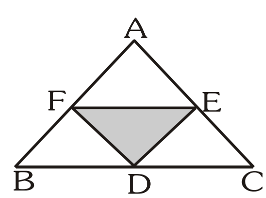
19. In the adjoining figure D, E and F are the mid-points of the sides BC, AC and AB respectively. ΔDEF is congruent to triangle :

(A) ABC

(B) AEF

(C) CDE , BFD

(D) AFE, BFD and CDE



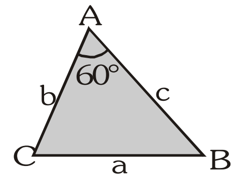
20. In the adjoining figure $\angle BAC = 60°$ and BC = a, AC = b and AB = c, then :

(A) $a^2 = b^2 + c^2$

(B) $a^2 = b^2 + c^2 - bc$

(C) $a^2 = b^2 + c^2 + bc$

(D) $a^2 = b^2 + 2bc$



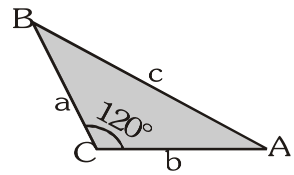
21. In the adjoining figure of ΔABC, $\angle BCA = 120°$ and AB = c, BC = a, AC = b then

(A) $c^2 = a^2 + b^2 + ba$

(B) $c^2 = a^2 + b^2 - ba$

(C) $c^2 = a^2 + b^2 - 2ba$

(D) $c^2 = a^2 + b^2 + 2ab$



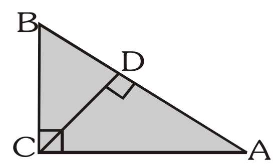
22. In a right angled ΔABC, $\angle C = 90°$ and CD is the perpendicular on the hypotenuse AB, AB = c, BC = a, AC = b and CD = p, then:

(A) $\frac{p}{a} = \frac{p}{b}$

(B) $\frac{1}{p^2} + \frac{1}{b^2} = \frac{1}{a^2}$

(C) $p^2 = b^2 + c^2$

(D) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$



23. If the medians of a triangle are equal, then the triangle is:

(A) right angled (B) isosceles (C) equilateral (D) scalene

24. The incentre of a triangle is determined by the:

(A) medians (B) angle bisectors

(C) perpendicular bisectors (D) altitudes

25. The circumcentre of a triangle is determined by the:

(A) altitudes (B) median (C) perpendicular bisectors (D) angle bisectors

26. The point of intersection of the angle bisectors of a triangle is :

(A) orthocentre (B) centroid (C) incentre (D) circumcentre

27. A triangle PQR is formed by joining the mid-points of the sides of a triangle ABC. 'O' is the circumcentre of ΔABC, then for ΔPQR, the point 'O' is :

(A) incentre (B) circumcentre (C) orthocentre (D) centroid

28. If in a ΔABC, 'S' is the circumcentre then:

(A) S is equidistant from all the vertices of a triangle

(B) S is equidistant from all the sides of a triangle

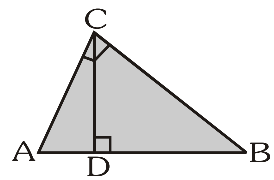
(C) AS, BS and CS are the angular bisectors

(D) AS, BS and CS produced are the altitudes on the opposite sides.

29. If AD, BE, CF are the altitudes of ΔABC whose orthocentre is H, then C is the orthocentre of :

(A) ΔABH (B) ΔBDH (C) ΔABD (D) ΔBEA

30. In a right angled ΔABC, $\angle C = 90°$ and CD is the perpendicular on hypotenuse AB. If BC = 15 cm and AC = 20 cm then CD is equal to :



(A) 18 cm

(B) 12 cm

(C) 17.5 cm

(D) can't be determined

31. In an equilateral ΔABC, if a, b and c denote the lengths of perpendiculars from A, B and C respectively on the opposite sides, then:

(A) $a > b > c$ (B) $a > b < c$ (C) $a = b = c$ (D) $a = c \neq b$

32. What is the ratio of side and height of an equilateral triangle?

(A) 2 : 1 (B) 1 : 1 (C) $2 : \sqrt{3}$ (D) $\sqrt{3} : 2$

33. The triangle is formed by joining the mid-points of the sides AB, BC and CA of ΔABC and the area of ΔPQR is 6 cm^2, then the area of ΔABC is :

(A) 36 cm^2 (B) 12 cm^2 (C) 18 cm^2 (D) 24 cm^2

34. One side other than the hypotenuse of right angle isosceles triangle is 6 cm. The length of the perpendicular on the hypotenuse from the opposite vertex is :

(A) 6 cm (B) $6\sqrt{2}$ cm (C) 4 cm (D) $3\sqrt{2}$ cm

35. Any two of the four triangles formed by joining the midpoints of the sides of a given triangle are:

(A) congruent (B) equal in area but not congruent

(C) unequal in area and not congruent (D) none of these

36. The internal bisectors of $\angle B$ and $\angle C$ of ΔABC meet at O. If $\angle A = 80°$ then $\angle BOC$ is :

(A) 50° (B) 160° (C) 100° (D) 130°

37. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :

(A) centroid (B) incentre (C) circumcentre (D) orthocentre

38. Incentre of a triangle lies in the interior of :

(A) an isosceles triangle only (B) a right angled triangle only

(C) any equilateral triangle only (D) any triangle

39. In a triangle PQR, PQ = 20 cm and PR = 6 cm, the side QR is :

(A) equal to 14 cm (B) less than 14 cm (C) greater than 14 cm (D) none of these

40. The four triangles formed by joining the pairs of mid· points of the sides of a given triangle are congruent if the given triangle is :

(A) an isosceles triangle (B) an equilateral triangle (C) a right angled triangle (D) of any shape

41. O is orthocentre of a triangle PQR, which is formed by joining the mid·points of the sides of a ΔABC, O is :

(A) orthocentre (B) incentre (C) circumcentre (D) centroid

42. In a ΔABC, a line PQ parallel to BC cuts AB at P and AC at Q. If BQ bisects $\angle PQC$, then which one of the following relations is always true:

(A) BC= CQ (B) BC = BQ (C) BC $\neq$ CQ (D) BC $\neq$ BQ

43. Which of the following is true, in the given figure, where AD is the altitude to the hypotenuse of a right angled ΔABC?

(i) ΔCAD and ΔABD are similar (ii) ΔCDA and ΔADB are congruent (iii) ΔADB and ΔCAB are similar

Select the correct answer using the codes given below:

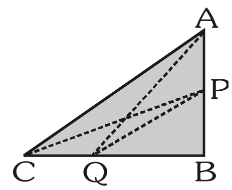
(A) 1 and 2 (B) 2 and 3 (C) 1 and 3 (D) 1, 2 and 3

44. If D is such a point on the side, BC of ΔABC that $\frac{AB}{AC} = \frac{BD}{CD}$, then AD must be a/an:

(A) altitude of ΔABC
(B) median of ΔABC
(C) angle bisector of ΔABC
(D) perpendicular bisector of ΔABC

45. In right angled ΔABC, ∠ABC = 90°, if P and Q are points on the sides AB and BC respectively, then:

(A) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$
(B) $AQ^2 + CP^2 = AC^2 + PQ^2$
(C) $(AQ^2 + CP^2) = \frac{1}{2}(AC^2 + PQ^2)$
(D) $(AQ + CP) = \frac{1}{2}(AC + PQ)$



46. If ABC is a right angled triangle at B and M, N are the mid-points of AB and BC, then $4(AN^2 + CM^2)$ is equal to –

(A) $4AC^2$
(B) $6AC^2$
(C) $5AC^2$
(D) $\frac{5}{4}AC^2$

47. If ΔABC and ΔDEF are so related that $\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}$, then which of the following is true?

(A) ∠A= ∠F and ∠B = ∠D
(B) ∠C = ∠F and ∠A = ∠D
(C) ∠B = ∠F and ∠C = ∠D
(D) ∠A = ∠E and ∠B = ∠D

48. ABC is a right angle triangle at A and AD is perpendicular to the hypotenuse. Then $\frac{BD}{CD}$ is equal to :

(A) $\left(\frac{AB}{AC}\right)^2$
(B) $\left(\frac{AB}{AD}\right)^2$
(C) $\frac{AB}{AC}$
(D) $\frac{AB}{AD}$

49. Let ABC be an equilateral triangle. Let BE ⊥ CA meeting CA at E, then $(AB^2 + BC^2 + CA^2)$ is equal to :

(A) $2BE^2$
(B) $3BE^2$
(C) $4BE^2$
(D) $6BE^2$

50. If D, E and F are respectively the mid-points of sides of BC, CA and AB of a ΔABC. If EF = 3 cm, FD = 4 cm, and AB = 10 cm, then DE, BC and CA respectively will be equal to :

(A) 6, 8 and 20 cm
(B) 4, 6 and 8 cm
(C) 5, 6 and 8 cm
(D) $\frac{10}{3}$, 9 and 12 cm

51. In the right angle triangle ∠C = 90°. AE and BD are two medians of a triangle ABC meeting at F. The ratio of the area of ΔABF and the quadrilateral FDCE is :

(A) 1 : 1
(B) 1 : 2
(C) 2 : 1
(D) 2 : 3

52. ABC is a triangle and DE is drawn parallel to BC cutting the other sides at D and E. If AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm, then AE is equal to :

(A) 1.4 cm
(B) 1.8 cm
(C) 1.2 cm
(D) 1.05 m

53. Consider the following statements:

(1) If three sides of a triangle are equal to three sides of another triangle, then the triangles are congruent.

(2) If three angles of a triangle are equal to three angles of another triangle respectively, then the two triangles are congruent. Of these statements:

(A) 1 is correct and 2 is false
(B) both 1 and 2 are false
(C) both 1 and 2 are correct
(D) 1 is false and 2 is correct

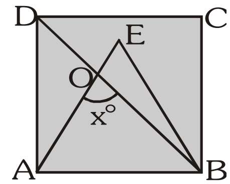
54. In the figure ΔABE is an equilateral triangle in a square ABCD. Find the value of angle x in degrees :

(A) 60°

(B) 45°

(C) 75°

(D) 90°



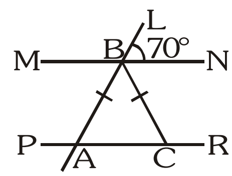
55. In the given diagram MN || PR and m ∠LBN = 70°, AB = BC. Find m ∠ABC :

(A) 40°

(B) 30°

(C) 35°

(D) 55°



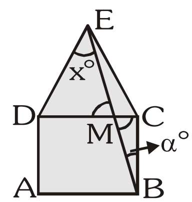
56. In the given diagram, equilateral triangle EDC surmounts square ABCD. Find the m ∠BED represented by x, where m ∠EBC = α°

(A) 45°

(B) 60°

(C) 30°

(D) None of these



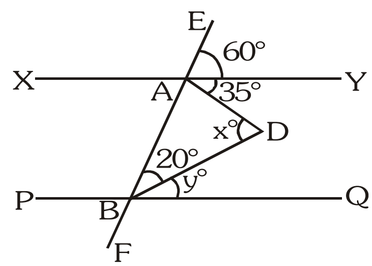
57. In the given diagram XY||PQ. Find m∠x° and m∠y° :

(A) 75° and 40°

(B) 45°, 60°

(C) 75°, 45°

(D) 60° and 45°



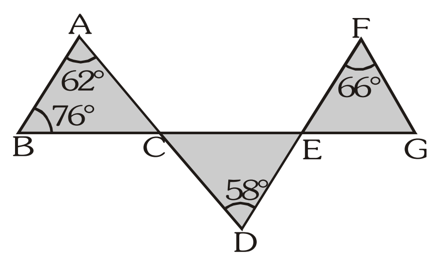
58. In the adjoining figure m ∠CAB = 62°, m ∠CBA = 76°, m ∠ADE = 58° and ∠DFG = 66°, find m ∠FGE:

(A) 44°

(B) 34°

(C) 36°

(D) none of these



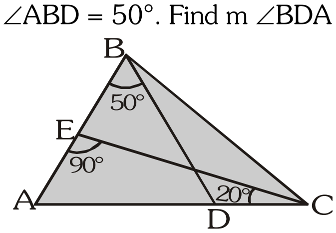
59. In the given figure CE ⊥ AB, m ∠ACE = 20° and m ∠ABD = 50°. Find m ∠BDA :

(A) 50°

(B) 60°

(C) 70°

(D) 80°



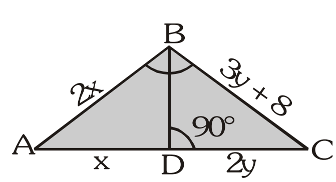
60. In the ΔABC, BD bisects ∠B, and is perpendicular to AC. If lengths of the sides of the triangle are expressed in terms of x and y as shown, find the value of x and y:

(A) 6, 12

(B) 10, 12

(C) 16, 8

(D) 8, 15



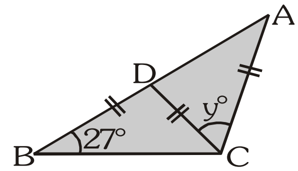
61. In the following figure ADBC, BD = CD = AC, m ∠ABC = 27° , m ∠ACD = y. Find the value of y.

(A) 27°

(B) 54°

(C) 72°

(D) 58°



62. ABC is an isosceles triangle with AB = AC. Side BA is produced to D such that AB = AD. Find m ∠BCD.

(A) 60° (B) 90° (C) 120° (D) can't be determined

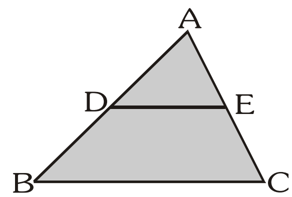
63. In ΔABC, AC = 5 cm. Calculate the length of AE where DE||BC, given that AD = 3 cm and BD = 7 cm.

(A) 2 cm

(B) 1 cm

(C) 1.5 cm

(D) 2.5 cm



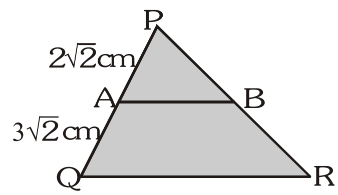
64. In ΔPQR, AP = $2\sqrt{2}$ cm, AQ = $3\sqrt{2}$ cm and PR = 10 cm, AB||QR. Find the length of BR.

(A) $6\sqrt{2}$ cm

(B) 6 cm

(C) $5\sqrt{2}$ cm

(D) none of these



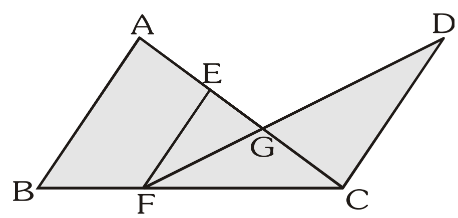
65. In the adjoining figure (not drawn to scale) AB, EF and CD are parallel lines. Given that EG = 5 cm, GC = 10 cm and DC = 18 cm. Calculate AC, if AB = 15 cm.

(A) 21 cm

(B) 25 cm

(C) 18 cm

(D) 28 cm



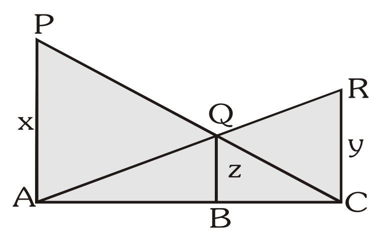
66. In the adjoining figure PA, QB and RC are each perpendicular to AC. Which one of the following is true?

(A) x + y = z

(B) xy = 2z

(C) $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$

(D) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$



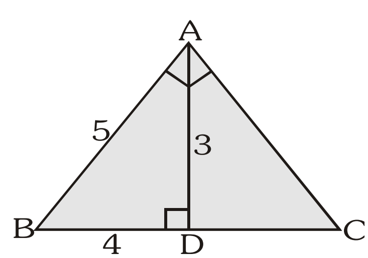
67. In the adjoining figure the ∠BAC and ∠ADB are right angles. BA = 5 cm, AD = 3 cm and BD = 4 cm, what is the length of DC ?

(A) 2.5

(B) 3

(C) 2.25

(D) 2



68. The areas of the similar triangles are in the ratio of 25 : 36. What is the ratio of their respective heights?

(A) 5 : 6 (B) 6 : 5 (C) 1 : 11 (D) 2 : 3

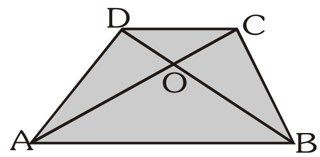
69. In the given diagram AB||CD, then which one of the following is true?

(A) $\frac{AB}{CD} = \frac{AO}{OC}$

(B) $\frac{AB}{CD} = \frac{BO}{OD}$

(C) $\Delta AOB \sim \Delta COD$

(D) all of these



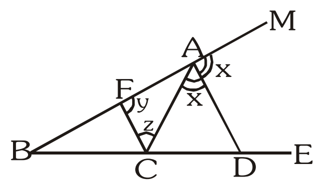
70. The bisector of the exterior ∠A of ΔABC intersects the side BC produced to D. Here CF is parallel to AD.

(A) $\frac{AB}{AC} = \frac{BD}{CD}$

(B) $\frac{AB}{AC} = \frac{CD}{BD}$

(C) $\frac{AB}{AC} = \frac{BC}{CD}$

(D) None of these



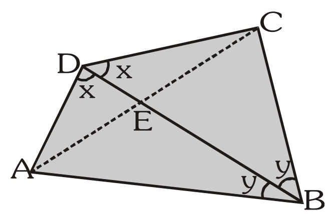
71. The diagonal BD of a quadrilateral ABCD bisects ∠B and ∠D, then:

(A) $\frac{AB}{CD} = \frac{AD}{BC}$

(B) $\frac{AB}{BC} = \frac{AD}{CD}$

(C) AB = AD Ч BC

(D) None of these



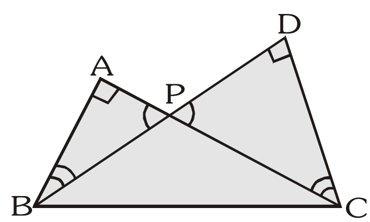
72. Two right triangles ABC and DBC are drawn on the same hypotenuse BC on the same side of BC. If AC and DB intersects at P, then

(A) $\frac{AP}{PC} = \frac{BP}{DP}$

(B) AP Ч DP = PC Ч BP

(C) AP Ч PC = BP Ч DP

(D) AP Ч BP = PC Ч PD



73. A man goes 150 m due east and then 200 m due north. How far is he from the starting point?

(A) 200 m (B) 350 m (C) 250 m (D) 175 m

74. From a point O in the interior of a ΔABC perpendiculars OD, OE and OF are drawn to the sides BC, CA and AB respectively, then which one of the following is true?

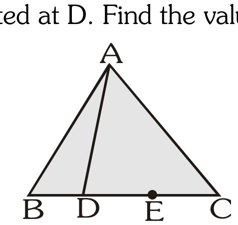
(A) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$ (B) $AB^2 + BC^2 = AC^2$

(C) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2$ (D) $AF^2 + BD^2 + CE^2 = OD^2 + OE^2 + OF^2$

75. In an equilateral triangle ABC, the side BC is trisected at D. Find the value of AD^2

(A) $\frac{9}{7}AB^2$ (B) $\frac{7}{9}AB^2$

(C) $\frac{3}{4}AB^2$ (D) $\frac{4}{5}AB^2$



76. ABC is a triangle in which ∠A = 90°. AN ⊥ BC, AC = 12 cm and AB = 5 cm. Find the ratio of the areas of ΔANC and ΔANB :

(A) 125 : 44 (B) 25 : 144 (C) 144 : 25 (D) 12 : 5

77. A vertical strick 15 cm long casts it shadow 10 cm long on the ground. At the same time a flag pole casts a shadow 60 cm long. Find the height of the flag pole.

(A) 40 cm (B) 45 cm (C) 90 cm (D) None of these

78. Vertical angles of two isoceles triangles are equal. Then corresponding altitudes are in the ratio 4 : 9. Find the ratio of their areas :

(A) 16 : 49 (B) 16 : 81 (C) 16 : 65 (D) None of these

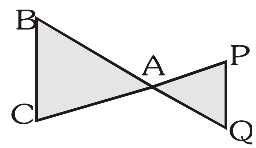
79. In the figure ΔACB ~ ΔAPQ. If BC = 8 cm, PQ = 4 cm, AP = 2.8 cm, find CA :

(A) 8 cm

(B) 6.5 cm

(C) 5.6 cm

(D) None of these



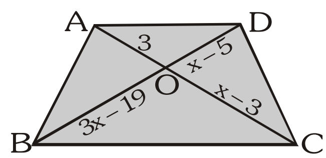
80. In the fig. BC||AD. Find the value of x :

(A) 9, 10

(B) 7, 8

(C) 10, 12

(D) 8, 9



81. In an equilateral triangle of side 2a, calculate the length of its altitude :

(A) $2a\sqrt{3}$ (B) $a\sqrt{3}$ (C) $a\frac{\sqrt{3}}{2}$ (D) None of these

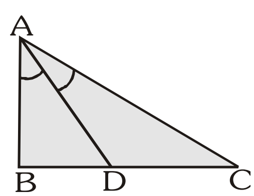
82. In fig. AD is the bisector of ∠BAC. If BD = 2 cm, CD = 3 cm and AB = 5 cm. Find AC :

(A) 6 cm

(B) 7.5 cm

(C) 10 cm

(D) 15 cm



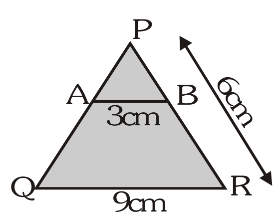
83. In the fig. AB||QR. Find the length of PB :

(A) 2 cm

(B) 3 cm

(C) 2.5 cm

(D) 4 cm



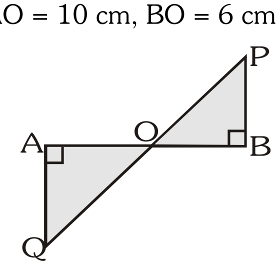
84. In the fig. QA and PB are perpendicular to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ :

(A) 8 cm

(B) 9 cm

(C) 15 cm

(D) 12 cm



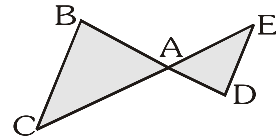
85. In the given figure AB = 12 cm, AC = 15 cm and AD = 6 cm. BC||DE, find the length of AE:

(A) 6 cm

(B) 7.5 cm

(C) 9 cm

(D) 10 cm



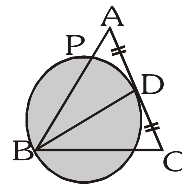
86. In the figure, ABC is a triangle in which AB = AC. A circle through B touches AC at D and intersects AB at P. If D is the mid·point of AC, Find the value of AB.

(A) 2AP

(B) 3AP

(C) 4AP

(D) none of the above



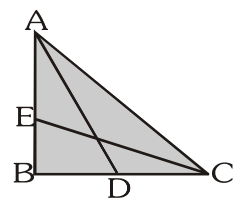
87. In figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE:

(A) $2\sqrt{5}$ cm

(B) 2.5 cm

(C) 5 cm

(D) $4\sqrt{2}$ cm



88. In a ΔABC, AB = 10 cm, BC = 12 cm and AC = 14 cm. Find the length of median AD. If G is the centroid, find length of GA :

(A) $\frac{5}{3}\sqrt{7}, \frac{5}{9}\sqrt{7}$ (B) $5\sqrt{7}, 4\sqrt{7}$ (C) $\frac{10}{\sqrt{3}}, \frac{8}{3}\sqrt{7}$ (D) $4\sqrt{7}, \frac{8}{3}\sqrt{7}$

89. ΔABC is a right angled triangle at A and AD is the altitude to BC. If AB = 7 cm and AC = 24 cm. Find the ratio of AD is to AM if M is the mid-point of BC.

(A) 25 : 41 (B) 32 : 41 (C) $\frac{336}{625}$ (D) $\frac{625}{336}$

90. Area of ΔABC = 30 cm^2. D and E are the mid-points of BC and AB respectively. Find ar (ΔBDE).

(A) 10 cm (B) 7.5 cm (C) 15 cm (D) none of these

91. The three sides of a triangles are given. Which one of the following is not a right triangle?

(A) 20, 21, 29 (B) 16, 63, 65 (C) 56, 90, 106 (D) 36, 35, 74

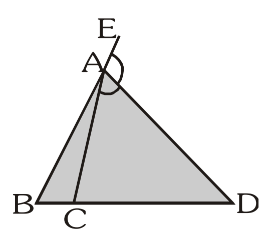
92. In the figure AD is the external bisector of ∠EAC, intersects BC produced to D. If AB = 12 cm, AC = 8 cm and BC = 4 cm, find CD.

(A) 10 cm

(B) 6 cm

(C) 8 cm

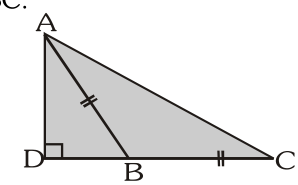
(D) 9 cm



93. In ΔABC, AB^2 + AC^2 = 2500 cm^2 and median AD = 25 cm, find BC.

(A) 25 cm (B) 40 cm (C) 50 cm (D) 48 cm

94. In the given figure, AB = BC and ∠BAC = 15°, AB = 10 cm. Find the area of ΔABC.

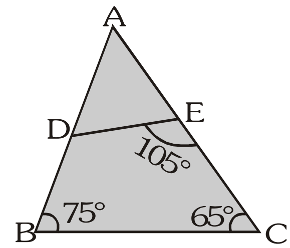


(A) 50 cm^2
(B) 40 cm^2
(C) 25 cm^2
(D) 32 cm^2

95. In ΔABC, G is the centroid, AB = 15 cm, BC = 18 cm and AC = 25 cm. Find GD, where D is the mid-point of BC.

(A) $\frac{1}{3}\sqrt{86}$ cm (B) $\frac{2}{3}\sqrt{86}$ cm (C) $\frac{8}{3}\sqrt{15}$ cm (D) none of these

96. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and if AE = 10 cm. Find AB.

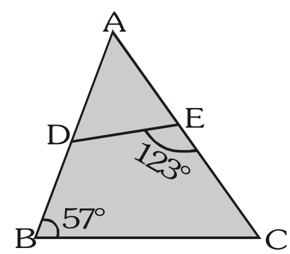


(A) 16 cm
(B) 12 cm
(C) 15 cm
(D) 18 cm

97. Find the maximum area that can be enclosed in a triangle of perimeter 24 cm.

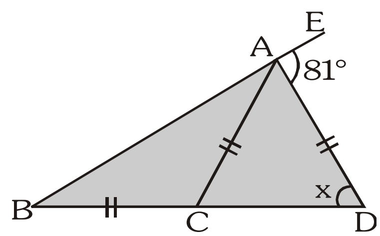
(A) 32 cm^2 (B) $16\sqrt{3}cm^2$ (C) $16\sqrt{2}\,cm^2$ (D) 27 cm^2

98. In the figure AD = 12 cm, AB = 20 cm and AE = 10 cm. Find EC.



(A) 14 cm
(B) 10 cm
(C) 8 cm
(D) 15 cm

99. In the given fig, BC = AC = AD, ∠EAD = 81°. Find the value of x.



(A) 45°
(B) 54°
(C) 63°
(D) 36°

100. What is the ratio of inradius to the circumradius of a right angled triangle?

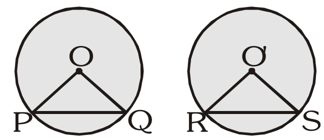
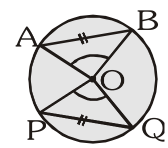
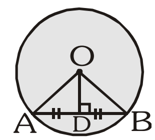
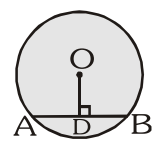
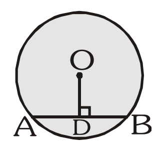
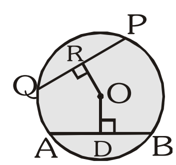
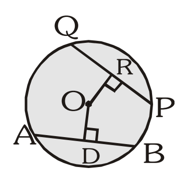
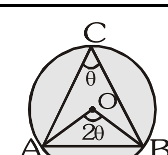
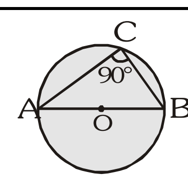
(A) 1 : 2 (B) 1 : $\sqrt{2}$ (C) 2 : 5 (D) Can't be determined

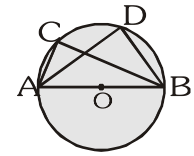
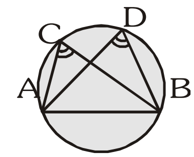
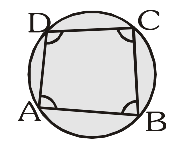
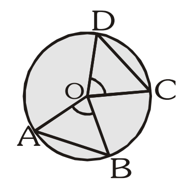
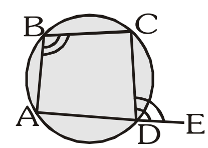
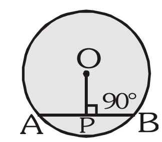
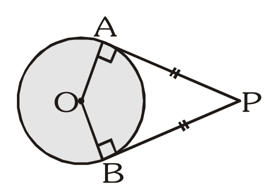
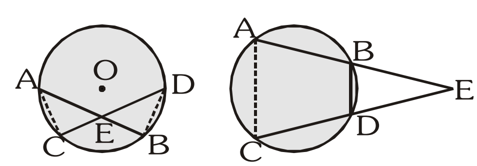
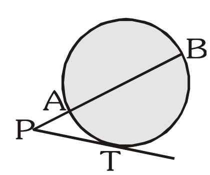
ANSWER KEY

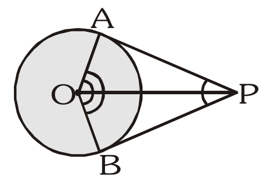
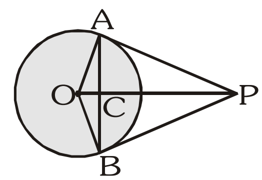
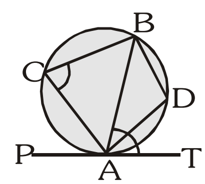
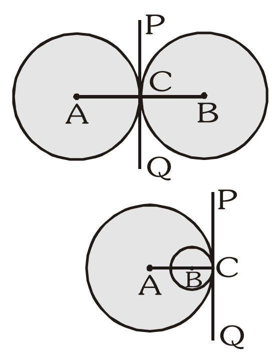
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	B	C	D	A	C	D	D	B	B	B	C	D	B	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	B	D	B	A	D	C	B	C	C	C	A	A	B
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	C	C	D	D	A	D	B	D	C	D	C	A	C	C	B
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	A	A	C	C	A	A	A	C	A	A	A	B	B	C
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	C	B	C	B	B	C	C	A	D	A	B	C	C	A	B
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	C	C	B	C	D	B	B	A	C	B	C	A	D	C	B
Que.	91	92	93	94	95	96	97	98	99	100					
Ans.	D	C	C	C	B	C	B	A	B	D					

CIRCLES

SOME IMPORTANT THEOREMS :

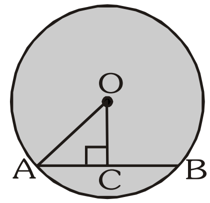
S.No.	Theorem	Diagram
1.	In a circle (or in congruent circles) equal chords are made by equal arcs. {OP = OQ} = {O'R = O'S} and $\overset{\frown}{PQ} = \overset{\frown}{RS}$ $\therefore$ PQ = RS	
2.	Equal arcs (or chords) subtend equal angles at the centre $\overset{\frown}{PQ} = \overset{\frown}{AB}$ (or PQ = AB) $\therefore$ $\angle POQ = \angle AOB$	
3.	The perpendicular from the centre of a circle to a chord bisects the chord i.e., if OD $\perp$ AB $\therefore$ AB = 2AD = 2BD	
4.	The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. $\because$ AD = DB $\therefore$ OD $\perp$ AB	
5.	Perpendicular bisector of a chord passes through the centre. i.e., if OD $\perp$ AB and AD = DB $\therefore$ O is the centre of the circle.	
6.	Equal chords of a circle (or of congruent circles) are equidistant from the centre. $\because$ AB = PQ $\therefore$ OD = OR	
7.	Chords which are equidistant from the centre in a circle (or in congruent, circles) are equal. $\because$ OD = OR $\therefore$ AB = PQ	
8.	The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. m$\angle$AOB = 2m$\angle$ACB.	
9.	Angle in a semicircle is a right angle.	

10.	Angle in the same segment of a circle are equal i.e., $\angle ACB = \angle ADB$	
11.	If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle. $\angle ACB = \angle ADB$ $\therefore$ Points A, C, D, B are concyclic i.e., lie on the circle	
12.	The sum of pair of opposite angles of a cyclic quadrilateral is 180°. $\angle DAB + \angle BCD = 180°$ and $\angle ABC + \angle CDA = 180°$ (converse of this theorem is also true)	
13.	Equal chords (or equal arcs) of a circle (or congruent circles) subtend equal angles at the centre. $AB = CD$ (or $\overset{\frown}{AB} = \overset{\frown}{CD}$) $\therefore$ $\angle AOB = \angle COD$ (converse of this theorem is also true)	
14.	If a side of a cylic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. $m\angle CDE = m\angle ABC$	
15.	A tangent at any point of a circle is perpendicular to the radius through the point of contact. (converse of this theorem is also true)	
16.	The lengths of two tangents drawn from an external point to a circle are equal. i.e., AP = BP	
17.	If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E) then AE Ч BE = CE Ч DE	
18.	If PB be a secant which intersects the circle at A and B and PT be a tangent at T then $PA \cdot PB = (PT)^2$	

19.	From an external point from which the tangents are drawn to the circle with centre O, then (a) They subtend equal angles at the centre. (b) They are equally inclined to the line segment joining the centre of that point. $\angle AOP = \angle BOP$ and $\angle APO = \angle BPO$	
20.	If P is an external point from which the tangents to the circle with centre O touch it at A and B then OP is the perpendicular bisector of AB. $OP \perp AB$ and $AC = BC$	
21.	**Alternate Segment Theorem :** If from the point of contact of a tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram. $\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$	
22.	The point of contact of two tangents lies on the straight line joining the two centres. (a) When two circles touch externally then the distance between their centres is equal to sum of their radii i.e. $AB = AC + BC$ (a) When two circles touch internally then the distance between their centres is equal to the difference between their radii i.e. $AB = AC - BC$	

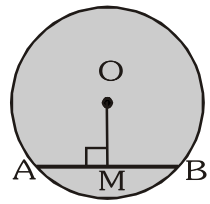
EXERCISE

1. In the given figure, O is the centre of the circle. Radius of the circle is 17 cm. If OC = 8 cm, then the length of the chord AB is :



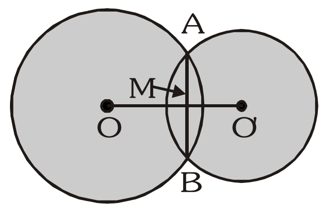
(A) 35 cm
(B) 30 cm
(C) 15 cm
(D) 18 cm

2. In the given figure OM ⊥ AB, radius of the circle is 5 cm and length of the chord AB = 8 cm. Find the measure of OM :



(A) 3 cm
(B) 2.5 cm
(C) 2 cm
(D) 6 cm

3. In the given figure, two circles with their respective centres intersect each other at A and B and AB intersects OO' at M, then m ∠OMA is :

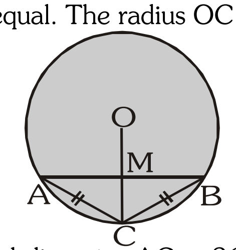


(A) 60°
(B) 80°
(C) 90°
(D) Can't be determined

4. In the above question (no. 3) what is the ratio of AM : BM?

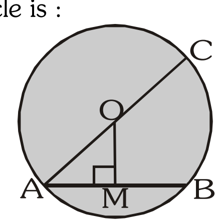
(A) 5 : 6 (B) 3 : 2 (C) 1 : 1 (D) can't be determined

5. In the given figure the two chords AC and BC are equal. The radius OC intersect AB at M, then AM : BM is :



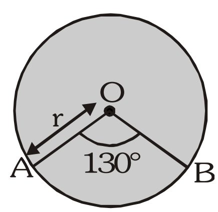
(A) 1 : 1
(B) $\sqrt{2}$: 3
(C) 3 : $\sqrt{2}$
(D) none of the above

6. In the adjoining figure, O is the centre of circle and diameter AC = 26 cm. If chord AB = 10 cm, then the distance between chord AB and centre O of the circle is :



(A) 24 cm
(B) 16 cm
(C) 12 cm
(D) none of the above

7. In the adjoining circle C (O, r) the degree measure of minor arc AB = 130°. Find the degree measure of major arc :

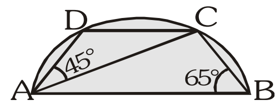


(A) 230°
(B) 260°
(C) 310°
(D) none of the above

8. If the diagonals of a cyclic quadrilateral are equal, then the quadrilateral is

(A) rhombus (B) square (C) rectangle (D) none of these

9. In the given figure, AB is diameter of the circle. C and D lie on the semicircle. ∠ABC = 65° and ∠CAD = 45°. Find m ∠DCA.



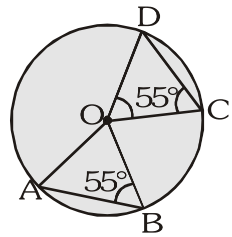
(A) 45°

(B) 25°

(C) 20°

(D) none of these

10. In the given figure, chords AB and CD are equal. If ∠OBA = 55° , then m ∠COD is:



(A) 65°

(B) 55°

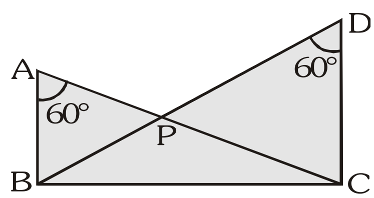
(C) 70°

(D) 50°

11. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is a:

(A) rectangle (B) square (C) parallelogram (D) cyclic quadrilateral

12. ΔABC and ΔDBC have a common base and drawn towards one sides. ∠BAC = ∠BDC = 60°. If AC and DB intersect at P, then :



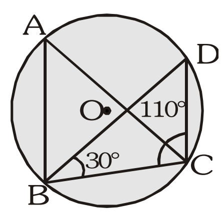
(A) AP Ч PC = BP Ч PD

(B) AP Ч BP = PC Ч PD

(C) AP Ч PD = PC Ч BP

(D) none of these

13. In the given figure, ∠BAC and ∠BDC are the angles of same segments. ∠DBC = 30° and ∠BCD = 110°. Find m ∠BAC is :



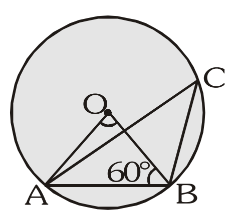
(A) 35°

(B) 40°

(C) 55°

(D) 60°

14. In the given figure, O is the centre of the circle. ∠ABO = 60°. Find the value of ∠ACB :



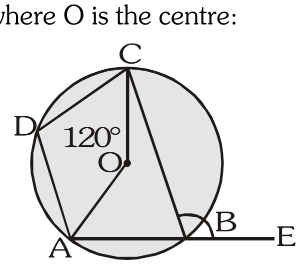
(A) 40°

(B) 60°

(C) 50°

(D) 30°

15. In the given figure, ∠AOC = 120°. Find m ∠CBE, where O is the centre:



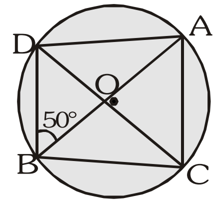
(A) 60°

(B) 100°

(C) 120°

(D) 150°

16. In the adjoining figure, O is the centre of the circle and ∠OBD = 50°. Find the m ∠BAD:



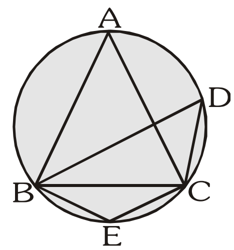
(A) 60°

(B) 40°

(C) 80°

(D) 45°

17. In the given figure, ΔABC is an equilateral triangle. Find m ∠BEC:



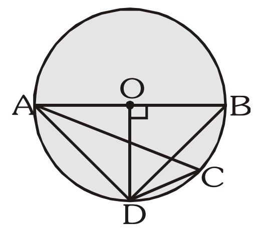
(A) 120°

(B) 60°

(C) 80°

(D) none of the above

18. In the given figure, AB is the diameter of the circle. Find the value of ∠ACD :



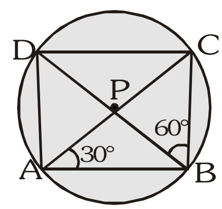
(A) 30°

(B) 60°

(C) 45°

(D) 25°

19. In the given figure, ABCD is a cyclic quadrilateral and diagonals bisect each other at P. If ∠DBC = 60° and ∠BAC = 30°, then ∠BCD is :



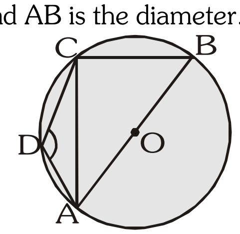
(A) 90°

(B) 60°

(C) 80°

(D) none of the above

20. In the given figure, ABCD is a cyclic quadrilateral and AB is the diameter. ∠ADC = 140°, then find m ∠BAC:



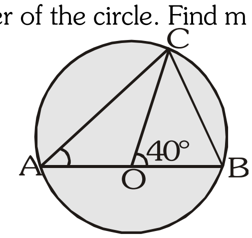
(A) 45°

(B) 40°

(C) 50°

(D) none of the above

21. In the given figure, ∠COB = 40°, AB is the diameter of the circle. Find m ∠CAB:



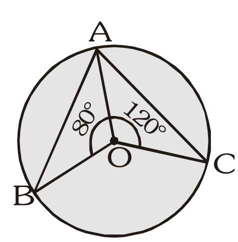
(A) 40°

(B) 20°

(C) 30°

(D) None of these

22. In the given figure, O is the centre of circle. ∠AOB = 80° and ∠AOC = 120°. Find m ∠BAC :



(A) 120°

(B) 80°

(C) 100°

(D) None of the above

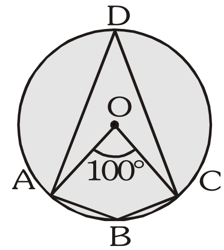
23. In the given figure, O is the centre of the circle and ∠AOC = 100°. Find the ratio of m ∠ADC : m ∠ABC :

(A) 5 : 6

(B) 1 : 2

(C) 5 : 13

(D) None of the above



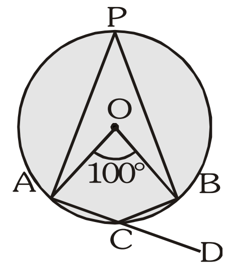
24. In the given figure, O is the centre of the circle and, ∠AOB = 100°. Find m ∠BCD :

(A) 80°

(B) 60°

(C) 50°

(D) 40°



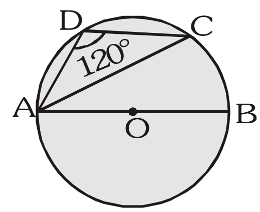
25. In the given figure, AB is the diameter of the circle. ∠ADC = 120°, Find m ∠CAB.

(A) 20°

(B) 30°

(C) 40°

(D) Can't be determined.



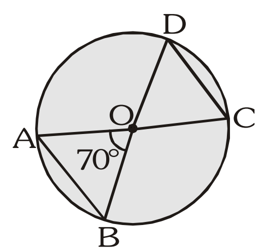
26. In the given figure, O is the centre of the circle. ∠AOB = 70°, find m ∠OCD :

(A) 70°

(B) 55°

(C) 65°

(D) 110°



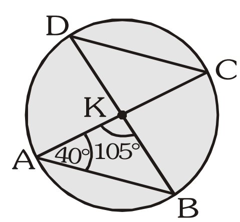
27. In the given figure, ∠CAB = 40° and ∠AKB = 105°. Find ∠KCD :

(A) 65°

(B) 35°

(C) 40°

(D) 72°



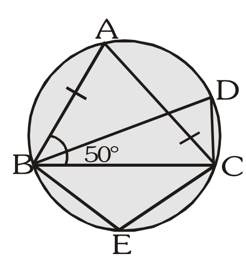
28. In the given figure, ABC is an isosceles triangle in which AB = AC and m ∠ABC = 50°, m ∠BDC:

(A) 80°

(B) 60°

(C) 65°

(D) 100°



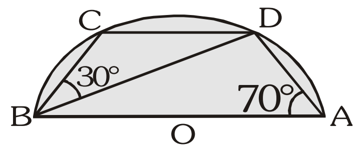
29. In the given figure, AB is the diameter, m ∠BAD = 70° and m ∠DBC = 30°. Find m ∠BDC :

(A) 25°

(B) 30°

(C) 40°

(D) 60°



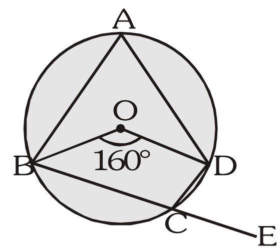
30. Find the value of ∠DCE :

(A) 100°

(B) 80°

(C) 90°

(D) 75°



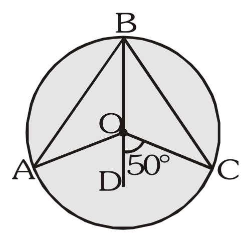
31. 'O' is the centre of the circle, line segment BOD is the angle bisector of ∠AOC, m ∠COD = 50°. Find m ∠ABC:

(A) 25°

(B) 50°

(C) 100°

(D) 120°



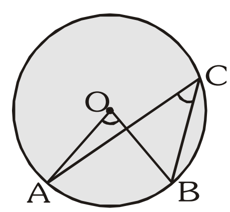
32. In the given figure, O is the centre of the circle and ∠ACB = 25°. Find ∠AOB :

(A) 25°

(B) 50°

(C) 75°

(D) 60°



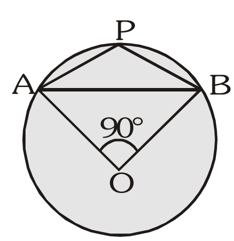
33. In the given figure, O is the centre of the circle. ∠AOB = 90°. Find m ∠APB :

(A) 130°

(B) 150°

(C) 135°

(D) can't be determined



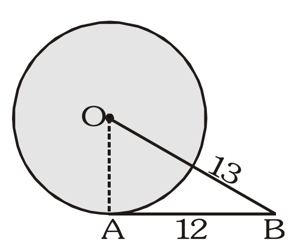
34. In the given figure, O is the centre of the circle. AB is tangent. AB = 12 cm and OB = 13 cm. Find OA:

(A) 6.5 cm

(B) 6 cm

(C) 5 cm

(D) none of these



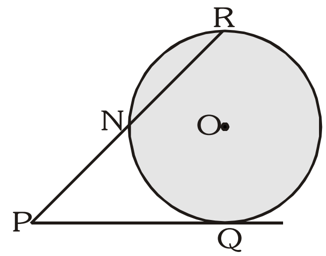
35. In the given figure, PQ is the tangent of the circle. Line segment PR intersects the circle at Nand R. PQ = 15 cm, PR = 25 cm, find PN:

(A) 15 cm

(B) 10 cm

(C) 9 cm

(D) 6 cm



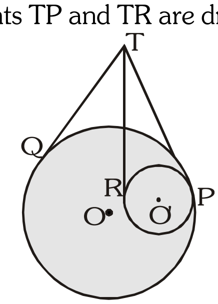
36. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find TQ : TR

(A) 8 : 7

(B) 7 : 8

(C) 5 : 4

(D) 1 : 1



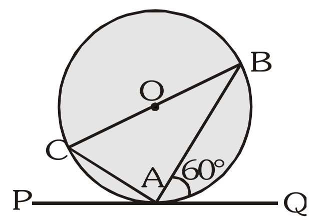
37. In the given figure, PAQ is the tangent. BC is the diameter of the circle. m ∠BAQ = 60°, find m ∠ABC :

(A) 25°

(B) 30°

(C) 45°

(D) 60°



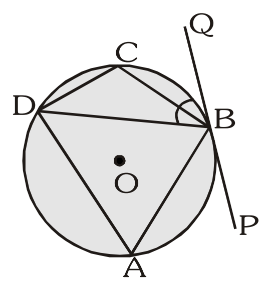
38. ABCD is a cyclic quadrilateral PQ is a tangent at B. If ∠DBQ = 65° , then ∠BCD is :

(A) 35°

(B) 85°

(C) 115°

(D) 90°



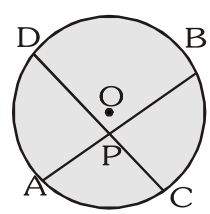
39. In the given figure, AP = 2 cm, BP = 6 cm and CP = 3 cm. Find DP :

(A) 6 cm

(B) 4 cm

(C) 2 cm

(D) 3 cm



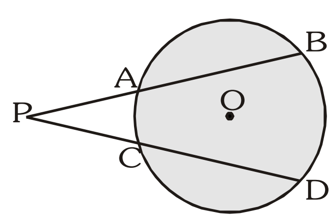
40. In the given figure, AP = 3 cm, BA = 5 cm and CP = 2 cm. Find CD :

(A) 12 cm

(B) 10 cm

(C) 9 cm

(D) 6 cm



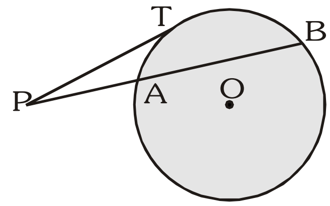
41. In the given figure, tangent PT = 5 cm, PA = 4 cm, find AB :

(A) $\frac{7}{4}$ cm

(B) $\frac{11}{4}$ cm

(C) $\frac{9}{4}$ cm

(D) can't be determined



42. Two circles of radii 13 cm and 5 cm touch internally each other. Find the distance between their centres :

(A) 18 cm (B) 12 cm (C) 9 cm (D) 8 cm

43. Three circles touch each other externally. The distance between their centre is 5 cm, 6 cm and 7 cm. Find the radii of the circles :

(A) 2 cm, 3 cm, 4 cm (B) 3 cm, 4 cm, 1 cm (C) 1 cm, 2.5 cm, 3.5 cm (D) 1 cm, 2 cm, 4 cm

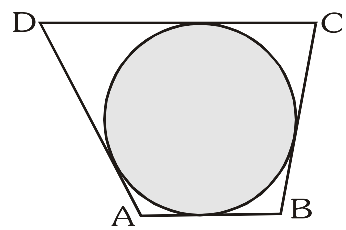
44. A circle touches a quadrilateral ABCD. Find the true statement:

(A) AB + BC = CD + AD

(B) AB + CD = BC + AD

(C) BD = AC

(D) none of the above



45. O and O' are the centres of two circles which touch each other externally at P. AB is a common tangent. Find ∠APO:

(A) 90° (B) 120° (C) 60° (D) data insufficient

46. If AB is a chord of a circle, P and Q are two points on the circle different from A and B, then:

(A) the angle subtended by AB at P and Q are either equal or supplementary.

(B) the sum of the angles subtended by AB at P and Q is always equal two right angles.

(C) the angles subtended at P and Q by AB are always equal.

(D) the sum of the angles subtended at P and Q is equal to four right angles.

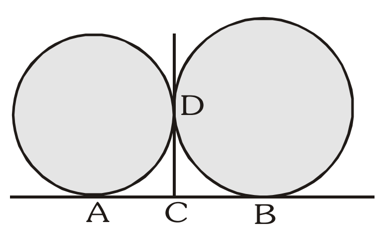
47. In the given figure, AB and CD are two common tangents to the two touching circles. If CD = 6 cm, then AB is equal to:

(A) 9 cm

(B) 15 cm

(C) 12 cm

(D) none of the above



48. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then:

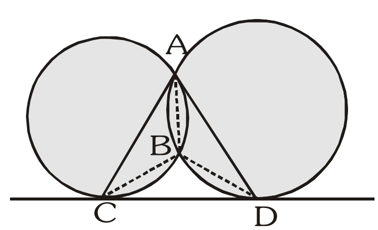
∠CAD + ∠CBD = ?

(A) 120°

(B) 90°

(C) 360°

(D) 180°



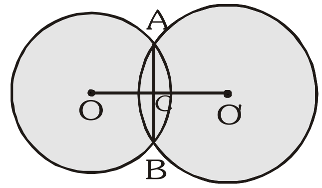
49. O and O' are the centres of circle of radii 20 cm and 37 cm. AB = 24 cm. What is the distance OO'?

(A) 51 cm

(B) 45 cm

(C) 35 cm

(D) 48 cm



50. In a circle of radius 5 cm, AB and AC are the two chords such that AB = AC = 6 cm. Find the length of the chord BC.

(A) 4.8 cm (B) 10.8 cm (C) 9.6 cm (D) none of these

51. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is :

(A) 23 cm (B) 30 cm (C) 15 cm (D) none of these

52. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :

(A) $\sqrt{3} : 2$ (B) $\sqrt{3} : 1$ (C) $\sqrt{5} : 1$ (D) none of these

53. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the other circle which is outside the inner circle, is of length :

(A) $2\sqrt{2}$ cm (B) $3\sqrt{2}$ cm (C) $2\sqrt{3}$ cm (D) $4\sqrt{2}$ cm

54. Through any given set of four points P,Q, R, S it is possible to draw :

(A) atmost one circle (B) exactly one circle (C) exactly two circles (D) exactly three circles

55. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is :

(A) 4 cm (B) 6 cm (C) 8 cm (D) 10 cm

56. The number of common tangents that can be drawn to two given circles is at the most :

(A) 1 (B) 2 (C) 3 (D) 4

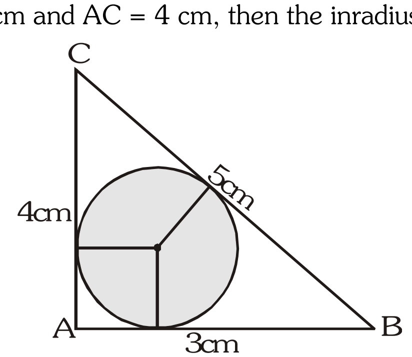
57. ABC is a right angled triangle AB = 3 cm, BC = 5 cm and AC = 4 cm, then the inradius of the circle is :

(A) 1 cm

(B) 1.25 cm

(C) 1.5 cm

(D) none of these



58. A circle has two parallel chords of lengths 6 cm and 8 cm. If the chords are 1 cm apart and the centre is on the same side of the chords, then a diameter of the circle is of length:

(A) 5 cm (B) 6 cm (C) 8 cm (D) 10 cm

59. Three equal circles of unit radius touch each other. Then, the area of the circle circumscribing the three circles is :

(A) $6\pi(2+\sqrt{3})^2$ (B) $\frac{\pi}{6}(2+\sqrt{3})^2$ (C) $\frac{\pi}{3}(2+\sqrt{3})^2$ (D) $3\pi(2+\sqrt{3})^2$

60. The radius of a circle is 20 cm. The radii (in cm) of three concentric circles drawn in such a manner that the whole area is divided into four equal parts, are:

(A) $20\sqrt{2}$, $20\sqrt{3}$, 20 (B) $\frac{10\sqrt{3}}{3}, \frac{10\sqrt{2}}{3}, \frac{10}{3}$ (C) $10\sqrt{3}$, $10\sqrt{2}$, 10 (D) 17,14, 9

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	C	C	A	C	A	C	C	C	D	A	B	D	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	A	C	A	C	B	B	C	C	B	B	B	A	C	B
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B	B	C	C	C	D	B	C	B	B	C	D	A	B	A
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	A	C	D	A	C	B	B	D	A	C	B	A	D	C	C

TRIGONOMETRY

The following concepts/formulae will assist you in solving the following questions :

1. Concept of similar triangles : If two triangles are similar, their sides are proportional and angles are equal.
2. $\sin(180° - \theta) = \sin\theta$

   $\cos(180° - \theta) = -\cos\theta$
3. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

   $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
4. sine rule : $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$
5. cosine rule :

   (i) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

   (ii) $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

   (iii) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

TRIGONOMETRY EXERCISE

1. If $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos B} = q$, then tan B is equal to :-

(A) $\pm\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$ (B) $\pm\sqrt{\frac{q^2-1}{1-p^2}}$ (C) $\pm\frac{p}{q}\sqrt{\frac{p^2-1}{1-q^2}}$ (D) None of these

2. If $T_n = \sin^n \theta + \cos^n \theta$, then $2T_6 - 3T_4 + 1$ is equal to

(A) 0 (B) $\sin\theta$ (C) $\cos\theta$ (D) $2\sin\theta\cos\theta$

3. If $T_n = \sin^n \theta + \cos^n \theta$, then $6T_{10} - 15T_8 + 10T_6 - 1$ is equal to

(A) 0 (B) 1 (C) $\sin^2\theta$ (D) $\sin^3\theta$

4. If $\sin\theta$ and $\cos\theta$ are the roots of $ax^2 + bx + c = 0$ $(ac \neq 0)$, then

(A) $a^2 + b^2 - 2ac = 0$ (B) $a^2 - b^2 + 2ac = 0$ (C) $(a + c)^2 = b^2 + c^2$ (D) None of these

5. A flagstaff 5 m high stands on a building 25 m high. At an observer at a height of 30 m, the flagstaff and the building subted equal angles. The distance of the observe from the top of the flagstaff is :-

[Hint : The angle of a triangle divides the opposite side in the ratio of the sides containing the angle]

(A) $\frac{5\sqrt{3}}{2}$ (B) $5\sqrt{\frac{3}{2}}$ (C) $5\sqrt{\frac{2}{3}}$ (D) None of these

6. If b = 3, c = 4 and $b = \frac{\pi}{3}$, then the number of triangles that can be constructed is :-

(A) One (B) Two (C) Infinite (D) None of these

7. In an equilateral triangle the inradius r and circumradius R are connected by : [Hint : $r = \frac{\Delta}{s}$]

(A) $r = \frac{R}{3}$ (B) $r = \frac{R}{2}$ (C) r = 4R (D) None of these

8. We are given b, c and sin B such that B is acute and b < c sin B, then :-

(A) One triangle is possible (B) Two triangles are possible

(C) A right angled triangle is possible (D) No triangle is possible

9. The perimeter of a triangle ABC is 6 times the A.M. of the sines of its angles. If a = 1, the angle A (acute) is ;-

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

10. If the radius of the circumcircle of an isosceles triangle PQR is equal to PQ(= PR), then the angle P is :-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

11. The angle of elevation of the top of two vertical towers as seen from the middle point of the line joining the feet of the towers are 60° and 30° respectively. The ratio of heights of the towers is :-

(A) 2 : 1 (B) $\sqrt{3}$: 1 (C) 3 : 2 (D) 3 : 1

12. A tower of height h standing at the centre of a square with sides of length a makes the same angle α at each of the four corners. Then $\frac{a^2}{h^2 \cot^2 \alpha}$ is :-

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4

13. AB is a vertical pole. The end A is on level ground C is the midpoint of AB. P is a point on level gound. The portion BC subtends an angle β at P. If AP = 5AB, then $\tan\beta$ =

(A) $\frac{5}{21}$ (B) $\frac{5}{51}$ (C) $\frac{5}{24}$ (D) $\frac{5}{26}$

14. Each side of an equilateral triangle subtends angle of 60° at the top of a tower of height h standing at the centre of the triangle. If 2a be the length of the side of the triangle, then $\frac{a^2}{h^2}$ =

(A) $\frac{8}{3}$ (B) $\frac{3}{8}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$

15. A tower subtends an angle of 30° at a point on the same level as the foot of tower. At a second point h m high above the first, the depression of the foot of tower is 60°. The horizontal distance of the tower from the point is:-

(A) $\frac{h}{\sqrt{3}}$ (B) $\frac{h\cot 60°}{\sqrt{3}}$ (C) $\frac{h\cot 60°}{3}$ (D) h cot 30°

16. The angle of elevation of a cloud at a height h above the level of water in a lake is α and the angle of depression of its image in the lake is β. The height of the cloud above the surface of the lake is equal to :-

(A) $\frac{h(\tan\beta + \tan\alpha)}{(\tan\beta - \tan\alpha)}$ (B) $\frac{h\cos(\alpha+\beta)}{\sin(\beta-\alpha)}$ (C) $\frac{h(\cot\alpha + \cot\beta)}{\cos\alpha - \cos\beta}$ (D) h

17. The elevation of a tower due north of a station A is α and of another station B due west of A is β. The height of tower is :-

(A) $\frac{AB\cos\alpha}{\sqrt{\cot^2\beta - \cot^2\alpha}}$ (B) $\frac{AB\sin\alpha\sin\beta}{\sqrt{\sin^2\alpha - \sin^2\beta}}$ (C) $\frac{AB\tan\alpha\cot\beta}{\sqrt{\tan^2\alpha - \tan^2\beta}}$ (D) $\frac{AB\sin\alpha\cos\beta}{\sqrt{\sin(\alpha+\beta)\sin(\alpha-\beta)}}$

18. A man observes that when he moves up a distance c metres on a slope, the angle of depression of a point on the horizontal plane from the base of the slope is 30°, and when he moves up further a distance c metres, the angle of depression of that point is 45°. The angle of inclination of the slope with the horizontal is :-

(A) 60° (B) 45° (C) 75° (D) 30°

19. A person walking along a straight road observes that at two points 1 km apart, the angles of elevation of a pole in front of him are 30° and 75°. The height of the pole is :-

(A) $250(\sqrt{3}+1)$m (B) $250(\sqrt{3}-1)$m (C) $225(\sqrt{2}-1)$m (D) $225(\sqrt{2}+1)$m

20. A flagstaff stands in the centre of a rectangular field whose diagonal is 1200 m and subtends angles 15° and 45° at the mid points of the sides of the field. The height of the flagstaff is :-

(A) 200 m (B) $300\sqrt{2+\sqrt{3}}$ m (C) $300\sqrt{2-\sqrt{3}}$ m (D) 400 m

21. Two flagstaffs stand on a horizontal plane. A and B are two points on the line joining their feet and between. The angles of elevation of the tops of the flagstaffs as seen from A are 30° and 60° and as seen from B are 60° and 45°. If AB is 30 m, the distance between the flagstaffs in metres is :-

(A) $30 + 15\sqrt{3}$ (B) $45 + 15\sqrt{3}$ (C) $60 - 15\sqrt{3}$ (D) $60 + 15\sqrt{3}$

22. In a cubical hall ABCDPQRS with side 10 m, G is the centre of the wall BCRQ and T is the mid point of the side AB. The angle of elevation of G at the point T is :-

(A) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (C) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

23. Each side of an equilateral triangle subtends an angle of 60° at the top of a tower h m high located at the centre of the triangle. If a is the length of each side of the triangle, then :-

(A) $3a^2 = 2h^2$ (B) $2a^2 = 3h^2$ (C) $a^2 = 3h^2$ (D) $3a^2 = h^2$

24. The angles of elevation of the top of a tower from two points on the ground at a distance a and 2a from its foot are α and β, respectively. If h is the height of the tower, then tan(α – β) =

(A) $\frac{ah}{a^2+h^2}$ (B) $\frac{ah}{2a^2+h^2}$ (C) $\frac{ah}{a^2+2h^2}$ (D) $\frac{h}{2a}$

25. The distances of three points on a pole from its foot are in A.P. If the angles of elevation of these points from a point on the ground are α, β and γ respectively, then tan α, tan β and tan γ are in :-

(A) AP (B) GP (C) HP (D) None of these

26. The angle of elevation of a cloud from a point x m above a take is θ and the angle of depression of its reflection in the take is 45°. The height of the cloud is :-

(A) $x \tan(45° - \theta)$ (B) $x \tan(45° + \theta)$ (C) $\frac{1}{x}\cot(45° - \theta)$ (D) $\frac{1}{x}\cot(45° + \theta)$

27. In a triangular plot ABC with BC = 7 m, CA = 8 m and AB = 9 m. A lamp post is situated at the middle point E of the side AC and subtends an angle $\tan^{-1} 3$ at the point B, the height of the lamp post is :-

(A) 21 m (B) 24 m (C) 27 m (D) 30 m

28. A vertical tower CP subtends the same angle θ, at point B on the horizontal plane through C, the foot of the tower and at point A in the vertical plane. If the triangle ABC is equilateral with length of each side equal to 4 m, the height of the tower is :-

(A) $8\sqrt{3}$ m (B) $\frac{4}{\sqrt{3}}$ m (C) $4\sqrt{3}$ m (D) $\frac{8}{\sqrt{3}}$ m

29. From a point on the horizontal plane, the elevation of the top of a hill is 45°. After walking 500 m towards its summit up a slope inclined at an angle of 15° to the horizon the elevation is 75°, the height of the hill is :-

(A) $500\sqrt{6}$ m (B) $500\sqrt{3}$ m (C) $250\sqrt{6}$ m (D) $250\sqrt{3}$ m

30. The elevation of a steeple at a place due south of it is 45° and at a place B due west of A the elevation is 15°. If AB = 2a, the height of the steeple is :-

(A) $\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)a$ (B) $\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)a$ (C) $a(3^{1/4} - 3^{-1/4})$ (D) $a(3^{1/4} + 3^{-1/4})$

31. A river flows due North, and a tower stands on its left bank. From a point A upstream and on the same bank as the tower, the elevation of the tower is 60° and from a point B just opposite A on the other bank the elevation is 45°. If the tower is 360 m high, the breadth of the river is :-

(A) $120\sqrt{6}$ m (B) $\frac{240}{\sqrt{3}}$ m (C) $240\sqrt{3}$ m (D) $240\sqrt{6}$ m

32. A balloon of radius r subtends an angle α at the eye of an observer and the elevation of the centre of the balloon from the eye is β, the height h of the centre of the balloon is given by :-

(A) $\frac{r\sin\beta}{\sin\alpha}$ (B) $r \sin\beta \sin\alpha$ (C) $\frac{r\sin\beta}{\sin(\alpha/2)}$ (D) $\frac{r\sin\alpha}{\sin(\beta/2)}$

33. Two poles of height a and b stand at the centres of two circular plots which touch each other externally at a point and the two poles subtend angles of 30° and 60° respectively at this point, then distance between the centres of these plots is :-

(A) a + b (B) $\frac{(3a+b)}{\sqrt{3}}$ (C) $\frac{a+3b}{\sqrt{3}}$ (D) $a\sqrt{3} + b$

34. A man standing on a level plane observes the elevation of the top of a pole to be α. He then walks a distance equal to double the height of the pole and finds that the elevation is now 2α. Then α =

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{12}$ (D) $\frac{\pi}{8}$

35. A vertical tower PQ subtends the same angle of 30° at each of the two places A and B, 60 m apart on the ground. If AB subtends an angle of 120° at P, the foot of the lower, the height of the tower is :-

(A) 10 m (B) 20 m (C) 30 m (D) 40 m

36. The angle of elevation of the top of a tree at a point B due south of it is 60° and at a point C due north of it is 30°. D is a point due north of C where the angle of elevation is 15°, then

(given $\sqrt{3} = 1\frac{8}{11}$ and BC Ч CD = 2^3 Ч 3^2 Ч 19 Ч 11) the height of the tree is :-

(A) 53 (B) 55 (C) 57 (D) 59

37. From the top of a cliff of height a, the angle of deppresion of the foot of a certain tower is found to be double the angle of elevation of the top of the tower of height h. If θ be the angle of elevation, then its value is :-

(A) $\cos^{-1}\sqrt{\frac{2h}{a}}$ (B) $\sin^{-1}\sqrt{\frac{2h}{a}}$ (C) $\sin^{-1}\sqrt{\frac{a}{2-h}}$ (D) $\tan^{-1}\sqrt{3-\frac{2h}{a}}$

38. The angle of elevation of the top of a tower from a point A due south of the tower is α and from a point B due east of the tower is β. If AB = d, then the height of the tower is :-

(A) $\frac{d}{\sqrt{\tan^2\alpha-\tan^2\beta}}$ (B) $\frac{d}{\sqrt{\tan^2\alpha+\tan^2\beta}}$ (C) $\frac{d}{\sqrt{\cot^2\alpha+\cot^2\beta}}$ (D) $\frac{d}{\sqrt{\cot^2\alpha-\cot^2\beta}}$

39. AB is vertical tower. The point A is on the ground and C is the middle point of AB. The part CB subtend an angle α at a point P on the ground. If AP = nAB, then tan α =

(A) $n(n^2+1)$ (B) $\frac{n}{2n^2-1}$ (C) $\frac{n^2}{2n^2+1}$ (D) $\frac{n}{2n^2+1}$

40. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation p and q respectively. The height of hill is :-

(A) $\frac{h\cot p}{\cot q-\cot p}$ (B) $\frac{h\cot p}{\cot p-\cot q}$ (C) $\frac{h\tan p}{\tan p-\tan q}$ (D) $\frac{h\tan p}{\tan q-\tan p}$

41. If A and B are two points on one bank of a straight river and C, D are two other points on the other bank of river. If direction from A to B is same as that from C to D and AB = a, ∠ CAD = α, ∠ DAB = β, ∠ CBA = γ, then CD =

(A) $\frac{a\sin\beta\sin\gamma}{\sin\alpha\sin(\alpha+\beta+\gamma)}$ (B) $\frac{a\sin\alpha\sin\gamma}{\sin\beta\sin(\alpha+\beta+\gamma)}$ (C) $\frac{a\sin\alpha\sin\beta}{\sin\gamma\sin(\alpha+\beta+\gamma)}$ (D) None of these

42. A vertical lamp post of height 9 metres stands at the corner of a rectangular field. The angle of elevation of its top from the farthest corner is 30°, while from another corner it is 45°. The area of the field is :-

(A) $81\sqrt{2}$ m² (B) $9\sqrt{2}$ m² (C) $81\sqrt{3}$ m² (D) $9\sqrt{3}$ m²

43. ABC is a triangular park with AB = AC = 100 m. A clock tower is situated at the midpoint of BC. The angles of elevation of top of the tower at A and B are $\cot^{-1}(3.2)$ and $\text{cosec}^{-1}(2.6)$ respectively. The height of tower is :-

(A) $\frac{25}{2}$ m (B) 25 m (C) 50 m (D) None of these

44. ABCD is a square plot. The angle of elevation of the top of a pole standing on D from A or C is 30° and that from B is θ, then tan θ is equal to :-

(A) $\sqrt{6}$ (B) $\frac{1}{\sqrt{6}}$ (C) $\frac{\sqrt{3}}{\sqrt{2}}$ (D) $\frac{\sqrt{2}}{\sqrt{3}}$

45. If a flagstaff subtends equal angles at four points A, B, C and D on the horizontal plane through the foot of the flagstaff, then A, B, C and D must be the vertices of :-

(A) Square (B) Cyclic quadrilateral (C) Rectangle (D) Parallelogram

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	A	B, C	B	D	B	D	D	D	D	C	B	B	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	B	C	A	C	D	A	B	B	A	B	A	B	C	C
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	A	C	B	C	B	C	D	C	D	B	B	A	B	B	B

SEQUENCE, SERIES & PROGRESSIONS

Sequence: A set of numbers occurring in a definite order or by a rule is called a sequence.

e.g. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ..., \frac{1}{n}, ...$ is a sequence, since the numbers occur in a definite order. A sequence can be finite or infinite according to the number of elements in it. If there are finite number of elements, then it is a finite sequence and if there are infinite number of elements in it, then it is an infinite sequence and is denoted by $[a_n]$.

Series: Let $u_1, u_2, u_3, ..., u_n$ be a given sequence of n numbers, where n is a specified positive integer. Then their sum i.e. $u_1 + u_2 + u_3 + ... + u_n$ is called a finite series and the numbers $u_1, u_2, u_3, ..., u_n$ are called terms of the series.

If $S_n = u_1 + u_2 + u_3 + ... + u_n$, then S_n is called sum to n terms of the series and is denoted by

$$\sum_{r=1}^{n} u_r \text{ or } \sum u_r$$

e.g $2 + 4 + 6 + ... + 30$ is a finite series and can be denoted as $\sum_{n=1}^{15} 2n$ $(\sum \rightarrow \text{Sigma, summation notation})$

Progressions: Sequences of special types are called progressions. The sequence $a_1, a_2, a_3, ..., a_n.......$ or $\{a_n\}$ is known when its nth term a_n is given..

e.g. If $a_n = 2n$, then the sequence is 2, 4, 6, 8, ...

But to define a sequence, we may not always have a formula for the nth term. For example, the sequence of the prime numbers is 2, 3, 5, 7, 11, 13, ... for which a formula for the nth term may not be given.

ARITHMETIC PROGRESSION (A.P.)

A sequence $a_1, a_2, a_3, ..., a_n$ is said to be in arithmetic progression, when $a_2 - a_1 = a_3 - a_2 = ...$ i.e when the quantities increase or decrease continuously by a common quantity and this common quantity is called the common difference of the A.P.

The general term or the nth term is denoted by T_n.

$$T_n = a + (n - 1)d$$

Arithmetic Mean (A.M.): When three quantities are in A.P., the middle one is called arithmetic mean of the other two. Let a, m, b are in A.P., then A.M. of a and b

i.e. $\quad m = \frac{a+b}{2}$

$\therefore \quad \text{A.M.} = \frac{a+b}{2}$

In general, if a, $m_1, m_2, m_3, ..., m_n$, b are in A.P. then the intermediate numbers $m_1, m_2, m_3, ..., m_n$ are called the n arithmetic means between a and b.

Also, the A.M. of $a_1, a_2, a_3, ..., a_n$, which are in A.P. is

$$\text{A.M.} = \left(\frac{a_1 + a_2 + a_3 + ...+ a_n}{n}\right)$$

To Insert n A.Ms between a and b: If a and b are the first and last term respectively between which we have to insert n arithmetic means $m_1, m_2, m_3, \ldots, m_n$ then common difference $d = \frac{b-a}{n+1}$

Thus,
$$m_1 = a+\frac{b-a}{n+1}$$
$$m_2 = a+2\left(\frac{b-a}{n+1}\right)$$
$$m_3 = a+3\left(\frac{b-a}{n+1}\right)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$m_n = a+n\left(\frac{b-a}{n+1}\right)$$

Properties of Arithmetic Progressions:

1. If a certain number is added or subtracted to each term of a given A.P., then the resulting sequence is also an A.P. and it has the same common difference as that of the given A.P.

   e.g., 1, 4, 7, 10, 13, 16, ..., then

   3, 6, 9, 12, 15, 18, ..., is also an A.P.

   and 2, 5, 8, 11, 14, 17,......, is also an A.P.

2. If each term of an A.P. is multiplied by a fixed constant (or divided by a non zero fixed constant), then the resulting sequence is also an A.P.

   e.g., 6, 12, 18, 24, 30, 36, then

   9, 18, 27, 36, 45, 54, ..., is also an A.P.

   3, 6, 9, 12, 15, 18, ..., is also an A.P.

3. If $m_1, m_2, m_3, \ldots, m_k$ and $n_1, n_2, n_3, \ldots, n_k$ are in A.P., then $(m_1 + n_1), (m_2 + n_2), (m_3 + n_3), \ldots, (m_k + n_k)$ are also in A.P.

4. If $m_1, m_2, m_3, \ldots, m_k$ and $n_1, n_2, n_3, \ldots, n_k$ are in A.P., then $(m_1 - n_1), (m_2 - n_2), (m_3 - n_3), \ldots, (m_k - n_k)$ are also in A.P.

5. If there are odd number of terms in an A.P. as $a_1, a_2, a_3, \ldots, a_n$ then the middle most term is the A.M. of the given sequence.

   e.g., 4, 10, 16, 22, 28

   then, $\frac{4+10+16+22+28}{5} = 16$

   also $\frac{4+28}{2} = \frac{10+22}{2} = 16\text{(A.M.)}$

6. If there are even number of terms in an A.P. as $a_1, a_2, a_3, \ldots, a_n$ then $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2}$ etc.

   e.g., 4, 10, 16, 22, 28, 34, 40, 46

   then, 4 + 46 = 10 + 40 = 16 + 34 = 22 + 28

   also $\frac{4+46}{2} = \frac{10+40}{2} = \frac{16+34}{2} = \frac{22+28}{2} = 25\text{(A.M.)}$

7. The sum of an A.P. with n terms is given by $S_n = n\text{(A.M.)}$

EXERCISE

1. If the sides of a right triangle are in A.P., then the ratio of its smallest side to the greatest side is :-
(A) 3 : 4 (B) 3 : 5 (C) 4 : 5 (D) None of these

2. Sum of the series : x + (x + 1) + (x + 2) + ... + y is :-
(A) $\frac{x+y}{2}(x-y+1)$ (B) $\frac{x+y}{2}(x+y+1)$ (C) $\frac{x+y}{2}(y-x+1)$ (D) None of these

3. Given that n A.M.'s are inserted between two sets of numbers a, 2b and 2a, b, where a, b $\in$ R. If the mth means in the two cases are same then ratio a : b is equal to :-
(A) n : (n – m + 1) (B) (n – m + 1) : m (C) (n – m + 1) : n (D) m : (n – m + 1)

4. The sum of all the 2 digit numbers, which leave remainder 1 when divided by 4, is :-
(A) 1212 (B) 1210 (C) 1012 (D) 1201

5. Sum of first n terms of an A.P. is an(n – 1). The sum of squares of these terms is :-
(A) $\frac{a^2}{6}n(n-1)(2n-1)$ (B) $\frac{2a^2}{3}n(n+1)(2n+1)$ (C) $a^2n^2(n-1)^2$ (D) $\frac{2a^2}{3}n(n-1)(2n-1)$

6. The n^{th} term of the series $1+\frac{1}{1+3}+\frac{1}{1+3+5}+\ldots$ is :-
(A) $\frac{2}{n(n+1)}$ (B) $\frac{1}{n^2}$ (C) n^2 (D) None of these

7. If a, b, c, d, e, f are A.M.s between 2 and 12, then a + b + c + d + e + f is equal to :-
(A) 14 (B) 84 (C) 42 (D) None of these

8. If A is the AM between a and b, then $\frac{A-2b}{A-a}+\frac{A-2a}{A-b}=$:-
(A) 4 (B) 2 (C) –4 (D) None of these

9. The n^{th} term of the sequence 2, 5, 11, 20, 32, ... is :-
(A) $\frac{3n^2+3n-4}{2}$ (B) $\frac{3n^2-3n+4}{2}$ (C) $\frac{3n^2+3n+4}{2}$ (D) None of these

10. For the A.P. a + (a + d) + (a + 2d) + ... + ℓ of n terms :-
(A) $S_n=\frac{n}{2}(a+\ell)$ (B) $S_n=\frac{n}{2}\{2a+(n-1)d\}$ (C) $S_n=\frac{n}{2}\{2\ell-(n-1)d\}$ (D) $S_n=\frac{(\ell-a+d)(a+\ell)}{2d}$

11. If a, b, c are in A.P. then :-
(A) the equation $(b-c)x^2+(c-a)x+(a-b)=0$, $b \neq c$ has equal roots
(B) a^2, b^2, c^2 are in A.P.
(C) $\lambda a+\mu, \lambda b+\mu, \lambda c+\mu$ are in A.P., $\lambda, \mu \in R$
(D) None of these

12. Which term of the sequence 8 – 6i, 7 – 4i, 6 – 2i, ... is a real number ?
(A) 7th (B) 6th (C) 5th (D) 4th

13. The sum of n terms of a series is $An^2 + Bn$, then the nth term is :-

(A) $A(2n - 1) - B$ (B) $A(1 - 2n) + B$ (C) $A(1 - 2n) - B$ (D) $A(2n - 1) + B$

14. In an A.P. sum of first p terms is equal to the sum of first q terms. Sum of it's first p + q terms is :-

(A) $-(p + q)$ (B) $p + q$ (C) 0 (D) None of these

15. 2, $\sqrt{6}$, 4.5 are the following terms of an A.P.

(A) 101st, 207th, 309th (B) 101st, 201st, 301st

(C) 2nd, 6th, 9th (D) None of these

16. The sum of 40 A.M's between two numbers is 120. The sum of 50 A.M's between them is equal to :-

(A) 130 (B) 160 (C) 150 (D) None of these

17. The sum of first n terms of an A.P. whose last term is ℓ and common difference is d is :-

(A) $\frac{n}{2}[2\ell + (n - 1) d]$ (B) $\frac{n}{2}[2\ell - (n - 1) d]$ (C) $\frac{n}{2}[\ell + (n - 1) d]$ (D) $\frac{n}{2}[\ell - (n - 1) d]$

18. In an A.P., sum of first n terms is $2n^2 + 3n$, it's common difference is :-

(A) 4 (B) 3 (C) 2 (D) 6

19. The number of terms common to the arithmetic progressions 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is :-

(A) 51 (B) 14 (C) 21 (D) 28

20. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ be the arithmetic mean between a and b, then the value of n is :-

(A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) -1

21. The ratio of first to the last of n A.M.'s between 5 and 35 is 1 : 4. The value of n is :-

(A) 9 (B) 11 (C) 10 (D) None of these

22. If the sum of first n terms of an A.P. is $Pn + Qn^2$ where P and Q are constants, then common difference of A.P. will be :-

(A) P + Q (B) P – Q (C) 2P (D) 2Q

23. Let the sequence $a_1, a_2, a_3, ..., a_n, ...$ form an A.P., then $a_1^2 - a_2^2 + a_3^2 - a_4^2 + ... + a_{2n-1}^2 - a_{2n}^2$ is equal to

(A) $\frac{n}{2n-1}\left(a_1^2 - a_{2n}^2\right)$ (B) $\frac{2n}{n-1}\left(a_{2n}^2 - a_1^2\right)$ (C) $\frac{n}{n+1}\left(a_1^2 + a_{2n}^2\right)$ (D) None of these

24. If x, y, z are in A.P., then $(x + y - z)(y + z - x)$ is equal to :-

(A) $8xy + 3y^2 - 4x^2$ (B) $8xy - 3y^2 - 4x^2$ (C) $8xy - 3x^2 + 4y^2$ (D) $8xy - 3y^2 + 4x^2$

25. If the ratio of 2nd to the 7th of n AM's between –7 and 65 is 1 : 7, then n is equal to :-

(A) 11 (B) 13 (C) 12 (D) None of these

26. If x, y, z are in A.P., then $(x + 2y - z)(x + z - y)(z + 2y - x)$ is equal to :-

(A) xyz (B) 2xyz (C) 4xyz (D) None of these

27. The value of n, for which $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is the A.M. between a and b is :-

(A) 0 (B) 1 (C) $-\frac{1}{2}$ (D) -1

28. If the A.M. of the roots of a quadratic equation is $\frac{8}{5}$ and the A.M. of their reciprocals is $\frac{8}{7}$, then the quadratic equation is :-

(A) $7x^2 + 16x + 5 = 0$
(B) $7x^2 - 16x + 5 = 0$
(C) $5x^2 - 16x + 7 = 0$
(D) $5x^2 - 8x + 7 = 0$

29. If S denotes the sum of first n terms of the A.P. a + (a + d) + (a + 2d) + ... whose nth term is ℓ, then the common 'd' of the A.P. is :-

(A) $\frac{\ell - a}{n}$
(B) $\frac{\ell^2 - a^2}{2S - a + \ell}$
(C) $\frac{\ell^2 - a^2}{2S - (a + \ell)}$
(D) None of these

30. For the A.P. x + (x + 1) + (x + 2) + ... + y

(A) C. D. is 1
(B) Numer of terms is y – x + 1
(C) Sum of the series is $\frac{y - x + 1}{2}(x + y)$
(D) None of these

31. If $\log_3 2$, $\log_3 (2^x - 5)$ and $\log_3(2^x - \frac{7}{2})$ are in A.P., then x =

(A) 2
(B) 3
(C) 4
(D) 2 or 3

32. If the angles A < B < C of a triangle are in A.P. then :-

(A) $c^2 = a^2 + b^2 - ab$
(B) $b^2 = a^2 + c^2 - ac$
(C) $c^2 = a^2 + b^2$
(D) None of these

33. If a, b, c are in A.P., then (a + 2b – c) (2b + c – a) (c + a – b) equals :-

(A) $\frac{1}{2}abc$
(B) abc
(C) 2abc
(D) 4abc

34. If 1, $\log_9(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P., then x equals :-

(A) $\log_3 4$
(B) $1 - \log_4 3$
(C) $1 - \log_3 4$
(D) $\log_4 3$

35. The last term in the n^{th} row of the following arrangement is :-

$$\begin{matrix} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & 5 & & 6 & \\ 7 & & 8 & & 9 & & 10 \end{matrix}$$

(A) n
(B) n(n + 1)
(C) $\frac{n(n+1)}{2}$
(D) None of these

36. For an A.P., $\frac{S_{kn}}{S_n}$ is independent of n. The value of $\frac{d}{a}$ for this A.P. is :-

(A) 1
(B) 2
(C) 3
(D) 4

37. If non-zero numbers $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ are in A.P. (whose common difference d is non-zero), then $a_1a_n\left(\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \frac{1}{a_3a_4} + \ldots\ldots + \frac{1}{a_{n-1}a_n}\right)$ is independent of :-

(A) n
(B) n and a_1
(C) n, a_1 and a_n
(D) a_1, a_n and d.

38. If an A.P., $S_m : S_n :: m^2 : n^2$. The ratio of the p^{th} term to q^{th} term is :-

(A) $\frac{p-1}{q-1}$ (B) $\frac{p}{q}$ (C) $\frac{2p-1}{2q-1}$ (D) None of these

39. The value of x, for which $\log_2(5.2^x + 1)$, $\log_4(2^{1-x} + 1)$ and 1 are in A.P. is :-

(A) $\log_2\left(\frac{2}{5}\right)$ (B) $\log_2\left(\frac{5}{2}\right)$ (C) $\log_2\left(\frac{3}{2}\right)$ (D) $\log_2\left(\frac{2}{3}\right)$

40. Sum of numbers in the n^{th} row of the arrangement

1
2 3 4
5 6 7 8 9
10 11 12 13 14 15 16 is

(A) $n^3 + (n-1)^3$ (B) $n^3 + (n+1)^3$ (C) $(2n-1)^3$ (D) $(2n+1)^3$

41. The sum of numbers in the nth row of the following arrangement is

1
3 5
7 9 11
13 15 17 19

(A) $n^2 (n + 1)$ (B) $n^3 - n$ (C) n^3 (D) None of these

42. When a, x, y, z, b are in A.P., x + y + z = 15 and when $\frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{b}$ are in A.P., $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$; then the number a and b are :-

(A) 8, 2 (B) 9, 1 (C) 7, 3 (D) None of these

43. The sums of first n terms of two A.P.'s are in the ratio (3n + 8) : (7n + 15). The ratio of their 12th terms is :-

(A) $\frac{4}{9}$ (B) $\frac{7}{16}$ (C) $\frac{3}{7}$ (D) None of these

44. The sum of all numbers from 1 to 1000 which are neither divisible by 2 nor by 5 is :-

(A) 200000 (B) 500500 (C) 250000 (D) None of these

45. The next term of the sequence 9, 16, 27, 42, ... is :-

(A) 53 (B) 61 (C) 57 (D) None of these

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	D	B	D	B	C	C	B	A,B,C,D	A, C	D	D	C	D
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	B	A	B	A	A	D	A	B	A	C	A	C	C	A,B,C
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B	B	D	C	C	B	D	C	A	A	C	B	B	A	B

GEOMETRIC PROGRESSION (G.P.)

A sequence $a_1, a_2, a_3, \ldots, a_n$ is said to be in geometric progression, when

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = r(\text{say})$$

and $a_1, a_2, a_3, \ldots, a_n$ are all non-zero numbers and r is said to be the common ratio of the G.P.

e.g. (i) $2, 4, 8, 16, \ldots \rightarrow r = 2$

(ii) $\frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \ldots \rightarrow r = 3$

The general term or the nth term is denoted by T_n.

$$T_n = a.r^{(n-1)}$$

Geometric Mean (G.M.): When three quantities are in G.P., the middle one is called geometric mean of the other two. Let a, m, b are in G.P., then m is the G.M. of a and b

and since $\frac{m}{a} = \frac{b}{m}$

$\therefore \quad m^2 = ab \quad \Rightarrow \quad m = \pm\sqrt{ab}$

In general if $a_1, a_2, a_3, \ldots, a_n$ are in G.P., then

$$\text{G.M.} = (a_1.a_2.a_3 \ldots a_n)^{1/n}$$

To Insert n G.Ms between a and b: If a and b are the first and last term respectively between which we have to insert n geometric means $m_1, m_2, m_3, \ldots, m_n$, then

common ratio, $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$$m_1 = a.\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$m_2 = a.\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$m_3 = a.\left(\frac{b}{a}\right)^{\frac{3}{n+1}}$$

… … … …

… … … …

$$m_n = a.\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Properties of Geometric Progressions:

1. If each term of a G.P. is multiplied or divided by a certain non-zero constant, then the resulting sequence is also a G.P.

   e.g. 3, 6, 12, 24, 48, 96, ... then

   9, 18, 36, 72, 144, 288, ... is also a G.P.

   $\frac{9}{2}$, 9, 18, 36, 72, 144, ... is also a G.P.

2. If a_1, a_2, a_3, ... and b_1, b_2, b_3, ...are two geometric progressions , then the sequence a_1b_1, a_2b_2, a_3b_3, ... is also a G.P.

   e.g. 2, 6, 18, 54, 162, ... and 1, 2, 4, 8, 16, ... then

   2, 12, 72, 432, 2592, is also a G.P.

3. If there are odd number of terms in a G.P. as a_1, a_2, a_3, ..., a_n then the middle most term is the G.M. of the given sequence.

   e.g., 2, 4, 8, 16, 32

   then, $(2 \times 4 \times 8 \times 16 \times 32)^{1/5} = 8$

   also $(2 \times 32)^{1/2} = 8$

4. If there are even number of terms in a G.P. as a_1, a_2, a_3, ..., a_n then

   $\sqrt{a_1.a_n} = \sqrt{a_2.a_{n-2}} = \sqrt{a_3.a_{n-3}} = G.M.$

5. If we have to take three terms in G.P., it is convenient to take them as $\frac{m}{k}$, m, mk, where $\frac{m}{k}$, m and mk are consecutive terms of G.P. and k is the common ratio.

   Similarly, we can take 5 terms of G.P. as $\frac{m}{k^2}, \frac{m}{k}$, m, mk, mk^2.

6. If we have to take four terms in G.P., it is convenient to take them as $\frac{m}{k^3}, \frac{m}{k}$, m, mk^3, where $\frac{m}{k^3}, \frac{m}{k}$, m and mk^3 are consecutive terms of G.P. and k^2 is the common ratio.

   Similarly, we can take 6 terms of G.P. as $\frac{m}{k^5}, \frac{m}{k^3}, \frac{m}{k}$, mk, mk^3, mk^5.

7. If a_1, a_2, a_3, ..., a_n are in G.P. then $a_1^m, a_2^m, a_3^m, ...a_n^m$ are also in G.P.

8. If a_1, a_2, a_3, ..., a_n are in G.P. (for every $a_i > 0$, $i \in I^+$) then $\log a_1$, $\log a_2$, $\log a_3$, ..., $\log a_n$ are in A.P.

9. The sum of an infinite G.P. is given as $S_\infty = \frac{a}{1-r}$ $|r|<1$, where a is the first term and r is the common ratio of the G.P. of infinite terms.

EXERCISE

1. Find the 22nd term of the G.P. –2, 2, –2.....
(A) –22 (B) –2 (C) 2 (D) None of these

2. Find the 7th term of the series $-\frac{1}{8}+\frac{1}{4}-\frac{1}{2}+1$......
(A) 8 (B) –16 (C) –8 (D) None of these

3 The 5th, 8th and 11th terms of G.P. are a, b, c respectively, then which one of the following is true?
(A) 2b = ac (B) b^2 = ac (C) a + b + c = 0 (D) None of these

4. The 5th and 12th terms of a G.P. are 32 and 4096 respectively. Find the nth term of the G.P.
(A) 2^n (B) n^2 (C) $2n^2$ (D) None of these

5. What is the least number of terms of the G.P. 5 + 10 + 20 + ... whose sum would surely exceed 10^6 ?
(A) 17 (B) 18 (C) 19 (D) 21

6. The A.M. of two positive nubmbers is 15 and their G.M. is 12. What is the smaller number?
(A) 8 (B) 12 (C) 6 (D) 24

7. The sum of three numbers in G.P. is 38 and their product is 1728. Find the greatest number.
(A) 24 (B) 18 (C) 16 (D) 8

8. The sum of first three terms of a G.P. is to the sum of the first six terms is 125:152. Find the common ratio of the G.P.
(A) $\frac{3}{5}$ (B) 3 (C) $\frac{2}{5}$ (D) $\frac{5}{8}$

9. The sum of three numbers in G.P. is 14. If the first two terms are each increased by 1 and the third term is decreased by 1, the resulting numbers are in A.P. Find the product of these three numbers.
(A) 125 (B) 64 (C) 216 (D) 124

10. The third term of G.P. is 4. The product of first five terms is :
(A) 4^3 (B) 4^4 (C) 4^5 (D) None of these

11. The sum of first three terms of a G.P. is 21 and sum of their squares is 189. Find the common ratio.
(A) $\frac{1}{2}$ or 2 (B) 3 or $\frac{1}{3}$ (C) 4 or $\frac{1}{4}$ (D) None of these

12. The sum of the first and the third term of a G.P. is 15 and that of the 5th and the 7th terms is 240. Find the 9th term.
(A) 678 (B) 786 (C) 867 (D) 768

13. Sum of three consecutive terms in a G.P. is 42 and their product is 512. Find the largest of these numbers.
(A) 28 (B) 16 (C) 32 (D) None of these

14. The sum of three numbers in G.P. is 70, if the two extremes be multiplied each by 4 and the mean by 5, the new numbers so formed are in A.P. Find the product of original numbers.
(A) 8000 (B) 6000 (C) 7000 (D) None of these

15. The sum of four terms in G.P. is 312. The sum of first and fourth terms is 252. Find the product of second and third term
(A) 500 (B) 150 (C) 60 (D) None of these

16. A bouncing tennis ball rebounds each time to a height equal to one half the height of the previous bounce. If it is dropped from a height of 16 m, find the total distance it has travelled when it hits the ground for the 10th time.
(A) $47\frac{15}{16}$ (B) $37\frac{5}{16}$ (C) $67\frac{11}{16}$ (D) None of these

17. Find the sum to n terms of the series 7 + 77 + 777 + 7777 + ...

(A) $\frac{7}{9}\{10(10^n - 1) - n\}$ (B) $\frac{7}{9}\{\frac{10}{9}(10^n - 1) - n\}$ (C) $7\{\frac{10}{9}(10^n - 1) - n\}$ (D) None of these

18. Find the sum to n terms of the series 0.8 + 0.88 + 0.888 + ...

(A) $\frac{8}{9}\left[\frac{1}{9}\{1-(0.1)^n\}\right]$ (B) $\frac{8}{9}\left[\frac{1}{10}\{1-(0.1)^n\}\right]$ (C) $\frac{8}{9}\left[n-\frac{1}{9}\{1-(0.1)^n\}\right]$ (D) None of these

19. If the pth, qth and rth terms of a G.P. be respectively a, b and c then the value of $a^{q-r} . b^{r-p} . c^{p-q}$ is :

(A) 0 (B) 1 (C) 3 (D) $\frac{abc}{pqr}$

20. If a, b, c are respectively the xth, yth and zth terms of a G.P. then the value of

$(y - z)\log a + (z - x)\log b + (x - y)\log c$

(A) 0 (B) 1 (C) 3 (D) None of these

21. There are four numbers such that the first three of them form an A.P. and the last three form a G.P. The sum of the first and the third is 2 and that of second and fourth is 26. What is the sum of first and the fourth number?

(A) $85\frac{13}{25}$ (B) $25\frac{73}{85}$ (C) 83 (D) None of these

22. Find the sum to n terms of the series 1 + 3 + 7 + 15 + ...

(A) $[(2^n - 1) - n]$ (B) $[(2^n - 1) + n]$ (C) $[2(2^n - 1) - n]$ (D) None of these

23. Find the sum of $\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+......+\frac{2n-1}{2^n}$

(A) $\left(\frac{2n-3}{2^n}\right)$ (B) $\left(3-\frac{2n+3}{2^n}\right)$ (C) $\left(\frac{2n+3}{2^n}\right)$ (D) None of these

24. Find the sum to n terms of the series 11 + 102 + 1003 + 10004 + ...

(A) $(10^n - 1) + \frac{n(n+1)}{2}$ (B) $\frac{10}{9}(10^n - 1) + \frac{n(n+1)}{2}$ (C) $10^n + n^2 - 1$ (D) None of these

25. Find the sum to first n groups of

(1) + (1 + 3) + (1 + 3 + 9) + (1 + 3 + 9 + 27) + ...

(A) $\frac{1}{2}(3^n - 1)$ (B) $\frac{3}{4}(3^n - 1) - \frac{n}{2}$ (C) $n^2 + 1$ (D) None of these

26. Find the sum to n terms of the following series : 2 + 5 + 14 + 41 + ...

(A) $\frac{3}{4}(3^{n-1}) + \frac{n}{2}$ (B) $\frac{3}{4}(3^n - 1) + \frac{n}{2}$ (C) (3n + 1) (D) None of these

27. Find the sum to n terms : $1 + 2x + 3x^2 + 4x^3 + ...; x \neq 1$

(A) $1 + nx^n$ (B) $\frac{x(x+1)}{2} + x^n$ (C) $\frac{1-x^n}{(1-x)^2}$ (D) $\left\{\frac{1-x^n}{(1-x)^2}-\frac{nx^n}{(1-x)}\right\}$

28. Find the sum to n terms of : $1 + 3x + 5x^2 + 7x^3 + ..., x \neq 1$

(A) $2(1 - x^{n-1})$ (B) $\frac{2x(1-x^{n-1})}{(1-x^2)}$

(C) $\frac{2x(1-x^{n-1})}{(1-x)^2}+\frac{1-(2n-1)x^n}{(1-x)}$ (D) None of these

29. Find the sum to first n terms : $1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+....$

(A) $\frac{3}{4}-\frac{3+2n}{4(3^{n-1})}$ (B) $\frac{9}{4}-\frac{3}{4}\left[\frac{3+2n}{3^n}\right]$ (C) $2n-\left(\frac{1}{n}\right)^2$ (D) None of these

30. Find the sum to n terms of $3.2 + 5.2^2 + 7.2^3 + ...$

(A) $2^{n+2} - 2^{n+1} - 2$ (B) $n.2^{n+2} - 2^{n+1} + 2$ (C) $n.2^{n+2} + 2^{n-2}$ (D) None of these

31. Find the sum of the series : $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+......$ to n terms.

(A) $\frac{35}{16}+\frac{7n}{5}$ (B) $\frac{12n-7}{16(5^{n-1})}$ (C) $\frac{35}{16}-\left[\frac{12n+7}{16(5^{n-1})}\right]$ (D) None of these

32. Find the sum of the series : $1.3^2 + 2.5^2 + 3.7^2 + ...$ to 20 terms.

(A) 12896 (B) 187898 (C) 98970 (D) 188090

33. Find the sum of the series : $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+......$ to 16 terms.

(A) 224 (B) 446 (C) 2356 (D) None of these

34. In a set of four numbers, the first three are in G.P. and the last three are in A.P. with common difference 6. If the first number is the same as the fourth, find the third number.

(A) –8 (B) 4 (C) 2 (D) can't be determined

35. The sum of an infinite G.P. is 16 and the sum of squares of its terms is $153\frac{3}{5}$. Find the fourth term of the progression :-

(A) $\frac{7}{16}$ (B) $\frac{5}{32}$ (C) $\frac{3}{16}$ (D) None of these

36. If $x = 1 + a + a^2 + ...$ to ∞ and $y = 1 + b + b^2 + ...$ to ∞, $|a| < 1$ and $|b| < 1$, then the value of : $1 + ab + a^2b^2 + ...$ to ∞ is :

(A) $\frac{xy}{x+y}$ (B) $\frac{x+y+1}{xy}$ (C) $\frac{xy}{x-y}$ (D) $\frac{xy}{x+y-1}$

37. A person is entitled to receive an annual payment which for each year is less by one-tenth of what it was for the year before. If the first payment is 100, then find the maximum possible payment which he can receive, however long he may live.

(A) 900 (B) 9999 (C) 1000 (D) None of these

38. Find the sum of the series $\frac{1}{4}-\frac{3}{16}+\frac{9}{64}-\frac{27}{256}+...$

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{7}$ (D) None of these

39. The sum of first two terms of a G.P. is $\frac{5}{3}$ and the sum to infinity of the series is 3. Find the first term.

(A) 1 (B) $\frac{2}{3}$ (C) 5 (D) Both (A) and (C)

40. A ball is dropped from a height of 96 feet and it rebounds $\frac{2}{3}$ of the height it falls. If it continues to fall and rebound, find the total distance that the ball can travel before coming to rest.

(A) 240 ft (B) 360 ft (C) 480 ft (D) None of these

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	B	A	B	C	B	A	B	C	A	D	C	A	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	B	C	B	A	D	C	B	B	B	B	D	C	B	B
Que.	31	32	33	34	35	36	37	38	39	40					
Ans.	C	D	B	C	C	D	C	C	D	C					

1. The points (k – 1, k + 2), (k, k + 1), (k + 1, k) are collinear for

(A) any value of k (B) k = –1/2 only

(C) no value of k (D) integral values of k only

2. The area of the triangle formed by the points (k, 2 – 2k), (– k + 1, 2k) and (– 4 – k, 6 – 2 k) is 70 units. For

(A) four real values of k (B) no integral value of k

(C) two integral values of k (D) only one integral value of k

3 The quadrilateral ABCD formed by the points A (0, 0), B (3, 4), C (7, 7) and D (4, 3) is a

(A) rectangle (B) square (C) rhombus (D) parallelogram

4. The triangle with vertices A (2, 7), B (4, y) and C (– 2, 6) is right angled at A if

(A) y = –1 (B) y = 0 (C) y = 1 (D) none of these

5. The join of the points (– 3, – 4) and (1, – 2) is divided by y-axis in the ratio.

(A) 1 : 3 (B) 2 : 3 (C) 3 : 1 (D) 3 : 2

6. The straight lines x + y – 4 = 0, 3x + y – 4 = 0 and x + 3y – 4 = 0 form a triangle which is

(A) isosceles (B) right angled (C) equilateral (D) none of these

7. The points P(a, b + c), Q(b, c + a) and R(c, a + b) are such that PQ = QR if

(A) a, b, c are in A.P. (B) a, b, c are in G.P. (C) a, b, c are in H.P. (D) none of these

8. If a, b, c are in A.P. then the points (a, x), (b, y) and (c, z) are collinear if

(A) $x^2 = y$ (B) $x = z^2$ (C) $y^2 = z$ (D) x, y, z are in A.P.

9. The centroid of a triangle lies at the origin and the coordinates of its two vertices are (–8, 7) and (9, 4). The area of the triangle is

(A) 95/6 (B) 285/2 (C) 190/3 (D) 285

10. The mid points of the sides AB and AC of a triangle ABC are (2, – 1) and (– 4, 7) respectively, then the length of BC is

(A) 10 (B) 20 (C) 25 (D) 30

11. If the vertices of a triangle ABC are A (– 4, – 1), B (1, 2) and C (4, – 3), then the coordinates of the circumcentre of the triangle are

(A) (1/3, – 2/3) (B) (0, – 4) (C) (0, – 2) (D) (– 3/2, 1/2)

12. The extremities of a diagonal of a parallelogram are the points (3, – 4) and (– 6, 5). If third vertex is (– 2, 1) then the coordinates of the fourth vertex are

(A) (1, 0) (B) (0, 0) (C) (1, 1) (D) none of these

13. The number of lines that can be drawn through the point (4, – 5) at a distance 12 from the point (– 2, 3) is

(A) 0 (B) 1 (C) 2 (D) Infinite

14. If O is the origin and the coordinates of A and B are (x_1, y_1) and (x_2, y_2) respectively then OA Ч OB cos $\angle$ AOB is equal to

(A) $x_1y_1 + x_2y_2$ (B) $x_1x_2 + y_1 y_2$ (C) $x_1y_2 + x_2y_1$ (D) $x_1x_2 - y_1y_2$

15. If the lines x + ay + a = 0, bx + y + b = 0 and cx + cy + 1 = 0 (a, b, c being distinct and ≠ 1) are concurrent, then the value of $\frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1}$ is

(A) – 1 (B) 0 (C) 1 (D) none of these

16. The line $\frac{x}{3} + \frac{y}{4} = 1$ meets the axis of y and axis of x at A and B respectively. A square ABCD is constructed on the line segment AB away from the origin, the coordinates of the vertex of the square farthest from the origin are

(A) (7, 3) (B) (4, 7) (C) (6, 4) (D) (3, 8)

17. ABCD is a quadrilateral. P (3, 7) and Q (7, 3) are the middle points of the diagonals AC and BD respectively. The coordinates of the mean point (or the centre of mean position) of the vertices of the quadrilateral are

(A) (0, 0) (B) (3, 3) (C) (5, 5) (D) (7, 7)

18. The area of a triangle, two of whose vertices are (2, 1) and (3, – 2) is 5. The coordinates of the third vertex can not be

(A) (6, – 1) (B) (4, 5) (C) (– 1, 20) (D) (2, 9)

19. If the circumcentre of a triangle lies at the point (a, a) and the centroid is the mid point of the line joining the points (2a + 3, a + 4) and (a – 4, 2a – 3) ; then the orthocentre of the triangle lies on the line

(A) y = x (B) (a – 1) x + (a + 1) y = 0

(C) (a – 1) x – (a + 1) y = 0 (D) (a + 1) x – (a – 1) y = 2a

20. If $A(at^2, 2at)$, $B(a/t^2, -2a/t)$ and S (a, 0) are three points, then $\frac{1}{SA} + \frac{1}{SB}$ is independent of

(A) a (B) t (C) both a and t (D) none of these

21. If the line $\sqrt{5}\,x = y$ meets the lines x = 1, x = 2, ... x = n at points $A_1, A_2, \ldots, A_n$ respectively, then $(OA_1)^2 + (OA_2)^2 + \ldots + (OA_n)^2$ is equal to

(A) $3n^2 + 3n$ (B) $2n^3 + 3n^2 + n$ (C) $3n^3 + 3n^2 + 2$ (D) $(3/2)(n^4 + 2n^3 + n^3)$

22. One diagonal of a square is the portion of the line 3x + 2y = 12 intercepted between the axes. The coordinates of the extremity of the other diagonals not lying in the first quadrant are

(A) (1, – 1) (B) (– 1, – 1) (C) (– 1, 1) (D) none of these

23. ABCD is a square in which A lies on the positive y-axis and B lies on the positive x-axis. If D is the point (12, 17), the coordinates of C are

(A) (17 , 12) (B) (17 , 5) (C) (14 , 16) (D) (15, 3)

24. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2AC. If the coordinates of D and M are (1, 1) and (2, – 1) respectively, the coordinates of A are

(A) (– 3, – 1/2) (B) (1, – 3/2) (C) (3/2, – 1) (D) (1/2, – 3)

25. If (0, 1), (1, 1) and (1, 0) are the mid points of the sides of a triangle, the coordinates of its incentre are

(A) $(2 + \sqrt{2}, 2 + \sqrt{2})$ (B) $((2 + \sqrt{2}), -(2 + \sqrt{2}))$

(C) $((2 - \sqrt{2}), (2 - \sqrt{2}))$ (D) $((2 - \sqrt{2}), -(2 - \sqrt{2}))$

26. The points (0, 8/3), (1, 3) and (82, 30) are the vertices of

(A) obtuse angled triangle (B) acute angled triangle (C) right angled triangle (D) none of these

27. Area of the rhombus enclosed by the lines $ax \pm by \pm c = 0$ is

(A) $2a^2/bc$ (B) $2b^2/ca$ (C) $2c^2/ab$ (D) none of these

28. The coordinates of the points A and B are respectively (– 3, 2) and (2, 3). P and Q are points on the line joining A and B such that AP = PQ = QB. A square PQRS is constructed on PQ as one side, the coordinates of R are

(A) (–4/3, 7/3) (B) (0, 13/3) (C) (1/3, 8/3) (D) (2/3, 1)

29. A ray of light coming from the point (1, 2) is reflected at a point A on the axis of x and then passes through the point (5, 3). The coordinates of the point A are

(A) (5/13, 0) (B) (–7, 0) (C) (13/5, 0) (D) (15, 0)

30. The incentre of the triangle with vertices $(1, \sqrt{3})$, (0, 0) and (2, 0) is

(A) $(1, \sqrt{3}/2)$ (B) $(2/3, 1/\sqrt{3})$ (C) $(2/3, \sqrt{3}/2)$ (D) $(1, 1/\sqrt{3})$

31. The distance between the orthocentre and the circumcentre of the triangle with vertices (0, 0), (0, a) and (b, 0) is

(A) $\sqrt{|a^2 - b^2|/2}$ (B) a + b (C) a – b (D) $\sqrt{a^2 + b^2}/2$

32. The centroid of a triangle lies at the origin and the coordinates of its two vertices are (– 8, 0) and (9, 11), the area of the triangle in sq. units is

(A) 11 / 8 (B) 8 / 11 (C) 88 (D) none of these

33. The line 3x + 2y = 24 meets the y-axis at A and the x-axis at B ; C is a point on the perpendicular bisector of AB such that the area of the triangle ABC is 91 sq. units. The coordinates of C can be

(A) (29/2, – 1) (B) (29/2, 13) (C) (– 13/2, 1) (D) (– 13/2, 13)

34. If the straight lines x + 2x – 9 = 0, 3x + 5y – 5 = 0 and ax + by + 1 = 0 are concurrent, then the straight line 35x – 22y – 1 = 0 passes through

(A) (a, b) (B) (b, a) (C) (a, – b) (D) (– a, b)

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	D	C	A	C	A	A	D	B	B	C	D	A	B	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	D	D	B	B	C	B	B	C	D	C	D	C	A
Que.	31	32	33	34											
Ans.	D	D	B	A											

POLYGONS

✯ DEFINITIONS & USEFUL CONCEPTS

(i) **Polygon:** It is a closed plane figure bounded by some straight Lines. A polygon is called a Triangle, Quadrilateral, Pentagon, Hexagon, Heptagon, Octagon. Nonagon and Decagon according as it contains 3,4,5,6, 7, 8, 9, 10 sides respectively.

(ii) **Convex & Concave Polygon :** A polygon in which none of its interior angles is more than 180° is known as a convex polygon.

On the other hand, if at least one angle of a polygon is more than 180°, then it is said to be concave.

(iii) **Regular Polygon:** A polygon having all sides equal and all angles equal is called a regular polygon.

I. Each exterior angle of a regular polygon = $\left(\frac{360}{\text{Number of sides}}\right)^0$

II. Each interior angle = 180° – (exterior angle).

(iv) In a convex polygon of n sides, we have:

I. Sum of all interior angles = (2n – 4) right angles;

II. Sum of all exterior angles = 4 right angles.

(v) Number of diagonals of a polygon of n sides = $\frac{n(n-1)}{2} - n$

EXERCISE

1. The sum of all interior angles of a convex polygon of n sides is :
(A) 4 right angles (B) 4n right angles (C) (n – 1) right angles (D) (2n - 4) right angles

2. The sum of all exterior angles of a convex polygon of n sides is :
(A) 4 right anlges (B) 2n right angles (C) (2n – 4) right angles (D) $\frac{n}{2}$ right angles

3. Each interior angle of a regular polygon is 135°. The number of sides of the polygon is :
(A) 6 (B) 8 (C) 5 (D) 9

4. Each angle of a regular pentagon will be :
(A) 72° (B) 90° (C) 108° (D) 120°

5. Each interior angle of a regular hexagon is :
(A) 60° (B) 120° (C) 108° (D) 90°

6. If one of the interior angles of a regular polygon is found to be equal to (9/8) times of one of the interior angles of a regular hexagon, then the number of sides of the polygon is :
(A) 4 (B) 5 (C) 7 (D) 8

7. The ratio of the measure of an angle of a regular octagon to the measure of its exterior angle is :
(A) 1 : 2 (B) 1 : 3 (C) 2 : 3 (D) 3 : 1

8. The sum of the interior angles of a hexagon is :
(A) 360° (B) 540° (C) 720° (D) 900°

9. The sum of the interior angles of a polygon is 1080°. The number of sides of the polygon is :
(A) 8 (B) 6 (C) 10 (D) 9

10. The angles of a pentagon are in the ratio 1 : 2 : 3 : 5 : 9. The smallest angle is :
(A) 72° (B) 45° (C) 54° (D) 27°

11. If each interior angle of a regular polygon is 11 times its exterior angle, the number of sides of the polygon is :
(A) 22 (B) 24 (C) 18 (D) 11

12. One angle of a hexagon is 100° and all the other five angles are equal. What is the measure of each one of the equal angles ?
(A) 36° (B) 144° (C) 124° (D) 72°

13. The ratio between the number of sides of two regular polygons is 1 : 2 and the ratio between their interior angles is 2 : 3. The number of sides of these polygons are respectively;
(A) 4, 8 (B) 5, 10 (C) 6, 12 (D) 8, 16

14. How many diagonals are there in a hexagon ?
(A) 6 (B) 4 (C) 11 (D) 9

15. How many diagonals are there in a decagon ?
(A) 19 (B) 29 (C) 35 (D) 45

16. A polygon has 27 diagonals. The number of sides of the polygon is :
(A) 9 (B) 10 (C) 11 (D) 12

17. The ratio of an interior angle to the exterior angle of a regular polygon is (7 : 2). The number of sides of the polygon is :
(A) 6 (B) 9 (C) 7 (D) 8

18. The sum of the interior angles of a regular polygon is twice the sum of its exterior angles. The polygon is :
(A) an octagon (B) a nonagon (C) a decagon (D) a hexagon

19. Each exterior angle of a regular polygon is 40°. The number of sides of the polygon is :
(A) 8 (B) 9 (C) 6 (D) 10

20. The difference between the interior and exterior angles of a regular polygon is 60°. The number of sides in the polygon is :
(A) 5 (B) 6 (C) 8 (D) 10

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	A	B	C	B	D	D	C	A	D	B	C	A	D	C
Que.	16	17	18	19	20										
Ans.	A	B	D	B	B										

LOGARITHMS

IMPORTANT FACTS AT A GLANCE

✯ LOGARITHM

If a is a positive real number, other than 1 and $a^m = x$, then m is called the logarithm of x to the base a, written as $\log_a x$.

Thus, $a^m = x \Leftrightarrow \log_a x = m$.

Ex. (i) $10^2 = 100 \Leftrightarrow \log_{10} 100 = 2$

(ii) $5^3 = 125 \Leftrightarrow \log_5 125 = 3$

(iii) $3^{-3} = \frac{1}{27} \Leftrightarrow \log_3 \left(\frac{1}{27}\right) = -3$

(vi) $(.1)^2 = .01 \Leftrightarrow \log_{.1} (.01) = 2.$

✯ PROPERTIES OF LOGARITHMS

(i) $\log_a (xy) = \log_a x + \log_a y$

(ii) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

(iii) $\log_a (x^k) = k (\log_a x)$

(iv) $\log_a 1 = 0 \quad [\because a^0 = 1]$

(v) $\log_x x = 1$

(vi) $\log_a x = \frac{1}{\log_x a}$

(vii) $\log_a x = \frac{\log_b x}{\log_b a}$. So, $\log_b x = \frac{\log x}{\log b}$

(viii) $a^{\log_a x} = x$; e.g. $5^{\log_5 7} = 7$.

(ix) If $a > 1$ and $x > 1$, then $\log_a x > 0$.

(x) If $0 < a < 1$ and $0 < x < 1$, then $\log_a x > 0$.

(xi) If $0 < a < 1$ and $x > 1$, then $\log_a x > 0$

(xii) If $a > 1$ and $0 < x < 1$, then $\log_a x < 0$.

Remark: When base is not mentioned, it will be taken as 10.

Common Logarithm: Logarithms to the base 10 are known as common logarithms.

SOLVED EXAMPLES

Ex.1 Evaluate:

(i) $\log_2 32$ (ii) $\log_3 \left(\frac{1}{243}\right)$ (iii) $\log_{100} (0.1)$

Sol. (i) $\log_2 32 = m \Leftrightarrow 2^m = 32 = 2^5 \Leftrightarrow m = 5.$

(ii) $\log_3 \left(\frac{1}{243}\right) = m \Leftrightarrow 3^m = \frac{1}{243} = \frac{1}{3^5} = 3^{-5} \Leftrightarrow m = -5$

(iii) $\log_{100} (0.1) = m \Leftrightarrow 100^m = 0.1 = \frac{1}{10} \Leftrightarrow 10^{2m} = 10^{-1}$

$\Leftrightarrow 2m = -1 \Leftrightarrow m = -\frac{1}{2}$

Ex.2 Evaluate:

(i) $36^{\log_6 4}$ (ii) $\log_a 1$ (iii) $\log_{25} 25$.

Sol. (i) We know that: $a^{\log_a x} = x$.

$\therefore$ $36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{2\log_6 4} = 6^{\log_6 (4^2)} = 6^{\log_6 (16)} = 16$

(ii) $\log_a 1 = m \Leftrightarrow a^m = 1 = a^0 \Leftrightarrow m = 0$.

(iii) We know that $\log_a a = 1$. So, $\log_{25} 25 = 1$.

Ex.3 Evaluate:

(i) $\log_9 27 - \log_{27} 9$ (ii) $\log_3 5 \,.\, \log_{25} 27$

Sol. (i) $\log_9 27 = m \Leftrightarrow 9^m = 27 \Leftrightarrow 3^{3m} = 3^3 \Leftrightarrow 3m = 3 \Leftrightarrow m = I$.

And, $\log_{27} 9 = n \Leftrightarrow (27)^n = 9 \Leftrightarrow 3^{3n} = 3^2 \Leftrightarrow 3n = 2 \Leftrightarrow n = \frac{2}{3}$.

$\therefore$ $\log_9 27 - \log_{27} 9 = \left(1 - \frac{2}{3}\right) = \frac{1}{3}$

(ii) We know that $\log_a b = \frac{\log b}{\log a}$.

$\therefore$ $\log_3 5 . \log_{25} 27 = \frac{\log 5}{\log 3} \times \frac{\log 27}{\log 25} = \frac{\log 5}{\log 3} \times \frac{\log 3^3}{\log 5^2} = \frac{\log 5}{\log 3} \times \frac{3\log 3}{2\log 5} = \frac{3}{2}$

Ex.4 If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find x.

Sol. $\log_{\sqrt{8}} x = \frac{10}{3} \Rightarrow x = (\sqrt{8})^{10/3} = (2^{3/2})^{10/3} = 2^{\left(\frac{3}{2} \times \frac{10}{3}\right)} = 2^5 = 32$

Ex.5 Evaluate : $\log_2 (\log_3 81)$.

Sol. $\log_3 81 = m \Leftrightarrow 3^m = 81 = 3^4 \Leftrightarrow m = 4$.

$\therefore$ $\log_2 (\log_3 81) = \log_2 4 = \log_2 (2^2) = 2 (\log_2 2) = 2$ Ч $1 = 2$.

Ex.6 If $\log_{10} 2 = 0.30103$, then find the number of digits in 2^{64}

Sol. $\log_{10} (2^{64}) = 64$ Ч $\log_{10} 2 = 64$ Ч $0.30103 = 19.26592$

$\therefore$ Its characteristic is 19.

Hence, the number of digits in 2^{64} is 20.

Ex.7 Evaluate: $\log_b a \ \log_c b \ \log_d c \ \log_a d$.

Sol. $\log_b a \,.\, \log_c b \,.\, \log_d c \,.\, \log_a d$

$$\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log d} \times \frac{\log d}{\log a} = 1$$

Ex.8. If $\log_{10} [98 + \sqrt{x^2 - 12x + 36}] = 2$, find x.

Sol. $\log_{10} [98 + \sqrt{x^2 - 12x + 36}] = 2 = \log_{10} 100$

$\Leftrightarrow 98 + \sqrt{x^2 - 12x + 36} = 100 \Leftrightarrow \sqrt{x^2 - 12x + 36} = 2$

$\Leftrightarrow x^2 - 12x + 36 = 4 \Leftrightarrow x^2 - 12x + 32 = 0$

$\Leftrightarrow (x - 8)(x - 4) = 0 \Leftrightarrow x = 8$ or $x = 4$.

Ex.9 Find the value of $\log_{10} 20 + \log_{10} 30 - \frac{1}{2} \log_{10} 36$.

Sol. $\log_{10} 20 + \log_{10} 30 - \frac{1}{2} \log_{10} 36$

$= \log_{10} 20 + \log_{10} 30 - \log_{10} (36^{1/2}) = \log_{10} 20 + \log_{10} 30 - \log_{10} 6$

$= \log_{10} \left(\frac{20 \times 30}{6}\right) = \log_{10} 100 = \log_{10} (10^2) = 2.$

Ex.10. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4717$, find :

(i) $\log_{10} 25$ (ii) $\log_{10} 4.5$

Sol. (i) $\log_{10} 25 = \log_{10} \left(\frac{100}{4}\right) = \log_{10} 100 - \log_{10} 4 = 2 - 2 \log_{10} 2 = (2 - 2$ Ч $0.3010) = 1.398$

(ii) $\log_{10} 4.5 = \log_{10} \left(\frac{9}{2}\right) = \log_{10} 9 - \log_{10} 2 = \log_{10} (3^2) - \log_{10} 2 = 2 \log_{10} 3 - \log_{10} 2$

$= (2$ Ч $0.4771 - 0.3010) = 0.6532.$

EXERCISE

1. If $a^x = b$, then:

(A) $\log_b x = a$ (B) $\log_a x = b$ (C) $\log_x a = b$ (D) $\log_a b = x$

2. If $\log_a b = c$, then:

(A) $b^c = a$ (B) $a^c = b$ (C) $a^b = c$ (D) $b^a = c$

3. $\log_a (pq)$ is equal to :

(A) $(\log_a p)(\log_a q)$ (B) $\log_a p + \log_a q$ (C) $\log_a p - \log_a q$ (D) $\log_p a + \log_q a$

4. $\log_a \left(\frac{p}{q}\right)$ is equal to :

(A) $\log_a p - \log_a q$ (B) $(\log_a p) \div (\log_a q)$ (C) $\log_a p + \log_a q$ (C) $\log_a q - \log_a p$

5. If $\log_a 4 = \frac{1}{4}$, then a is equal to :

(A) 16 (B) 64 (C) 128 (D) 256

6. The value of $\log_{27} 9$ is :

(A) $\frac{1}{3}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) 3

7. The value of $\log_5 \left(\frac{1}{625}\right)$ is :

(A) 4 (B) – 4 (C) $\frac{1}{4}$ (D) $-\frac{1}{4}$

8. The value of $\log_{\sqrt{2}} 16$ is :

(A) 4 (B) 8 (C) 16 (D) $\frac{1}{8}$

9. If $\log_8 x = \frac{2}{3}$, then the value of x is :

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) 4 (D) 3

10. If $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2}$, then the value of x is :

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$ (C) $\frac{81}{256}$ (D) $\frac{256}{81}$

11. If $\log_{10} x = -2$, then x is :

(A) $\sqrt{10}$ (B) $\frac{1}{\sqrt{10}}$ (C) $\frac{1}{20}$ (D) $\frac{1}{100}$

12. If $\log_{10000} x = -\frac{1}{4}$, then the value of x is :

(A) $\frac{1}{10}$ (B) $\frac{1}{100}$ (C) $\frac{1}{1000}$ (D) $\frac{1}{10000}$

13. The value of $\log_{.01}(1000)$ is :

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

14. The value of $\log_2(\log_5 625)$ is :

(A) 2 (B) 5 (C) 10 (D) 15

15. The value of $\log_{10} 0.00001$ is :

(A) – 4 (B) – 5 (C) $-\frac{1}{4}$ (D) $-\frac{1}{5}$

16. If $\log_x 0.1 = -\frac{1}{3}$, then the value of x is :

(A) 10 (B) 100 (C) 1000 (D) $\frac{1}{1000}$

17. The value of $25^{\log_5 4}$ is :

(A) 16 (B) 32 (C) 20 (D) 625

18. If $\log_{10}[\log_{10}(\log_{10}x)] = 0$, then the value of x is :

(A) 10 (B) 10^2 (C) 10^3 (D) 10^{10}

19. The value of $\log_2[\log_2 \log_2 \log_2 (65536)]$ is :

(A) 0 (B) 1 (C) 2 (D) 4

20. If $\log_2[\log_3(\log_2 x)] = 1$, then x is equal to :

(A) 512 (B) 128 (C) 12 (D) 0

21. If $\log_{10} 2x = 1$, the value of x is :

(A) $\frac{1}{5}$ (B) 100 (C) 5 (D) 20

22. $[\log_{10} 10 + \log_{10} 100 + \log_{10} 1000 + \log_{10} 10000 + \log_{10} 100000]$ is :

(A) 15 (B) $\log_1 11111$ (C) $\log_{10} 1111$ (D) $14 \log_{10} 100$

23. The value of $\left(\log\frac{3}{5}+\log\frac{5}{36}+\log 12\right)$ is equal to :

(A) log 5 (B) log 3 (C) log 2 (D) 0

24. $\left(\log\frac{11}{5}+\log\frac{14}{3}-\log\frac{22}{15}\right)$ is equal to :

(A) log 2 (B) log 3 (C) log 5 (D) log 7

25. The value of $\left(\frac{1}{3}\log_{10}125-2\log_{10}4+\log_{10}32\right)$ is :

(A) 0 (B) $\frac{4}{5}$ (C) 2 (D) None of these

26. The value of $7\log\left(\frac{16}{15}\right) + 5\log\left(\frac{25}{24}\right) + 3\log\left(\frac{81}{80}\right)$ is :

(A) log 2 (B) log 4 (C) log 6 (D) log 8

27. $\log_{-1/3} 81$ is equal to :

(A) – 27 (B) – 4 (C) 4 (D) 27

28. $\log_{2\sqrt{3}} 1728$ is equal to :

(A) 3 (B) 5 (C) 6 (D) 9

29. If $\log_{10} x + \log_{10} y = z$, then x is equal to :

(A) $\frac{z}{y}$ (B) $\frac{10^z}{y}$ (C) yz (D) $(10^z)\,y$

30. $\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ac}\right) + \log\left(\frac{c^2}{ab}\right)$ is :

(A) 1 (B) 0 (C) 2 (D) abc

31. $\frac{1}{(\log_a bc)+1} + \frac{1}{(\log_b ca)+1} + \frac{1}{(\log_c ab)+1}$ is equal to :

(A) 1 (B) 2 (C) 3 (D) $\frac{3}{2}$

32. $\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}$ is equal to :

(A) 1 (B) 2 (C) 3 (D) 4

33. The value of ($\log_b a$ Ч $\log_c b$ Ч $\log_a c$) is :

(A) 0 (B) 1 (C) abc (D) a + b + c

34. ($\log_b a$) Ч ($\log_a b$) is equal to :

(A) a (B) b (C) 0 (D) 1

35. Which of the following statements is not correct ?

(A) $\log_{10} 1 = 0$ (B) log (1 + 2 + 3) = log 1 + log 2 + log 3

(C) $\log_{10} 10 = 1$ (D) log (2 + 3) = log (2 Ч 3)

36. If log (x + y) = log x + log y and x = 1.15683, then the value of y is :

(A) $\bar{7}.736$ (B) 7.376 (C) 3.456 (D) 1.234

37. Given that $\log_{10} 2 = 0.3010$, the value of $\log_{10} 5$ is :

(A) 0.3241 (B) 0.6911 (C) 0.6990 (D) 0.7525

38. If $\log_{10} 2 = 0.30103$, then the value of $\log_{10} 50$ is :

(A) .69897 (B) 1.30103 (C) 1.69897 (D) 2.30103

39. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 80$ is :

(A) 1.9030 (B) 1.6020 (C) 3.9030 (D) None of these

40. The value of ($\log_9 27 + \log_8 32$) is :

(A) 4 (B) 7 (C) $\frac{7}{2}$ (D) $\frac{19}{6}$

41. If $\log_{10} 2 = 0.3010$, then the value of $\log_{10} 25$ is :

(A) 1.5050 (B) 1.3980 (C) 1.2040 (D) 0.6020

42. If log 2 = 0.3010 and $5^x = 400$, then x is equal to :

(A) 2.40 (B) 3.72 (C) 4.36 (D) 1

43. ($\log_{10} 40000 - \log_{10} 4$) equals :

(A) 4 (B) 10000 (C) $\log_{10} 39996$ (D) 39996

44. Consider the following statements :

1. $\log_{10}(0.1)^2 + \log_{10} 10 \cdot \log_{10} 100 = 0$
2. $\log_{10} \log_{10} 10 = 1$
3. $\log_{10} \sqrt{10} + \log_{10} \sqrt{10} = 1$

Choose the correct answer:

(A) 1 and 3 are correct (B) 2 & 3 are correct (C) 1 and 2 are correct (D) all are correct

45. ($\log_5 3$) Ч ($\log_3 625$) equals :

(A) 1 (B) 2 (C) 3 (D) 4

46. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{100}(7.2)$ is :

(A) $\bar{1}.9286$ (B) $\bar{1}.8573$ (C) 1.8572 (D) .9286

47. The value of $\log_{10} 0.02$ lies between :

(A) 0 and 1 (B) – 2 and – 1 (C) 0 & – 1 (D) – 2 & – 3

48. If a, b, c are three consecutive integers, then log (ac + 1) has the value :

(A) $(\log b)^2$ (B) log b (C) 2 log b (D) log 2b

49. Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{10} 1.5$ is:

(A) 0.7161 (B) 0.1761 (C) 0.7116 (D) 0.7611

50. Given log 2 = 0.30103, the number of digits in 5^{20} is :

(A) 14 (B) 16 (C) 18 (D) 25

51. Value of $\log_{10} 1$ is :

(A) 1 (B) 0 (C) 0.1 (D) 0.01

52. Which one of the following is not true in general ?

(A) $\log x + \log y = \log xy$ (B) $\log x + \log y = \log (x + y)$

(C) $\log x - \log y = \log \left(\frac{x}{y}\right)$ (D) $\log_x y = y^{\log x}$

53. Which of the following is correct ?

(A) $\log m^n = (\log m)^n$ (B) ($\log_a b$) Ч ($\log_b a$) = 1

(C) log (m Ч n) = (log m Ч log n) (D) $\log (m - n) = \log m - \log n$

54. The value of $(\log_b a)(\log_c b)(\log_a c)$ is :

(A) 0 (B) 1 (C) 10 (D) log (abc)

55. If $\log_e x + \log_e (1 + x) = 0$, then:

(A) $x^2 + x - 1 = 0$ (B) $x^2 + x + 1 = 0$ (C) $x^2 + x - e = 0$ (D) $x^2 + x + e = 0$

56. If $\log_4 (x^2 + x) - \log_4 (x + 1) = 2$, then the value of x is :

(A) 2 (B) 4 (C) 5 (D) 16

57. If $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$, then x is equal to :

(A) 12 (B) 16 (C) 18 (D) 24

58. The value of $\left(\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}\right)$ is :

(A) 5 (B) 0 (C) 60 (D) 1

59. If $\log_{10} 125 + \log_{10} 8 = x$, then x is equal to :

(A) – 3 (B) 3 (C) $\frac{1}{3}$ (D) .064

60. If $\log a + \log b = \log (a + b)$, then:

(A) $a = b$ (B) $b = \frac{1}{a}$ (C) $b = \left(\frac{a-1}{a}\right)$ (D) $b = \left(\frac{a}{a-1}\right)$

61. If $\log \frac{m}{n} + \log \frac{n}{m} = \log (m + n)$, then:

(A) $m + n = 1$ (B) $\frac{m}{n} = 1$ (C) $m - n = 1$ (D) $m^2 - n^2 = 1$

62. The value of : $(\log_3 4) (\log_4 5) (\log_5 6) (\log_6 7) (\log_7 8) (\log_8 9)$ is :

(A) 2 (B) 7 (C) 8 (D) 33

63. If $\log (x + 1) + \log (x - 1) = \log 3$, then x is equal to :

(A) 1 (B) 2 (C) 3 (D) 4

64. If $\frac{\log 8}{\log 2} = x$, then x is equal to :

(A) 2 (B) 3 (C) 4 (D) $\frac{1}{2}$

65. The value of x satisfying $\log_{32} x = 0.8$ is :

(A) 25.6 (B) 16 (C) 10 (D) 12.8

66. If $\log_{10} x = 1.9675$, then $\log_{10} (1000 x)$ is :

(A) 19.675 (B) 4.9675 (C) 1.9675 Ч 3 (D) 1967.5

67. The mantissa of log 3274 is .5150. The value of log 0.3274 is :

(A) 0.5150 (B) 1.5150 (C) $\bar{1}.5150$ (D) $\bar{2}.5150$

68. (log tan 1° · log tan 2° .... log tan 50°) is :

(A) 1 (B) – 1 (C) 0 (D) None of these

69. The characteristic in log (6.7432 Ч 10^{-5}) is:

(A) – 5 (B) – 4 (C) 1 (D) 5

70. If $\log_{10} 2 = .3010$, then $\log_2 10$ is :

(A) .3322 (B) 3.2320 (C) 3.3222 (D) 5

71. If $\log_{10} (.1) = -1$, then $\log_{10} (.001)$ is :

(A) – 1.3 (B) – 2 (C) – 2.3 (D) – 3

72. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then the value of $a^a b^b c^c$ is:

(A) abc (B) $\frac{1}{abc}$ (C) 1 (D) log (abc)

73. $16^{\log_4 5}$ equals :

(A) 5 (B) 16 (C) 25 (D) $\frac{5}{64}$

74. $\log_5 5 \cdot \log_4 9 \cdot \log_3 2$ simplifies to :

(A) 2 (B) 1 (C) 5 (D) $\frac{3}{2}$

75. $\log_2 7$ is :

(A) an integer (B) a prime number (C) a rational number (D) an irrational number

76. If log a, log b, log c are in A.P., then :

(A) a, b, c are in G.P. (B) a^2, b^2, c^2 are in G.P.

(C) a, b, c are in A.P. (D) $\log a^2, \log b^2, \log c^2$ are in G.P .

77. The value of $\left[\frac{1}{\log_{(p/q)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x}\right]$ is :

(A) 3 (B) 2 (C) 1 (D) 0

78. If $\log_4 x + \log_2 x = 6$, then the value of x is :

(A) 2 (B) 4 (C) 8 (D) 16

79. The value of $3^{-\frac{1}{2}\log_3 9}$ is :

(A) – 1 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $-\frac{3}{2}$

80. If $10^x = 1.73$ and $\log_{10} 1730 = 3.2380$, then x equals:

(A) 1.2380 (B) 0.2380 (C) 2.380 (D) 2.2380

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	B	B	A	D	C	B	B	C	D	D	A	D	A	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	A	D	B	A	C	A	D	D	A	A	B	C	B	B
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	A	B	B	D	D	B	C	C	A	D	B	B	A	A	D
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	A	B	C	B	A	B	B	B	B	A	D	A	D	B	D
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	A	A	B	B	B	B	C	C	A	C	D	C	C	B	D
Que.	76	77	78	79	80										
Ans.	A	D	D	C	B										

LOCI

★ **Locus :** If a point moves according to some given geometrical conditions, then the path traced out by the moving point is called its locus.

Ex.1 If a point moves in such a way that its distance from a given fixed point is always constant, then the locus of the path is a circle.

Ex.2 The locus of a point equidistant from two given points A and B is the perpendicular bisector of AB.

Ex.3 The locus of a point equidistant from two intersecting lines is the angle bisector of the angle made by these lines.

Ex.4 If ℓ and m be two lines such that $\ell \parallel m$. If a point moves in such a way that it is equidistant from ℓ and m , then locus is a line in just mid way between ℓ and m and parallel to each one of them.

Ex.5 Locus of the centres of all circles passing through two given points A and B is the perpendicular bisector of AB.

Ex.6 If the bisectors of $\angle B$ and $\angle C$ of a quadrilateral ABCD intersect in P, then P is equidistant from AB and CD .

Ex.7 If AB is a fixed line segment, then the locus of a point P such that $\angle APB = 90°$, is a circle with AB as diameter.

Ex.8 The locus of the vertices of isosceles triangles having a common base is the perpendicular bisector of the base.

Ex.9 AB is a fixed line. The locus of a point P so that $AB^2 = AP^2 + BP^2$ is a circle with AB as diameter.

Ex.10 The locus of mid-points of equal chords of a circle is the circle concentric with the given circle and radius equal to the distance of equal chords from the centre.

Ex.11 The locus of mid points of parallel chords of a circle is 'the diameter of the circle perpendicular to the parallel chords.

EXERCISE

1. Consider a point which moves such that its distances from two given points A and B are equal. Then, the locus of the point P is :

   (A) a circle with centre at A.

   (B) a circle with centre at B.

   (C) a straight line passing through either A or B

   (D) a straight line, which is the right bisector of AB.

2. The locus of a point P which moves in such a way that its distance from a fixed straight line is constant, is :

   (A) a circle.

   (B) a straight line perpendicular to the given line.

   (C) a straight line parallel to the given line.

   (D) a straight line intersecting the given straight line at an acute angle.

3. The locus of a point equidistant from two intersecting lines, is :

   (A) a single line (B) a pair of lines (C) a circle (D) a parabola

4. Which of the following statements is incorrect ?

   (A) A circle is the locus of a point.

   (B) The locus of the points which are equidistant from three non-collinear points, is the centre of the circle passing through the given points.

   (C) The locus of the points which are equidistant from three distinct points on a line, is a line parallel to the given line.

   (D) None of these

5. The locus of a point inside a circle and equidistant from two given points on the circle :

   (A) is the chord joining the two points.

   (B) is a circle with the line joining the two points, as its diameter.

   (C) is the perpendicular bisector of the chord joining the two points.

   (D) does not exist.

6. The locus of the mid points of equal chords of a circle is :

   (A) a circle (B) a diameter

   (C) a line not passing through the centre of the circle (D) none of these

7. If AB is a line segment, then the locus of a point P such that $\angle APB = 90°$, is:

   (A) a circle with AB as diameter. (B) the triangle APB.

   (C) a semi circle with AB as diameter. (D) the perpendicular bisector of AB.

8. The locus of the mid points of parallel chords of a circle, is :

   (A) a circle. (B) the line joining the mid points.

   (C) the diameter perpendicular to the given chords. (D) none of these

9. In a ΔABC, the point on BC which is equidistant from sides AB and AC, is the one where:

   (A) the median from A meets BC (B) the bisector of $\angle BAC$ meets BC

   (C) the altitude on BC meets BC (D) the perpendicular bisector of BC intersects BC.

10. Two fixed circles are in the same plane. If P is a point in that plane and the tangents from P to the two circles are equal, then the locus of P :

(A) is a straight line perpendicular to the line joining the centres.

(B) is a circle

(C) is a parabola

(D) cannot be determined

11. Let C be a circle. A point P moves such that the tangents from P to C, include an angle of 60°. The locus of P is:

(A) a straight line

(B) a circle concentric with C

(C) a circle touching C

(D) a circle intersecting C at two points

12. A number of triangles on the same base and on the same side of it are of equal areas. Then, the locus of their vertices is :

(A) a straight line parallel to the base

(B) a circle with the base as the diameter

(C) a straight line parallel to the altitude of a triangle on the given base

(D) None of these

13. P is a point in a rectangle ABCD such that PA + PD = PC + PB. The locus of P is :

(A) diagonal AC or BD.

(B) perpendicular bisector of AB or BC

(C) a circle

(D) none of these

14. The locus of the vertices of the isosceles triangles formed on a given base BC is:

(A) a circle

(B) a line perpendicular to BC

(C) bisector of BC

(D) right bisector of BC

15. If a point P moves such that the sum of the squares of its distances from two fixed points A and B is a constant, then the locus of P is :

(A) a straight line

(B) a circle

(C) the perpendicular bisector of AB

(D) an arbitrary curve

16. A wheel of diameter 80 cm rolls on a plane straight road. The locus of the centre of the wheel is :

(A) a circle of radius 40 cm

(B) a circle of radius 20 cm

(C) a straight line parallel to the path of the wheel and 40 cm distant from it

(D) a straight line parallel to the path of the wheel and 80 cm distant from it

17. The locus of the mid points of the equal chords of a given circle is :

(A) the concentric circle with radius equal to the distance of the chords from the centre of the given circle

(B) the concentric circle with radius equal to half the distance of the chords from the centre of the given circle

(C) the largest equilateral triangle inscribed in the given circle

(D) None of these

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	C	B	C	C	B	A	C	B	A	B	A	B	D	B
Que.	16	17													
Ans.	C	A													