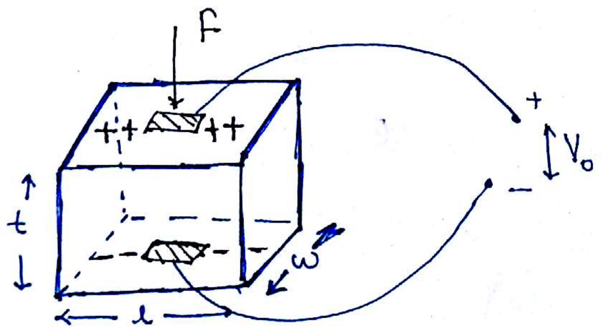
Piezo - electric sensor :-

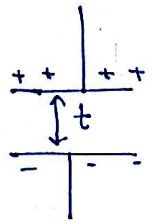
Piezo electric sensors works on the principle piezo - electric property.

Piezo - electric property :- When we apply external force on piezo - electric crystals charge will be develope one the surface of the piezo - electric crystal.

The developed charge can be converted into Voltage by using the basic of capcitance.

Piezo-electric crystals are made up of quartz material.





$$C = \frac{\epsilon A}{t}$$

$$q = CV$$

$$q \propto F$$

$$q = dF \quad - (1)$$

↓

Charge Sensitivity (C/N)

$$d = q/F$$

$$q = C.V_0 \quad - (2)$$

From (1) & (2)

$$dF = C.V_0$$

$$V_0 = \frac{d}{C}.F$$

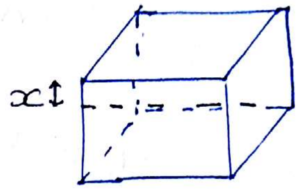
$$V_0 = \left(\frac{d}{\epsilon}\right) \times t \times \left(\frac{F}{A}\right)$$

$$\frac{\left(\frac{V_0}{t}\right)}{\left(F/A\right)} = \frac{d}{\epsilon} = 'g' \quad \leftarrow \text{Constant}$$

Note:- If crystal dimension change by an amount x then the charge developed is directly perpotional to 'x'

$$q \propto x$$

$$q = K_q \cdot x$$



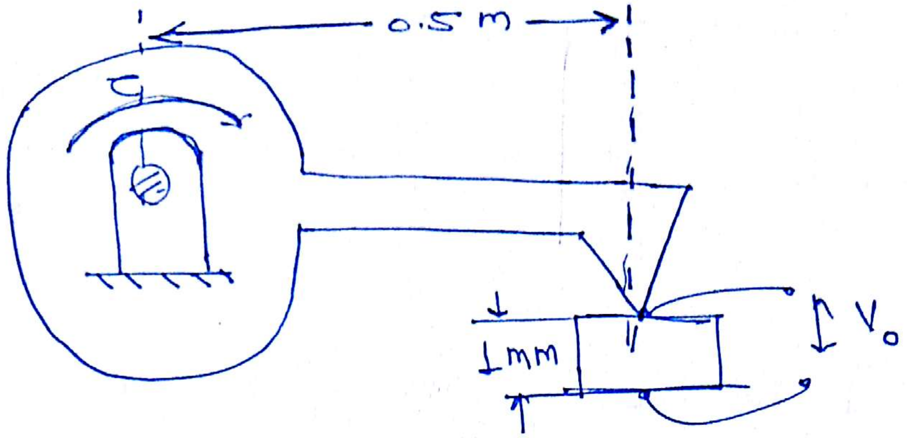
$x = \Delta t$ = change in thickness.

Ques:- A dynmometer makes contact with piezo-electric load cell as shown. The g constant of piezo-electric material is $g = 50\times10^{-3}\left(\frac{V m}{N}\right)$ and the surface area of the load cell is $A = 4 cm^2$, if a torque of $\tau = 20$ N-m is applied to dynmometer the output voltage V_0 of load cell is $V_0 = ?$

Sol^n

$$\frac{V_0}{1\times10^{-3}} = \frac{20}{0.5} \times \frac{50\times10^{-3}}{4\times10^{-4}\times10^{-1}}$$

$$= \frac{40}{4} \times \frac{50}{100} = 5 V$$



$g = 50 \times 10^{-3} \frac{Vm}{N}$

$A = 4\ cm^2$

$\tau = 20\ N\text{-}m$

$$g = \frac{(V_0/t)}{(F/A)} \Rightarrow V_0 = g \times t \times \frac{F}{A}$$

$$V_0 = 50 \times 10^{-3} \times 1 \times 10^{-3} \times \frac{20}{0.5} \times \frac{1}{4 \times 10^{-4}}$$

$$V_0 = 5\ V$$

$$\tau \propto F \propto q \propto V_0$$

Que A quartz crystal has a youngs modulas of $9 \times 10^{10}\ N/m^2$ and has all piezo-electric properties. if the diameter of crystal is 10 mm and thickness is 2 mm the Voltage sensitivity $4500 \frac{V}{km}$ if the output Voltage is 127.3 V then the applied load is — ?

Sol^n

$E = 9\times10^{10}\ N/m^2$

$D = 10\ mm$

$t = 2\ mm$

Voltage Sensitivity = $\frac{4500\ V}{1\mu m}$

$V_0 = 127.3$ Volt

$1\ \mu m \longrightarrow 4500\ V$ (1 μm dim^n change for 4500 Volt)

$\frac{127.3\times10^{-6}}{4500} \longleftarrow 127.3\ V$

↑

Δt

$$E = \frac{F/A}{\Delta t/t} = \frac{\sigma}{e}$$

$$F = E \times A \times \frac{\Delta t}{t}$$

$$F = 9\times10^{10} \times \frac{\pi}{4}(10)^2\times10^{-6} \times \frac{127.3\times10^{-6}}{4500\times2\times10^{-3}}$$

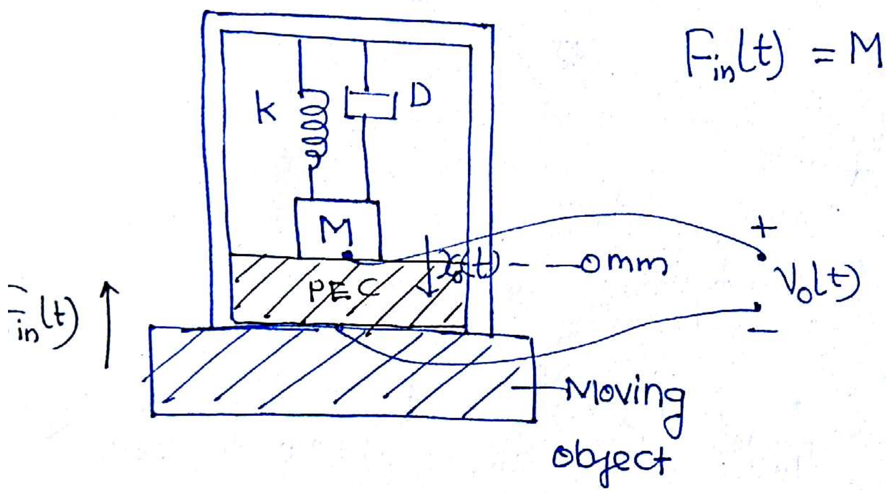
$F \simeq 100\ N$

Piezo-electric Accelerometer:-

It is used to measure accelaration of a moving object.

It converts input acc^n into output Voltage

Piezo-electric accelerometer consist of Piezo electric crystal, which is supported by mass, damper spring system as shown below



$$F_{in}(t) = M.\frac{d^2x_0(t)}{dt^2} + D\frac{dx_0(t)}{dt} + Kx_0(t)$$

at steady state

$$F_{in}(t) = M.\frac{d^2x_0(t)}{dt^2} + D.\frac{dx_0(t)}{dt} + kx_0(t)$$

$$F_{in}(t) = K.x_0(t)$$

$$x_0(t) = \frac{1}{K}.F_{in}(t) \quad — ①$$

For piezo electric crystall (PEC) $\Rightarrow q = K_q.x$

$$q(t) = K_q x_0(t)$$

$$q(t) = c \cdot V_o(t)$$

$$V_o(t) = \frac{1}{c} q(t)$$

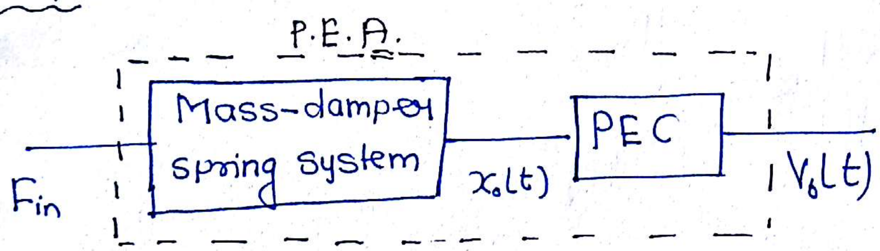
$$V_o(t) = \frac{1}{c} \cdot K_q \cdot x_o(t)$$

$$V_o(t) = \frac{K_q}{CK} \cdot F_{in}(t)$$

$$\boxed{V_o(t) = \frac{K_q}{CK} \cdot M \cdot a_i(t)}$$

$$\boxed{V_o(t) \propto a_i(t)}$$

Note:-



Every accelerometer by default Consist of mass damper and spring system

Note:- if $D = 0$

$$F_{in}(t) = m \cdot \frac{d^2 x_o(t)}{dt^2} + K x_o(t)$$

apply Laplace transfor both side

$$F_{in}(s) = M \cdot s^2 X_o(s) + K X_o(s)$$

$$F_{in}(s) = (Ms^2 + K)\, X_o(s)$$

$$\frac{X_o(s)}{F_{in}(s)} = \frac{1}{Ms^2 + K}$$

$$\frac{X_o(s)}{F_{in}(s)} = \frac{(1/M)}{s^2 + \left(\sqrt{K/m}\right)^2}$$

$$\frac{X_o(s)}{F_{in}(s)} = \frac{(1/M) \times \sqrt{\frac{M}{K}} \times \sqrt{\frac{K}{M}}}{s^2 + \left(\sqrt{K/m}\right)^2}$$

$$\frac{X_o(s)}{F_{in}(s)} = \frac{1}{\sqrt{MK}} \cdot \frac{\sqrt{K/m}}{s^2 + \left(\sqrt{K/m}\right)^2}$$

$$\frac{X_o(s)}{F_{in}(s)} = \frac{1}{\sqrt{MK}} \cdot \sin\left(\sqrt{\frac{K}{m}} \cdot t\right)$$

Natural Frequency $\left(\omega_n = \sqrt{\frac{k}{m}} \text{ rad/Sec.}\right)$

Quest. A quartz crystal of dimension $(10 \times 10 \times 10)$ mm is subjected to a dynamic load of $F(t) = 10^6 \sin(100t)$ Find the amplitude of charge and voltage generated Given that charge sensitivity $d = 2 \times 10^{-12} (C/N)$ and young's modulas $E = 6 \times 10^{10} (N/m^2)$ and permitivity $\epsilon = 42 \times 10^{-12} (F/m)$.

Soln

$$q \propto F$$

$$q = d F$$

$$q(t) = d\, F(t)$$

$$q(t) = 2 \times 10^{-12} \times 10^{6} \sin(100t)$$

$$q(t) = 2 \times 10^{-6} \sin(100t)$$

amplitude of charge $A = 2 \times 10^{-6}$ **Ans**

$$q = C V$$

$$q(t) = C V_o(t)$$

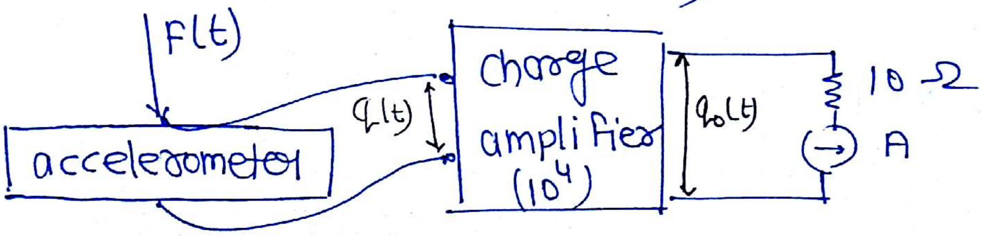
$$V_o(t) = \frac{1}{C} \cdot q(t)$$

$$V_o(t) = \frac{1}{\frac{\epsilon A}{t}} \cdot q(t) = \frac{t}{\epsilon A} \cdot q(t)$$

$$V_o(t) = \frac{10 \times 10^{-3}}{42 \times 10^{-12} \times (10 \times 10^{-3})^2} \times 2 \times 10^{-6} \sin(100t)$$

$$V_o(t) = 4.76 \times 10^{6} \sin(100t)$$ **Ans**

Ques:- A P.E.A. has a mass of $m = 0.005\,kg$ is subjected a i/p accen $a_i(t) = 10^8 \sin 2t$ if o/p of the accelerometer is amplified by a charge amplifier of gain $= 10^4$, then find the amplitude of current indicated by ameter which is as shown below.
charge sensitivity $d = 2 \times 10^{-12}$ (C/N)



Soln

$q \propto F \Rightarrow q = d.F$

$q_o(t) = 10^4\, q(t)$ (Gain)

V_o~~(t) ∝ a₀(t)~~)

$q_o(t) = 10^4 \left[d\, F(t) \right]$

$q_o(t) = 10^4 \left[2 \times 10^{-12} \times 0.005 \times 10^8 \sin 2t \right]$

$q_o(t) = 0.01 \sin 2t$

$$I_o(t) = \frac{dq_o(t)}{dt}$$

$I_o(t) = 0.01 \cos 2t \cdot 2$

$I_o(t) = 0.02 \cos 2t$