10. Direct and Inverse Variations

EXERCISE 10(A)

Question 1.

In which ofthe following tables, x and y vary directly:

(*i*)

x	3	5	8	11
y	4.5	7.5	12	16.5

(*ii*)

x	16	30	40	56
y	32	60	80	84

(*iii*)

x	27	45	54	75
y	81	180	216	225

Solution:

(*i*) $\frac{x_1}{y_1} = \frac{3}{4.5} = \frac{1}{1.5}$

$\frac{x_2}{y_2} = \frac{5}{7.5} = \frac{1}{1.5}$

$\frac{x_3}{y_3} = \frac{8}{12} = \frac{1}{1.5}$

$\frac{x_4}{y_4} = \frac{11}{16.5} = \frac{1}{1.5}$

$\Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = \frac{x_4}{y_4}$

Yes, x and y vary directly.

(*ii*) $\frac{x_1}{y_1} = \frac{16}{32} = \frac{1}{2}$

$\frac{x_2}{y_2} = \frac{30}{60} = \frac{1}{2}$

$$\frac{x_3}{y_3} = \frac{40}{80} = \frac{1}{2}$$

$$\frac{x_4}{y_4} = \frac{56}{84} = \frac{28}{42} = \frac{14}{21} = \frac{7}{3}$$

$$\Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} \neq \frac{x_4}{y_4}$$

x and y are not in direct variation.

(*iii*) $\frac{x_1}{y_1} = \frac{27}{81} = \frac{3}{9} = \frac{1}{3}$

$$\frac{x_2}{y_2} = \frac{45}{150} = \frac{15}{50} = \frac{3}{10}$$

$$\frac{x_3}{y_3} = \frac{54}{216} = \frac{18}{72} = \frac{1}{4}$$

$$\frac{x_4}{y_4} = \frac{75}{225} = \frac{25}{45} = \frac{5}{9}$$

$$\Rightarrow \frac{x_1}{y_1} \neq \frac{x_2}{y_2} \neq \frac{x_3}{y_3} \neq \frac{x_4}{y_4}$$

x and y are not in direct variation.

Question 2.

If x and y vary directly, find the values of x, y and z:

x	3	x	y	10
y	36	60	96	z

Solution:

x and y are in direct variation

$$\therefore \frac{3}{36} = \frac{x}{60} = \frac{y}{96} = \frac{10}{z}$$

$$\Rightarrow \frac{3}{36} = \frac{x}{60}, \frac{3}{36} = \frac{y}{96}, \frac{3}{36} = \frac{10}{z}$$

$$x = \frac{3}{36} \times 60, y = \frac{3}{36} \times 96,$$

$$z = 10 \times \frac{36}{3}$$

$$\Rightarrow x = 5, y = 8, z = 120$$

x	3	5	8	10
y	36	60	96	120

Question 3.

A truck consumes 28 litres of diesel for moving through a distance of 448 km. How much distance will it cover in 64 litres of diesel?

Solution:

Let the truck cover x km in 64 litres of diesel.

Diesel (in litres)	28	64
Distance (in km)	448	x

It is the case of direct variation

$$\Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{28}{448} = \frac{64}{x}$$

i.e., $28x = 64 \times 448$

$$x = \frac{64 \times 448}{28} = 1024 \text{ km}$$

Question 4.

For 100 km, a taxi charges Rs. 1,800. How much will it charge for a journey of 120 km?

Solution:

Let a charges of car is ₹ x in 120 km

Distance in (km)	1800	x
Taxi charges (₹)	100	120

Since it is the case of direct variation

$$\Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{1800}{100} \times \frac{x}{120}$$

$$\Rightarrow 100x = 1800 \times 120$$

$$x = \frac{1800 \times 120}{100} = ₹2160$$

Question 5.

If 27 identical articles cost Rs. 1,890, how many articles can be bought for Rs. 1,750?

Solution:

Let x number of articles be purchased in ₹1750

Cost (₹)	1890	1750
No. of articles	27	x

Since, it is a case of direct variation

$$\Rightarrow \frac{1890}{27} = \frac{1750}{x}$$

$$\Rightarrow x = \frac{1750 \times 27}{1890}$$

$$= 25 \text{ articles}$$

Question 6.

7 kg of rice costs Rs. 1,120. How much rice can be bought for Rs. 3,680?

Solution:

Rice : Cost :: Rice : Cost

7 kg : ₹1120 :: x kg : ₹3680

$$\therefore\ x = \frac{7 \times 3680}{1120} = 23 \text{ kg}$$

Question 7.

6 note-books cost Rs. 156, find the cost of 54 such note-books.

Solution:

Notebooks : Cost :: Notebooks : Cost

6 : ₹156 :: 54 : ₹ x

$$\therefore\ x = \frac{156 \times 54}{6} = ₹1404$$

Question 8.

22 men can dig a 27 m long trench in one day. How many men should be employed for digging 135 m long trench of the same type in one day?

Solution:

Men : Length trench :: Men : Length of trench

22 : 27 m :: x : 135 m

$$x = \frac{22 \times 135}{27} = 110 \text{ men}$$

Question 9.

If the total weight of 11 identical articles is 77 kg, how many articles of the same type would weigh 224 kg?

Solution:

No. of articles : Weight :: No. of articles : Weight

11 : 77 kg :: x : 224 kg

$$x = \frac{11 \times 224}{77} = 32 \text{ articles}$$

Question 10.

A train is moving with uniform speed of 120 km per hour.

(i) How far will it travel in 36 minutes?

(ii) In how much time will it cover 210 km?

Solution:

(*i*) Speed of train in 60 minutes = 120 km

i.e. distance covered in 60 minutes = $\frac{120}{60}$

Distance covered in 36 minutes = $\frac{120 \times 36}{60}$

$= 2 \times 36 = 73$ km

(*ii*) If distance covered is 120 km then time taken

= 60 minutes

If distance covered is 1 km then time taken

$= \frac{60}{120}$

If distance covered is 210 km then time taken

$= \frac{60}{120} \times 210 = 105$ minutes

= 1 hour 45 minutes

EXERCISE 10(B)

Question 1.

(i)

x	4	3	12	1
y	6	8	2	24

(ii)

x	30	120	60	24
y	60	30	30	75

(iii)

x	10	30	60	10
y	90	30	20	90

x and y are inversely proportional.
Then xy are equal.

(i) xy = 4 x 6 = 24
xy = 3 x 8 = 24
xy = 12 x 2 = 24
xy = 1 x 24 = 24
xy in each case is equal.
x and y are inversely proportional
(ii) xy = 30 x 60 = 1800
xy= 120 x 30 = 3600
xy = 60 x 30= 1800
xy = 24 x 75 = 1800
xy in each case is not equal.
x and y are not inversely proportional.
(iii) xy = 10 x 90 = 900
= 30 x 30 = 900
= 60 x 20= 1200
= 10 x 90 = 900
xy in each case is not equal.
x and y are not inversely proportional.

Question 2.

If x and y vary inversely, find the values of l, m and n :

(i)

x	4	8	2	32
y	4	l	m	n

(ii)

x	24	32	m	16
y	l	12	8	n

Solution:

$\because$ x and y are inversely proportional

$\therefore$ xy is equal

Now,

(*i*) $xy = 4 \times 4 = 16$

$$8 \times l = 16 \Rightarrow l = \frac{16}{8} = 2$$

$$2 \times m = 16 \Rightarrow m = \frac{16}{2} = 8$$

$$32 \times n = 16 \Rightarrow n = \frac{16}{32} = 0.5$$

(*ii*) $xy = 32 \times 12 = 384$

$$24 \times l = 384 \Rightarrow l = \frac{384}{24} = 16$$

$$m \times 8 = 384 \Rightarrow m = \frac{384}{8} = 48$$

$$16 \times n = 384 \Rightarrow n = \frac{384}{16} = 24$$

Question 3.

36 men can do a piece of work in 7 days. How many men will do the same work in 42 days?

Solution:

Men : Days :: Men : Days

36 : 7 :: x : 42

$\therefore$ By inverse proportional

$36 \times 7 = x \times 42$

$$\Rightarrow x = \frac{36 \times 7}{42} = 6 \text{ men}$$

Question 4.

12 pipes, all of the same size, fill a tank in 42 minutes. How long will it take to fill the same tank, if 21 pipes of the same size are used?

Solution:

Pipes : Time :: Pipes : Time

12 : 2x :: 21 : 42

∴ By inverse proportion

$12 \times 42 = 21 \times x$

$$\Rightarrow x = \frac{12 \times 42}{21} = 24 \text{ minutes}$$

Question 5.

In a fort 150 men had provisions for 45 days. After 10 days, 25 men left the fort. How long would the food last at the same rate?

Solution:

After 10 days :

For 150 men, provision will last (45 – 10) days = 35 days

⇒ For 1 man, the provisions will last

= 150 × 35 days

And for (150 – 25) = 125 men, the provisions will last for $= \frac{150 \times 35}{125} = 42$ days

Question 6.

72 men do a piece of work in 25 days. In how many days will 30 men do the same work?

Solution:

Men : Days :: Men : Days

72 : 25 :: 30 : x

∴ By inverse proportion

$72 \times 25 = 30 \times x$

$$\Rightarrow x = \frac{25 \times 72}{30}$$

$= 60$ days

Question 7.

If 56 workers can build a wall in 180 hours, how many workers will be required to do the

same work in 70 hours?

Solution:

Workers : Hours :: Workers : Hours

56 : 180 :: x : 70

$\therefore$ By inverse proportion

$56 \times 180 = x \times 70$

$$\Rightarrow x = \frac{180 \times 56}{70}$$

$= 144$ workers

Question 8.

A car takes 6 hours to reach a destination by travelling at the speed of 50 km per hour. How long will it take when the car travels at the speed of 75 km per hour?

Solution:

Time : Distance :: Time : Distance

6 : 50 km :: x : 75

$\therefore$ By inverse proportion

$6 \times 50 = x \times 75$

$$\Rightarrow x = \frac{50 \times 6}{75}$$

$= 4$ hours

EXERCISE 10(C)

Question 1.
Cost of 24 identical articles is Rs. 108, Find the cost of 40 similar articles.

Solution:

$$\text{Cost of 24 identical articles} = \text{Rs. } 108$$

$$\therefore \text{Cost of 1 identical articles} = \text{Rs. } \frac{108}{24}$$

$$\therefore \text{Cost of 40 similar articles} = \text{Rs. } \left(\frac{108}{24} \times 40\right)$$

$$= \text{Rs. } \frac{108}{3} \times 5 = \text{Rs. } 36 \times 5 = \text{Rs. } 180$$

Question 2.
If 15 men can complete a piece of work in 30 days, in how many days will 18 men complete it?

Solution:

Since 15 men can complete a piece of work in = 30 days

$\therefore$ 1 man can do the work in = 30×15 days

$$\therefore \text{18 men can do the work in} = \frac{30 \times 15}{18} = \frac{5 \times 15}{3}$$

$$= 5 \times 5 = 25 \text{ days}$$

Hence 18 men can do the work in 25 days.

Question 3.
In order to complete a work in 28 days, 60 men are required. How many men will be required if the same work is to be completed in 40 days ?

Solution:
Let x be number of men required 60 men can do the work in = 28 days
1 man can do the work in = 28 x 60 days
x man can do the work in = $\frac{28 \times 60}{x}$ day

$$\text{But } \frac{28 \times 60}{x} = 40 \text{ day} \qquad \text{(Given)}$$

$$\therefore x = \frac{28 \times 60}{40} = 7 \times 6 = 42$$

Required number of men = 42

Question 4.

A fort had provisions for 450 soldiers for 40 days. After 10 days, 90 more soldiers come to the fort. Find in how many days will the remaining provisions last at the same rate ?

Solution:

After 10 days :

For 450 soldiers, provision are sufficient for (40 – 10) days = 30 days

For 1 soldier, provision are sufficient for 30 x 450 days

For 540 soldiers, the provision are sufficient for

$$= \frac{30 \times 450}{540} = \frac{30 \times 50}{60} = \frac{50}{2} = 25 \text{ days}$$

Question 5.

A garrison has sufficient provisions for 480 men for 12 days. If the number of men is reduced by 160; find how long will the provisions last.

Solution:

Strength of garrison = 480 men

Strength reduced by = 160 men

New strength of garrison = 480 – 160 = 320

Provision for 480 men lasts for = 12 days

Provision for 1 man last for = 480 × 12 days

$$\text{Provision for 320 men lasts for} = \frac{480 \times 12}{320} \text{ days}$$

$$= \frac{30 \times 12}{2} = 180 \text{ days}$$

Question 6.

$\frac{3}{5}$ quintal of wheat costs Rs.210. Find the cost of :

(i) 1 quintal of wheat

(ii) 0.4 quintal of wheat

Solution:

(i) $\frac{3}{5}$ quintal of wheat costs = Rs.210

1 quintal of wheat costs = 210 x $\frac{3}{5}$ = 70 x 5 = Rs.350

(ii) 1 quintal of wheat costs = Rs.350

0.4 quintal of wheat costs = 350 x 0.4 = Rs. 140.0 = Rs.140

Question 7.

If $\frac{2}{9}$ of a property costs Rs.2,52,000; find the cost of $\frac{4}{7}$ of it.

Solution:

$\frac{2}{9}$ of a property costs = Rs.252000

1 of a property costs = $\text{Rs.}252000 \times \frac{9}{2}$

$= \text{Rs.}\frac{2268000}{2}$

$= \text{Rs.}1134000$

$\frac{4}{7}$ of a property costs $= 1134000 \times \frac{4}{7}$

$= 162000 \times 4$

$= \text{Rs.}6,4,8000$

Question 8.

4 men or 6 women earn Rs. 360 in one day. Find, how much will:
(i) a man earn in one day ?
(ii) a woman earn in one day ?
(iii) 6 men and 4 women earn in one day ?

Solution:

4 men earn Rs. 360 in one day

(i) 1 man can earn = Rs. $\frac{360}{4}$ = Rs. 90 in one day

(ii) 6 women earn Rs. 360 in one day

1 woman can earn Rs. $\frac{360}{6}$ = Rs. 60 in one day

(iii) ∴ 6 men and 4 women earn in one day

$= 6 \times 90 + 4 \times 60 = 540 + 240 = \text{Rs. } 780$

Question 9.

16 boys went to canteen to have tea and snacks together. The bill amounted to Rs. 114.40. What will be the contribution of a boy who pays for himself and 5 others ?

Solution:

16 boys pay for bill = Rs. 114.40

1 boy pays for bill = Rs. $\frac{114.40}{16}$ = Rs. 7.15

∴ Contribution of a boy who pays for himself and 5 others = 6 × 7.15 = Rs. 42.90

Question 10.

50 labourers can dig a pond in 16 days. How many labourers will be required to dig an another pond, double in size in 20 days ?

Solution:

In 16 days for diging a pond labour reqd. = 50

In 1 day, labour reqd. = 50 × 16

In 20 days, labour reqd. = $\frac{50 \times 16}{20}$

In 20 days, with double work, labour reqd.

$= \frac{50 \times 16 \times 2}{20} = 5 \times 8 \times 2 = 80$

Hence labourers required = 80

Question 11.

If 12 men or 18 women can complete a piece of work in 7 days, in how many days can 4 men and 8 women complete the same work?

Solution:

12 men

= 18 women

$\therefore$ 4 m = $\frac{18}{12} \times 4$

= 6 women

Total women in second case = 4 men + 8 women

6 + 8 = 14 women

18 women can do a piece of work in 7 days

Let 14 women will do the same work in x days

$\therefore$ 18 : 14 : : 7 : x

(Less women, more days)

$\Rightarrow$ 18 : 14 : : x : 7

(Using inverse proportion)

$x = \frac{18 \times 7}{14}$

= 9

$\therefore$ They will complete the work in 9 days

Question 12.

If 3 men or 6 boys can finish a work in 20 days, how long will 4 men and 12 boys take to finish the same work ?

Solution:

3 men = 6 boys

4 men = $\frac{6}{3}$ x 4 = 8

Total boys in second case :

= 4 men + 12 boys = 8 + 12 = 20 boys

6 boys can do a piece of work in 20 days

Then let 20 boys will do the same work in x days

$\therefore$ 6 : 20 : : 20 ; x

(More boys , less days)

$\Rightarrow$ 6 : 20 : : x : 20

(By inverse proportion)

$$x = \frac{20 \times 6}{20}$$

= 6 days

$\therefore$ They will do the work in 6 days

Question 13.

A particular work can be completed by 6 men and 6 women in 24 days; whereas the same work can be completed by 8 men and 12 women in 15 days. Find :
(i) according to the amount of work done, one man is equivalent to how many women.
(ii) the time taken by 4 men and 6 women to complete the same work.

Solution:

6 men + 6 women can finish the work in = 24 days
144 men + 144 women can finish it in = 1 day
8 m + 12 women can finish the work in = 15 days
120 men + 180 women can finish it in = 1 day
(i) 144 men + 144 women = 120 men + 180 women
=> 144 men – 120 men
= 180 – women – 144 women
=> 24 men = 36 women

1 man = $\frac{36}{24}$

$= \frac{3}{2}$ women

$= 1\frac{1}{2}$ women

(*ii*) In first case,

$6 \times \frac{3}{2} + 6 = 9 + 6$

= 15 women

In second case,

$4 \times \frac{3}{2} + 6$

$= 6 + 6$

= 12 women

Now, 15 women can do a piece of work in = 24 days

1 woman will do it in = 24 × 15 days

and 12 women will do it in

$= \frac{24 \times 15}{12}$

= 30 days

Question 14.

If 12 men and 16 boys can do a piece of work in 5 days and, 13 men and 24 boys can do it in 4 days, how long will 7 men and 10 boys take to do it ?

Solution:

12 men + 16 boys can do a piece of work in = 5 days

60 men + 80 boys can do a piece of work in = 1 day ……. (i)

and 13 men + 24 boys can do the same work in = 4 days

52 men + 96 boys can do the same work in = 1 day ……(ii)

From (i) and (ii)

60 men + 80 boys = 52 men + 96 boys

=> 60 men – 52 men = 96 boys – 80 boys

=> 8 men = 16 boys
1 men = $\frac{16}{8}$ = 2 boys
Now, in first case,
12 men + 16 boys = 12 x 2 + 16 = 24+16 = 40 boys
In the second case,
7 men + 10 boys = 7 x 2 + 10 = 14 + 10 = 24 boys
Now 40 boys can do a piece of work in = 5 days
1 boy can do the same work in = 5 x 40 days
and 24 boys will do the same work in

$$= \frac{5 \times 40}{24} \text{ days}$$

$$= \frac{25}{3}$$

$$= 8\frac{1}{3} \text{ days}$$

EXERCISE 10(D)

Question 1.
Eight oranges can be bought for Rs. 10.40. How many more can be bought for Rs. 16.90?
Solution:
Number of oranges bought for Rs. 10.40 = 8

$$\text{Number of oranges bought for Re.1} = \frac{8}{10{\cdot}40}$$

Number of oranges bought for Rs.16·90

$$= \frac{8}{10{\cdot}40} \times 16{\cdot}90 = \frac{8 \times 1690}{1040} = \frac{13520}{1040} = 13$$

$\therefore$ Number of more oranges which can be bought for Rs.16·90 = 13 – 8 = 5

Question 2.
Fifteen men can build a wall in 60 days. How many more men are required to build another wall of same size in 45 days ?
Solution:
In 60 days a wall can be built by = 15 men
In 1 day a wall can be built by = 15 x 60 men

In 45 days a wall can be built by = $\frac{15 \times 60}{45} = \frac{900}{45}$ = 20 men
No. of more men required to build the wall in 45 days = 20 – 15 = 5 men

Question 3.
Six taps can fill an empty cistern in 8 hours. How much more time will be taken, if two taps go out of order ? Assume, all the taps supply water at the same rate.
Solution:
Total no. of taps = 6
Out of order taps = 2
Taps in working condition =6 – 2 = 4
6 taps can fill an empty cistern in = 8 hours
1 tap can fill an empty cistern in = 6 x 8 hours
4 taps can fill an empty cistern in = $\frac{48}{4}$ = 12 hours
More time taken when 2 taps are out of order = 12 – 8 = 4 hour

Question 4.
A contractor undertakes to dig a canal, 6 kilometres long, in 35 days and employed 90 men. He finds that after 20 days only 2 km of canal have been completed. How many more men must be employed to finish the work in time ?
Solution:
Length of canal = 6 km
In 20 days canal made = 2 km
Remaining length of canal = 6 – 2 = 4 km
Remaining time = 35 – 20 = 15 days
In 20 days 2 km canal is made by = 90 men
In 1 day 2 km canal is made by = 90 x 20 men
In 15 days 2 km canal is made by = $\frac{90 \times 20}{15}$ men
In 15 days 1 km canal is made by = $\frac{90 \times 20}{15 \times 2}$ men
In 15 days 4 km canal is made by = $\frac{90 \times 20 \times 4}{15 \times 2}$ men = 6 x 10 x 4 men = 240 men
Number of more men to be employed to finish the work in time = 240-90 = 150 men

Question 5.

If 10 horses consume 18 bushels in 36 days. How long will 24 bushels last for 30 horses ?

Solution:

10 horses consume 18 bushels in $= 36$ days

1 horse consumes 18 bushels in $= 36 \times 10$ days

$= 360$ days

30 horses consume 18 bushels in $= \frac{360}{30}$ days

$= 12$ days

30 horses consume 1 bushel in $= \frac{12}{18}$ days

30 horses consume 24 bushels in

$= \frac{12}{18} \times 24$ days

$= \frac{2}{3} \times 24$ days

$= 16$ days

Question 6.

A family of 5 persons can be main¬tained for 20 days with Rs.2,480. Find, how long Rs.6944 maintain a family of 8 persons

Solution:

A family of 5 persons can be maintained with Rs.2480 for = 20 days

A family of 5 persons can be maintained with

Re.1 for $= \frac{20}{2480}$ days

A family of 5 persons can be maintained with

Rs.6944 $= \frac{20}{2480} \times 6944 = \frac{1}{124} \times 6944$

$= 56$ days

A family of 1 person can be maintained

$= 56 \times 5$ days

A family of 8 persons can be maintained

$= \frac{56 \times 5}{8}$ days

$= 7 \times 5$ days

$= 35$ days

Question 7.

90 men can complete a work in 24 days working 8 hours a day. How many men are required to complete the same work in 18 days working 7 hours a day ?

Solution:

In 24 days, working 8 hours a day, work can be completed by = 90 men

In 1 day, working 8 hours a day, work can be completed by = 90×24 men

In 18 days, working 8 hours a day, work can be completed by = $\frac{90 \times 24}{18}$ men

" " " " 1 hour " " " " = $\frac{90 \times 24}{18} \times 8$ men

" " $7\frac{1}{2}$ *i.e.* $\frac{15}{2}$ " " = $\frac{90 \times 24 \times 8}{18} \times \frac{2}{15}$ men

$= \frac{90 \times 24 \times 8 \times 2}{18 \times 15} = \frac{1 \times 24 \times 8 \times 2}{3} = 128$ men

Question 8.
Twelve typists, all working with same speed, type a certain number of pages in 18 days working 8 hours a day. Find, how many hours per day must sixteen typists work in order to type the same number of pages in 9 days ?

Solution:
12 typists can type in 18 days with number of working hours in day = 8 hours
1 typist can type in 18 days = 8 x 12 hour
1 typist can type in 9 days = 2 (8 x 12) hour
16 typist can type in a day = $\frac{2(8\times12)}{16}$ = 12 hours

Question 9.
If 25 horses consume 18 quintal in 36 days, how long will 28 quintal last for 30 horses ?

Solution:
25 horses consume 18 quintal in = 36 days

1 horse consume 1 quintal in = $\frac{36\times25}{18}$ days

30 horses consume 28 quintal in

$$= \frac{36\times25\times28}{18\times30} \text{ days}$$

$$= \frac{2\times5\times28}{6} = \frac{140}{3} = 46\frac{2}{3} \text{ days}$$

Question 10.
If 70 men dig 15,000 sq. m of a field in 5 days, how many men will dig 22,500 sq. m field in 25 days ?

Solution:
70 men dig 15,000 sq. m of a field in 5 days

1 men can dig 15,000 sq. m of a field in

$$= \frac{5\times70}{15{,}000}$$

Men required to dig 22,500 sq. m. field in 25 days $= \frac{5\times70\times22500}{15{,}000\times25} = \frac{70\times900}{3000} = \frac{63}{3} = 21$

Hence required to dig 22,500 sq. m field in 25 days = 21

Question 11.
A contractor undertakes to build a wall 1000 m long in 50 days. He employs 56 men, but at the end of 27 days, he finds that only 448 m of wall is built. How many extra men must the contractor employ so that the wall is completed in time ?
Solution:
Number of men employed in the beginning = 56
Length of wall = 1000 m No. of days = 50
In the time of 27 days, only 448 m of wall was completed
Remaining period = 50 – 27 = 23 days
and length of wall to be completed = 1000 – 448 = 552
Now in 27 days, 448 m long wall was completed by = 1000 m
in 1 day, 448 m long was completed by = 56 x 27
inf 1 day, 1 m long wall will be completed by

$$= \frac{56 \times 27}{448} \text{ m}$$

and in 23 days 552 m long wall be completed

$$\text{by} = \frac{56 \times 27 \times 552}{448 \times 23} \text{ men}$$

$$= \frac{1 \times 27 \times 24}{8} = 81 \text{ men}$$

$\therefore$ No. of extra men required
$= 81 - 56 = 25$ men

Question 12.
A group of labourers promises to do a piece of work in 10 days, but five of them become absent. If the remaining labourers complete the work in 12 days, find their original number in the group.
Solution:
Total period = 10 days
But work completed in = 12 days
No. of men were absent = 5
Let the number of men in the beginning = x
Now x men can do a piece work in = 10 days

1 man will do it in = 10x x days

and $(x-5)$ will do it in $= \frac{10 \times x}{x-5}$ days

$\therefore \quad \frac{10x}{x-5} = 12 \Rightarrow 10x = 12x - 60$

$\Rightarrow 12x - 10x = 60 \Rightarrow 2x = 60$

$\Rightarrow x = \frac{60}{2} = 30$

$\therefore$ No. of men in the beginning = 30

Question 13.

Ten men, working for 6 days of 10 hours each, finish $\frac{5}{21}$ of a piece of work. How many men working at the same rate and for the same number of hours each day, will be required to complete the remaining work in 8 days ?

Solution:

Work do one $= \frac{5}{21}$

Remaining work $= 1 - \frac{5}{21} = \frac{16}{21}$

$\frac{5}{21}$ of a work can be done in 6 days working

10 hours a day by = 10 m

1 work can be done in 6 days working 10

hours a day by $= \frac{10 \times 21}{5}$

1 work can be done in 1 day working 10

hours a day by $= \frac{10 \times 21 \times 6}{5}$ men

$\frac{16}{21}$ work can be done in 8 days working 10

hours a day by $= \frac{10 \times 21 \times 6 \times 16}{5 \times 21 \times 8} = 24$ men

EXERCISE 10(E)

Question 1.

A can do a piece of work in 10 days and B in 15 days. How long will they take together to finish it ?

Solution:

$$\text{A's 1 day work} = \frac{1}{10}$$

$$\text{B's 1 day work} = \frac{1}{15}$$

$$\text{(A+B)'s 1 day work} = \frac{1}{10}+\frac{1}{15}$$

$$= \frac{3+2}{30} = \frac{5}{30}$$

$$= \frac{1}{6}$$

$\therefore$ Both A and B together can finish the work in

= 6 days

Question 2.

A and B together can do a piece of work in $6\frac{2}{3}$ days ; but B alone can do it in 10 days. How long will A take to do it alone ?

Solution:

$$\text{A and B can do the work in} = 6\frac{2}{3}\text{ days}$$

$$= \frac{20}{3}\text{ days}$$

$$\text{B can " " " } = 10 \text{ days}$$

$$\therefore \quad \text{(A+B)'s 1 day work} = \frac{3}{20}$$

$$\text{B's " " " " } = \frac{1}{10}$$

$$\text{A's " " " " } = \frac{3}{20}-\frac{1}{10}$$

$$= \frac{3-2}{20} = \frac{1}{20}$$

$\therefore$ A can do the work in = 20 days

Question 3.

A can do a work in 15 days and B in 20 days. If they together work on it for 4 days ; what fraction of the work will be left ?

Solution:

$$\text{A's 1 day work} = \frac{1}{15}$$

$$\text{B's 1 day work} = \frac{1}{20}$$

$$\text{(A+B)'s 1 day work} = \frac{1}{15} + \frac{1}{20}$$

$$= \frac{4+3}{60} = \frac{7}{60}$$

$$\text{(A+B)'s 4 days work} = \frac{7}{60} \times 4 = \frac{7}{15}$$

$$\text{Remaining work} = 1 - \frac{7}{15} = \frac{15-7}{15}$$

$$= \frac{8}{15}$$

Question 4.

A, B and C can do a piece of work in 6 days, 12 days and 24 days respectively. In what time will they all together do it ?

Solution:

A can do a piece of work in = 6 days

B can do a piece of work in = 12 days

C can do a piece of work in = 24 days

$$\therefore \text{A's 1 day work} = \frac{1}{6}$$

$$\therefore \text{B's 1 day work} = \frac{1}{12}$$

$$\therefore \text{C's 1 day work} = \frac{1}{24}$$

$$\text{(A+B+C)'s 1 day work} = \frac{1}{6} + \frac{1}{12} + \frac{1}{24}$$

$$= \frac{4+2+1}{24} = \frac{7}{24}$$

$$\therefore \text{(A+B+C) can do the work in} = \frac{24}{7} \text{ days}$$

$$= 3\frac{3}{7} \text{ days}$$

Question 5.

A and B working together can mow a field in 56 days and with the help of C, they could have mowed it in 42 days. How long would C take by himself ?

Solution:

$(A+B)$ can mow a field in $= 56$ days

$(A+B+C)$ can mow a field in $= 42$ days

$\therefore$ $(A+B)$'s 1 day work $= \frac{1}{56}$

$(A+B+C)$'s 1 day work $= \frac{1}{42}$

C's 1 day work $= \frac{1}{42} - \frac{1}{56}$

$= \frac{4-3}{168}$

$= \frac{1}{168}$

2	42, 56
7	21, 28
	3, 4

L.C.M. $= 2 \times 7 \times 3 \times 4 = 168$

$\therefore$ C can do the work in $= 168$ days

Question 6.

A can do a piece of work in 24 days, A and B can do it in 16 days and A, B and C in 10 $\frac{2}{3}$ days. In how many days can A and C do it ?

Solution:

A can do a piece of work in = 24 days

(A+B) can do a piece of work in = 16 days

(A+B+C) can do a piece of work in

$$= 10\frac{2}{3} \text{ days} = \frac{32}{3} \text{ days}$$

$$\therefore \quad \text{A's 1 day work} = \frac{1}{24}$$

$$\text{(A+B)'s 1 day work} = \frac{1}{16}$$

$$\text{(A+B+C)'s 1 day work} = \frac{3}{32}$$

$$\text{C's 1 day work} = \frac{3}{32} - \frac{1}{16}$$

$$= \frac{3-2}{32} = \frac{1}{32}$$

$$\text{(A+C)'s 1 day work} = \frac{1}{24} + \frac{1}{32}$$

$$= \frac{4+3}{96} = \frac{7}{96}$$

$$\therefore \text{ (A+C) can do the work in} = \frac{96}{7} \text{ days}$$

$$= 13\frac{5}{7} \text{ days}$$

Question 7.
A can do a piece of work in 20 days and B in 15 days. They worked together on it for 6 days and then A left. How long will B take to finish the remaining work ?
Solution:

A can do a piece of work in = 20 days

B can do a piece of work in = 15 days

$\therefore$ A's 1 day work $= \frac{1}{20}$

B's 1 day work $= \frac{1}{15}$

(A+B)'s 1 day work $= \frac{1}{20} + \frac{1}{15}$

$= \frac{3+4}{60} = \frac{7}{60}$

(A+B)'s 6 days work $= \frac{7}{60} \times 6 = \frac{7}{10}$

Remaining Work $= 1 - \frac{7}{10}$

$= \frac{10-7}{10} = \frac{3}{10}$

B can do 1 work in = 15 days

B can do $\frac{3}{10}$ work in $= 15 \times \frac{3}{10}$ days

$= \frac{45}{10}$ days $= \frac{9}{2}$ days $= 4\frac{1}{2}$ days

Question 8.
A can finish a piece of work in 15 days and B can do it in 10 days. They worked together for 2 days and then B goes away. In how many days will A finish the remaining work?

Solution:

A can finish a piece of work in = 15 days

B can finish a piece of work in = 10 days

$\therefore$ A's 1 day work $= \frac{1}{15}$

B's 1 day work $= \frac{1}{10}$

(A+B)'s 1 day work $= \frac{1}{15} + \frac{1}{10}$

$= \frac{2+3}{30} = \frac{5}{30} = \frac{1}{6}$

(A+B)'s 2 days work $= \frac{1}{6} \times 2 = \frac{1}{3}$

Remaining work which will be done by A alone

$= 1 - \frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$

$\therefore$ A can finish 1 work in = 15 days

A can finish $\frac{2}{3}$ work in $= 15 \times \frac{2}{3}$ days

$= \frac{30}{3}$ days $= 10$ days

Question 9.
A can do a piece of work in 10 days ; B in 18 days; and A, B and C together in 4 days. In what time would C alone do it ?

Solution:

A can do a piece of work in = 10 days

B can do a piece of work in = 18 days

(A+B+C) can do a piece of work in = 4 days

$\therefore$ A's 1 day work $= \frac{1}{10}$

B's 1 day work $= \frac{1}{18}$

(A+B)'s 1 day work $= \frac{1}{10} + \frac{1}{18} = \frac{9+5}{90}$

$= \frac{14}{90} = \frac{7}{45}$

(A+B+C)'s 1 day work $= \frac{1}{4}$

$\therefore$ C's 1 day work $= \frac{1}{4} - \frac{7}{45}$

$= \frac{45-28}{180} = \frac{17}{180}$

$\therefore$ C can do the piece of work in $= \frac{180}{17}$ days

$= 10\frac{10}{17}$ days

Question 10.

A can do $\frac{1}{4}$ of a work in 5 days and B can do $\frac{1}{3}$ of the same work in 10 days. Find the number of days in which both working together will complete the work.

Solution:

A can do $\frac{1}{4}$ of work in = 5 days

A can do 1 work in = 5×4 days

= 20 days

B can do $\frac{1}{3}$ of work in = 10 days

B can do 1 work in = 10×3 days

= 30 days

A's 1 day work = $\frac{1}{20}$

B's 1 day work = $\frac{1}{30}$

(A+B)'s 1 day work = $\frac{1}{20} + \frac{1}{30}$

$= \frac{3+2}{60}$

$= \frac{5}{60}$

$= \frac{1}{12}$

$\therefore$ A and B working together will complete the work in = 12 days

Question 11.
One tap can fill a cistern in 3 hours and the waste pipe can empty the full cistern in 5 hours. In what time will the empty cistern be full, if the tap and the waste pipe are kept open together ?

Solution:

One tap can fill a cistern in = 3 hours

Waste pipe can empty a cistern in = 5 hours

$\therefore$ One tap 1 hour work $= \frac{1}{3}$

Waste pipe 1 hour work $= \frac{1}{5}$

$\therefore$ One tap and waste pipe together 1 hour work

$$= \frac{1}{3} - \frac{1}{5} = \frac{5-3}{15}$$

$$= \frac{2}{15}$$

$\therefore$ Empty cistern will be full in $= \frac{15}{2}$ hours

$$= 7\frac{1}{2} \text{ hours}$$

Question 12.
A and B can do a work in 8 days; B and C in 12 days, and A and C in 16 days. In what time could they do it, all working together ?

Solution:
A and B can do a work in = 8 days
B and C can do a work in = 12 days
A and C can do a work in = 16 days
(A+B)'s 1 day work = $\frac{1}{8}$

(B+C)'s 1 day work = $\frac{1}{12}$

$$(A+C)\text{'s 1 day work} = \frac{1}{16}$$

$$\therefore [(A+B)+(B+C)+(A+C)]\text{'s 1 day work}$$

$$= \frac{1}{8}+\frac{1}{12}+\frac{1}{16}$$

$$i.e.\ [A+B+B+C+A+C]\text{'s 1 day work}$$

$$= \frac{1}{8}+\frac{1}{12}+\frac{1}{16}$$

$$= \frac{6+4+3}{48} = \frac{13}{48}$$

$$i.e.\quad 2(A+B+C)\text{'s 1 day work} = \frac{13}{48}$$

$$i.e.\quad (A+B+C)\text{'s 1 day work} = \frac{13}{48}\times\frac{1}{2} = \frac{13}{96}$$

$$\therefore (A+B+C) \text{ can do the work in} = \frac{96}{13} \text{ days}$$

$$= 7\frac{5}{13} \text{ days}$$

Question 13.

A and B complete a piece of work in 24 days. B and C do the same work in 36 days ; and A, B and C together finish it in 18 days. In how many days will (i) A alone,
(ii) C alone,
(iii) A and C together, complete the work ?

Solution:

A and B complete a piece of work in = 24 days
B and C complete a piece of work in = 36 days
(A+B+C) complete a piece of work in = 18 days
(A+B)'s 1 day work = $\frac{1}{24}$
(B+C)'s 1 day work = $\frac{1}{36}$

(A+B+C)'s 1 day work = $\frac{1}{18}$

$$(i) \quad \text{A's 1 day work} = \frac{1}{18} - \frac{1}{36}$$

$$= \frac{2-1}{36} = \frac{1}{36}$$

$\therefore$ A will complete the work in = 36 days

$$(ii) \quad \text{C's 1 day work} = \frac{1}{18} - \frac{1}{24}$$

$$= \frac{4-3}{72} = \frac{1}{72}$$

$\therefore$ C will complete the work in = 72 days

$$(iii) \quad (A+C)\text{'s 1 day work} = \frac{1}{36} + \frac{1}{72}$$

$$= \frac{2+1}{72} = \frac{3}{72}$$

$$= \frac{1}{24}$$

$\therefore$ (A+C) will complete the work in = 24 days

Question 14.

A and B can do a piece of work in 40 days; B and C in 30 days; and C and A in 24 days.
(i) How long will it take them to do the work together ?
(ii) In what time can each finish it working alone ?

Solution:

A and B can do a piece of work in = 40 days
B and C can do a piece of work in = 30 days
C and A can do a piece of work in = 24 days
(A+B)'s 1 day work = $\frac{1}{40}$
(B+C)'s 1 day work = $\frac{1}{30}$
(C+A)'s 1 day work = $\frac{1}{24}$
(i) [(A+B)+(B+C)+(C+A)]'s 1 day work = $\frac{1}{40} + \frac{1}{30} + \frac{1}{24}$

$$= \frac{3+4+5}{120}$$

$$= \frac{12}{120} = \frac{1}{10}$$

i.e. (A+B+B+C+C+A)'s 1 day work $= \frac{1}{10}$

i.e. 2 (A+B+C)'s 1 day work $= \frac{1}{10}$

$\therefore$ (A+B+C)'s 1 day work $= \frac{1}{10} \times \frac{1}{2}$

$$= \frac{1}{20}$$

$\therefore$ (A+B+C) can do the work in = 20 days

(*ii*) A's 1 day work $= \frac{1}{20} - \frac{1}{30}$

$$= \frac{3-2}{60} = \frac{1}{60}$$

$\therefore$ A can do the work in 60 days

B's 1 day work $= \frac{1}{20} - \frac{1}{24}$

$$= \frac{6-5}{120} = \frac{1}{120}$$

$\therefore$ B can do the work in 120 days

Now, C's 1 day work $= \frac{1}{20} - \frac{1}{40}$

$$= \frac{2-1}{40} = \frac{1}{40}$$

C can do the work in 40 days
Hence A can do the work in = 60 days
B can do the work in = 120 days
C can do the work in = 40 days

Question 15.
A can do a piece of work in 10 days, B in 12 days and C in 15 days. All begin together but A leaves the work after 2 days and B leaves 3 days before the work is finished. How long did the work last ?

Solution:

$$\text{A's one day's work} = \frac{1}{10}$$

$$\text{B's one day's work} = \frac{1}{12}$$

$$\text{C's one day's work} = \frac{1}{15}$$

$$\text{A, B and C's one day's work} = \frac{1}{10} + \frac{1}{12} + \frac{1}{15}$$

$$= \frac{6+5+4}{60} = \frac{15}{60} = \frac{1}{4}$$

Let the work completed in x days

$\therefore$ A's 2 days work + B's $(x - 3)$ days work + C's x days work = 1

$$\Rightarrow 2 \times \frac{1}{10} + (x-3) \times \frac{1}{12} + x \times \frac{1}{15} = 1$$

$$\Rightarrow \frac{1}{5} + \frac{x-3}{12} + \frac{x}{15} = 1$$

$$\frac{12+5x-15+4x=60}{60}$$

(L.C.M. of 5, 12, 15 = 60)

$\Rightarrow 9x = 60 - 12 + 15 \Rightarrow 9x = 63$

$$\Rightarrow x = \frac{63}{9} = 7$$

$\therefore$ Work will last for 7 days

Question 16.

Two pipes P and Q would fill an empty cistern in 24 minutes and 32 minutes respectively. Both the pipes being opened together, find when the first pipe must be turned off so that the empty cistern may be just filled in 16 minutes.

Solution:

P's one minutes's work = $\frac{1}{24}$

Q's one minutes work = $\frac{1}{32}$

Let the first pipe must turn off after x minutes

The cistern filled in 16 minutes

Then P's x minutes work + Q's 16 minutes work = 1

$$\Rightarrow \frac{1}{24} \times x + \frac{1}{32} \times 16 = 1$$

$$\frac{x}{24} + \frac{1}{2} = 1$$

$$\Rightarrow \frac{x}{24} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \frac{x}{24} = \frac{1}{2}$$

$$\Rightarrow x = \frac{24}{2} = 12$$

$\therefore$ After 12 minutes pipe P would turned off