ASSIGNMENT (NCJPS/GSA/X/MATHS/2011-12/01)

Chapter: - Real Numbers and Introduction to Trigonometry

Q1. Show that any positive odd integer is of the form 4q +1 or 4q+3, where q is some integer.

Q2. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m+1, where m is some integer.

Q3.Using Euclid's division algorithm, find which of the following pairs of numbers are co-prime:-

(i) 231, 396 (ii) 847, 2160 Ans. (ii).

Q4. Show that the square of a positive integer is of the form 8m+1, for some whole integer m.

Q5. Prove that (i) $\sqrt{2}+\sqrt{3}$ and (ii) $\sqrt{3}+\sqrt{5}$ is irrational.

Q6. Show that cube of any positive integer is of the form 4q, 4q +1 or 4q+3, where q is some integer.

Q7. Show that the square of any positive integer cannot be of the form 5m+2 or 5m+3 for any integer m.

Q8. Show that the square of any positive integer cannot be of the form 6m+2 or 6m+5 for any integer m.

Q9. If n are odd integers then show that (n^2-1) is divisible by 8.

Q10 Prove that if x and y are both positive odd integer, then x^2 +$y^2$is even but not divisible by4.

Q11.Using Euclid's division algorithm to find the HCF of 441, 567, 69 Ans.63

Q12. Using Euclid's division algorithm to find the HCF of 210 and 55 If HCF is expressible in the form 210x +55y, find x and y. Ans. 5, 5, -19,

Q13. Using Euclid's division algorithm to find the largest number that divides 1251, 9377 and 15628 leaving remainders 1,2and 3 respectively. Ans. 625.

Q14.Show that 12^n cannot end with the digit 0 or 5 for any Natural number n.

Q15. Prove that $\sqrt{p}+\sqrt{q}$ is irrational where p and q are primes.

Q16. Write the denominator of the rational number $\frac{257}{5000}$ in the form 2^m x5^n, where m, n are non-negative integers. Hence write its decimal expansion, without actual division. Ans. m=3,n=4,0.0514

Q17. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? Ans. 2520cm.

Q18.Show that the cube of a positive integer of the form 6q +r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m+r.

Q19.Prove that one and only one out of n, n+2 and n+4 is divisible by 3, where n is any positive integer.

Q20. Prove that one of any three consecutive positive integers must be divisible by 3.

Q21.Prove that (n^3-n)is divisible by 6, where n is any positive integer.

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Q22. Show that one and only one out of n, n+4,n+8, n+12 and n+16 is divisible by 5, where n is any positive integer.

Q23.If tanα= $\frac{\sqrt{3}-1}{\sqrt{3}+1}$, find sinα and cosα. Ans. $\frac{\sqrt{3}-1}{2\sqrt{2}}, \frac{\sqrt{3}+1}{2\sqrt{2}}$,

Q24. If qcosα = $\sqrt{q^2 - p^2}$,then prove that qsinα =p.

Q25.If sin(A+B) =1 and cos(A-B) = $\frac{\sqrt{3}}{2}$, find A and B. Ans. $60^0, 30^0$.

Q26.Evaluate each of the following without using Trigonometric tables:-

(i) $\frac{\tan A \cot(90^0 - A) - \sec A \operatorname{cosec}(90^0 - A) + \sin^2 35^0 + \sin^2 55^0}{\tan 10^0 . \tan 20^0 . \tan 45^0 . \tan 70^0 . \tan 80^0}$. Ans. 0.

(ii) $\sin^2 65^0 + \sin^2 25^0 + \sqrt{3}$ ($\tan 5^0 \tan 15^0 \tan 30^0 \tan 75^0 \tan 85^0$). Ans. 2.

(iii) $\frac{\sec^2 54^0 - \cot^2 36^0}{\operatorname{cosec}^2 57^0 - \tan^2 33^0}$ + $2\sin^2 38^0 \sec^2 52^0 - \sin^2 45^0$. Ans. $\frac{5}{2}$.

(iv) (1 + cotA- cosecA) (1 + tanA + secA) . Ans. 2.

(v)$3(\sin x - \cos x)^4 + 4(\sin^6 x + \cos^6 x) + 6(\sin x + \cos x)^2$Ans. 13.

Q27.If sinα + cosα = $\sqrt{2}$ sin (90^0-α), find cotα. Ans. $\sqrt{2} + 1$.

Q28.If sec 4A =cosec (A-20^0), where 4A is an acute angle, find the value of A. Ans. 22^0.

Q29.If 3 sinx + 5cosx = 5, show that 5 sinx -3cosx = ±3

Q30.Prove the following trigonometric identities:-

(i).$\sin^8 x - \cos^8 x = (\sin^2 x - \cos^2 x)(1 - 2\sin^2 x \cos^2 x)$. (ii).$\frac{1+\cos x + \sin x}{1 + \cos x - \sin x} = \frac{1 + \sin x}{\cos x}$.

(iii).$\frac{1}{\sec x + \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x - \tan x}$. (iv).$\frac{1 + \cos x - \sin^2 x}{\sin x(1 + \cos x)} = \cot x$.

(v).$\frac{\tan x + \sin x}{\tan x - \sin x} = \frac{\sec x + 1}{\sec x - 1}$ (vi).$\frac{\cos^2 x}{\sin x} - \operatorname{cosec} x + \sin x = 0$

Q31.If $\frac{\cos^4 x}{\cos^2 y} + \frac{\sin^4 x}{\sin^2 y} = 1$ then prove that (i). $\sin^4 x + \sin^4 y = 2\sin^2 x \sin^2 y$ (ii). $\frac{\cos^4 y}{\cos^2 x} + \frac{\sin^4 y}{\sin^2 x} = 1$

Q32.If $\tan^2 x = 1 - e^2$, prove that $\sec x + \tan^3 x \operatorname{cosec} x = (2 - e^2)^{3/2}$

Q33.Eliminate x between the equations asecx + btanx + c= 0 and psecx + qtanx + r = 0 .

Ans.$(br-qc)^2-(pc-ar)^2 =(aq-pb)^2$.

Q34.If cosecx –sinx =m, and secx –cosx =n, eliminate x. Ans. $(m^2 n)^{\frac{2}{3}} + (mn^2)^{\frac{2}{3}} = 1$.

---**Best of Luck**---