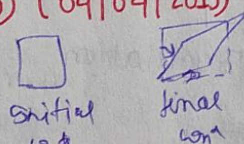
Rate of shear strain :- (lecture 9) (04/04/2019)

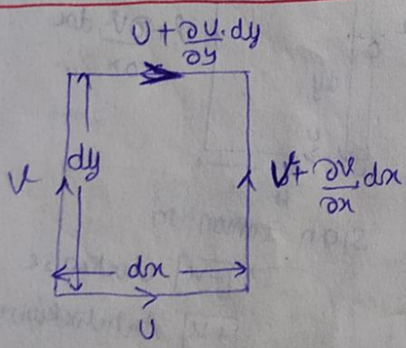
In x-y plane $= \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$



circulation: (Γ) [Capital Gamma]

It is defined as the line integration of velocity vector taken along a close loop

$\boxed{\text{Vorticity : } 2\vec{\omega}}$

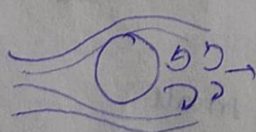


$\Gamma = \oint \vec{V}.\vec{dr}$

In x-y Plane

$\Gamma = u\,dx + \left(v + \frac{\partial v}{\partial x}.dx\right)dy - \left(u + \frac{\partial u}{\partial y}.dy\right)dx$
$- (v\,dy)$

$$\boxed{\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx.dy}$$



Pb-(22) $u = 2x + 3y$ sol^n $\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx.dy$

$v = 2y$

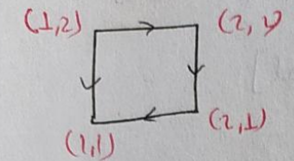
$r = 2$ units

$= (0 - 3)(\pi r^2)$

$= -3 \times \pi (2)^2$

(Pb) $U = x^2$

$V = -2xy$



Sol^n $\Gamma = \int_{x=1}^{x=2} u.dx + \int_{y=1}^{y=2} (v.dy) + \int_{x=2}^{x=1} u.dx + \int_{y=2}^{y=1} v.dy$

$\boxed{y=1}$ $\boxed{x=2}$ $\boxed{y=2}$ $\boxed{x=1}$

$= \int_1^2 x^2\,dx + \int_1^2 -2(2)y.dy + \int_2^1 x^2.dx + \int_2^1 -2(1).y\,dy$

$= \cancel{\int_1^2 x^2 dx} - 4\int_1^2 y.dy + \cancel{\int_2^1 x^2.dx} - 2\int_2^1 y.dy$

$= -4\left[\frac{y^2}{2}\right]_1^2 - 2\left[\frac{y^2}{2}\right]_2^1$

$= -\frac{4}{2}[4-1] - \frac{2}{2}[1-4]$

$= -2(3) - 1(-3)$

$= -6+3 = -3$

→ velocity Potential fun^n (ϕ)

→ stream fun^n (ψ)

(V.P.F) (ϕ) In general V.P.F is defined as a fun^n of space and time in a flow such that the negative derivative of this fun^n W.R.T any dir^n gives the velocity of fluid flow in that dir^n.

$$\begin{bmatrix} U = -\frac{\partial \phi}{\partial x} \\ V = -\frac{\partial \phi}{\partial y} \\ W = -\frac{\partial \phi}{\partial z} \end{bmatrix}$$

Boundation with ϕ. continuity eq^n For 2D'- steady, incompressible flow →

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) = 0$$

$$-\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

→ $\boxed{\nabla^2 \phi = 0}$

laplace eq^n

ϕ must satisfy laplace eq^n

(Pb) $\phi = 2x + 3y$

$\boxed{\nabla^2 \phi = 0}$

ϕ = valid

irrotational Flow

(ii) $\phi = 4x^2 - 5y^2$

$\nabla^2 \phi = 2$

$\neq 0$

ϕ = invalid

Physical significance

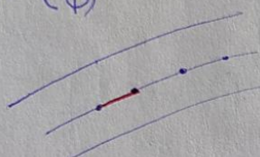
$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right)\right]$$

$$= 0$$

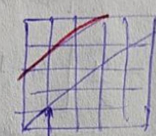
[ϕ exist $\rightarrow$ irrotational]

Equⁿ of equipotential funⁿ line:

These line (curve) are formed by Joining the different point ~~adding~~ having the same value of velocity potential funⁿ (ϕ)



ϕ = constant



domain

ϕ = constant

Ex: $2x+3y$

for 2D steady incompressible flow

$$\phi = f^n(x, y)$$

$$d\phi = \frac{\partial \phi}{\partial x}.dx + \frac{\partial \phi}{\partial y}.dy = -u\,dx - v\,dy$$

$$0 = -u\,dx - v\,dy, \qquad u\,dx + v\,dy = 0$$

$$\boxed{\left.\frac{dy}{dx}\right|_{\phi = const} = -\frac{u}{v}}$$

Stream funⁿ: (ψ) In general

it is a funⁿ of space and time in a 2D (x-y plane) define in such a way i.e. continuity eqⁿ satisfy and flow is posibble.

continuity eqⁿ

For 2D - steady, incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

If
$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

then, $\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0$

$(0 = 0)$

continuity satisfied

check

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \psi}{\partial y}\right)\right] = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

$$= \frac{1}{2}\left|\nabla^2 \psi\right|$$

[If $\nabla^2 \psi = 0$, Irrotational flow]

[If $\nabla^2 \psi \neq 0$, Rotational flow]

Pb- 34 $\psi = px^2 + qy^2$

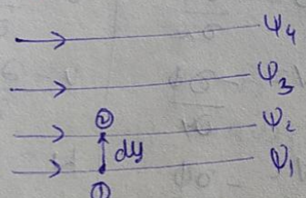
If ϕ exist means irrotational flow

$\nabla^2 \phi = 0$

$2p + 2q = 0$

$\boxed{p = -q}$

Physical significance :



Assume width = 1

For 2D - steady incompressible Flow

$\psi = f^n(x, y)$

Math, $d\psi = \left(\frac{\partial \psi}{\partial x}\right) dx + \left(\frac{\partial \psi}{\partial y}\right) dy$ ($\frac{\partial \psi}{\partial x} = v$, $\frac{\partial \psi}{\partial y} = -u$)

$d\psi = -u(dy.1)$

$-d\psi = dQ$ per unit width

Int it,

$-\int_1^2 d\psi = Q$ per unit width

$\boxed{\psi_1 - \psi_2 = Q \text{ per unit width}}$

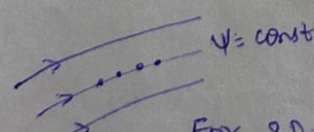
Pb 30 $\psi_1 = \frac{3}{2}(9-1)$

$\psi_2 = 0$

$|\psi_1 - \psi_2| = 12$ units

discharge (+ve)

Eq^n of Equi stream fun^n line :- these line (curves) are formed by joining the different Point having the same value of stream fun^n (ψ).



For 2D - steady, incompressible flow,

$\psi = f^n(x, y)$

Math, $\overset{0}{\cancel{d\psi}} = \left(\frac{\partial \psi}{\partial x}\right) dx + \left(\frac{\partial \psi}{\partial y}\right).dy$ ($\frac{\partial \psi}{\partial x} = v$, $\frac{\partial \psi}{\partial y} = -u$)

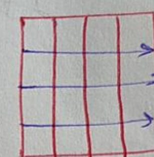
$0 = v.dx - u.dy$

$U \cdot dy = v\, dx$ $\boxed{\dfrac{dx}{U} = \dfrac{dy}{v}}$

$$\left.\frac{dy}{dx}\right|_{\psi \to const} = \frac{v}{U}$$

Note: $\left.\dfrac{dy}{dx}\right|_{\phi = const} \times \left.\dfrac{dy}{dx}\right|_{\psi \to const.} = -\dfrac{U}{V} \times \dfrac{V}{U} = -1$

[Equi-potential lies and equi stream funn lies are orthogonal to each other]



Flow net

Cauchy-Rieman eqn:

For irrotational flow

$$U = \boxed{-\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}} \quad ***$$

$$v = \boxed{-\frac{\partial \phi}{\partial y} = +\frac{\partial \psi}{\partial x}}$$

Sign $U = -\dfrac{\partial \phi}{\partial x}$ | $U = \dfrac{\partial \phi}{\partial x}$

$V = -\dfrac{\partial \phi}{\partial y}$ | $V = \dfrac{\partial \phi}{\partial y}$

$W = -\dfrac{\partial \phi}{\partial z}$ | $W = \dfrac{\partial \phi}{\partial z}$

$U = -\dfrac{\partial \psi}{\partial y}$ | continuity eqn | $U = \dfrac{\partial \psi}{\partial y}$

$U = \dfrac{\partial \psi}{\partial x}$ | $V = \dfrac{\partial \psi}{\partial x}$

$$\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) = 0$$

Pb (48) For 2D Incompressible flow field

$U = \dfrac{y^3}{3} + 2x - x^2 y$

$v = xy^2 - 2y - \dfrac{x^3}{3}$

Soln For 2D incompressible flow

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$(2 - 2xy) + (2xy - 2) = 0$$

$$0 = 0$$

continuity satisfied

Flow is Possible

Obtain ψ

$$-\frac{\partial \psi}{\partial y} = U$$

$$\frac{\partial \psi}{\partial y} = -\frac{y^3}{3} - 2x + x^2 y$$

Int it $\psi = \frac{-y^4}{12} - 2xy + \frac{x^2y^2}{2} + f(x) + \text{constant}$

$\frac{\partial \psi}{\partial x} = v$

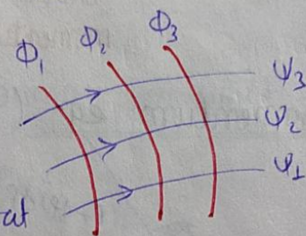
$\frac{\partial \psi}{\partial x} = xy^2 - 2y - \frac{x^3}{3}$

Int it :

$\psi = \frac{x^2y^2}{2} - 2xy - \frac{x^4}{12} + f(y) + \text{const.}$

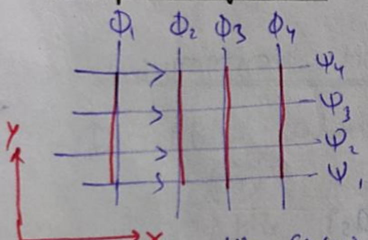
$$\boxed{\psi = \frac{x^2y^2}{2} - 2xy - \frac{y^4}{12} - \frac{x^4}{12} + \text{const}}$$

Flow Net: for irrotational flow.



It is a graphical representation of equipotential equi, func of line stream

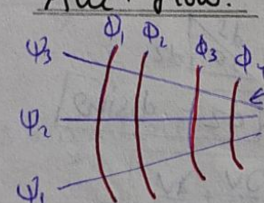
Uniform flow →



$\psi = f^n(y)$

$\phi = f^n(x)$

Accn flow:



← velocity ↑

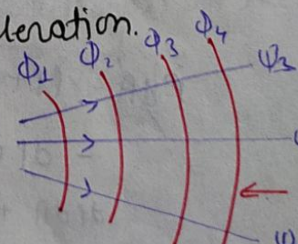
Extra

$|\psi_1 - \psi_3| = Q_{\text{per unit width}}$

$Q = \text{constant}$

$Q = A.V$

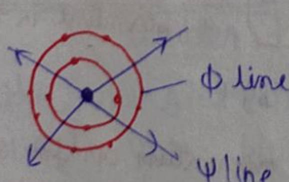
Deceleration.



← velocity ↓

Source

(Radially outward)



(ψ line never cut ϕ line)

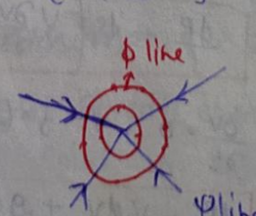
$\psi = f^n(\theta)$

$\phi = f^n(\ln r)$

radius

Sink

(Radially inward)



Fluid Dynamic (chapter)

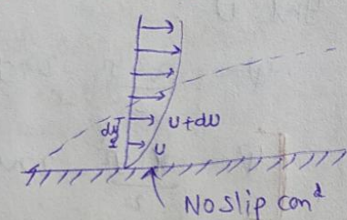
Study of motion of fluid with considering the basic cause of motion ($F_{external}$)

Flow over a flat plate:

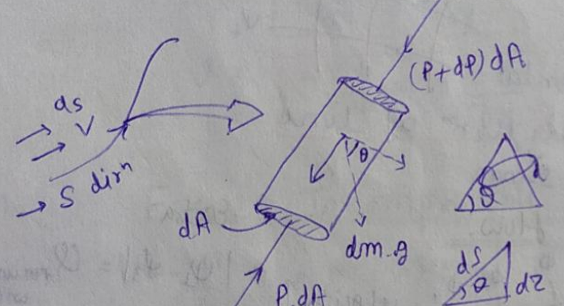
Invisid Region of fluid flow [$F_{vis} \approx 0$]

$F_p + F_g = F_i$ (Euler momentum eqn)

$F_p + F_g + F_{vis} = F_i$ (Navier's stoke momentum eqn)



Euler momentum eqn (flow along stream line)



$\boxed{dz = ds\sin\theta}$

$$\boxed{a_s = V.\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}} \quad **$$

$$\cancel{P}dA - (\cancel{P}+dP)dA - dm.g\sin\theta = dm\,a_s$$

$$-dP.dA - dm.g\sin\theta = dm\,a_s \quad 0$$

$$dP\,dA + \rho(dA.ds)\left[V.\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right] + \rho(dA\,ds\,g\sin\theta) = 0$$

dz

Divide by ρ

$$\boxed{\frac{dP}{\rho} + ds\left[V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right] + g\,dz = 0}$$

" Euler momentum eqn along a streamline in the dirn of Flow

→ steady Flow $\frac{\partial V}{\partial t} = 0$, $V = f^n(s,t)$ multiply by s

$$\boxed{\frac{dP}{\rho} + V.dV + g\,dz = 0}$$

Euler momentum eqn for steady flow along a stream line in the flow dirn

Int. it

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = const$$

Incompressible Flow → $\rho \rightarrow$ const

$$\frac{P}{\rho} + \frac{V^2}{2} + g.z = \text{constant}$$

$$\boxed{P + \frac{1}{2}\rho V^2 + \rho g.z = \text{const}}$$ Bernoullis eqn