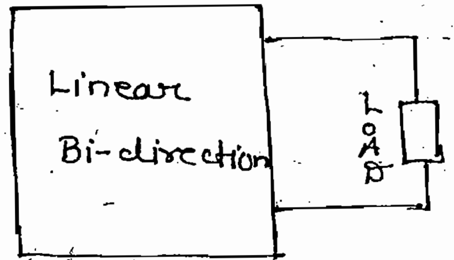
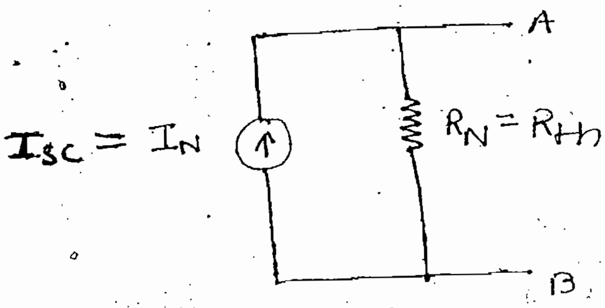
Lecture-7

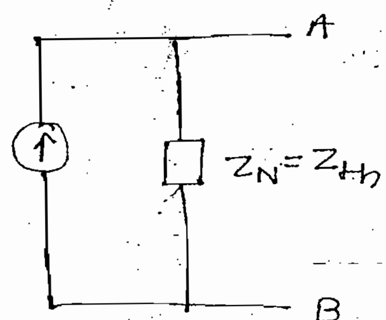
Norton's theorem:-

In any linear bidirection circuit having more no. of active and passive element, it can be replaced by single equivalent. ckt consisting of equivalent current source (I_N) in parallel with equivalent resistance (R_N)



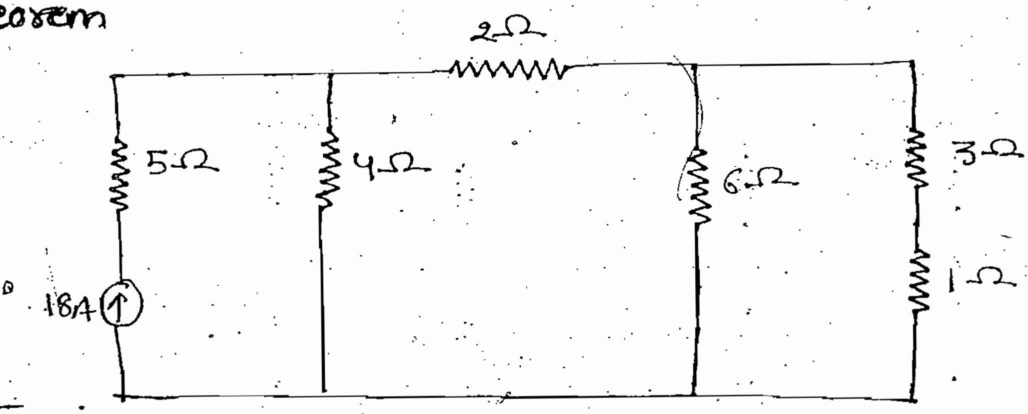


DC



AC

Ques:- Find current in 1Ω resistor using Norton's theorem



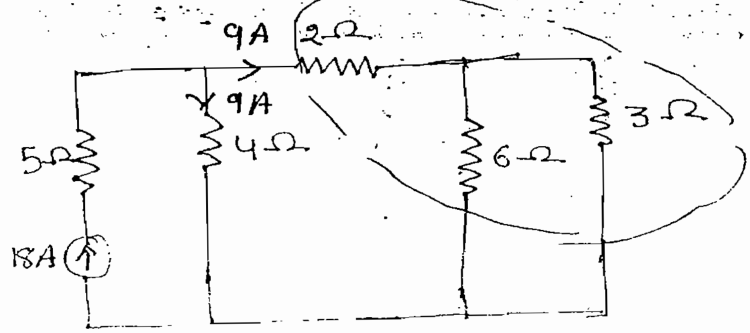
Soln:-

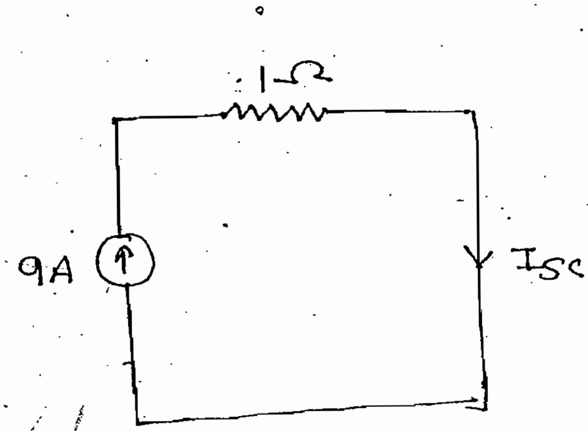
Case-(1):-

Replace the load resistor by S.C and find SC current

$$I_{sc} = 9 \times \frac{6}{6+3}$$

$$= 6A$$



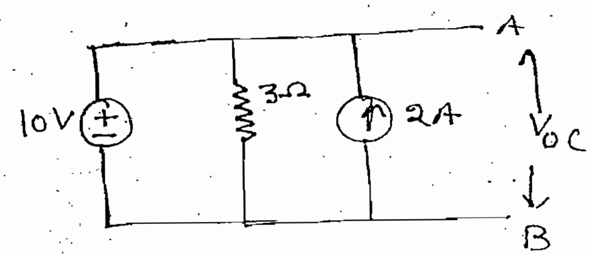


$I_{sc} = 9A$, Ans.

Ques:- Find o.c voltage and SC current w.r.t A & B

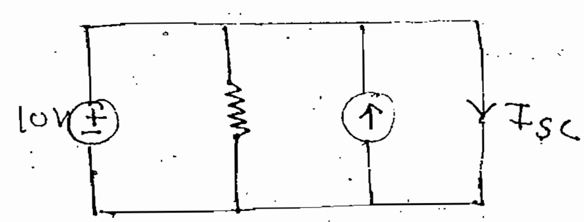
Sol^n:-

$V_{oc} = 10V$



For I_{sc}:-

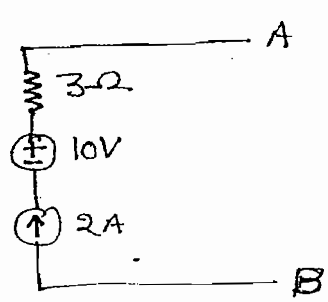
I_{sc} = not possible

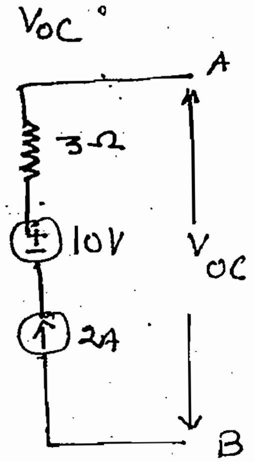


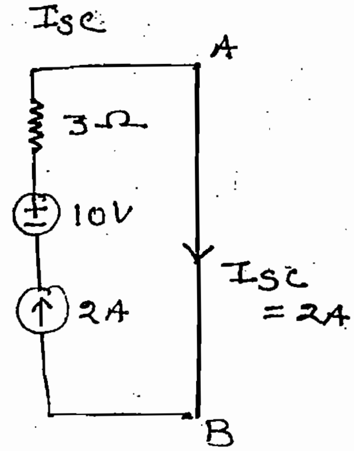
Note:-

In the above circuit it is not possible to find I_{sc} since it is not satisfying KVL

Ques:- Find V_{oc} and I_{sc} w.r.t A and B





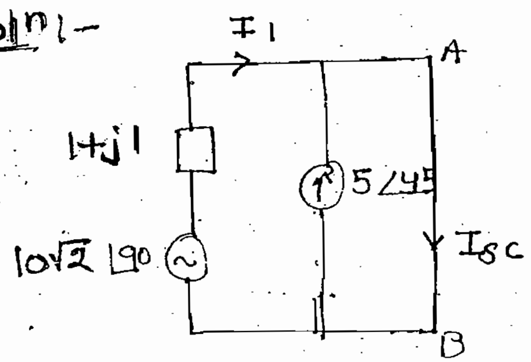


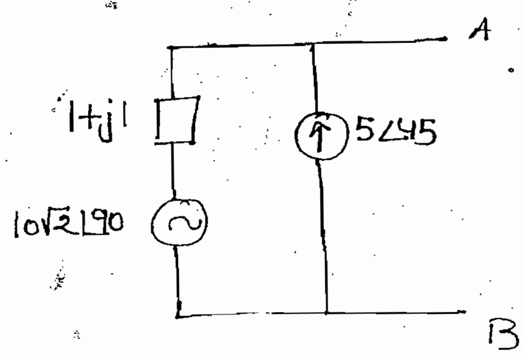
Note:-

In above circuit it is not possible to find O.C voltage since it is not satisfying KCL

Ques:- Find I_{sc} w.r.t A and B

Soln:-





$$I_1 = \frac{10\sqrt{2}\angle 90}{\sqrt{2}\angle 45} = 10\angle 45$$

$$I_1 + 5\angle 45 = I_{sc}$$

$$10\angle 45 + 5\angle 45 = I_{sc}$$

$$\Rightarrow \quad I_{sc} = 15\angle 45^\circ$$

Ques:- Find current in 4Ω resistor using the following data:

V	60	0
I	0	10A

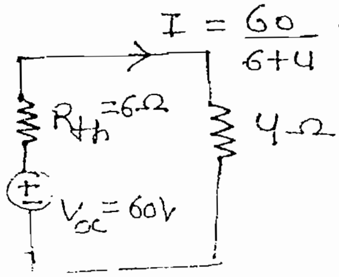
Soln:-

$V_{oc} = 60$

$I_{sc} = 10A$

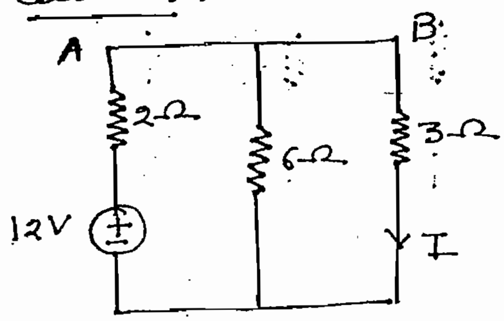
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{60}{10} = 6\Omega$$

$$I = \frac{60}{6+4} = 6A$$



Reciprocity theorem :-

Case-(1) :-



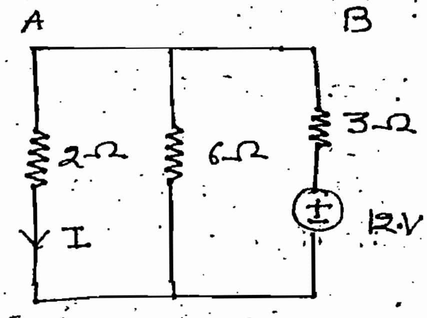
$$R_{eq} = 2 + \frac{6\times3}{6+3} = 4$$

$$I_T = \frac{12}{4} = 3A$$

$$I = 3\,\frac{6}{6+3} = 2A$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{2}{12} = \frac{1}{6}$$

Case-(II) :-



$$R_{eq} = 3 + \frac{6\times2}{6+2}$$

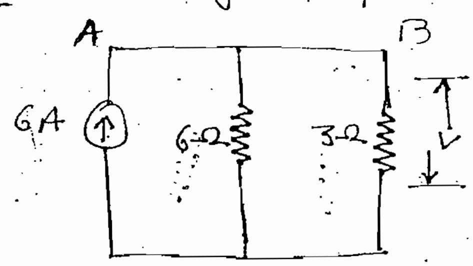
$$I_T' = \frac{12}{R_{eq}}$$

$$I = I_T'\,\frac{6}{6+2} = 2A$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{2}{12} = \frac{1}{6}$$

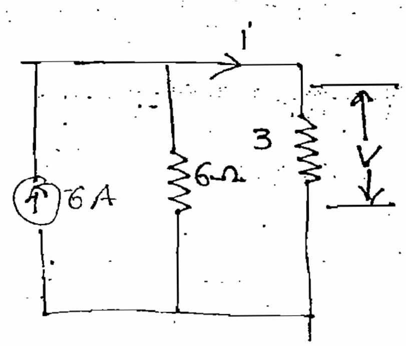
In above N/w after interchanging position of response and excitation the ratio of response to excitation is constant. Hence given N/w satisfy the reciprocity (Linear bi-directional element)

Ques:- Verify Reciprocity theorem of the circuit shown.



Soln:- Case-(1) :-

$$I' = 6\times\frac{6}{6+3} = 4A$$



$$V = 3\times4 = 12$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{12}{6} = 2$$

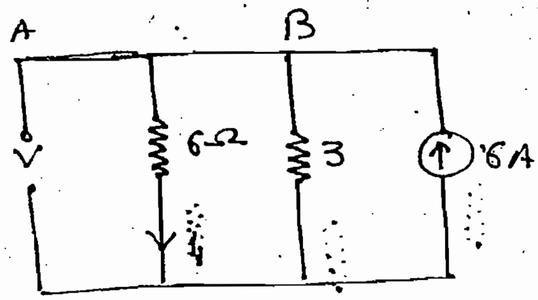
Case-(II):-

$i_1 = 6 \frac{3}{3+6} = 2A$

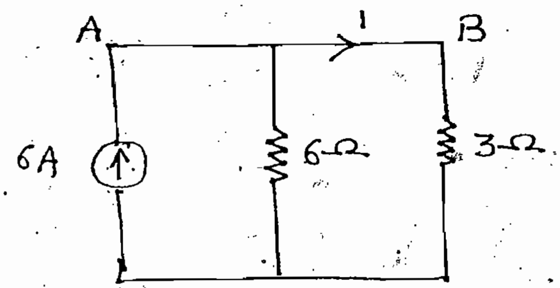
$V = 6 \times 2 = 12$

$\frac{\text{Response}}{\text{Excitation}} = \frac{12}{6} = 2$

$\longrightarrow$ satisfy Reciprocity



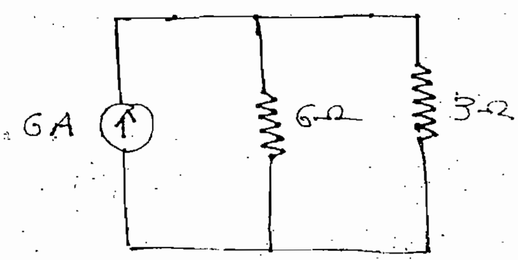
Ques:- Verify Reciprocity theorem of the ckt shown:-



Soln:-

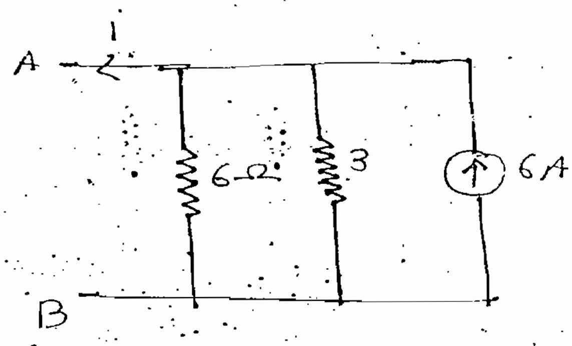
Case-(I)

$i = 6 \frac{6}{6+3} = 4A$



Case-(II)

$i = 0$



Note:- For above problem

(I) $\frac{\text{Response}}{\text{Excitation}} = \frac{I}{V_s} = \text{mho}$

(II) $\frac{\text{Response}}{\text{Excitation}} = \frac{V}{I_s} \longrightarrow \Omega$

(III) $\frac{\text{Response}}{\text{Excitation}} = i/I_s$

(IV) $\frac{\text{Response}}{\text{Excitation}} = \frac{V}{V_s}$

No units

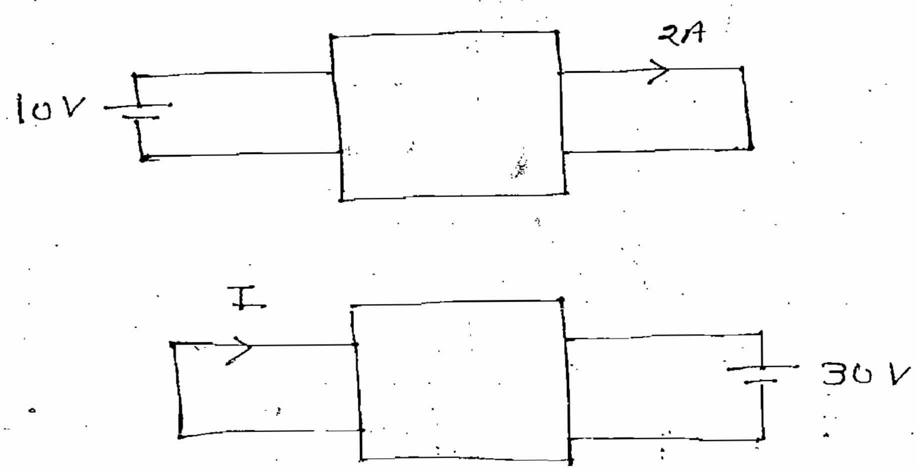
Note:-

For the above N/w Reciprocity theorem can't be applied since unit of response to excitation should be neither mho or Ω

Note:-

1- To apply Reciprocity theorem unit of Res/Exc. should be either mho or ohm
2. While applying reciprocity theorem N/w should consist of only one independent source
3. While applying reciprocity theorem N/w should not consist of any dependent sources

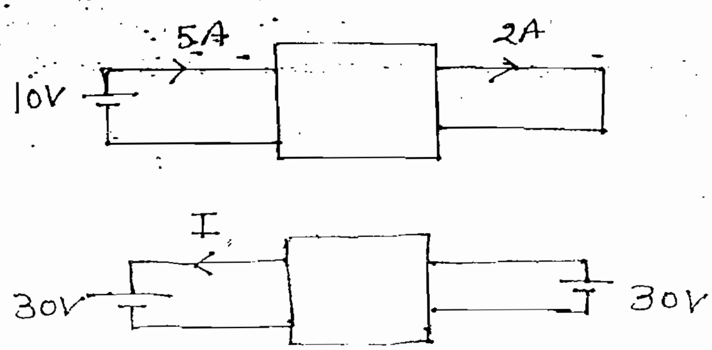
Ques:- When given N/w satisfy the Reciprocity then find the value of I



Soln:-

$$\frac{\text{Response}}{\text{Excitation}} = \frac{2}{10} = \frac{-I}{30} \Rightarrow I = -6A$$

Ques:- When given N/w satisfy the Reciprocity then find the value of I

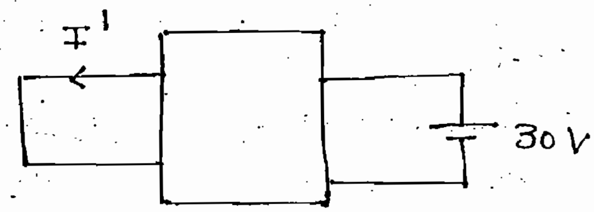


Solⁿ:- Solⁿ by using Superposition & Reciprocity
(∵ more than one independent sources are present)

Case-(I) (30V):-

$$\frac{\text{Response}}{\text{Excitation}} = \frac{I'}{30} = \frac{2}{10}$$

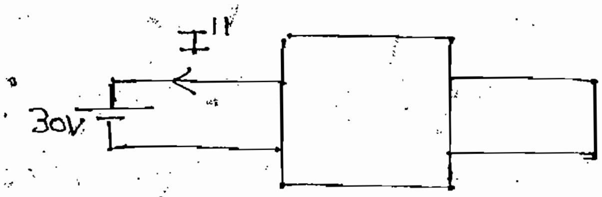
$$\Rightarrow I' = 6A$$



Case-(II) (30V):-

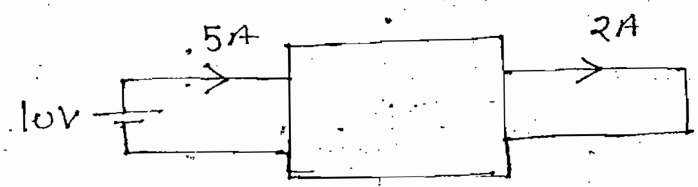
$$\frac{\text{Response}}{\text{Excitation}} = \frac{-I''}{30} = \frac{5}{10}$$

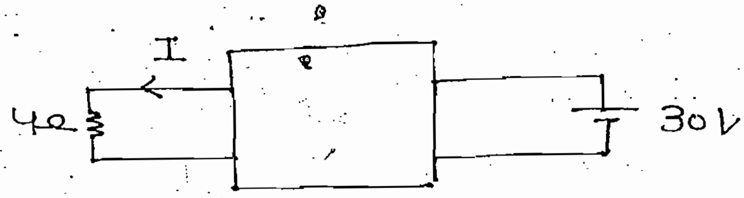
$$\Rightarrow I'' = -15$$



$$I = I' + I'' = 6 - 15 = -9A$$

Ques:- When given N/w satisfy the Reciprocity find the value of I

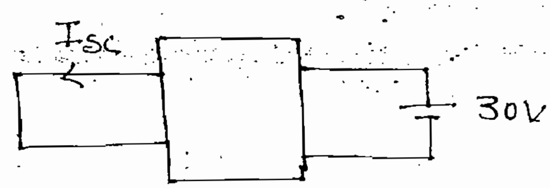




Solⁿ:- Case-(I) (I_{sc}):- (Norton's theorem)

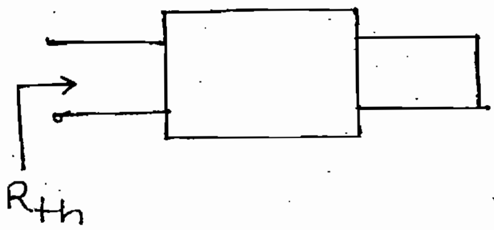
$$\frac{\text{Response}}{\text{Excitation}} = \frac{I_{sc}}{30} = \frac{2}{10}$$

$$I_{sc} = 6A$$

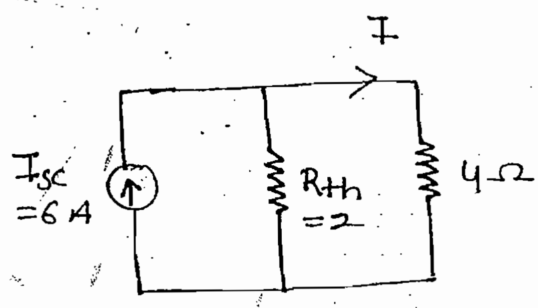


Case 2 (R_{th}) :-

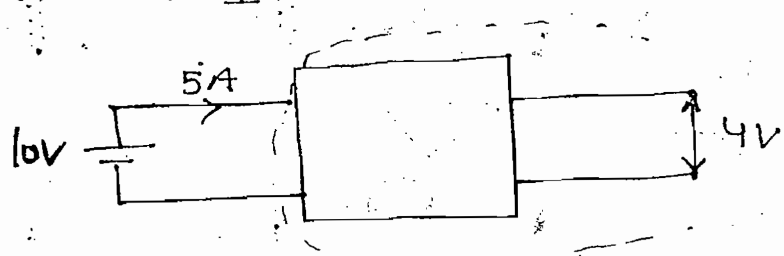
$$R_{th} = \frac{10}{5} = 2\Omega$$

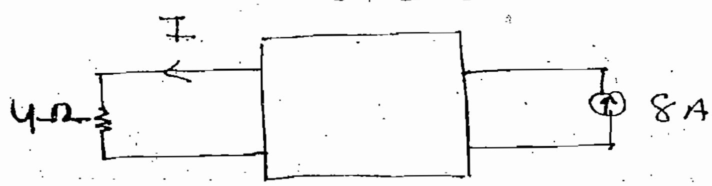


$$I = 6 \times \frac{2}{2+4} = 2A$$



Ques:- When given N/ω satisfy the Reciprocity Find the value of I :

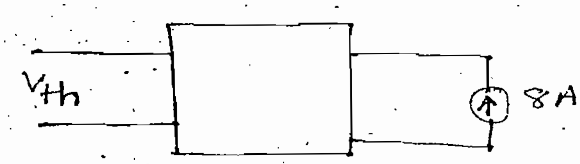




Soln:- Case (1) (V_{th}) :-

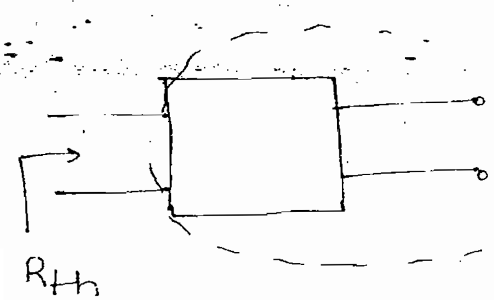
$$\frac{\text{Response}}{\text{Excitation}} = \frac{V_{th}}{8} = \frac{4}{5}$$

$$\Rightarrow V_{th} = \frac{32}{5}$$

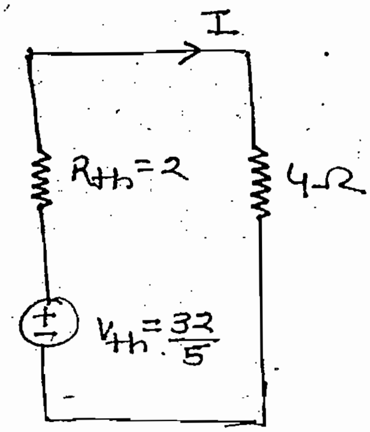


Case (II) (R_{th}) :-

$$R_{th} = \frac{10}{5} = 2\Omega$$



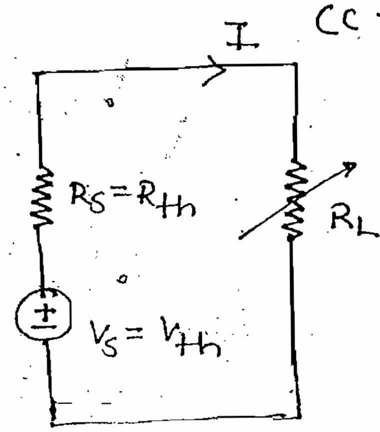
$$I = \frac{32/5}{2+4} = \frac{\cancel{32}^{16}}{5} \times \frac{1}{\cancel{6}_{3}} = \frac{16}{15} A$$



Maximum Power Transfer theorem :-

(variable = constant)

(C = V) X



$$I = \frac{V_S}{R_S + R_L}$$

$$P_L = I^2 R_L$$

$$\Rightarrow P_L = \left(\frac{V_S}{R_S + R_L}\right)^2 R_L \quad \text{—— (1)}$$

Diff eq-(1) w.r.t 'R_L' and equated to zero, we get

$$R_L = R_S$$

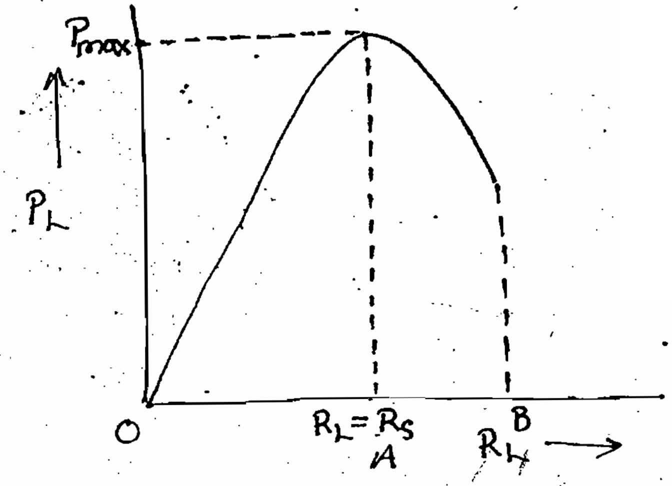
$$P_{max} = \frac{V_S^2}{(R_L + R_L)^2} R_L$$

$$\Rightarrow \boxed{P_{max} = \frac{V_S^2}{4R_L}}$$

$$\eta = \frac{O/P}{I/P} \times 100$$

$$\eta = \frac{I^2 R_L}{I^2 (R_L + R_S)} \times 100 \Rightarrow \eta = \frac{R_L}{R_L + R_L} \times 100$$

$$\Rightarrow \boxed{\eta = 50\%}$$



<u>Case – (I)</u> :–

OA $\Rightarrow$ $R_S > R_L$

$$\boxed{\eta < 50\%}$$

<u>Case – (II)</u> :–

At point A $\Rightarrow$ $\cancel{R_S} = R_L = R_S$

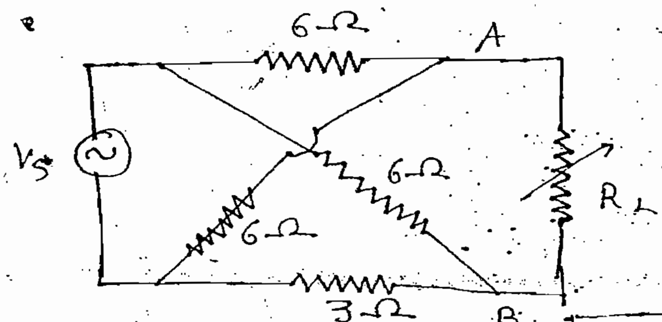
$$\boxed{\eta = 50\%}$$

<u>Case – (III)</u> :–

AB $\Rightarrow$ $R_L > R_S$

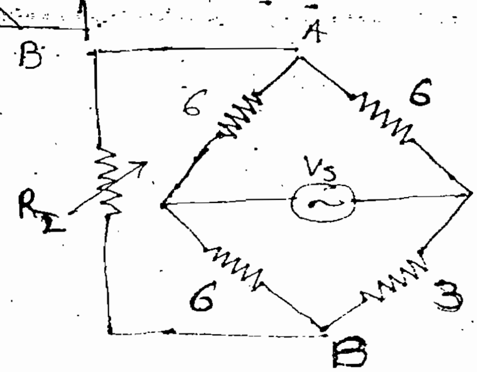
$$\boxed{\eta > 50\%}$$

<u>Ques</u> :– Find R_L to obtain max. power from source to load

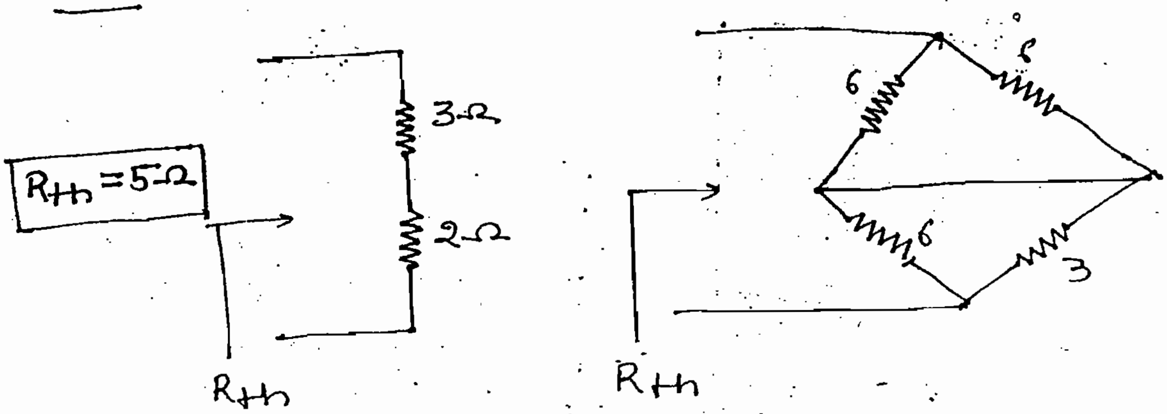


<u>Soln</u> :–

$\Rightarrow$

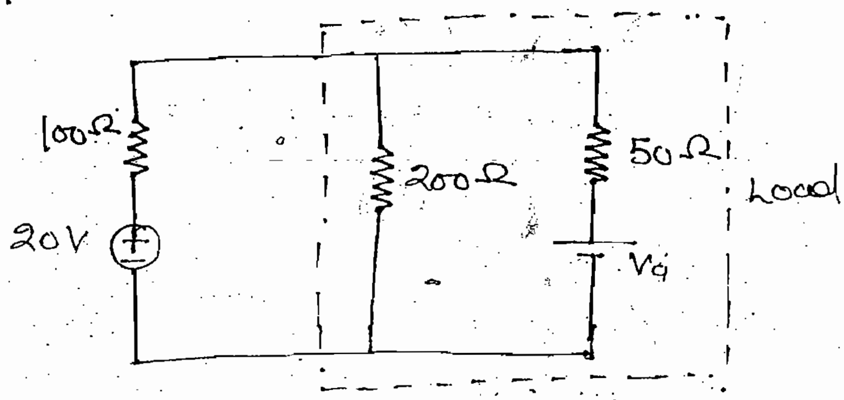


Rth :-

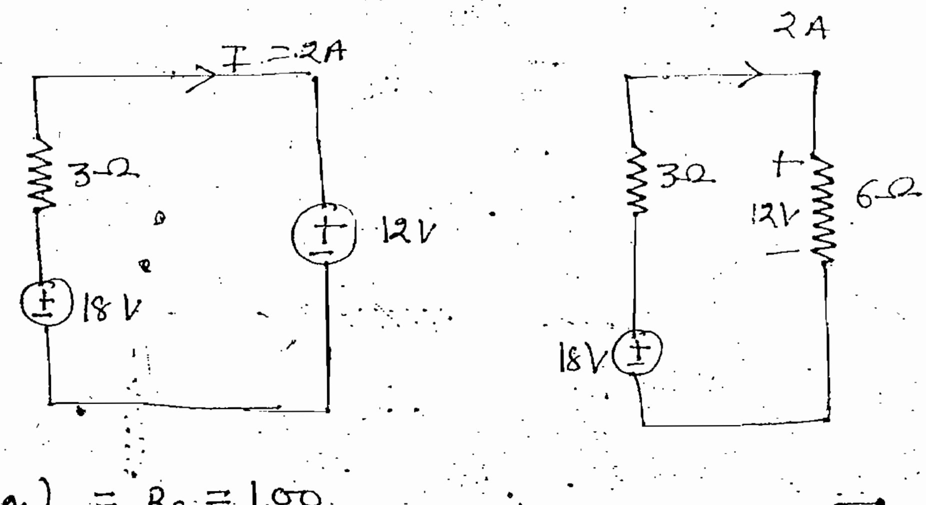


$$\boxed{R_L = R_{th} = 5\Omega}$$

Ques:- Find V_a to obtain max. power from source to load.



Note:-

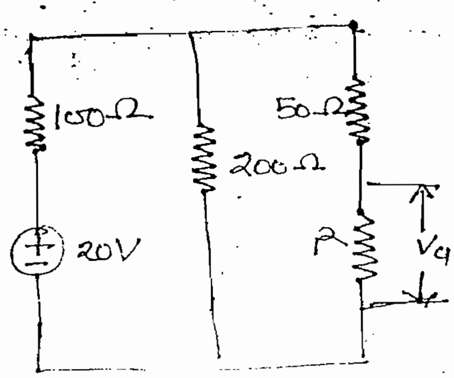


Soln:- $(R_{eq})_L = R_S = 100$

$$(R_{eq})_L = \frac{200(50+R)}{200+50+R}$$

$$\Rightarrow 100 = \frac{200(50+R)}{250+R}$$

$$\Rightarrow \boxed{R = 150\Omega}$$

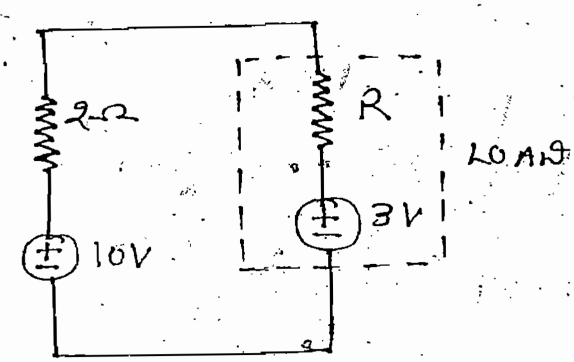


$$I_T = \frac{20}{R_S + (R_{eq.})_L}$$

$$I_T = \frac{20}{100+100} = \frac{1}{10} A$$

$$V_a = \frac{I_T}{2} R = 7.5 \text{ volts}, \text{ Ans}$$

Ques:- Find resistance R to obtain maximum power from source to load



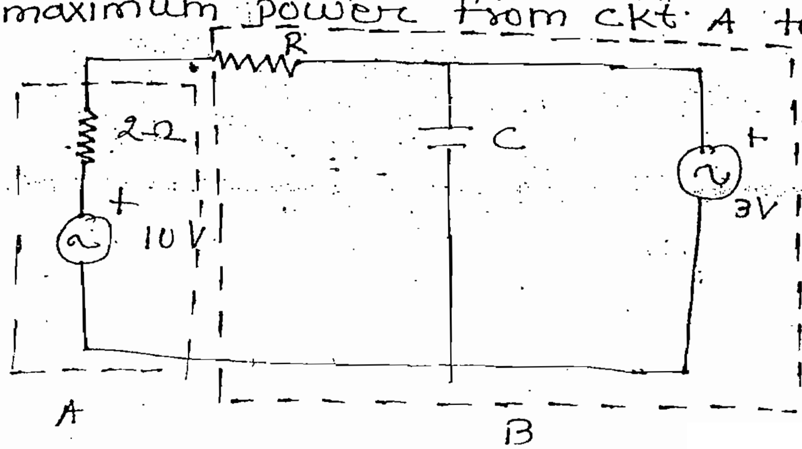
Soln:- $(R_{eq.})_L = R_S = 2\Omega$

$$I_T = \frac{10}{R_S + (R_{eq.})_L}$$

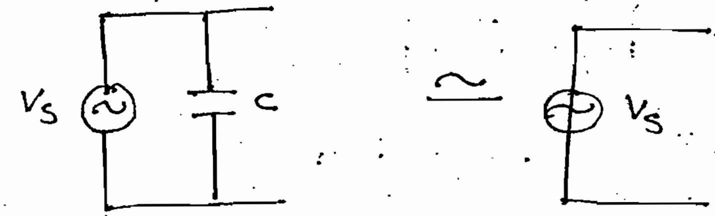
$$I_T = \frac{10}{2+2} = 2.5A$$

$$I_T = \frac{10-3}{2+R} = 2.5 \Rightarrow \boxed{R = 0.8\Omega} \text{ Ans}$$

Ques:- In the figure shown find resistance R to obtain maximum power from ckt. A to ckt. B

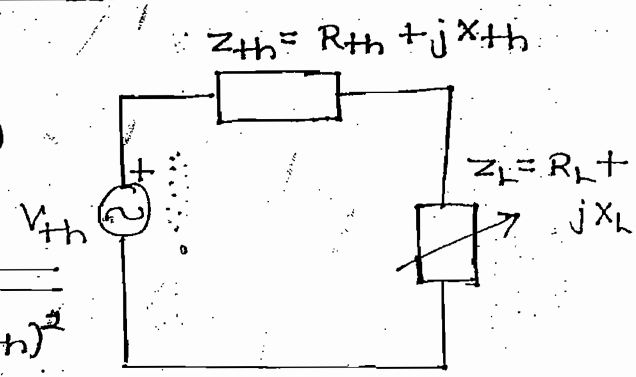


Note:-



Soln:- $R = 0.8\,\Omega$

Maximum Power Transfer theorem (For AC):-



$$i = \frac{V_{th}}{(R_L + R_{th}) + j(X_L + X_{th})}$$

$$i = \frac{V_{th}}{\sqrt{(R_L+R_{th})^2 + (X_L + X_{th})^2}}$$

$$P_L = i^2 R_L$$

$$P_L = \frac{V_{th}^2 R_L}{(R_L+R_{th})^2 + (X_L+X_{th})^2} \quad \text{—(1)}$$

→ Max Power transfer is applicable only for active power

Case (1):-

Both R_L & X_L are variable

(i) Diff. eq-(1) w.r.t R_L and equated to zero

(ii) Diff. eq-(ii) w.r.t X_L and equated to zero

we get

$$\boxed{\begin{aligned} R_L + jX_L &= R_{th} - jX_{th} \\ Z_L &= Z_{th}^* \end{aligned}}$$

Putting these values in eq-(1), we get

$$\boxed{\begin{aligned} P_{max} &= \frac{V_{th}^2}{4R_L} \\ \eta &= 50\% \end{aligned}}$$

Case (II) :—

Only R_L is variable . (X_L = constant)

→ Diff. eq-(1) w.r.t R_L and equated to zero

$$R_L = |Z_{th} + jX_L| = |R_{th} + j(X_{th} + X_L)|$$

$$R_L = \sqrt{R_{th}^2 + (X_L + X_{th})^2}$$

$$\eta = \frac{R_L}{R_L + R_{th}} \times 100$$

$$\boxed{\eta \geq 50\%}$$

Case (III) :—

Load impedance is only resistive ($X_L = 0$)

$$P_L = \frac{V_{th}^2 R_L}{(R_L + R_{th})^2 + X_{th}^2} \quad \text{—(II)}$$

Diff eq-(II) w.r.t R_L and equated to zero, we get

$$R_L = \sqrt{R_{th}^2 + X_{th}^2}$$

$$R_L = |Z_{th}| \qquad R_L > R_{th}$$

$$\boxed{\eta > 50}$$

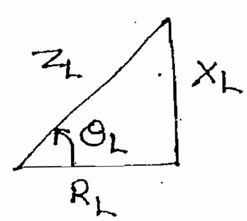
Case-(IV) :— Both R_L & X_L → Variable But load impedance angle → constant

$$Z_L = R_L + jX_L$$

load impedance angle $= \theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right) = \text{constant}$

$$R_L = Z_L \cos\theta_L$$

$$X_L = Z_L \sin\theta_L$$



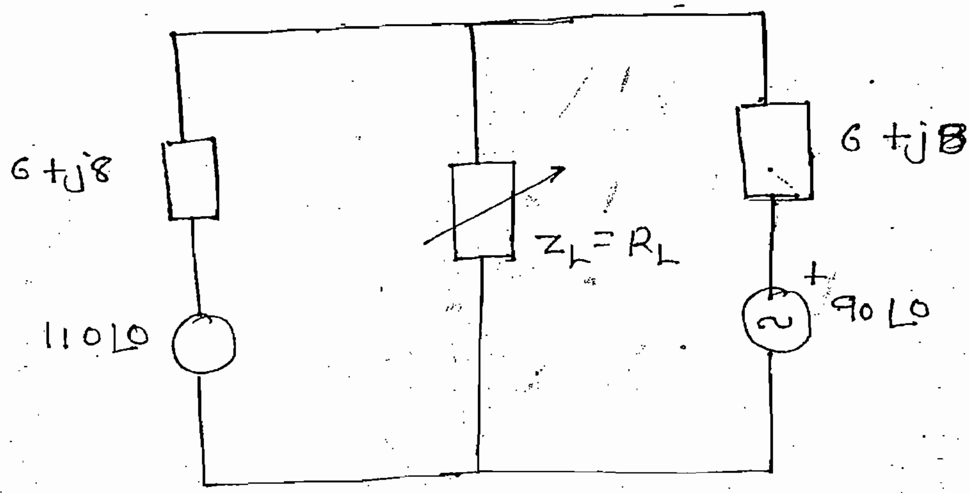
4A

$$P_L = \frac{V_{th}^2 Z_L \cos\theta_L}{(Z_L\cos\theta_L + R_{th})^2 + (Z_L \sin\theta_L + X_{th})^2} \quad \text{--- (iii)}$$

Diff. eq-(iii) w.r.t Z_L and equated to zero, we get

$$\boxed{Z_L = Z_{th}}$$

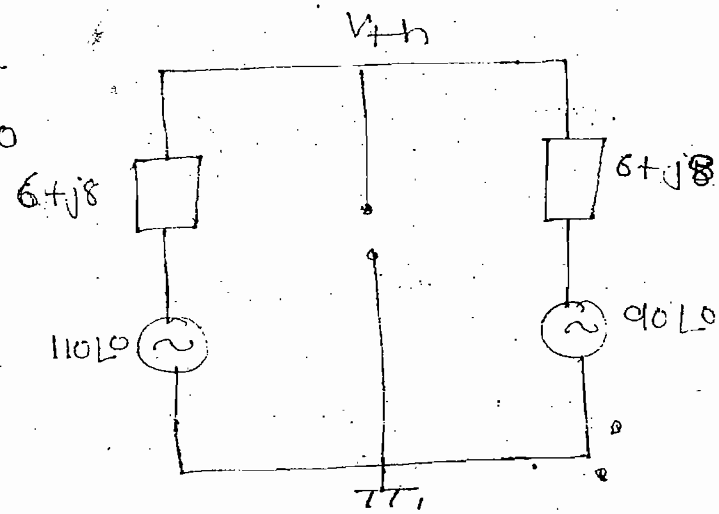
Ques:- Find max. power dissipation in load impedance



Soln:- Case-(i):- (V_{th})

$$\frac{V_{th} - 110\angle 0^\circ}{6+j8} + \frac{V_{th} - 90\angle 0^\circ}{6+j8} = 0$$

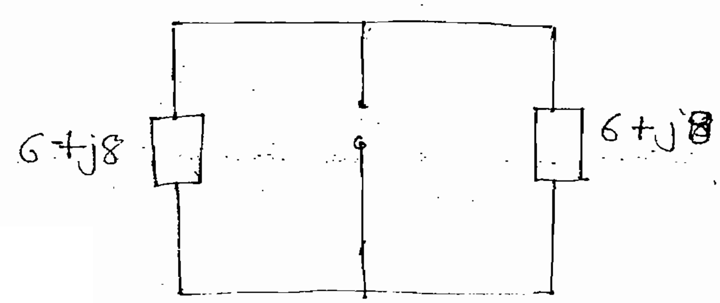
$$V_{th} = 100\angle 0$$



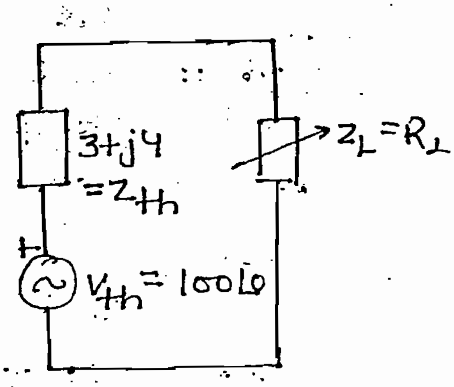
Case-(ii) (Z_{th}):-

$$Z_{th} = \frac{6+j8}{2}$$

$$Z_{th} = 3+j4$$



$$R_L = \sqrt{R_{th}^2 + X_{th}^2} = \sqrt{3^2+4^2} = 5$$

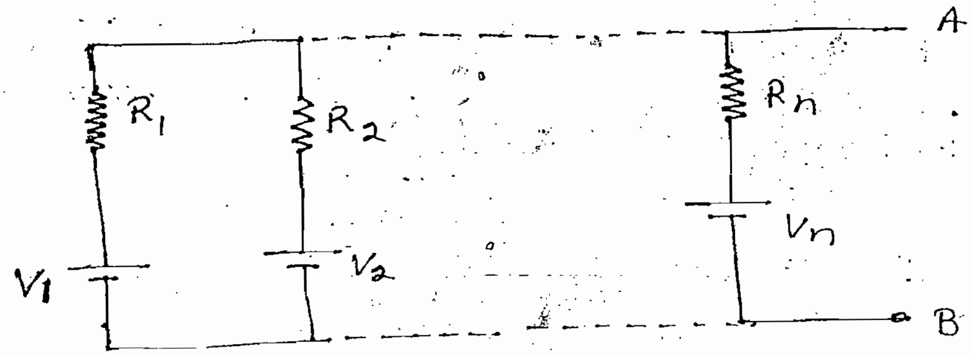


$$i = \frac{100\angle 0}{3+j4+5} = \frac{100}{8+j4}$$

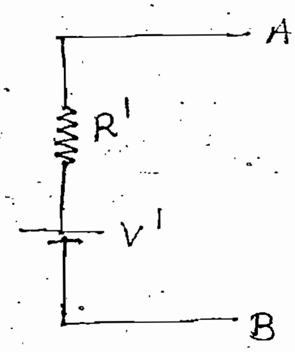
$$= \frac{100}{\sqrt{8^2+4^2}}$$

$$P_L = i^2 R_L \Rightarrow \boxed{P_L = 625\,W}$$

Millman's theorem :-



$$R' = R_{th}$$



$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots\cdots + \frac{1}{R_n}$$

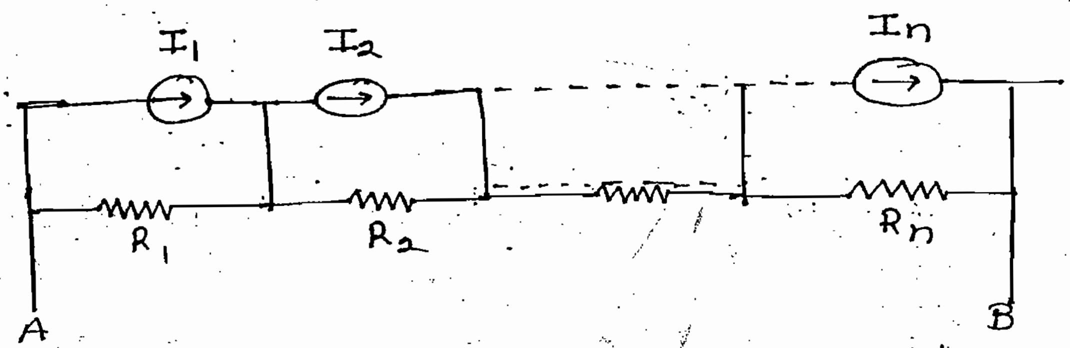
$$\boxed{\begin{aligned} R' &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} \\ R' &= \frac{1}{G_1 + G_2 + \cdots\cdots + G_n} \end{aligned}}$$

$$V_{th} = I_{sc} R_{th}$$

$$V' = I' R'$$

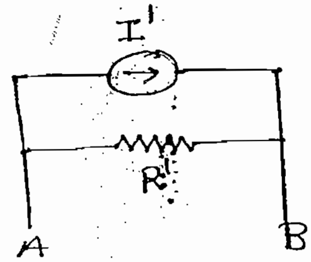
49 $$V' = \frac{V_1/R_1 + V_2/R_2 + \cdots\cdots + V_n/R_n}{1/R_1 + 1/R_2 + \cdots\cdots + 1/R_n}$$

$$\boxed{V' = \frac{V_1 G_1 + V_2 G_2 + \cdots\cdots + V_n G_n}{G_1 + G_2 + \cdots\cdots + G_n}}$$



$$R' = R_{th}$$

$$\boxed{R' = R_1 + R_2 + \cdots\cdots + R_{th}}$$

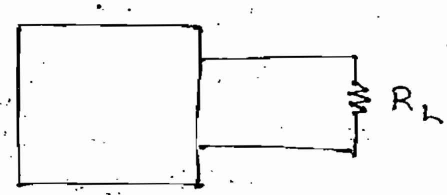


$$I_{sc} = \frac{V_{oc}}{R_{th}}$$

$$I' = \frac{V'}{R'}$$

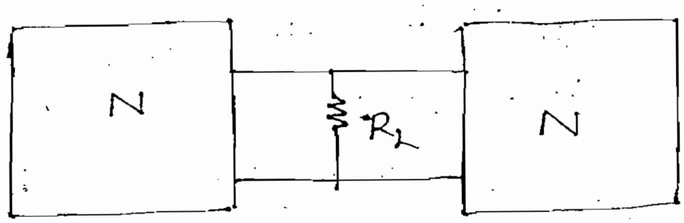
$$\boxed{I' = \frac{I_1 R_1 + I_2 R_2 + \cdots\cdots + I_n R_n}{R_1 + R_2 + \cdots\cdots + R_n}}$$

Ques:- When complex N/w of N is connected to load resistor power dissipation in the load resistor is P Watts. When two identical complex N/w of N are connected to same load resistor. Find power dissipation in the load resistor



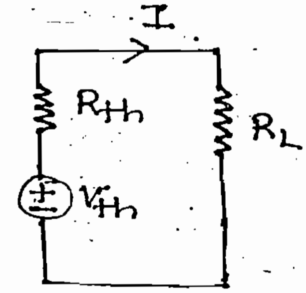
(a) P (b) 2P

(c) 4P (d) P to 4P



Soln:-

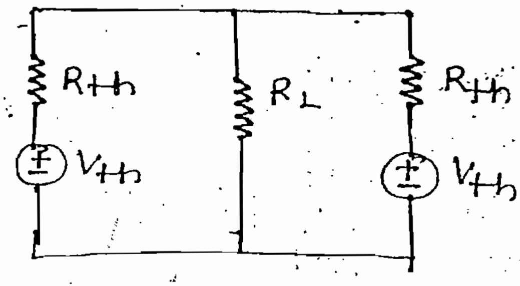
$$I = \frac{V_{th}}{R_L + R_{th}}$$



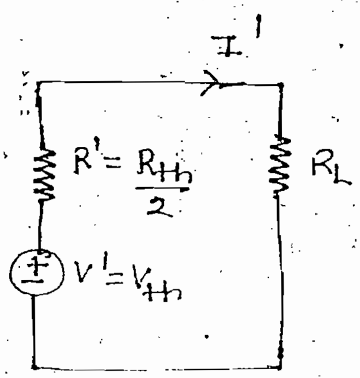
$$P = I^2 R_L$$

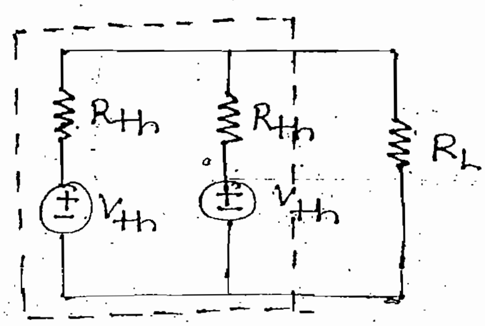
$$P = \left(\frac{V_{th}}{R_L + R_{th}}\right)^2 R_L \quad \text{— (1)}$$

$$V' = \frac{\frac{V_{th}}{R_{th}} + \frac{V_{th}}{R_{th}}}{\frac{1}{R_{th}} + \frac{1}{R_{th}}} = V_{th}$$



$$R' = \frac{1}{1/R_{th} + 1/R_{th}} = \frac{R_{th}}{2}$$



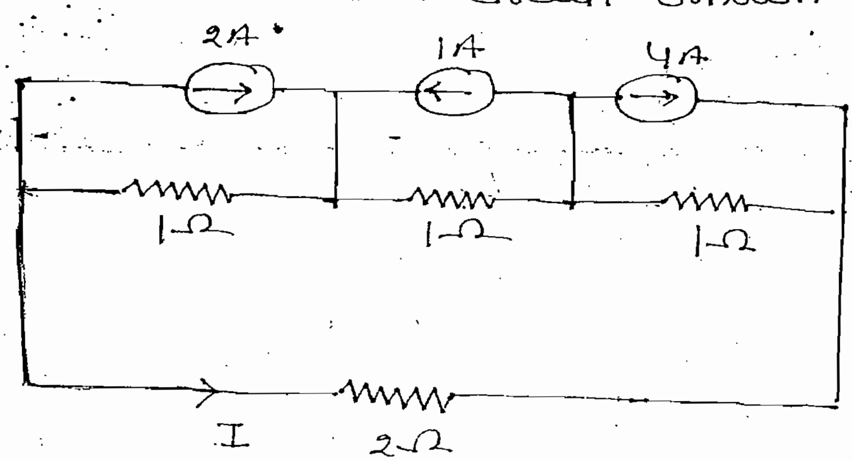


$$I' = \frac{V_{th}}{R_L + \frac{R_{th}}{2}} = \frac{2V_{th}}{2R_L + R_{th}}$$

$$P' = I'^2 R_L = \left(\frac{2V_{th}}{2R_L + R_{th}}\right)^2 R_L \quad \text{— (II)}$$

Comparing eq-(1) & (II) we get option-d

Ques:- Find I of the circuit shown

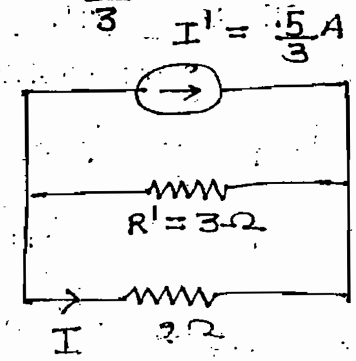


soln:-

$$I' = \frac{(2\times1) - (1\times1) + (4\times1)}{1+1+1} = \frac{5}{3}$$

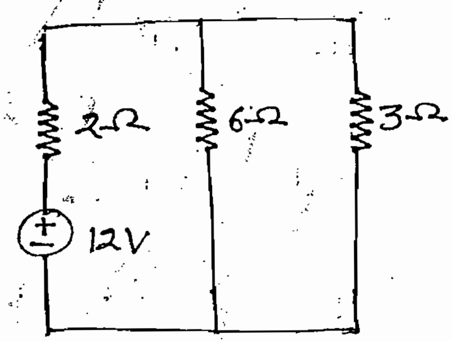
$$R' = 1+1+1 = 3\,\Omega$$

$$I = -\frac{5}{3}\,\frac{3}{3+2} = -1\,A$$



Tellegen's theorem:-

$$\sum_{k=1}^{n} V_k i_k = 0$$



Tellegen's theorem states that algebraic sum of the powers in any circuit at any instant is equal to zero (linear, non-linear, unidirectional, bidirectional, time variant and time invariant elements)

$$R_{eq} = 2 + \frac{6\times3}{6+3} = 4$$

$$I_T = \frac{12}{4} = 3A$$

$$I_6 = 3\,\frac{3}{3+6} = 1A \qquad I_3 = 3-1 = 2A$$

$$V_2 = I_T \times 2 = 6$$

$$V_6 = V_3 = 2\times3 = 6$$

$$V_2 I_T + V_6 I_6 + V_3 I_3 - V_S I_T$$

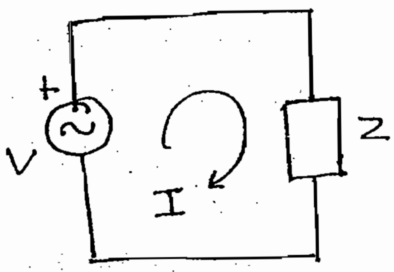
$$18 + 6 + 12 - 12\times3 = 0$$

Note:-

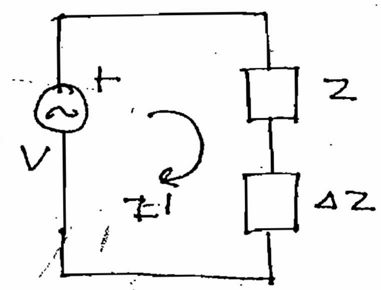
→ Tellegen's theorem is verified by using KCL and KVL equations

→ Tellegen's theorem works based on the principle of law of conservation of energy.

Compensation theorem:–



$$I = \frac{V}{Z}$$



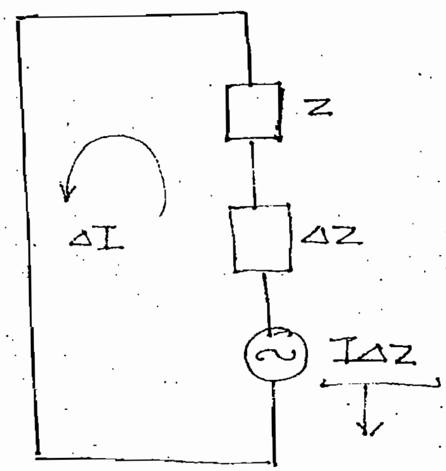
$$I' = \frac{V}{Z+\Delta Z}$$

$$\Delta I = |I - I'|$$

(Clockwise)

$$\boxed{\Delta I = \frac{-I\Delta Z}{Z+\Delta Z}}$$

Modified Circuit:–



Compensation emf
or
opposing emf