EXERCISE 13.3 - PROBABILITY.

Q No 1 An usn ball contains 5 red and 5 black balls. A ball is drawn at random, it's colour is noted and is returned to urn. Moreover, 2 additional balls of colour drawn are put in the urn and then a ball is drawn at random. What is the probability that 2nd ball is red?

Sol. No. of Red balls = 5 ; No. of black balls = 5

Let A: Second drawn ball is red.

B: Drawing first ball as red and adding two red balls.

C: Drawing first ball as black and adding two black ball.

By Total Probability theorem, Required probability

$$= P(A) = P(B)\,P(A|_B) + P(C)\,P(A|_C)$$

$$= \frac{5}{10} \times \frac{7}{12} + \frac{5}{10} \times \frac{5}{12}$$

$$= \frac{35}{120} + \frac{25}{120} = \frac{35+25}{120} = \frac{60}{120} = \frac{1}{2}$$

Q No 2. A bag contains 4 Red and 4 black balls another bag contains 2 red and 6 black balls. One of two bags is selected at random and a ball is drawn from bag which is found to be red. find the probability that ball is drawn from first bag.

Soln: Let E: Ball drawn is Red.

E_1: First bag is selected

E_2: Second bag is selected.

Bag I	Bag II
4R	2R
4B	6B

Since E_1, E_2 are mutually exclusive and exhaustive

with $P(E_1) = P(E_2) = \frac{1}{2}$

$P(E|_{E_1}) = P($ Drawing Red ball from first bag $= \frac{4}{8} = \frac{1}{2}$

$P(E|_{E_2}) = P($ Drawing Black ball from Second bag $= \frac{2}{8} = \frac{1}{4}$.

∴ By Baye's Theorem

$$P(E_1|_E) = \frac{P(E|_{E_1})P(E_1)}{P(E|_{E_1})\cdot P(E_1) + P(E|_{E_2})P(E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{8}} = \frac{2}{2+1} = \frac{2}{3}$$

Q No 3 Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars. Previous year result report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random and he has A grade, what is probability that the student is a hostlier?

Sol. Let E_1 : A student residing in Hostel

E_2 : A student is day scholar.

E : A student attains A Grade.

Then $E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$ (Mutually Exclusive and exhaustive)

and $P(E_1) = \frac{60}{100} = \frac{3}{5}$

$$P(E_2) = \frac{40}{100} = \frac{2}{5}$$

$$P(E|E_1) = \frac{30}{100} = \frac{3}{10} \quad P(E|E_2) = \frac{20}{100} = \frac{2}{10}$$

By Baye's thm, Required Probability

$$P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2) \cdot P(E_2)}$$

$$= \frac{\frac{3}{10} \times \frac{3}{5}}{\frac{3}{10} \times \frac{3}{5} + \frac{2}{10} \times \frac{2}{5}} = \frac{9}{9+4} = \frac{9}{13}$$

Q No 4. In answering a question on multiple choise test, a student either knows the answer or guesses. Let $3/4$ be the probability that he knows the answer and $1/4$ be the probability that he guesses. Assuming that a student who guesses at answer will be correct with probability $\frac{1}{4}$. What is the probability that he knows the ans. given that he answered it correctly.

Sol. Let E_1: The student knows the answer
E_2: The student guesses the answer
E: The answer is correct.

The E_1 and E_2 are mutually exclusive and exhaustive.
and $P(E_1) = \frac{3}{4}$; $P(E_2) = \frac{1}{4}$

and The $P(E|E_1) = 1$ ($\because$ When student knows the ans it is sure that ans. is correct)

$P(E|E_2) = \frac{1}{4}$

Then By Baye's theorem, Required probability

$$= P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{1 \times \frac{3}{4}}{1 \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}} = \frac{\frac{3}{4}}{\frac{12+1}{16}} = \frac{3}{4} \times \frac{16}{13} = \frac{12}{13}$$

QNo 5 A laboratory blood test is 99% effective in detecting a certain disease when it is infact, present. However the test also yields a false +ve result for 0.5% of healthy person tested (ie if a healthy person is tested, then the probability 0.005, the test will imply he has the disease). If 0.1 percent of population actually has the disease, what is the probability that a person has disease given that the test result is +ve?

Soln. Let E_1: A person is suffering from disease
E_2: A person is not suffering from disease.
E: Test yields a +ve result.

Now Since E_1 and E_2 are mutually exclusive and exhaustive with $P(E_1) = 0.1\% = \frac{1}{1000}$

$P(E_2) = 1 - \frac{1}{1000} = \frac{999}{1000}$

$P(E|E_1) = 99\% = \frac{99}{100}$ $P(E|E_2) = 0.5\% = \frac{5}{1000}$

∴ By Baye's theorem, Required Probability
= P(diagosed person has disease when test result is given to be +ve) = $P(E_1|E)$

$$= \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)} = \frac{\frac{99}{100} \times \frac{1}{1000}}{\frac{99}{100} \times \frac{1}{1000} + \frac{5}{1000} \times \frac{999}{1000}}$$

$$= \frac{990}{990 + 4995} = \frac{990}{5985} = \frac{22 \times 45}{133 \times 45} = \frac{22}{133}$$

QNo6 There are three coins, One is two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows head, what is the probability that it is two headed coin.

Sol. Let E : tossed coin shows head.
E_1 : two headed coin is chosen.
E_2 : biased head showing head 75% times is chosen
E_3 : Unbiased coin is chosen.
Here E_1, E_2, E_3 are mutually exclusive and exhaustive
and $P(E_1) = P(E_2) = P(E_3) = 1/3$

and $P(E|E_1) = P$(Two headed coin showing up head) = 100%
$= 1$
$P(E|E_2) = P$(biased coin showing up head) = $75\% = \frac{75}{100}$
$= 3/4$
$P(E|E_3) = P$(Unbiased coin showing up head) = $1/2$

∴ Required Prob. = P(tossed coin is two headed when it is given that head shows up)

$$P\left(E_1|E\right) = \frac{P(E|E_1)\,P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2) + P(E|E_3)P(E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{3}{4} + \frac{1}{3}\times\frac{1}{2}}$$

$$= \frac{1}{1+\frac{3}{4}+\frac{1}{2}} = \frac{4}{4+3+2} = \frac{4}{9}$$

Q No 7. An Insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The prob. of an accident are 0.01, 0.03, 0.15 respectively. One of the insured person meets with an accident. What is the prob. that he is scooter driver?

Sol. Let E_1 : Insured person is scooter driver
E_2 : Insured person is car driver
E_3 : Insured person is truck driver.
E : Insured person meets with an accident.

Since E_1, E_2, E_3 are mutually exclusive and exhaustive

and $P(E_1) = \frac{2000}{2000+6000+4000} = \frac{2}{12} = \frac{1}{6}$

$P(E_2) = \frac{4000}{12000} = \frac{1}{3}$ and $P(E_3) = \frac{6000}{12000} = \frac{1}{2}$

$P(E|E_1) = 0.01 = \frac{1}{100}$

$P(E|E_2) = 0.03 = \frac{3}{100}$

$P(E|E_3) = 0.15 = \frac{15}{100}$

$\therefore$ Required Prob. = P(Insured person who meets an accident is a scooter driver)

$$P(E_1|E) = \frac{P(E|E_1)\cdot P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2) + P(E|E_3)P(E_3)}$$

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100} + \frac{1}{3}\times\frac{3}{100} + \frac{1}{2}\times\frac{15}{100}} = \frac{1}{1+6+45} = \frac{1}{52}$$

Q No 8 A factory has two machines, A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one one stockpile and then one item is chosen at random from this and is found to be defective. What is the prob. that it was produced by machine B?

Sol. Let E_1 : Item produced by machine A

E_2 : Item produced by machine B

E : Item chosen is defective.

Now since E_1 and E_2 are mutually exclusive and exh. with

$$P(E_1) = \frac{60}{100} = \frac{3}{5}$$

$$P(E_2) = \frac{40}{100} = \frac{2}{5}$$

$$P(E|E_1) = \frac{2}{100} \qquad P(E|E_2) = \frac{1}{100}$$

$$\therefore \text{Required Prob.} = P(E_2|E) = \frac{P(E|E_2)\,P(E_2)}{P(E|E_2)\,P(E_2) + P(E|E_1)\,P(E_1)}$$

$$= \frac{\frac{1}{100} \times \frac{2}{5}}{\frac{2}{100} \times \frac{3}{5} + \frac{1}{100} \times \frac{2}{5}} = \frac{2}{6+2} = \frac{1}{4}$$

Q No 9. Two groups are competing for position on the Board of directors of a corporation. The probabilities that the first and the second group will win are 0.6 and 0.4 resp. Further, if the 1st group wins the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second gp. wins. Find the prob. that new product introduced was by 2nd gp.

Sol. Let E_1 : First gp wins

E_2 : Second gp wins.

E : New product is introduced.

E_1 and E_2 are mutually exclusive and exhaustive.

and $P(E_1) = 0.6 = \frac{6}{10}$; $P(E_2) = 0.4 = \frac{4}{10}$

$P(E|E_1) = 0.7 = \frac{7}{10}$; $P(E|E_2) = 0.3 = \frac{3}{10}$

Then Required Prob.

$$= P(E_2|E) = \frac{P(E|E_2).P(E_2)}{P(E|E_2)(P(E_2) + P(E|E_1)P(E_1)}$$

$$= \frac{\frac{3}{10} \times \frac{4}{10}}{\frac{7}{10}\times\frac{6}{10} + \frac{3}{10}\times\frac{4}{10}} = \frac{12}{42+12} = \frac{12}{54} = \frac{2}{9}$$

QNo10 Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the prob. that she threw 1, 2, 3 or 4 with die.

Sol. Let E_1 : Die shows 1, 2, 3 or 4.

E_2 : Die shows 5 or 6.

E : Exactly one head shows up.

Now E_1 and E_2 are mutually exclusive and exhaustive

$P(E_1) = \frac{4}{6} = \frac{2}{3}$; $P(E_2) = \frac{2}{6} = \frac{1}{3}$

$P(E|E_1) = \frac{1}{2}$; $P(E|E_2) = P(HTT, THT, TTH) = \frac{3}{8}$

$\therefore$ Required probability $= P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$

$$= \frac{\frac{1}{2}\times\frac{2}{3}}{\frac{1}{2}\times\frac{2}{3} + \frac{3}{8}\times\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{8}} = \frac{\frac{1}{3}}{\frac{8+3}{24}} = \frac{8}{11}$$

Q.No 11. A manufaturer has three machine operator A, B and C. The first operator A produces 1% defective item, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on job for 50% of the time B is on job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Sol. Let E_1 : an item is produced by operator A
E_2 : an item is produced by operator B
E_3 : an item is produced by operator C
E : A defective item is produced.

Since E_1, E_2 and E_3 are mutually exclusive and exhaustive

$$P(E_1) = 50\% = \frac{50}{100} = \frac{5}{10}$$

$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(E_3) = 20\% = \frac{20}{100} = \frac{2}{10}$$

$P(E|E_1) =$ P(an item produced by operator A is defective)

$$= 1\% = \frac{1}{100}$$

$$P(E|E_2) = 5\% = \frac{5}{100}$$

$$P(E|E_3) = 7\% = \frac{7}{100}$$

$\therefore$ Required probability = P(an item, given to be defective, is produced by operator A)

$$P(E_1|E) = \frac{P(E|E_1)\,P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2) + P(E|E_3).P(E_3)}$$

$$= \frac{\frac{5}{10}\times\frac{1}{100}}{\frac{5}{10}\times\frac{1}{100} + \frac{3}{10}\times\frac{5}{100} + \frac{2}{10}\times\frac{7}{100}} = \frac{5}{5+15+14} = \frac{5}{34}.$$

QNo 12. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of lost card being diamond.

Sol. Let E_1 : Lost card is a diamond

E_2 : Lost card is a heart

E_3 : Lost card is a spade

E_4 : Lost card is a club

E : The card drawn are diamonds

$$P(E_1) = P(E_2) = P(E_3) = P(E_4) = 13/52 = 1/4$$

$$P(E|E_1) = \frac{12}{51} \times \frac{11}{50} = \frac{22}{425}$$

$$P(E|E_2) = \frac{13}{51} \times \frac{12}{50} = \frac{26}{425}$$

$$P(E|E_3) = \frac{13}{51} \times \frac{12}{50} = \frac{26}{425}$$

$$P(E|E_4) = \frac{13}{51} \times \frac{12}{50} = \frac{26}{425}$$

Then Required probability $= P(E_1|E)$

$$= \frac{P(E|E_1) \cdot P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2) + P(E|E_3)P(E_3) + P(E|E_4)P(E_4)}$$

$$= \frac{\frac{1}{4} \times \frac{22}{425}}{\frac{1}{4} \times \frac{22}{425} + \frac{1}{4} \times \frac{26}{425} + \frac{1}{4} \times \frac{26}{425} + \frac{1}{4} \times \frac{26}{425}}$$

$$= \frac{22}{100} = \frac{11}{50}.$$

QNo 13. The prob. that A speaks truth is 4/5. A coin is tossed. A reports that 'a head' appears. The probability that actually there was head is

(A) $\frac{4}{5}$ (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$

Sol.. Let E_1 : Head comes up.
E_2 : Tail comes up.
E : A reports that head appears.

Clearly $P(E_1) = P(E_2) = \frac{1}{2}$.

$P(E|E_1) = P(\text{Head comes up and A speaks truth})$
$= 4/5$

$P(E|E_2) = P(\text{tail comes up and A tells a lie})$
$= \frac{1}{5}$

$\therefore$ Required prob. $= P(E_1|E)$

$$= \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$$

$$= \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{1}{2} + \frac{1}{5} \times \frac{1}{2}} = \frac{4}{4+1} = \frac{4}{5}$$

$\therefore$ (A) is correct option.

QNo 14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$ then which of following is correct.

(A) $P(A|B) = \frac{P(B)}{P(A)}$ (B) $P(A|B) < P(A)$ (C) $P(A|B) \geq P(A)$ (D) None of these

Sol.. When $A \subset B$ the $A \cap B = A$

$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \geq P(A)$

$\because 0 < P(B) \leq 1$

$\Rightarrow P(B) \leq 1$

$\Rightarrow \frac{1}{P(B)} \geq 1$

$\Rightarrow \frac{P(A)}{P(B)} \geq P(A)$

$\therefore$ (C) is Correct option

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