"Cams"

Cams:- A cam is rotating or raciprocating member of a mechanism use to impact rotating, reciprocating or oscillating motion to another member called follower through direct contect.

→ Cam and Follower constitutes a higher pair.

Necessary element of a cam mechanism are.

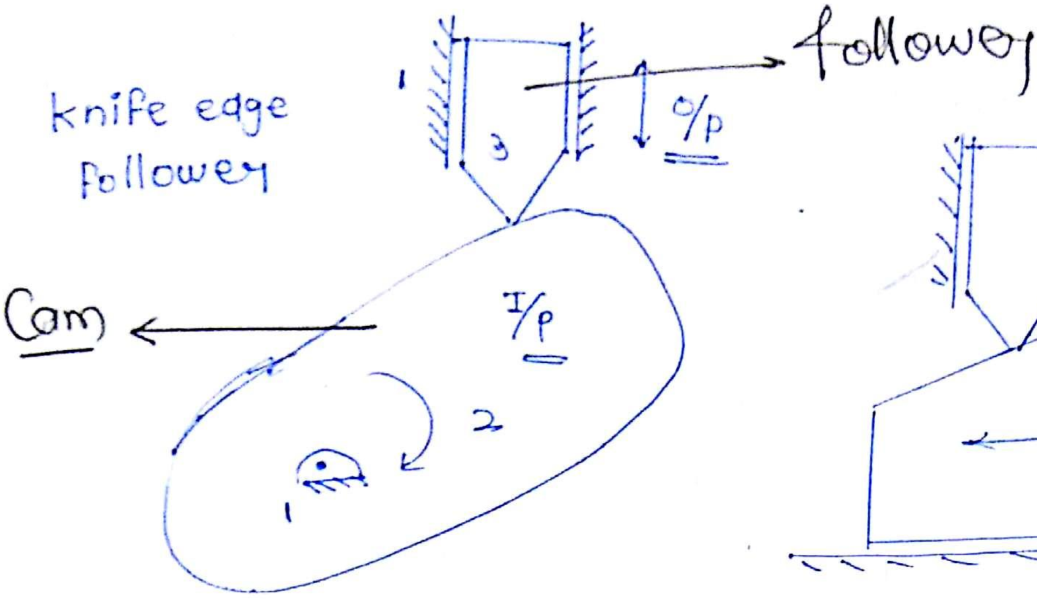
(1) A drive member Cam
(2) A driven member called follower.
(3) A frame which supports the cam and guides the follower

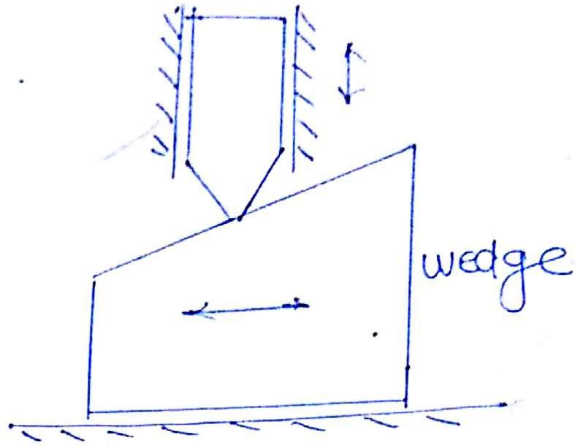
Advantages

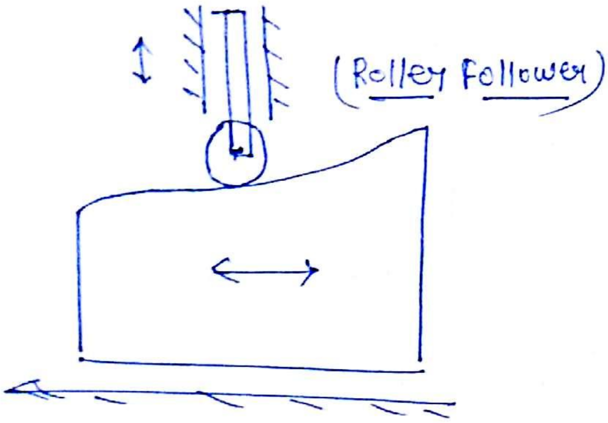
(1) less space require than the lower pair adjustment
(2) Input & output link are directly Connected so has to govern the motion of o/p only single i/p have to be designed,

Disadvantage

(1) No reversible
(2) High wear, because area of Contact where friction force is acting is very small



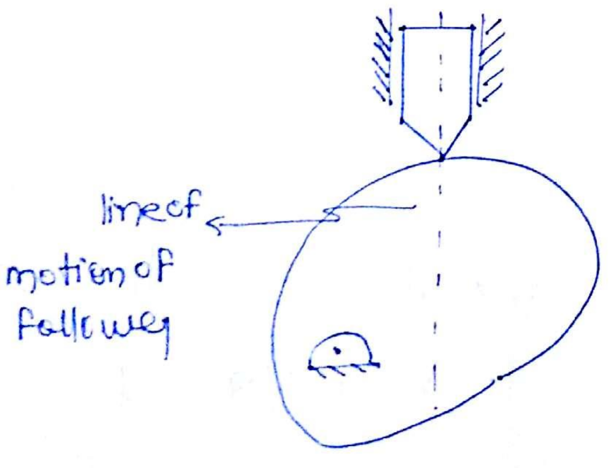




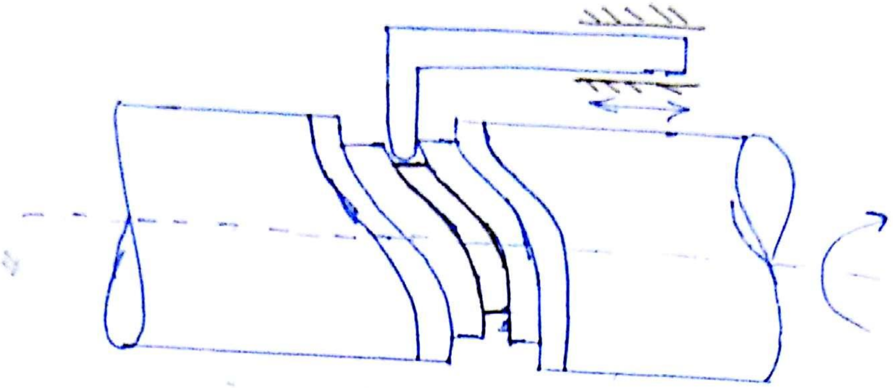
→ Types of cams
→ Types of follower

Types of Cams :-

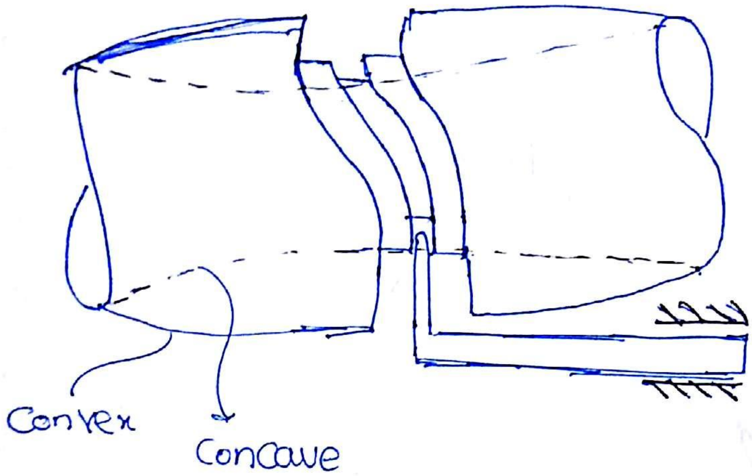
1) Radial/ Disc cam :- line of motion of follower is ⊥er to the axis of rotation of cam.



(ii) Cylinder Cam:- line of moment/Motion of follower is parallel to the axis of rotation of cam.

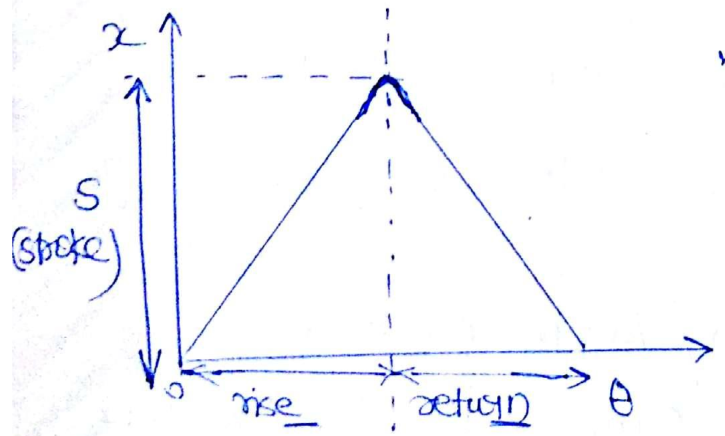


Globoidal Cam — Concave / Convex

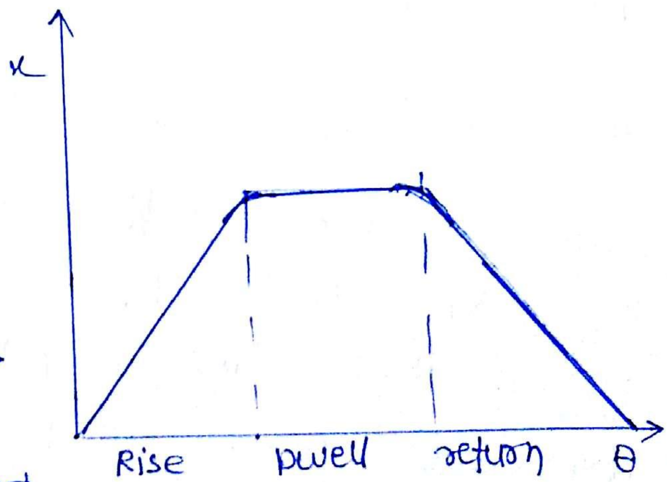


According to Follower movement:-

i) Rise- Return Cam :-



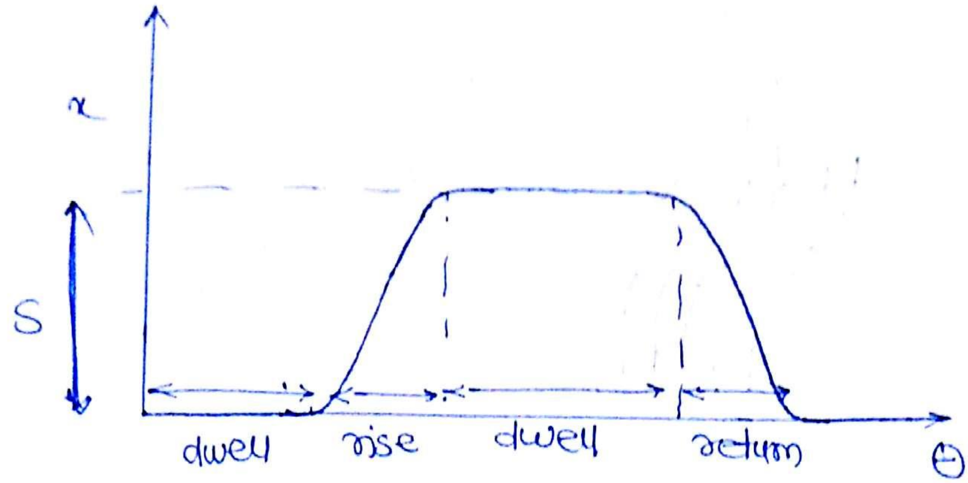
(ii) Rise - Dwell - Return Cam :



x - follower displacement

θ - Angle of Cam rotation.

(iii) Dwell - rise - Dwell - return cam :-



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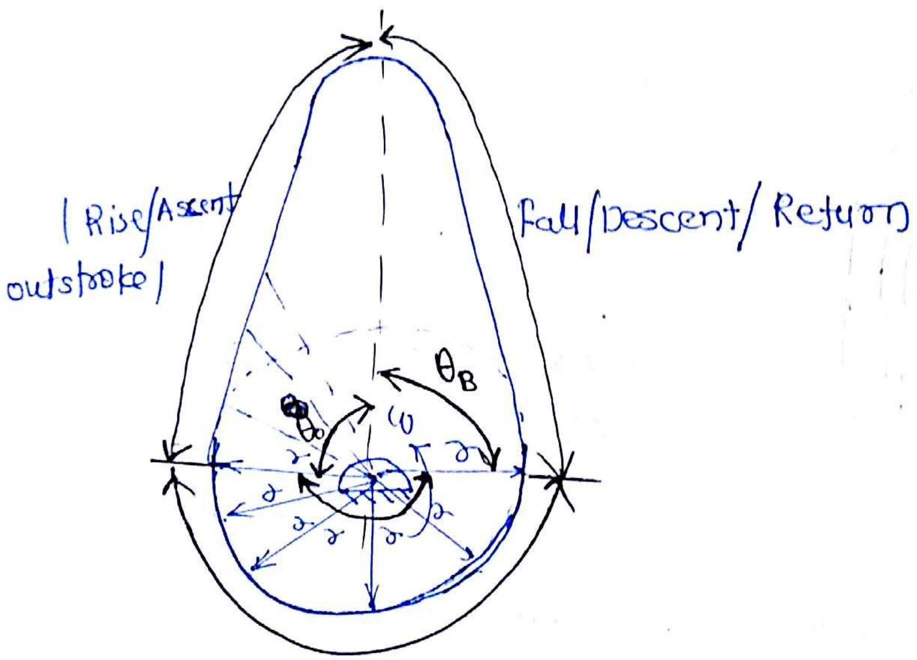


Angles of Cam :-

1) Outstroke Angle :- (θ_o) / Angle of ascent:

Angle through which cam rotates for the rise of follower.

2) Return stroke Angle (θ_R) / Angle of Descent

The angle through which cam rotates for the fall/return of follower.

(iii) Dwell (δ) :- The angle of cam rotation for which follower is in rest

(iv) Angle of Action (θ_a) :- it is total angle moved by the cam during the time b/w begaining of rise and the end of retuon of follower.

$$\theta_a \le 2\pi$$

Classification of Follower.

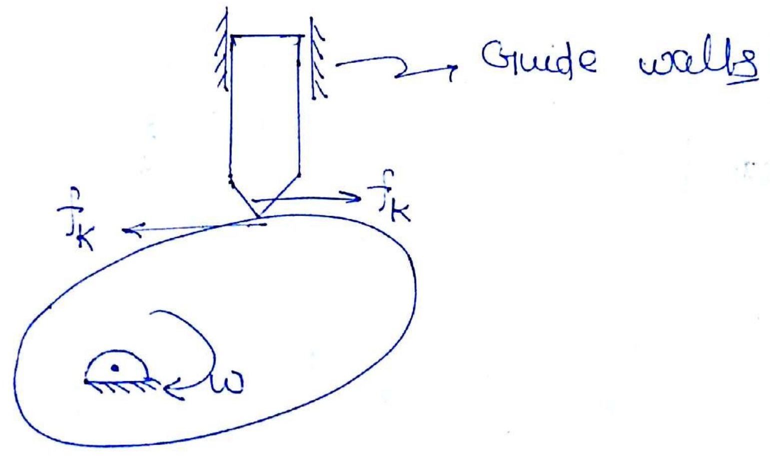
① Type of ~~moment~~ movement

→ Reciprocation

→ Oscilating

② Acc. to type of Contact

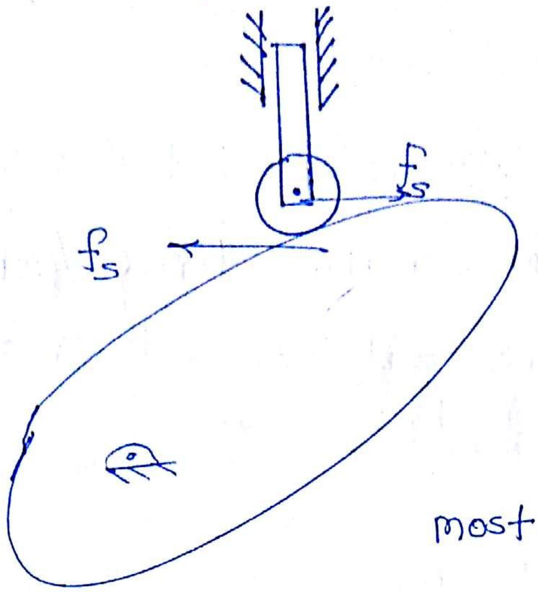
(a) Knife edge :- Not practically used due to excessive wear.



Disadvantage :- → Excessive wear because of very sharp Contact point

→ Heavy side thrust because of f_k (kinetic friction) against the guide wall of the follower.

(b) Roller follower :-



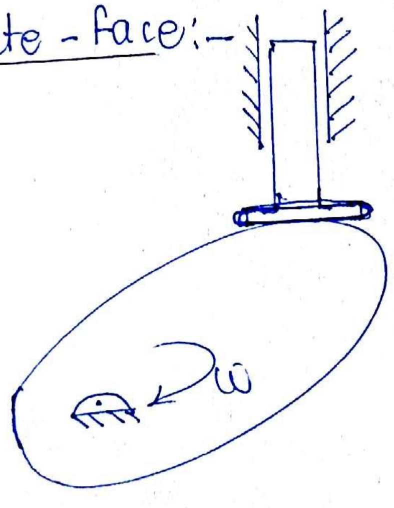
→ wear excesively reduced.
→ side thrust is reduced as compared to knife edge follower but considarable because $f_s < f_k$ for most of it's range.

→ wide application in aircraft engines.

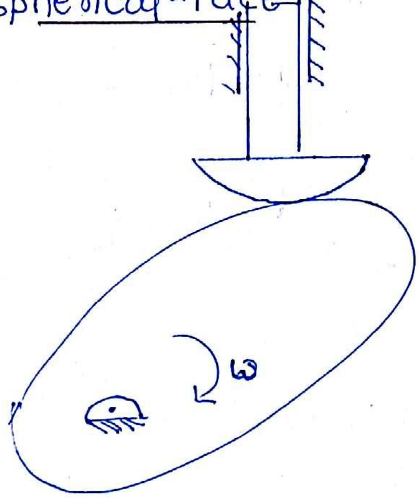
it requires more space because of roller dimension i.e. why not used in IC engine

(C) Mushroom follower :-

Flate - face :-



spherical - Face



Flate face :- Contact area as compare to knife edge is increase so wear is highly reduced compared to knife edge. but surface stress are generated.

disadvantage: surface stresses on cam profile which result in distortion of cam profile thats why insteade to flate-face, spherical face follower are used

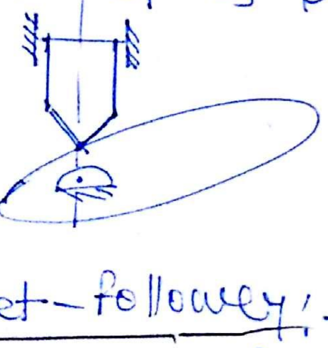
Spherical :- There are design to reduce surface contact which intorn to reduce surface stess.

- surface stess are minimise.
- wear slightly more than flate face follower (less contact area)

Note: Further slight reduction can be obtaine by offsetting the follower. (side thrust also reduce)

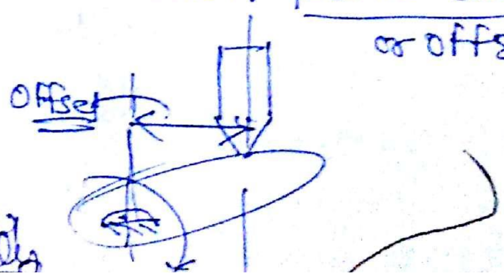
③ According to location of line of movement of follower

(a) Radial follower :- If line of motion passes through centre of rotation of cam, such a follower is known as radial follower.



(b) offset-follower :- if line of motion passes away or offset of from the centre of rotation of cam.

- avoids jamming of follower
- reduces wear, distributes wear uniformally



Cam terminology:-

Base Circle:- The smallest circle drawn from the centre of rotation of cam forming the part of the cam profile is known as base circle the radius of this circle is called to least radius of cam.

* The size of any cam is p specified by the size of base circle

Trace point and Pitch/Trace curve: The point on the follower which is required to trace the cam profile is known as trace point and the curve traced by the trace point by keeping the cam stationary and revolving the follower around it is known as pitch curve/Trace curve.

trace point

- knife edge follower → (knife edge)
- roller follower → (center of roller)

Pressure Angle:- (ϕ) It is the angle b/w the normal to the pitch curve at a point and line of motion of follower. It represent the stiffness of the cam profile

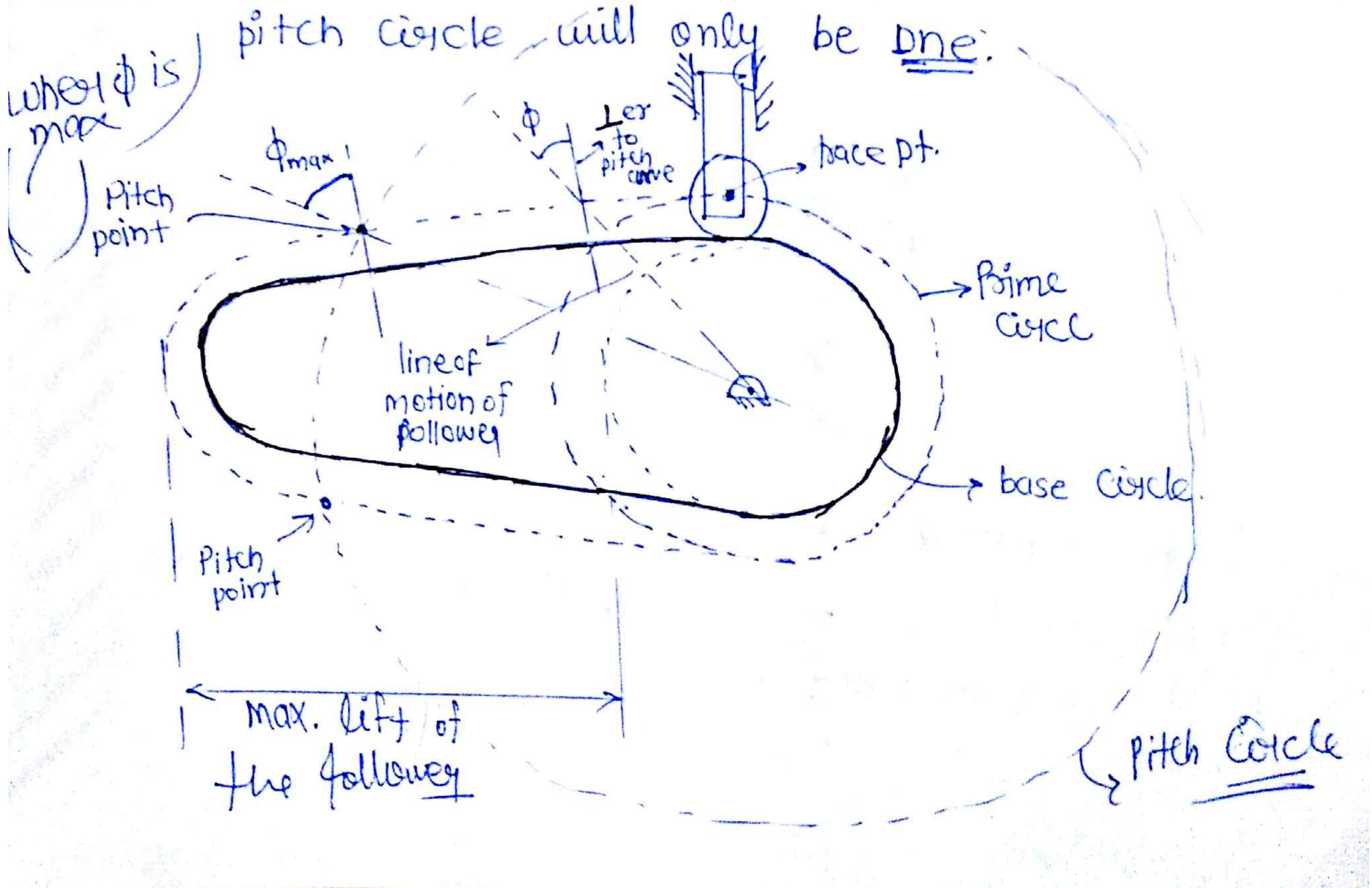
Note: ϕ varies in magnitude at all instant of follower motion a high value of max. ϕ is not desired (usually $\phi < 30^\circ$) as it might jam the follower in the bearing.

Pitch point:- The point on the pitch curve where ϕ is maximum is called pitch point.

Prime circle:- Minimum radius circle of pitch curve from the ~~vertical~~ centre of rotation of cam is known as prime circle.

Pitch circle:- It is the circle passing through the pitch point and concentric with the base circle.

Note: Pitch point can be more than one but pitch circle will only be one.



Follower Derivative:

$x \rightarrow \theta$ curve

$\omega \rightarrow$ angular vel. of Cam - constant

1st order derivative

$\frac{dx}{d\theta}$ → slope of $x-\theta$ curve

$\frac{dx}{d\theta}$ → $\frac{dx}{dt} \times \frac{dt}{d\theta} = \frac{V}{\omega}$

$\frac{dx}{dt} \propto V$ (vel. of follower)

Velocity of Cam not be very-very high that means slope of $x-\theta$ curve should not be very-very high.

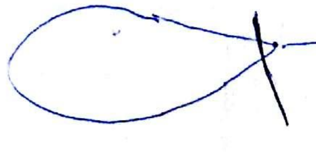
2nd order derivative

$\frac{d^2x}{d\theta^2}$ → $(x-\theta)$ curve proportion to inverse of radius of curvature of $x-\theta$ curve $\left\{\frac{1}{R} \propto \frac{d^2y}{dx^2}\right\}$

$\frac{d^2x}{d\theta^2}$ → $\frac{d^2x}{d\theta^2} \propto a$ (accn of follower)

$$\frac{d^2x}{d\theta^2} = \frac{d^2x}{(dt)^2} \times \frac{(dt)^2}{d^2\theta} = \frac{a}{\omega^2}$$

if accn of follower is ∞, then R of $x-\theta$ curve will be zero that means sharp point on cam profile

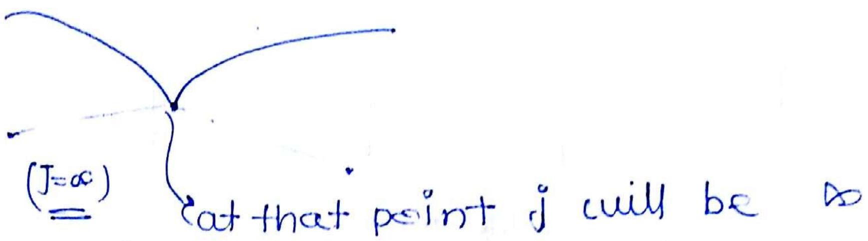


3rd order derivative

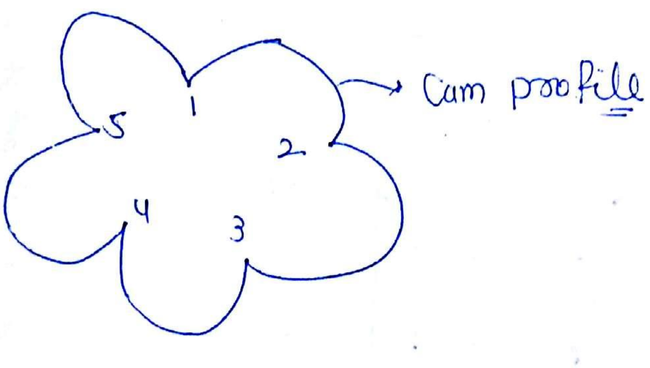
$\frac{d^3x}{d\theta^3}$ → represents $\frac{d}{d\theta}\left(\frac{d^2x}{d\theta^2}\right)$ → represent change of ~~rate of c~~ radius of curvature wrt θ

$\frac{d^3x}{d\theta^3}$ → $\frac{d^3x}{d\theta^3} \propto j$ (jerk of follower)

Jerk should not be ∞ level because if j is ∞, radius of curvature is changing at ∞ level that means profile will be like

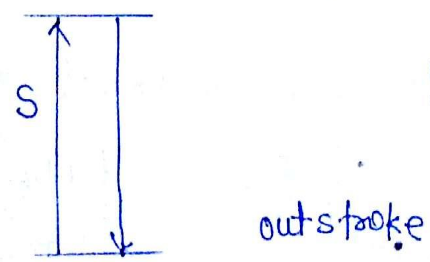


→ say 5 point are of ∞ jerk –



Types of Follower Motion :–

① Uniform Velocity :–



ω – avg. velocity of cam

θ_o → angle of outstroke

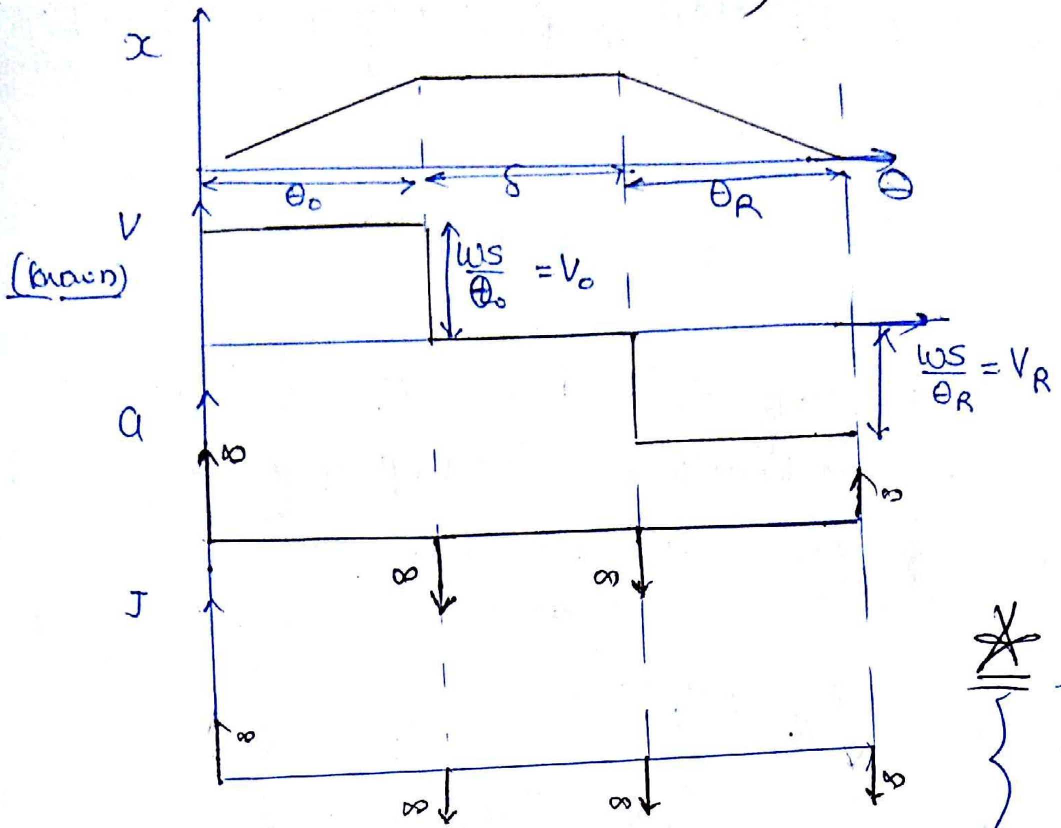
θ_R → angle of return stroke

$t_o = \theta_o / \omega$

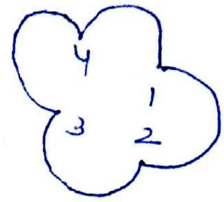
$V_o = \frac{S}{t_o} = \frac{\omega S}{\theta_o} = (V_{max})_{outstroke}$, $a_o = 0$

$V_R = \frac{\omega S}{\theta_R}$, $a_r = 0$

(ideal uniform Velocity)



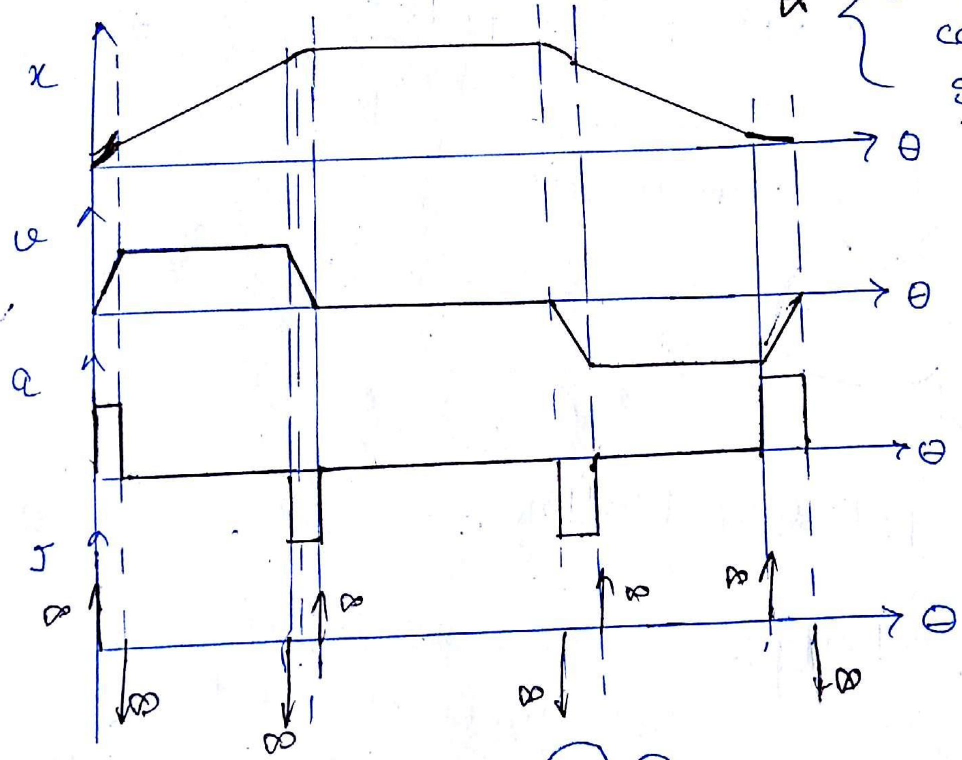
⇒ 4 point of interferance.

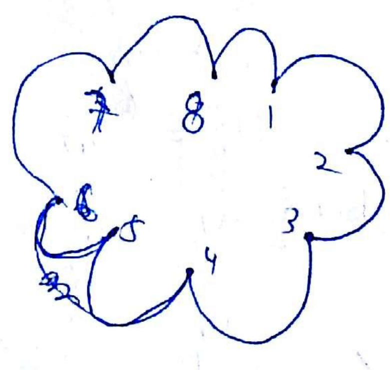


✱ This type of cam not used practically

✱ or else move the cam very very slow speed.

(Actual uniform velocity)



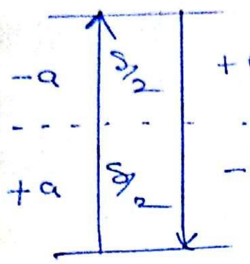


8 point of interferance

② Uniform accⁿ & Retardation (parabolic)

This type of Cam not used practically else moves the Cam very slow Speed

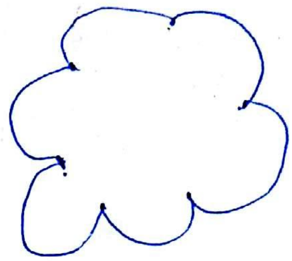
$$t_o = \frac{\theta_o}{\omega}$$

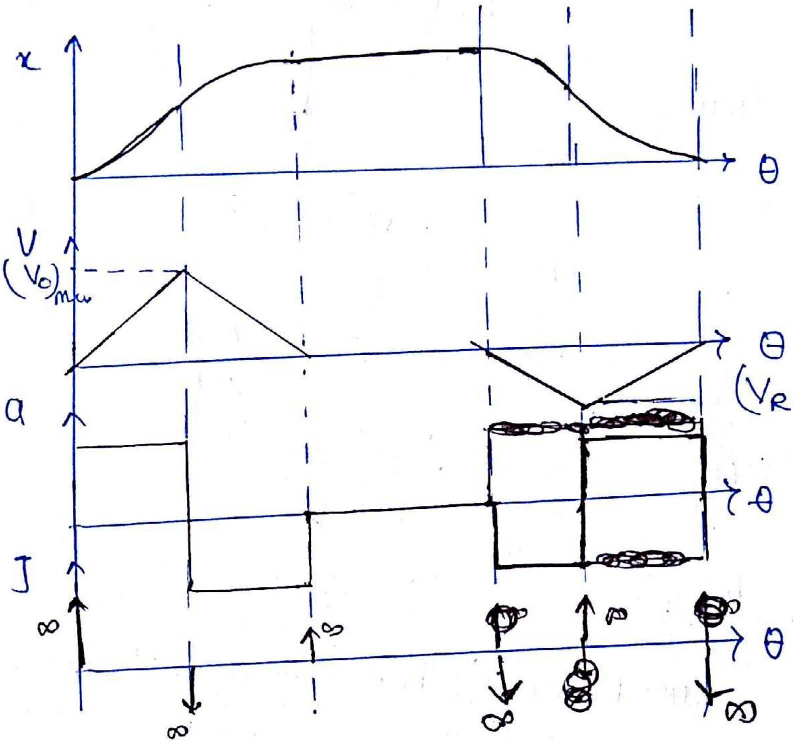


$$(V_o)_{mean} = \frac{S}{t_o} = \frac{\omega S}{\theta_o}$$

$$(V_o)_{mean} = \frac{V_{max} + V_{min}}{2}$$

$$V_{max} = 2(V_{o\,mean}) = \frac{2\omega S}{\theta_o}$$





$$\frac{2\omega S}{\theta_o} = (V_o)_{max}$$

$$(V_R)_{max} = 2(V_R)_{mean} = \frac{2\omega S}{\theta_R}$$

$$S = ut + \frac{1}{2}at^2$$

$$\frac{S}{2} = \frac{1}{2}a\left(\frac{t_o}{2}\right)^2 = \frac{1}{2}a\left(\frac{\theta_o}{2\omega}\right)^2$$

$$\boxed{a = \frac{4\omega^2 S}{\theta_o^2}}$$

$$V = u + at \qquad t = t_o/2$$

$$(V_o)_{max} = \frac{4\omega^2 S}{\theta_o^2} \times \left(\frac{\theta_o}{2\omega}\right) \qquad (V_{max}) = \frac{2\omega S}{\theta_o}$$

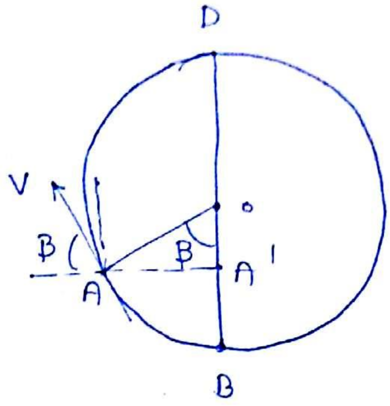
12/12/16

3. SHM :- when a particle moves in circular path with uniform angular velocity, its projection on diameter will make SHM.

$\vec{a} = \vec{a}_r$

$\vec{a}_t = 0$, $BD = S$

$V = $ Const.



$V_{A'} = V \sin\beta = V_{follower}$

$V = \frac{\pi S}{2 t_o}$ $\quad t_o = \frac{\theta_o}{\omega}$ $\quad V = \frac{\pi S}{2 t_o} = \frac{\pi S \omega}{2 \theta_o}$

$V = \frac{\pi \omega S}{2 \theta_o}$

$V = \frac{\pi \omega S.}{2 \theta_o}$

Now let in time t

↳ Point on circumference has moved through angle β

↳ follower has displaced from point B to A'

↳ Cam has rotate through an angle θ

θ_o (cam) ⟶ π (particle)

1 ⟶ $\frac{\pi}{\theta_o}$

θ ⟶ $\frac{\pi \theta}{\theta_o} = \beta$ $\Rightarrow$ $\boxed{\beta = \frac{\pi \theta}{\theta_o}}$ ★

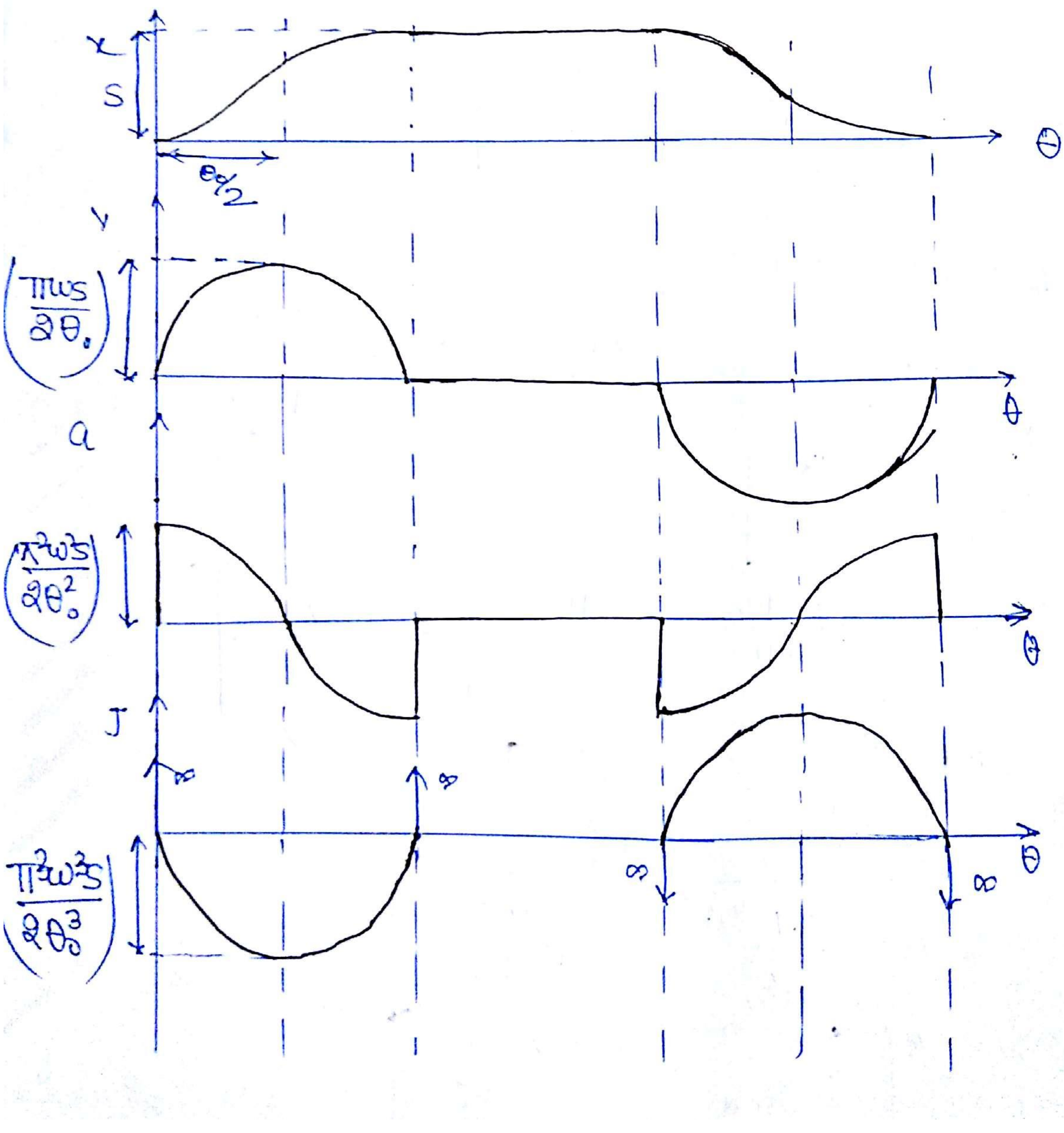
So $V_{A'}$ / $V_{outstroke}$ / $V_{follower}$ $= V_o = \frac{\pi \omega S}{2 \theta_o} \sin\left(\frac{\pi \theta}{\theta_o}\right)$

$$\boxed{(V_0)_{max} = \frac{\pi \omega S}{2\theta_0}}$$, when $\frac{\pi\theta}{\theta_0} = \frac{\pi}{2} \Rightarrow \theta = \frac{\theta_0}{2}$

$$a_0 = \left(\frac{dV}{d\theta}\right)\omega = \frac{\pi^2\omega^2 S}{2\theta_0^2}\cos\left(\frac{\pi\theta}{\theta_0}\right)$$

$$\boxed{(a_0)_{max} = \frac{\pi^2\omega^2 S}{2\theta_0^2}}$$, when $\theta = 0°$

$$J_0 = \frac{\pi^3\omega^3 S}{2\theta_0^3}\left(-\sin\frac{\pi\theta}{\theta_0}\right) \qquad \boxed{(J_0)'_{max} = \frac{\pi^3\omega^3 S}{2\theta_0^3}}$$



V= ~~fa~~ drawn start from here

+ Four infinites point so this motion can be used for medium speeds.

This is batter than uniform velocity and uniform acc^n retardation.

4 Cycloidal Motion :-

$$\Rightarrow x_0 = S\left[\frac{\theta}{\theta_0} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$V_0 = \frac{dx_0}{d\theta}\times\omega = \omega S\left[\frac{1}{\theta_0} - \frac{1}{\theta_0}\cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$V_\theta = \frac{\omega S}{\theta_0}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$V_0 \rightarrow max$, when $\cos\left(\frac{2\pi\theta}{\theta_0}\right) = -1$

$$\frac{2\pi\theta}{\theta_0} = \pi \Rightarrow \boxed{\theta = \frac{\theta_0}{2}}$$

$\rightarrow a = 0$

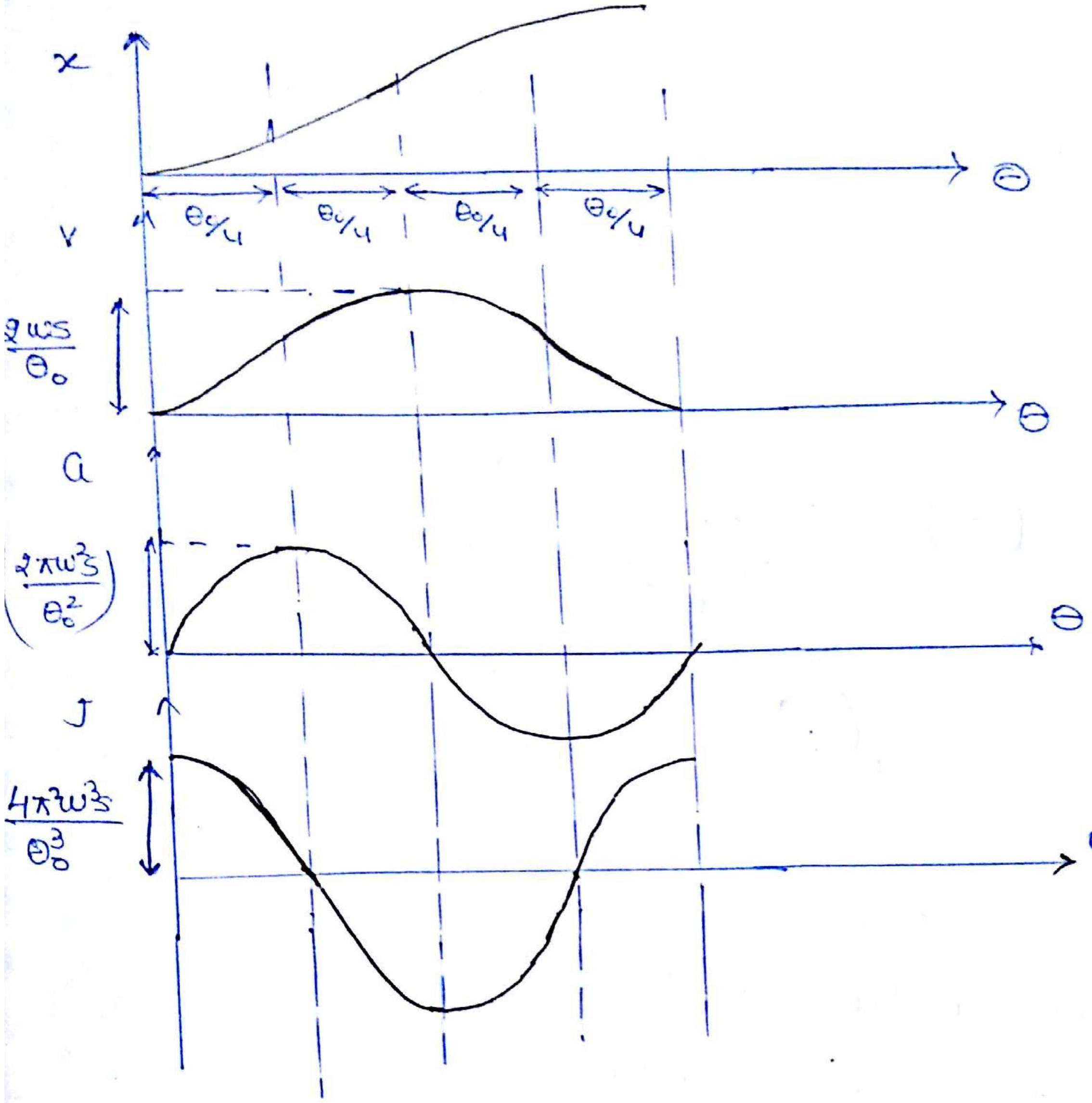
$$\boxed{(V_0)_{max} = \frac{2\omega S}{\theta_0}}$$

$$a_0 = \omega\left(\frac{dV_0}{d\theta}\right) = \frac{2\pi\omega^2 S}{\theta_0^2}\left[\sin\left(\frac{2\pi.\theta}{\theta_0}\right)\right]$$

$$\boxed{(a_0)_{max} = \frac{2\pi\omega^2 S}{\theta_0^2}}$$

$$J_0 = \frac{4\pi^2\omega^3 S}{\theta_0^3}\cos\left(\frac{2\pi\theta}{\theta_0}\right)$$

$$\boxed{(J_0)_{max} = \frac{4\pi^2\omega^3 S}{\theta_0^3}}$$

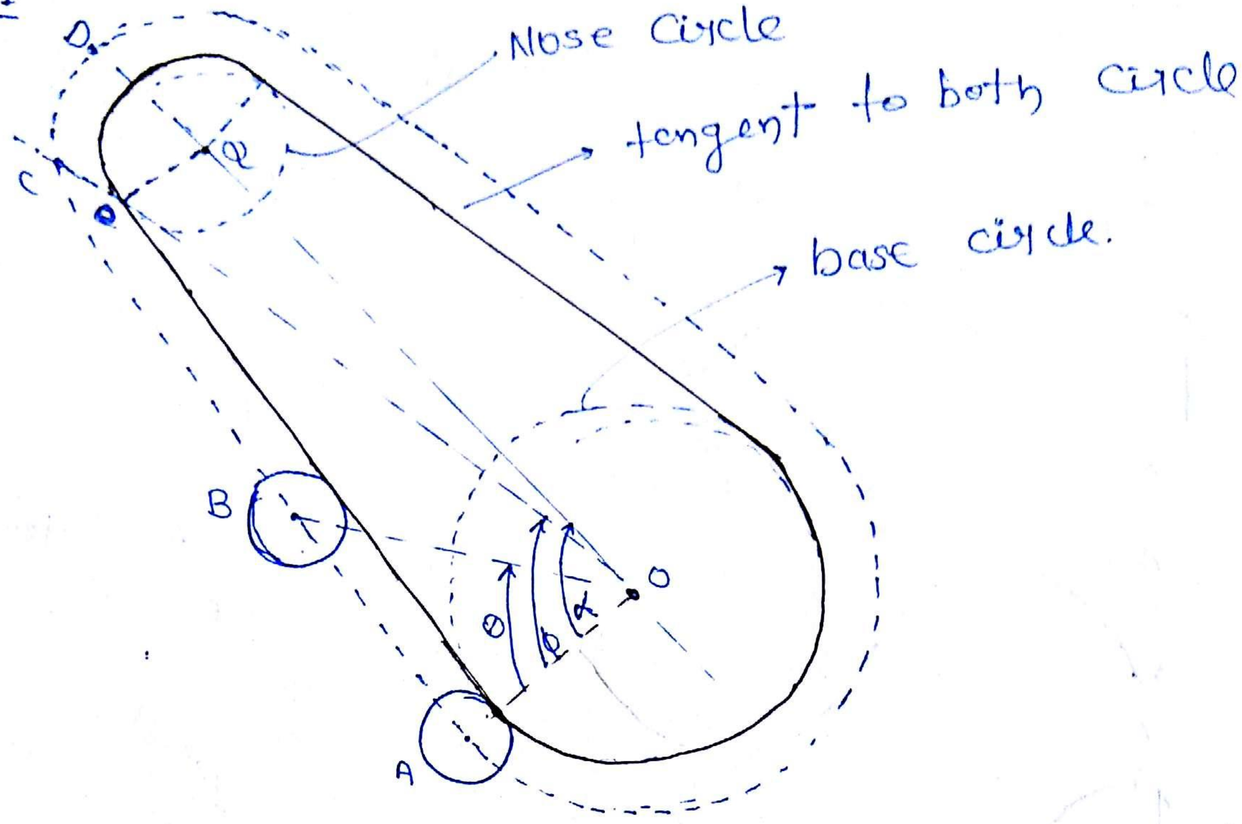


Cams with specified Contours :-

(a) Tangent cam with roller follower :-

when flank of cam is straight and tangential to base circle and nose circle it is known as tangent cam.

Case-1



⇒ r - distance b/w base circle & nose circle = OQ

⇒ r_1 - radius of base circle

⇒ r_2 - radius of nose circle

⇒ r_3 - radius of roller

⇒ $r_2 + r_3 = l$

⇒ α - angle of ascent

(i) When Roller is on tangent Plank:-

$$0 \leq \theta \leq \phi$$

Follower ~~roller~~ displacement $x = OB - OA$

ΔOAB $\quad OB \cos\theta = OA$

$$x = \frac{OA}{\cos\theta} - OA$$

$$x = OA\left(\frac{1-\cos\theta}{\cos\theta}\right)$$

$$\boxed{x = (r_1 + r_3)\left[\frac{1-\cos\theta}{\cos\theta}\right]}$$

$$V = \frac{dx}{dt} = \omega\frac{dx}{d\theta} = \omega(r_1+r_3)\left(\frac{\sin\theta}{\cos^2\theta}\right)$$

$$\boxed{V_{max} = \omega(r_1+r_3)\left(\frac{\sin\phi}{\cos^2\phi}\right)}$$

when $\theta = \phi$

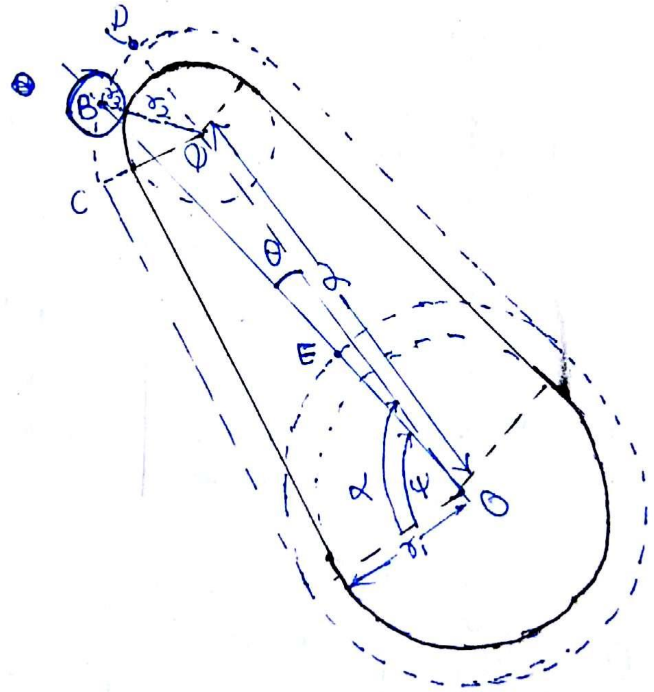
\# Velocity depends on **Roller follower** and **base** ~~part~~ Circle radius.

$$a = \omega\frac{dv}{d\theta} = \omega^2(r_1+r_3)\frac{(2-\cos^2\theta)}{(\cos^3\theta)}$$

$$\boxed{a_{max} = \omega^2(r_1+r_3)\left[\frac{2-\cos^2\phi}{\cos^3\phi}\right]}$$

Case-2

$r_2 + r_3 = l$



(ii) When roller on Nose part :-

$$\phi \le \varphi < \alpha$$

$$x = OB - OA = OB - OE$$

$$x = r\left[(1-\cos\theta) + \left(n - \sqrt{n^2 - \sin^2\theta}\right)\right]$$

$$n = \frac{l}{r} = \frac{r_2 + r_3}{r}$$

$$x = r\left[(1-\cos\theta) + \frac{r_2+r_3}{r} - \sqrt{\left(\frac{r_2+r_3}{r}\right)^2 - \sin^2\theta}\right]$$

$$x = \left[r(1-\cos\theta) + l - \sqrt{l^2 - r^2\sin^2\theta}\right]$$

$$\because l = r_2 + r_3$$

$$V = \omega\frac{dx}{d\theta} = \omega\left(r\sin\theta + \frac{(r)^2\sin 2\theta}{2\sqrt{l^2 - r^2\sin^2\theta}}\right)$$

$$V = r\omega\left[\sin\theta + \frac{r\sin 2\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right]$$

* Velocity depends on centre distance & nose radius

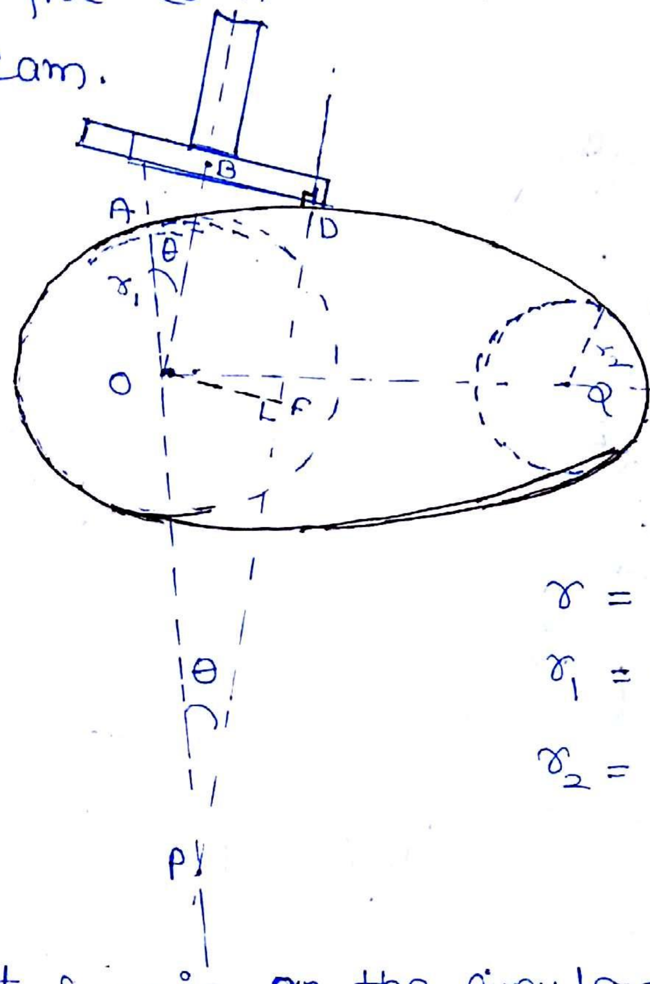
$$a = \omega\frac{dV}{d\theta} \Rightarrow a = \omega^2 r\left[\cos\theta + \frac{\left(l^2 r\cos 2\theta + r^3\sin^4\theta\right)}{\left(l^2 - r^2\sin^2\theta\right)^{3/2}}\right]$$

$$\boxed{\theta = \varphi - \psi}$$

* when the roller is on the nose part OQB behaves as a slider crank chain in which OQ is crank, QB the connecting rod and B is the slider. OB is the line of action.

b) Circular arc Cam with Flat - Face Follower :-

when the flanks of the cam are of convex circular arce and tengential to base circle and nose circle than the cam will be know as circular arc cam.



$r = OQ$

r_1 = radius of base circle

r_2 = radius of nose circle.

(i) Flat face is on the circulor flank :-

$x = OB - OA$

OB & PD are parallel

$x = FD - OA = (PD - PF) - OA$

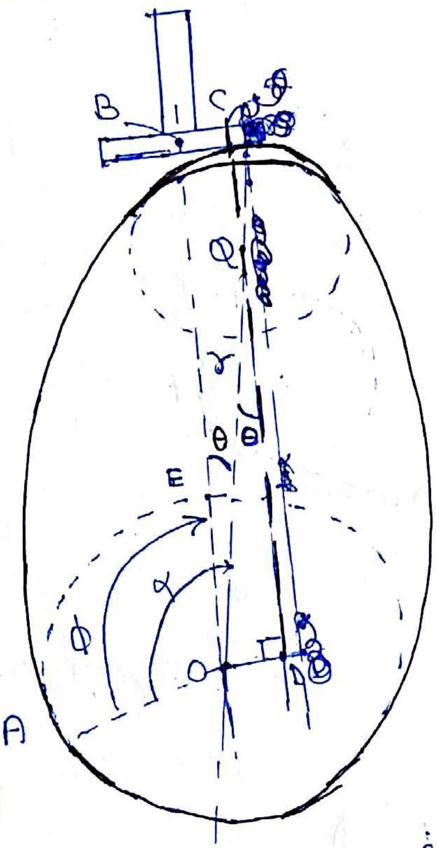
$x = (PA - OA) - PF$

$x = OP - OP\cos\theta$

$$\begin{bmatrix} x = (R - r_1)(1 - \cos\theta) \\ v = \omega (R - r_1)(\sin\theta) \\ a = \omega^2 (R - r_1)\cos\theta \end{bmatrix}$$

* Velocity depends on base circle radius and circular arc radius.

(ii) Flat face follower on the nose



$x = OB - OA$

OB & CD are parallel

$x = CD - OA$

$x = DQ + QC - OA$

in ΔODQ

$DQ = OQ\cos\theta = r\cos\theta$

* Velocity depend on base, nose circle radius and centre distance

$$\begin{bmatrix} x = r\cos\theta + r_2 - r_1 \\ v = \omega r(-\sin\theta) \\ a = -\omega^2 r\cos\theta \end{bmatrix}$$