Short Answer Questions (PYQ)

[2 Mark]

Q.1. Find $\frac{dy}{dx}$ at $t = \frac{2\pi}{3}$ **when *x* = 10(*t* – sin *t*) and *y* = 12 (1 – cos *t*).**

Ans.

Given

$$x = 10\ (t - \sin\ t) \text{ and } y = 12\ (1 - \cos\ t)$$

$$\because \quad x = 10\ (t - \sin\ t)$$

Differentiating w.r.t. *t* we get

$$\frac{dx}{dt} = 10\ (1 - \cos\ t)$$

Again $y = 12\ (1 - \cos\ t)$

$$\Rightarrow \quad \frac{dy}{dt} = 12\ (0 + \sin\ t) = 12\ \sin\ t$$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{12\ \sin\ t}{10\ (1 - \cos\ t)}$

$$\because \quad \left.\frac{dy}{dx}\right]_{t=\frac{2\pi}{3}} = \frac{12\ \sin\ \frac{2\pi}{3}}{10\left(1 - \cos\ \frac{2\pi}{3}\right)}$$

$$= \frac{6}{5} \times \frac{\sin\left(\pi - \frac{\pi}{3}\right)}{\left(1 - \cos\ \left(\pi - \frac{\pi}{3}\right)\right)} = \frac{6}{5} \times \frac{\sin\frac{\pi}{3}}{1 + \cos\frac{\pi}{3}}$$

$$= \frac{6}{5} \times \frac{\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}} = \frac{6\sqrt{3}}{5 \times 3} = \frac{2\sqrt{3}}{5}$$

Q.2. Find $\frac{dy}{dx}$ at $x = 1,\ y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

Ans.

$$\sin^2 y + \cos xy = K$$

Differentiating w.r.t. x, we get

$$2\sin y.\ \cos y\ \frac{dy}{dx} + (-\sin xy)(x.\ \frac{dy}{dx} + y) = 0$$

$$\sin 2y.\frac{dy}{dx} - x\ \sin xy.\ \frac{dy}{dx} - y\ \sin xy = 0$$

$$\frac{dy}{dx} = \frac{y\ \sin\ xy}{(\sin\ 2y - x\ \sin\ xy)}$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right]_{x=1,\ y=\frac{\pi}{4}} = \frac{\frac{\pi}{4}.\ \sin\frac{\pi}{4}}{\sin\frac{\pi}{2} - \sin\ \frac{\pi}{4}} = \frac{\frac{\pi}{4}.\frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = \frac{\pi}{4(\sqrt{2}-1)}$$

Short Answer Questions (OIQ)

[2 Mark]

Q.1. Write the derivative of $f(x) = |x^3|$ at $x = 0$.

Ans.

We have,

$$f(x) = |x^3| = \begin{cases} x^3 & ,x \geq 0 \\ -x^3 & ,x < 0 \end{cases}$$

$$f'(0) = \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}$$

Now, $f'(0^-) = \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} \lim_{x\to 0} \frac{-x^3 - 0}{x} = -\lim_{x\to 0} x^2 = 0$

and $f'(0^+) = \lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0} \lim_{x\to 0^+} \frac{+x^3 - 0}{x} = \lim_{x\to 0^+} x^2 = 0$

Thus, $f'(0) = f'(0-) = f'(0^+) = 0$

Q.2. Find $\frac{dy}{dx}$ if $y + \sin y = \cos x$.

Ans.

Given, $y + \sin y = \cos x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} + \cos y.\ \frac{dy}{dx} = -\sin x$$

$$\Rightarrow \quad \frac{dy}{dx}(1 + \cos y) = -\sin x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-\sin x}{1+\cos y} \qquad [\because \quad y \neq (2n+1)\pi]$$

Q.3. Differentiate: $2\sqrt{\cot\left(x^2\right)}$ **w.r.t. *x***

Ans.

Let $y = 2\sqrt{\cot\left(x^2\right)}$

Differentiating w.r.t. x both sides, we get

$$\frac{dy}{dx} = 2 \times \frac{1}{2\sqrt{\cot\left(x^2\right)}} \times -\operatorname{cosec}^2\left(x^2\right) \times 2x$$

$$= \frac{-2x \operatorname{cosec}^2\left(x^2\right)}{\sqrt{\cot\left(x^2\right)}}$$

Q.4. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Ans.

Let $x = \tan\theta$

$\Rightarrow \quad \theta = \tan^{-1} x.$

Now, $\quad y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow \quad y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow \quad y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow \quad y = \sin^{-1}(\sin 2\theta) \qquad \left[\because \ \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$

$\Rightarrow \quad y = 2\theta \qquad [\because \sin^{-1}(\sin 2\theta) = 2\theta]$

$\Rightarrow \quad y = 2\tan^{-1} x \qquad [\because \ \theta = \tan^{-1} x]$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 2.\frac{1}{1+x^2} \qquad = \frac{2}{1+x^2}$$

Q.5. Find $\frac{dy}{dx}$, if $y = \frac{\cos x}{\log x}$, $x > 0$.

Ans.

Given, $y = \frac{\cos x}{\log x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\log x \times (-\sin x) - \cos x \times \frac{1}{x}}{(\log x)^2}$$

$$= \frac{-\sin x \log x - \frac{\cos x}{x}}{(\log x)^2} = \frac{-x \sin x . \log x - \cos x}{x(\log x)^2}$$

Q.6. Differentiate w.r.t. x: $e^x + e^{x^2} + \ldots\ldots\ldots + e^{x^5}$

Ans.

Let $y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = e^x + e^{x^2}.\ 2x + e^{x^3}.3x^2 + e^{x^4}.4x^3 + e^{x^5}.5x^4$$
$$= e^x + 2xe^{x^2} + 3x^2 e^{x^3} + 4x^3 e^{x^4} + 5x^4 e^{x^5}$$

Q.7. If $y = x^{\sin x}$, $x > 0$, then find $\frac{dy}{dx}$.

Ans.

Given $y = x^{\sin x}$

Taking log of both sides, we get

$$\log y = \log x\sin x$$

$\Rightarrow$ $\log y = \sin x. \log x$

$\Rightarrow$ $\frac{1}{y}.\frac{dy}{dx} = \sin x.\frac{1}{x} + \log x.\cos x$ [Differentiating both sides w.r.t. x]

$\Rightarrow$ $\frac{dy}{dx} = y\left\{\frac{\sin x}{x} + \cos x \log x\right\} = x^{\sin x}\left\{\frac{\sin x}{x} + \cos x \log x\right\}$

Q.8. Find $\frac{dy}{dx}$, if $x = 2at^2$, $y = at^4$.

Ans.

$\because$ $x = 2at^2$ $\Rightarrow$ $\frac{dx}{dt} = 4at$

Again, $y = at^4$

$$\frac{dy}{dt} = 4at^3$$

Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$$= \frac{4at^3}{4at} = t^2$$

Q.9.

If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then prove that [*HOTS*]

(*i*) $C_1 + 2C_2 + ... + nC_n = n \,.\, 2^{n-1}$ (*ii*) $C_1 - 2C_2 + 3C_3 - .... + (-1)^n\, nC_n = 0$

Ans.

We have, $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$

Differentiating both sides with respect to *x*, we have

$$n(1 + x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + ... + nC_nx^{n-1}$$

Putting $x = 1$ and $x = -1$ successively, we have

$$C_1 + 2C_2 + 3C_3 + ... + nC_n = n \,.\, 2^{n-1}$$

and $$C_1 - 2C_2 + 3C_3 - .... + (-1)^n\, nC_n = 0$$