Equation of a Line

Exercise 14A

Question 1.
Find, which of the following points lie on the line $x - 2y + 5 = 0$:
(i) (1, 3) (ii) (0, 5)
(iii) (-5, 0) (iv) (5, 5)
(v) (2, -1.5) (vi) (-2, -1.5)

Solution:

The given line is $x - 2y + 5 = 0$.

(i) Substituting $x = 1$ and $y = 3$ in the given equation, we have:
L.H.S. = $1 - 2 \times 3 + 5 = 1 - 6 + 5 = 6 - 6 = 0$ = R.H.S.
Thus, the point (1, 3) lies on the given line.

(ii) Substituting $x = 0$ and $y = 5$ in the given equation, we have:
L.H.S. = $0 - 2 \times 5 + 5 = -10 + 5 = -5 \neq$ R.H.S.
Thus, the point (0, 5) does not lie on the given line.

(iii) Substituting $x = -5$ and $y = 0$ in the given equation, we have:
L.H.S. = $-5 - 2 \times 0 + 5 = -5 - 0 + 5 = 5 - 5 = 0$ = R.H.S.
Thus, the point (-5, 0) lie on the given line.

(iv) Substituting $x = 5$ and $y = 5$ in the given equation, we have:
L.H.S. = $5 - 2 \times 5 + 5 = 5 - 10 + 5 = 10 - 10 = 0$ = R.H.S.
Thus, the point (5, 5) lies on the given line.

(v) Substituting $x = 2$ and $y = -1.5$ in the given equation, we have:
L.H.S. = $2 - 2 \times (-1.5) + 5 = 2 + 3 + 5 = 10 \neq$ R.H.S.
Thus, the point (2, -1.5) does not lie on the given line.

(vi) Substituting $x = -2$ and $y = -1.5$ in the given equation, we have:
L.H.S. = $-2 - 2 \times (-1.5) + 5 = -2 + 3 + 5 = 6 \neq$ R.H.S.
Thus, the point (-2, -1.5) does not lie on the given line.

Question 2.

State, true or false:

(i) the line $\frac{x}{2} + \frac{y}{3} = 0$ passes through the point (2, 3).

(ii) the line $\frac{x}{2} + \frac{y}{3} = 0$ passes through the point (4, -6).

(iii) the point (8, 7) lies on the line $y - 7 = 0$.

(iv) the point (-3, 0) lies on the line $x + 3 = 0$.

(v) if the point (2, a) lies on the line $2x - y = 3$, then $a = 5$.

Solution:

(i) The given line is $\frac{x}{2} + \frac{y}{3} = 0$

Substituting $x = 2$ and $y = 3$ in the given equation,

$$\text{L.H.S.} = \frac{x}{2} + \frac{y}{3} = 1 + 1 = 2 \neq \text{R.H.S.}$$

Thus, the given statement is false.

(ii) The given line is $\frac{x}{2} + \frac{y}{3} = 0$

Substituting $x = 4$ and $y = -6$ in the given equation,

$$\text{L.H.S.} = \frac{4}{2} + \frac{-6}{3} = 2 - 2 = 0 = \text{R.H.S.}$$

Thus, the given statement is true.

(iii) L.H.S = $y - 7 = 7 - 7 = 0$ = R.H.S.

Thus, the point (8, 7) lies on the line $y - 7 = 0$.

The given statement is true.

(iv) L.H.S. = $x + 3 = -3 + 3 = 0$ = R.H.S

Thus, the point (-3, 0) lies on the line $x + 3 = 0$.

The given statement is true.

(v) The point (2, a) lies on the line $2x - y = 3$.

$\therefore\ 2(2) - a = 3$

$4 - a = 3$

$a = 4 - 3 = 1$

Thus, the given statement is false.

Question 3.

The line given by the equation $2x - \frac{y}{3} = 7$ passes through the point (k, 6); calculate the value of k.

Solution:

Given, the line given by the equation $2x - \frac{y}{3} = 7$ passes through the point (k, 6).

Substituting x = k and y = 6 in the given equation, we have:

$2k - \frac{6}{3} = 7$

$2k - 2 = 7$

$2k = 9$

$k = \frac{9}{2} = 4.5$

Question 4.

For what value of k will the point (3, -k) lie on the line 9x + 4y = 3?

Solution:

The given equation of the line is 9x + 4y = 3.
Put x = 3 and y = -k, we have:
9(3) + 4(-k) = 3
27 − 4k = 3
4k = 27 − 3 = 24
k = 6

Question 5.

The line $\frac{3x}{5} - \frac{2y}{3} + 1 = 0$ contains the point (m, 2m - 1); calculate the value of m.

Solution:

The equation of the given line is $\frac{3x}{5} - \frac{2y}{3} + 1 = 0$

Putting x = m, y = 2m - 1, we have:

$$\frac{3m}{5} - \frac{2(2m-1)}{3} + 1 = 0$$
$$\frac{3m}{5} - \frac{4m-2}{3} = -1$$
$$\frac{9m - 20m + 10}{15} = -1$$
$$9m - 20m + 10 = -15$$
$$-11m = -25$$
$$m = \frac{25}{11} = 2\frac{3}{11}$$

Question 6.
Does the line 3x – 5y = 6 bisect the join of (5, -2) and (-1, 2)?

Solution:

The given line will bisect the join of A (5, -2) and B (-1, 2), if the co-ordinates of the mid-point of AB satisfy the equation of the line.
The co-ordinates of the mid-point of AB are

$$\left(\frac{5-1}{2}, \frac{-2+2}{2}\right) = (2, 0)$$

Substituting x = 2 and y = 0 in the given equation, we have:
L.H.S. = 3x - 5y = 3(2) - 5(0) = 6 - 0 = 6 = R.H.S.
Hence, the line 3x - 5y = 6 bisect the join of (5, -2) and (-1, 2).

Question 7.
(i) The line y = 3x – 2 bisects the join of (a, 3) and (2, -5), find the value of a.
(ii) The line x – 6y + 11 = 0 bisects the join of (8, -1) and (0, k). Find the value of k.

Solution:

(i) The given line bisects the join of A (a, 3) and B (2, -5), so the co-ordinates of the mid-point of AB will satisfy the equation of the line.
The co-ordinates of the mid-point of AB are

$$\left(\frac{a+2}{2}, \frac{3-5}{2}\right) = \left(\frac{a+2}{2}, -1\right)$$

Substituting x = $\frac{a+2}{2}$ and y = -1 in the given equation, we have:

$$y = 3x - 2$$

$$-1 = 3 \times \frac{a+2}{2} - 2$$

$$3 \times \frac{a+2}{2} = 1$$

$$a + 2 = \frac{2}{3}$$

$$a = \frac{2}{3} - 2 = \frac{2-6}{3} = \frac{-4}{3}$$

(ii) The given line bisects the join of A (8, -1) and B (0, k), so the co-ordinates of the mid-point of AB will satisfy the equation of the line.
The co-ordinates of the mid-point of AB are

$$\left(\frac{8+0}{2}, \frac{-1+k}{2}\right) = \left(4, \frac{-1+k}{2}\right)$$

Substituting x = 4 and y = $\frac{-1+k}{2}$ in the given equation, we have:

$$x - 6y + 11 = 0$$

$$4 - 6\left(\frac{-1+k}{2}\right) + 11 = 0$$

$$6\left(\frac{-1+k}{2}\right) = 15$$

$$\frac{-1+k}{2} = \frac{15}{6}$$

$$\frac{-1+k}{2} = \frac{5}{2}$$

$$-1 + k = 5$$

$$k = 6$$

Question 8.
(i) The point (-3, 2) lies on the line ax + 3y + 6 = 0, calculate the value of a.
(ii) The line y = mx + 8 contains the point (-4, 4), calculate the value of m.

Solution:
(i) Given, the point (-3, 2) lies on the line ax + 3y + 6 = 0.
Substituting x = -3 and y = 2 in the given equation, we have:
a(-3) + 3(2) + 6 = 0
-3a + 12 = 0
3a = 12
a = 4

(ii) Given, the line y = mx + 8 contains the point (-4, 4).
Substituting x = -4 and y = 4 in the given equation, we have:

4 = -4m + 8
4m = 4
m = 1

Question 9.
The point P divides the join of (2, 1) and (-3, 6) in the ratio 2: 3. Does P lie on the line x – 5y + 15 = 0?

Solution:

Given, the point P divides the join of (2, 1) and (-3, 6) in the ratio 2: 3.
Co-ordinates of the point P are

$$\left(\frac{2\times(-3)+3\times 2}{2+3}, \frac{2\times 6+3\times 1}{2+3}\right)$$

$$= \left(\frac{-6+6}{5}, \frac{12+3}{5}\right)$$

$$= (0, 3)$$

Substituting x = 0 and y = 3 in the given equation, we have:
L.H.S. = 0 - 5(3) + 15 = -15 + 15 = 0 = R.H.S.
Hence, the point P lies on the line x - 5y + 15 = 0.

Question 10.
The line segment joining the points (5, -4) and (2, 2) is divided by the point Q in the ratio 1: 2. Does the line x – 2y = 0 contain Q?

Solution:

Given, the line segment joining the points (5, -4) and (2, 2) is divided by the point Q in the ratio 1: 2.
Co-ordinates of the point Q are

$$\left(\frac{1\times 2+2\times 5}{1+2}, \frac{1\times 2+2\times(-4)}{1+2}\right)$$

$$= \left(\frac{2+10}{3}, \frac{2-8}{3}\right)$$

$$= (4, -2)$$

Substituting x = 4 and y = -2 in the given equation, we have:
L.H.S. = x - 2y = 4 - 2(-2) = 4 + 4 = 8 $\neq$ R.H.S.
Hence, the given line does not contain point Q.

Question 11.
Find the point of intersection of the lines:
$4x + 3y = 1$ and $3x - y + 9 = 0$. If this point lies on the line $(2k - 1)x - 2y = 4$; find the value of k.

Solution:
Consider the given equations:
$4x + 3y = 1$ ….(1)
$3x - y + 9 = 0$ ….(2)

Multiplying (2) with 3, we have:
$9x - 3y = -27$ ….(3)
Adding (1) and (3), we get,
$13x = -26$
$x = -2$

From (2), $y = 3x + 9 = -6 + 9 = 3$
Thus, the point of intersection of the given lines (1) and (2) is (-2, 3).

The point (-2, 3) lies on the line $(2k - 1)x - 2y = 4$.
$(2k - 1)(-2) - 2(3) = 4$

$-4k + 2 - 6 = 4$
$-4k = 8$
$k = -2$

Question 12.
Show that the lines $2x + 5y = 1$, $x - 3y = 6$ and $x + 5y + 2 = 0$ are concurrent.

Solution:
We know that two or more lines are said to be concurrent if they intersect at a single point.

We first find the point of intersection of the first two lines.
$2x + 5y = 1$ ….(1)
$x - 3y = 6$ ….(2)

Multiplying (2) by 2, we get,
$2x - 6y = 12$ ….(3)
Subtracting (3) from (1), we get,
$11y = -11$
$y = -1$

From (2), $x = 6 + 3y = 6 - 3 = 3$

So, the point of intersection of the first two lines is (3, -1).
If this point lie on the third line, i.e., $x + 5y + 2 = 0$, then the given lines will be concurrent.

Substituting $x = 3$ and $y = -1$, we have:
L.H.S. = $x + 5y + 2 = 3 + 5(-1) + 2 = 5 - 5 = 0$ = R.H.S.

Thus, (3, -1) also lie on the third line.
Hence, the given lines are concurrent.

Exercise 14B

Question 1.

Find the slope of the line whose inclination is:
(i) $0°$ (ii) $30°$
(iii) $72° 30'$ (iv) $46°$

Solution:

(i) Slope = $\tan 0° = 0$
(ii) Slope = $\tan 30° = \frac{1}{\sqrt{3}}$
(iii) Slope = $\tan 72° 30' = 3.1716$
(iv) Slope = $\tan 46° = 1.0355$

Question 2.

Find the inclination of the line whose slope is:
(i) 0 (ii) $\sqrt{3}$
(iii) 0.7646 (iv) 1.0875

Solution:

(i) Slope = $\tan\theta = 0$
$\Rightarrow \theta = 0°$
(ii) Slope = $\tan\theta = \sqrt{3}$
$\Rightarrow \theta = 60°$
(iii) Slope = $\tan\theta = 0.7646$
$\Rightarrow \theta = 37° 24'$
(iv) Slope = $\tan\theta = 1.0875$
$\Rightarrow \theta = 47° 24'$

Question 3.
Find the slope of the line passing through the following pairs of points:

(i) (-2, -3) and (1, 2)
(ii) (-4, 0) and origin
(iii) (a, -b) and (b, -a)

Solution:

We know:

Slope = $\frac{y_2 - y_1}{x_2 - x_1}$

(i) Slope = $\frac{2+3}{1+2} = \frac{5}{3}$

(ii) Slope = $\frac{0-0}{0+4} = \frac{0}{4} = 0$

(iii) Slope = $\frac{-a+b}{b-a} = 1$

Question 4.
Find the slope of the line parallel to AB if:

(i) A = (-2, 4) and B = (0, 6)
(ii) A = (0, -3) and B = (-2, 5)

Solution:

(i) Slope of AB = $\frac{6-4}{0+2} = \frac{2}{2} = 1$

Slope of the line parallel to AB = Slope of AB = 1

(ii) Slope of AB = $\frac{5+3}{-2-0} = \frac{8}{-2} = -4$

Slope of the line parallel to AB = Slope of AB = -4

Question 5.
Find the slope of the line perpendicular to AB if:

(i) A = (0, -5) and B = (-2, 4)
(ii) A = (3, -2) and B = (-1, 2)

Solution:

(i) Slope of AB = $\frac{4+5}{-2-0} = \frac{-9}{2}$

Slope of the line perpendicular to AB = $\frac{-1}{\text{Slope of AB}} = \frac{-1}{\frac{-9}{2}} = \frac{2}{9}$

(ii) Slope of AB = $\frac{2+2}{-1-3} = \frac{4}{-4} = -1$

Slope of the line perpendicular to AB = $\frac{-1}{\text{Slope of AB}} = 1$

Question 6.
The line passing through (0, 2) and (-3, -1) is parallel to the line passing through (-1, 5) and (4, a). Find a.

Solution:

Slope of the line passing through (0, 2) and (-3, -1) = $\frac{-1-2}{-3-0} = \frac{-3}{-3} = 1$

Slope of the line passing through (-1, 5) and (4, a) = $\frac{a-5}{4+1} = \frac{a-5}{5}$

Since, the lines are parallel.

$\therefore\ 1 = \frac{a-5}{5}$

$a - 5 = 5$

$a = 10$

Question 7.
The line passing through (-4, -2) and (2, -3) is perpendicular to the line passing through (a, 5) and (2, -1). Find a.

Solution:

Slope of the line passing through (-4, -2) and (2, -3) = $\frac{-3+2}{2+4} = \frac{-1}{6}$

Slope of the line passing through (a, 5) and (2, -1) = $\frac{-1-5}{2-a} = \frac{-6}{2-a}$

Since, the lines are perpendicular.

$\therefore\ \frac{-1}{6} = \frac{-1}{\frac{-6}{2-a}}$

$$\frac{-1}{6} = \frac{2-a}{6}$$
$$2 - a = -1$$
$$a = 3$$

Question 8.
Without using the distance formula, show that the points A (4, -2), B (-4, 4) and C (10, 6) are the vertices of a right-angled triangle.

Solution:

The given points are A (4, -2), B (-4, 4) and C (10, 6).

$$Slope\ of\ AB = \frac{4+2}{-4-4} = \frac{6}{-8} = \frac{-3}{4}$$

$$Slope\ of\ BC = \frac{6-4}{10+4} = \frac{2}{14} = \frac{1}{7}$$

$$Slope\ of\ AC = \frac{6+2}{10-4} = \frac{8}{6} = \frac{4}{3}$$

It can be seen that:

$$Slope\ of\ AB = \frac{-1}{Slope\ of\ AC}$$

Hence, AB $\perp$ AC.

Thus, the given points are the vertices of a right-angled triangle.

Question 9.
Without using the distance formula, show that the points A (4, 5), B (1, 2), C (4, 3) and D (7, 6) are the vertices of a parallelogram.

Solution:

The given points are A (4, 5), B (1, 2), C (4, 3) and D (7, 6).

$$Slope\ of\ AB = \frac{2-5}{1-4} = \frac{-3}{-3} = 1$$

$$Slope\ of\ CD = \frac{6-3}{7-4} = \frac{3}{3} = 1$$

Since, slope of AB = slope of CD

Therefore AB || CD

$$Slope\ of\ BC = \frac{3-2}{4-1} = \frac{1}{3}$$

$$Slope\ of\ DA = \frac{5-6}{4-7} = \frac{-1}{-3} = \frac{1}{3}$$

Since, slope of BC = slope of DA

Therefore, BC || DA

Hence, ABCD is a parallelogram

Question 10.
(-2, 4), (4, 8), (10, 7) and (11, -5) are the vertices of a quadrilateral. Show that the quadrilateral, obtained on joining the mid-points of its sides, is a parallelogram.

Solution:

Let the given points be A (-2, 4), B (4, 8), C (10, 7) and D (11, -5).

Let P, Q, R and S be the mid-points of AB, BC, CD and DA respectively.

Co-ordinates of P are

$$\left(\frac{-2+4}{2}, \frac{4+8}{2}\right) = (1, 6)$$

Co-ordinates of Q are

$$\left(\frac{4+10}{2}, \frac{8+7}{2}\right) = \left(7, \frac{15}{2}\right)$$

Co-ordinates of R are

$$\left(\frac{10+11}{2}, \frac{7-5}{2}\right) = \left(\frac{21}{2}, 1\right)$$

Co-ordinates of S are

$$\left(\frac{11-2}{2}, \frac{-5+4}{2}\right) = \left(\frac{9}{2}, \frac{-1}{2}\right)$$

$$Slope\ of\ PQ = \frac{\frac{15}{2} - 6}{7-1} = \frac{\frac{15-12}{2}}{6} = \frac{3}{12} = \frac{1}{4}$$

$$Slope\ of\ RS = \frac{\frac{-1}{2} - 1}{\frac{9}{2} - \frac{21}{2}} = \frac{\frac{-1-2}{2}}{\frac{9-21}{2}} = \frac{-3}{-12} = \frac{1}{4}$$

Since, slope of PQ = Slope of RS, PQ || RS.

$$Slope\ of\ QR = \frac{1 - \frac{15}{2}}{\frac{21}{2} - 7} = \frac{\frac{2-15}{2}}{\frac{21-14}{2}} = \frac{-13}{7}$$

$$Slope\ of\ SP = \frac{6 + \frac{1}{2}}{1 - \frac{9}{2}} = \frac{\frac{12+1}{2}}{\frac{2-9}{2}} = \frac{13}{-7} = \frac{-13}{7}$$

Since, slope of QR = Slope of SP, QR || SP.

Hence, PQRS is a parallelogram.

Question 11.
Show that the points P (a, b + c), Q (b, c + a) and R (c, a + b) are collinear.

Solution:

The points P, Q, R will be collinear if slope of PQ and QR is the same.

$$Slope\ of\ PQ = \frac{c + a - b - c}{b - a} = \frac{a - b}{b - a} = -1$$

$$Slope\ of\ QR = \frac{a + b - c - a}{c - b} = \frac{b - c}{c - b} = -1$$

Hence, the points P, Q, and R are collinear.

Question 12.

Find x, if the slope of the line joining (x, 2) and (8, -11) is $\frac{-3}{4}$.

Solution:

Let A = (x, 2) and B = (8, -11)

Slope of AB = $\frac{-11-2}{8-x}$

$\frac{-11-2}{8-x} = \frac{-3}{4}$ (Given)

$\frac{13}{8-x} = \frac{3}{4}$

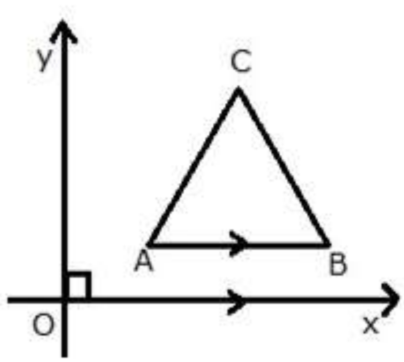
$52 = 24 - 3x$

$3x = 24 - 52 = -28$

$x = \frac{-28}{3}$

Question 13.

The side AB of an equilateral triangle ABC is parallel to the x-axis. Find the slope of all its sides.



Solution:

We know that the slope of any line parallel to x-axis is 0.

Therefore, slope of AB = 0

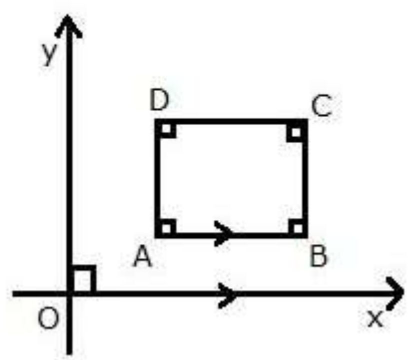
Since, ABC is an equilateral triangle, $\angle A = 60°$

Slope of AC = tan 60^{o} = $\sqrt{3}$

Slope of BC = -tan 60^{o} = -$\sqrt{3}$

Question 14.

The side AB of a square ABCD is parallel to the x-axis. Find the slopes of all its sides.
Also, find:
(i) the slope of the diagonal AC,
(ii) the slope of the diagonal BD.



Solution:

We know that the slope of any line parallel to x-axis is 0.
Therefore, slope of AB = 0

As CD || BC, slope of CD = Slope of AB = 0

As BC $\perp$ AB, slope of BC = $-\frac{1}{\text{Slope of AB}} = \frac{-1}{0}$ = not defined

As AD $\perp$ AB, slope of AD = $-\frac{1}{\text{Slope of AB}} = \frac{-1}{0}$ = not defined

(i) The diagonal AC makes an angle of 45° with the positive direction of x axis.

$\therefore$ Slope of AC = $\tan 45^{\circ} = 1$

(ii) The diagonal BC makes an angle of -45° with the positive direction of x axis.

$\therefore$ Slope of BC = $\tan(-45^{\circ}) = -1$

Question 15.

A (5, 4), B (-3, -2) and C (1, -8) are the vertices of a triangle ABC. Find:

(i) the slope of the altitude of AB,

(ii) the slope of the median AD, and

(iii) the slope of the line parallel to AC.

Solution:

Given, A (5, 4), B (-3, -2) and C (1, -8) are the vertices of a triangle ABC.

(i) Slope of AB = $\frac{-2-4}{-3-5} = \frac{-6}{-8} = \frac{3}{4}$

Slope of the altitude of AB = $\frac{-1}{\text{Slope of AB}} = \frac{-1}{\frac{3}{4}} = \frac{-4}{3}$

(ii) Since, D is the mid-point of BC.

Co-ordinates of point D are

$\left(\frac{-3+1}{2}, \frac{-2-8}{2}\right) = (-1, -5)$

Slope of AD = $\frac{-5-4}{-1-5} = \frac{-9}{-6} = \frac{3}{2}$

(iii) Slope of AC = $\frac{-8-4}{1-5} = \frac{-12}{-4} = 3$

Slope of line parallel to AC = Slope of AC = 3

Question 16.

The slope of the side BC of a rectangle ABCD is $\frac{2}{3}$. Find:

(i) the slope of the side AB,

(ii) the slope of the side AD.

Solution:

(i) Since, BC is perpendicular to AB,

Slope of AB = $\frac{-1}{\text{Slope of BC}} = \frac{-1}{\frac{2}{3}} = \frac{-3}{2}$

(ii) Since, AD is parallel to BC,

Slope of AD = Slope of BC = $\frac{2}{3}$

Question 17.

Find the slope and the inclination of the line AB if:

(i) A = (-3, -2) and B = (1, 2)

(ii) A = (0, $-\sqrt{3}$) and B = (3, 0)

(iii) A = (-1, 2$\sqrt{3}$) and B = (-2, $\sqrt{3}$)

Solution:

(i) A = (-3, -2) and B = (1, 2)

Slope of AB = $\frac{2+2}{1+3} = \frac{4}{4} = 1 = \tan\theta$

Inclination of line AB = $\theta = 45^\circ$

(ii) A = (0, $-\sqrt{3}$) and B = (3, 0)

Slope of AB = $\frac{0+\sqrt{3}}{3-0} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \tan\theta$

Inclination of line AB = $\theta = 30^\circ$

(iii) A = (-1, 2$\sqrt{3}$) and B = (-2, $\sqrt{3}$)

Slope of AB = $\frac{\sqrt{3}-2\sqrt{3}}{-2+1} = \frac{-\sqrt{3}}{-1} = \sqrt{3} = \tan\theta$

Inclination of line AB = $\theta = 60^\circ$

Question 18.
The points (-3, 2), (2, -1) and (a, 4) are collinear. Find a.

Solution:

Given, points A (-3, 2), B (2, -1) and C (a, 4) are collinear.

∴ Slope of AB = Slope of BC

$$\frac{-1-2}{2+3} = \frac{4+1}{a-2}$$

$$\frac{-3}{5} = \frac{5}{a-2}$$

$$-3a + 6 = 25$$

$$-3a = 25 - 6 = 19$$

$$a = \frac{-19}{3} = -6\frac{1}{3}$$

Question 19.
The points (K, 3), (2, -4) and (-K + 1, -2) are collinear. Find K.

Solution:

Given, points A (K, 3), B (2, -4) and C (-K + 1, -2) are collinear.

∴ Slope of AB = Slope of BC

$$\frac{-4-3}{2-K} = \frac{-2+4}{-K+1-2}$$

$$\frac{-7}{2-K} = \frac{2}{-K-1}$$

$$7K + 7 = 4 - 2K$$

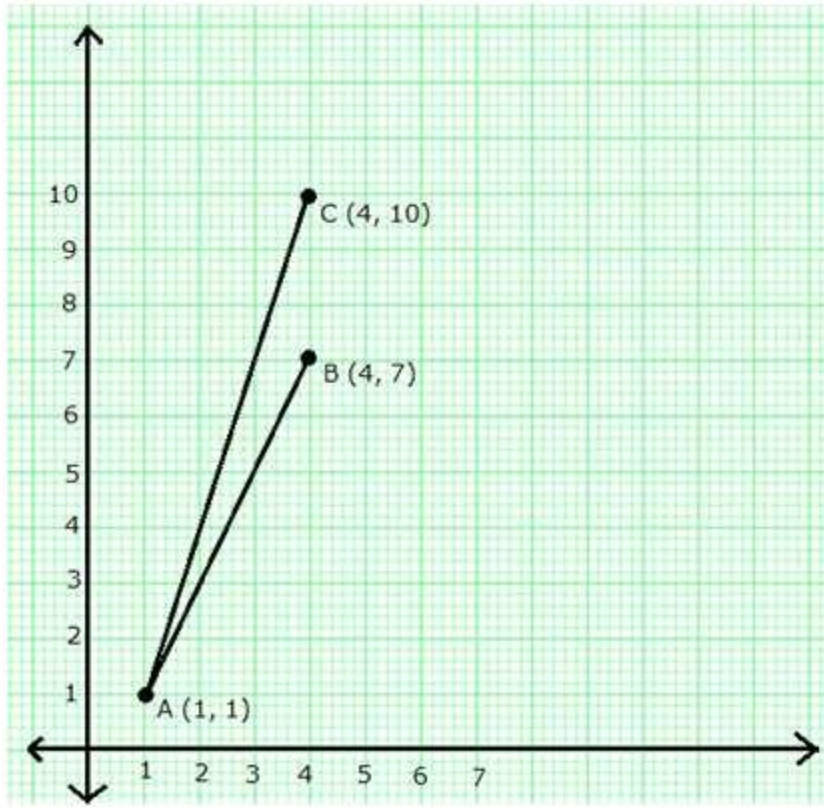
$$9K = -3$$

$$K = \frac{-1}{3}$$

Question 20.
Plot the points A (1, 1), B (4, 7) and C (4, 10) on a graph paper. Connect A and B, and also A and C.

Which segment appears to have the steeper slope, AB or AC?
Justify your conclusion by calculating the slopes of AB and AC.

Solution:



From the graph, clearly, AC has steeper slope.

Slope of AB = $\frac{7-1}{4-1} = \frac{6}{3} = 2$

Slope of AC = $\frac{10-1}{4-1} = \frac{9}{3} = 3$

The line with greater slope is steeper. Hence, AC has steeper slope.

Question 21.

Find the value(s) of k so that PQ will be parallel to RS. Given:
(i) P (2, 4), Q (3, 6), R (8, 1) and S (10, k)
(ii) P (3, -1), Q (7, 11), R (-1, -1) and S (1, k)
(iii) P (5, -1), Q (6, 11), R (6, -4k) and S (7, k^2)

Solution:

Since, PQ || RS,
Slope of PQ = Slope of RS

(i) Slope of PQ = $\frac{6-4}{3-2} = 2$

Slope of RS = $\frac{k-1}{10-8} = \frac{k-1}{2}$

$\therefore 2 = \frac{k-1}{2}$

$k - 1 = 4$

$k = 5$

(ii) Slope of PQ = $\frac{11+1}{7-3} = \frac{12}{4} = 3$

Slope of RS = $\frac{k+1}{1+1} = \frac{k+1}{2}$

$\therefore 3 = \frac{k+1}{2}$

$k + 1 = 6$

$k = 5$

(iii) Slope of PQ = $\frac{11+1}{6-5} = \frac{12}{1} = 12$

Slope of RS = $\frac{k^2+4k}{7-6} = k^2 + 4k$

$\therefore 12 = k^2 + 4k$

$k^2 + 4k - 12 = 0$

$(k + 6)(k - 2) = 0$

$k = -6$ and 2

Exercise 14C

Question 1.
Find the equation of a line whose:
y-intercept = 2 and slope = 3.

Solution:
Given, y-intercept = c = 2 and slope = m = 3.

Substituting the values of c and m in the equation y = mx + c, we get,
y = 3x + 2, which is the required equation.

Question 2.

Find the equation of a line whose:
y-intercept = -1 and inclination = 45^o.

Solution:

Given, y-intercept = c = -1 and inclination = 45^o.
Slope = m = tan 45^o = 1
Substituting the values of c and m in the equation y = mx + c, we get,
y = x - 1, which is the required equation.

Question 3.

Find the equation of the line whose slope is $\frac{-4}{3}$ and which passes through (-3, 4).

Solution:

Given, slope = $\frac{-4}{3}$
The equation passes through (-3, 4) = (x_1, y_1)
Substituting the values in $y - y_1 = m(x - x_1)$, we get,
$y - 4 = \frac{-4}{3}(x + 3)$
3y - 12 = -4x - 12
4x + 3y = 0, which is the required equation.

Question 4.

Find the equation of a line which passes through (5, 4) and makes an angle of 60^o with the positive direction of the x-axis.

Solution:

Slope of the line = tan 60^o = $\sqrt{3}$
The line passes through the point (5, 4) = (x_1, y_1)
Substituting the values in $y - y_1 = m(x - x_1)$, we get,
$y - 4 = \sqrt{3}(x - 5)$
$y - 4 = \sqrt{3}x - 5\sqrt{3}$
$y = \sqrt{3}x + 4 - 5\sqrt{3}$, which is the required equation.

Question 5.

Find the equation of the line passing through:
(i) (0, 1) and (1, 2) (ii) (-1, -4) and (3, 0)

Solution:

(i) Let (0, 1) = (x_1, y_1) and (1, 2) = (x_2, y_2)

$\therefore$ Slope of the line = $\frac{2-1}{1-0} = 1$

The required equation of the line is given by:

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 0)$

$y - 1 = x$

$y = x + 1$

(ii) Let (-1, -4) = (x_1, y_1) and (3, 0) = (x_2, y_2)

$\therefore$ Slope of the line = $\frac{0+4}{3+1} = \frac{4}{4} = 1$

The required equation of the line is given by:

$y - y_1 = m(x - x_1)$

$y + 4 = 1(x + 1)$

$y + 4 = x + 1$

$y = x - 3$

Question 6.

The co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively. Find:

(i) the gradient of PQ;

(ii) the equation of PQ;

(iii) the co-ordinates of the point where PQ intersects the x-axis.

Solution:

Given, co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively.

(i) Gradient of PQ = $\frac{5-6}{-3-2} = \frac{-1}{-5} = \frac{1}{5}$

(ii) The equation of the line PQ is given by:

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{1}{5}(x - 2)$

$5y - 30 = x - 2$

$5y = x + 28$

(iii) Let the line PQ intersects the x-axis at point A (x, 0).

Putting y = 0 in the equation of the line PQ, we get,

$0 = x + 28$

$x = -28$

Thus, the co-ordinates of the point where PQ intersects the x-axis are A (-28, 0).

Question 7.
The co-ordinates of two points A and B are (-3, 4) and (2, -1). Find:
(i) the equation of AB;
(ii) the co-ordinates of the point where the line AB intersects the y-axis.

Solution:

(i) Given, co-ordinates of two points A and B are (-3, 4) and (2, -1).

Slope = $\frac{-1-4}{2+3} = \frac{-5}{5} = -1$

The equation of the line AB is given by:

$y - y_1 = m(x - x_1)$

$y + 1 = -1(x - 2)$

$y + 1 = -x + 2$

$x + y = 1$

(ii) Let the line AB intersects the y-axis at point (0, y).

Putting x = 0 in the equation of the line, we get,

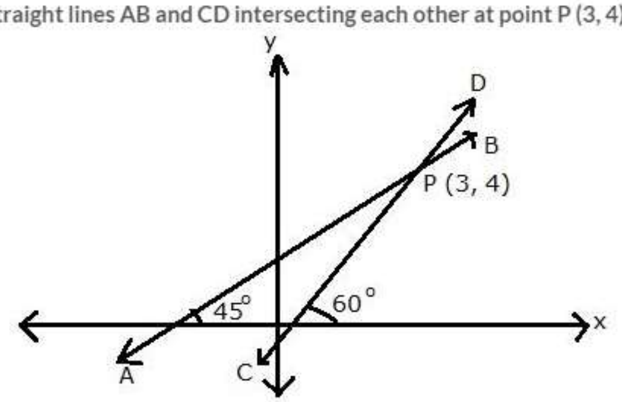
$0 + y = 1$

$y = 1$

Thus, the co-ordinates of the point where the line AB intersects the y-axis are (0, 1).

Question 8.

The figure given below shows two straight lines AB and CD intersecting each other at point P (3, 4). Find the equation of AB and CD.



Solution:

Slope of line AB = tan 45° = 1

The line AB passes through P (3, 4). So, the equation of the line AB is given by:

$y - y_1 = m(x - x_1)$

$y - 4 = 1(x - 3)$

$y - 4 = x - 3$

$y = x + 1$

Slope of line CD = tan 60° = $\sqrt{3}$

The line CD passes through P (3, 4). So, the equation of the line CD is given by:

$y - y_1 = m(x - x_1)$

$y - 4 = \sqrt{3}(x - 3)$

$y - 4 = \sqrt{3}x - 3\sqrt{3}$

$y = \sqrt{3}x + 4 - 3\sqrt{3}$

Question 9.

In ΔABC, A = (3, 5), B = (7, 8) and C = (1, -10). Find the equation of the median through A.

Solution:

The vertices of ΔABC are A(3, 5), B(7, 8) and C(1, − 10).

Coordinates of the mid-point D of BC $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{7 + 1}{2}, \frac{8 + (-10)}{2}\right)$

$= \left(\frac{8}{2}, \frac{-2}{2}\right)$

$= (4, -1)$

Slope of AD $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{4 - 3} = \frac{-6}{1} = -6$

Now, the equation of median is given by

$y - y_1 = m(x - x_1)$

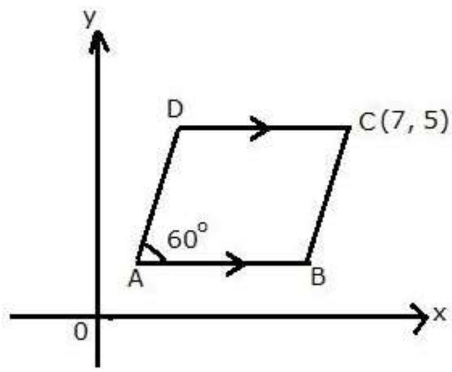
$\Rightarrow y - 5 = -6(x - 3)$

$\Rightarrow y - 5 = -6x + 18$

$\Rightarrow 6x + y = 23$

Question 10.

The following figure shows a parallelogram ABCD whose side AB is parallel to the x-axis, $\angle A = 60^\circ$ and vertex C = (7, 5). Find the equations of BC and CD.



Solution:

Since, ABCD is a parallelogram,

$\angle A + \angle B = 180^\circ$

$\angle B = 180^\circ - 60^\circ = 120^\circ$

Slope of BC = tan 120° = tan (90° + 30°) = cot30° = $\sqrt{3}$

Equation of the line BC is given by:

$y - y_1 = m(x - x_1)$

$y - 5 = \sqrt{3}(x - 7)$

$y - 5 = \sqrt{3}x - 7\sqrt{3}$

$y = \sqrt{3}x + 5 - 7\sqrt{3}$

Since, CD || AB and AB || x-axis, slope of CD = Slope of AB = 0

Equation of the line CD is given by:

$y - y_1 = m(x - x_1)$

$y - 5 = 0(x - 7)$

$y = 5$

Question 11.

Find the equation of the straight line passing through origin and the point of intersection of the lines x + 2y = 7 and x − y = 4.

Solution:

The given equations are:

x + 2y = 7 ....(1)

x - y = 4 ....(2)

Subtracting (2) from (1), we get,

3y = 3

y = 1

From (2), x = 4 + y = 4 + 1 = 5

The required line passes through (0, 0) and (5, 1).

$$Slope\ of\ the\ line = \frac{1-0}{5-0} = \frac{1}{5}$$

Required equation of the line is given by:

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 0 = \frac{1}{5}(x - 0)$$
$$\Rightarrow 5y = x$$
$$\Rightarrow x - 5y = 0$$

Question 12.
In triangle ABC, the co-ordinates of vertices A, B and C are (4, 7), (-2, 3) and (0, 1) respectively. Find the equation of median through vertex A.
Also, find the equation of the line through vertex B and parallel to AC.

Solution:

Given, the co-ordinates of vertices A, B and C of a triangle ABC are (4, 7), (-2, 3) and (0, 1) respectively.
Let AD be the median through vertex A.
Co-ordinates of the point D are

$$\left(\frac{-2+0}{2}, \frac{3+1}{2}\right)$$

$$(-1, 2)$$

$\therefore$ Slope of AD = $\frac{2-7}{-1-4} = \frac{-5}{-5} = 1$

The equation of the median AD is given by:
$y - y_1 = m(x - x_1)$
$y - 2 = 1(x + 1)$
$y - 2 = x + 1$
$y = x + 3$
The slope of the line which is parallel to line AC will be equal to the slope of AC.

Slope of AC = $\frac{1-7}{0-4} = \frac{-6}{-4} = \frac{3}{2}$

The equation of the line which is parallel to AC and passes through B is given by:

$y - 3 = \frac{3}{2}(x + 2)$

$2y - 6 = 3x + 6$
$2y = 3x + 12$

Question 13.
A, B and C have co-ordinates (0, 3), (4, 4) and (8, 0) respectively. Find the equation of the line through A and perpendicular to BC.

Solution:

Slope of BC = $\frac{0-4}{8-4} = \frac{-4}{4} = -1$

Slope of line perpendicular to BC = $\frac{-1}{\text{Slope of BC}} = 1$

The equation of the line through A and perpendicular to BC is given by:
$y - y_1 = m(x - x_1)$
$y - 3 = 1(x - 0)$
$y - 3 = x$
$y = x + 3$

Question 14.
Find the equation of the perpendicular dropped from the point (-1, 2) onto the line joining the points (1, 4) and (2, 3).

Solution:

Let A = (1, 4), B = (2, 3), and C = (-1, 2).

Slope of AB = $\frac{3-4}{2-1} = -1$

Slope of equation perpendicular to AB = $\frac{-1}{\text{Slope of AB}} = 1$

The equation of the perpendicular drawn through C onto AB is given by:

$y - y_1 = m(x - x_1)$

$y - 2 = 1(x + 1)$

$y - 2 = x + 1$

$y = x + 3$

Question 15.

Find the equation of the line, whose:

(i) x-intercept = 5 and y-intercept = 3

(ii) x-intercept = -4 and y-intercept = 6

(iii) x-intercept = -8 and y-intercept = -4

Solution:

(i) When x-intercept = 5, corresponding point on x-axis is (5, 0)

When y-intercept = 3, corresponding point on y-axis is (0, 3).

Let $(x_1, y_1) = (5, 0)$ and $(x_2, y_2) = (0, 3)$

Slope = $\frac{3-0}{0-5} = \frac{-3}{5}$

The required equation is:

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-3}{5}(x - 5)$

$5y = -3x + 15$

$3x + 5y = 15$

(ii) When x-intercept = -4, corresponding point on x-axis is (-4, 0)

When y-intercept = 6, corresponding point on y-axis is (0, 6).

Let $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (0, 6)$

Slope = $\frac{6-0}{0+4} = \frac{6}{4} = \frac{3}{2}$

The required equation is:
$y - y_1 = m(x - x_1)$
$y - 0 = \frac{3}{2}(x + 4)$
$2y = 3x + 12$

(iii) When x-intercept = -8, corresponding point on x-axis is (-8, 0)
When y-intercept = -4, corresponding point on y-axis is (0, -4).
Let $(x_1, y_1) = (-8, 0)$ and $(x_2, y_2) = (0, -4)$
Slope = $\frac{-4-0}{0+8} = \frac{-4}{8} = \frac{-1}{2}$

The required equation is:
$y - y_1 = m(x - x_1)$
$y - 0 = \frac{-1}{2}(x + 8)$
$2y = -x - 8$
$x + 2y + 8 = 0$

Question 16.

Find the equation of the line whose slope is $\frac{-5}{6}$ and x-intercept is 6.

Solution:

Since, x-intercept is 6, so the corresponding point on x-axis is (6, 0).
Slope = m = $\frac{-5}{6}$
Required equation of the line is given by:
$y - y_1 = m(x - x_1)$
$y - 0 = \frac{-5}{6}(x - 6)$
$6y = -5x + 30$
$5x + 6y = 30$

Question 17.
Find the equation of the line with x-intercept 5 and a point on it (-3, 2).

Solution:

Since, x-intercept is 5, so the corresponding point on x-axis is (5, 0).
The line also passes through (-3, 2).

$\therefore$ Slope of the line = $\frac{2-0}{-3-5} = \frac{2}{-8} = \frac{-1}{4}$

Required equation of the line is given by:

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-1}{4}(x - 5)$

$4y = -x + 5$

$x + 4y = 5$

Question 18.
Find the equation of the line through (1, 3) and making an intercept of 5 on the y-axis.

Solution:

Since, y-intercept = 5, so the corresponding point on y-axis is (0, 5).
The line passes through (1, 3).

$\therefore$ Slope of the line = $\frac{3-5}{1-0} = \frac{-2}{1} = -2$

Required equation of the line is given by:

$y - y_1 = m(x - x_1)$

$y - 5 = -2(x - 0)$

$y - 5 = -2x$

$2x + y = 5$

Question 19.
Find the equations of the lines passing through point (-2, 0) and equally inclined to the co-ordinate axis.

Solution:

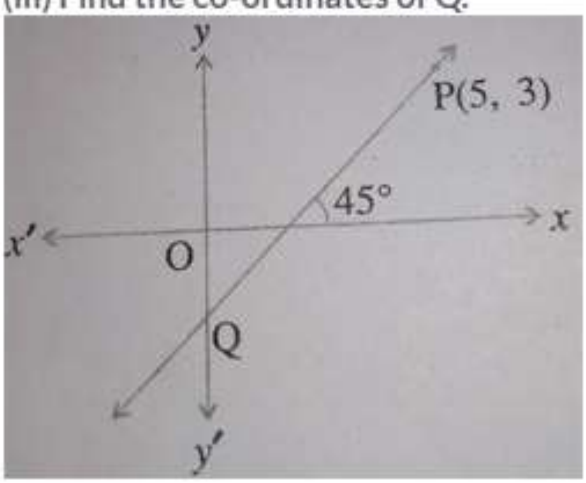
Let AB and CD be two equally inclined lines.
For line AB:

Slope = m = tan 45^o = 1
$(x_1, y_1) = (-2, 0)$
Equation of the line AB is:
$y - y_1 = m(x - x_1)$
$y - 0 = 1(x + 2)$
$y = x + 2$
For line CD:

Slope = m = tan (-45^o) = -1
$(x_1, y_1) = (-2, 0)$
Equation of the line CD is:
$y - y_1 = m(x - x_1)$
$y - 0 = -1(x + 2)$
$y = -x - 2$
$x + y + 2 = 0$

Question 20.

The line through P(5, 3) intersects y-axis at Q.
(i) Write the slope of the line.
(ii) Write the equation of the line.
(iii) Find the co-ordinates of Q.



Solution:

(i)
The equation of the y-axis is $x = 0$
Given that the required line through $P(5, 3)$
intersects the y-axis at Q and the angle
of inclination is 45°.
Therefore slope of the line $PQ = \tan 45° = 1$.

(ii)
The equation of a line passing through the point
$A(x_1, y_1)$ with slope 'm' is
$y - y_1 = m(x - x_1)$
Therefore the equation of the line passing
through the point P(5, 3) with slope 1 is
$y - 3 = 1 \times (x - 5)$
$\Rightarrow y - 3 = x - 5$
$\Rightarrow x - y = 2$

(iii)
From subpart (ii), the equation of the line PQ
is $x - y = 2$.
Given that the line intersects with the y-axis, $x = 0$
Thus, substituting $x = 0$ in the equation $x - y = 2$
we have, $0 - y = 2$
$\Rightarrow y = -2$
Thus, the coordinates point of intersection Q
are $Q(0, -2)$

Question 21.

Write down the equation of the line whose gradient is $\frac{-2}{5}$ and which passes through point P, where P divides the line segment joining A (4, -8) and B (12, 0) in the ratio 3: 1.

Solution:

Given, P divides the line segment joining A (4, -8) and B (12, 0) in the ratio 3: 1.
Co-ordinates of point P are

$$\left(\frac{3\times 12+1\times 4}{3+1}, \frac{3\times 0+1\times(-8)}{3+1}\right)$$

$$=\left(\frac{36+4}{4}, \frac{-8}{4}\right)$$

$$=(10,-2)$$

Slope = m = $\frac{-2}{5}$ (Given)

Thus, the required equation of the line is

$y - y_1 = m(x - x_1)$

$y + 2 = \frac{-2}{5}(x - 10)$

$5y + 10 = -2x + 20$

$2x + 5y = 10$

Question 22.
A (1, 4), B (3, 2) and C (7, 5) are vertices of a triangle ABC, Find:
(i) the co-ordinates of the centroid of triangle ABC.
(ii) the equation of a line, through the centroid and parallel to AB.

Solution:

(i) Co-ordinates of the centroid of triangle ABC are

$$\left(\frac{1+3+7}{3}, \frac{4+2+5}{3}\right)$$

$$=\left(\frac{11}{3}, \frac{11}{3}\right)$$

(ii) Slope of AB = $\frac{2-4}{3-1} = \frac{-2}{2} = -1$

Slope of the line parallel to AB = Slope of AB = -1

Thus, the required equation of the line is

$y - y_1 = m(x - x_1)$

$$y-\frac{11}{3}=-1\left(x-\frac{11}{3}\right)$$

$3y - 11 = -3x + 11$

$3x + 3y = 22$

Question 23.
A (7, -1), B (4, 1) and C (-3, 4) are the vertices of a triangle ABC. Find the equation of a line through the vertex B and the point P in AC; such that AP: CP = 2: 3.

Solution:

Given, AP: CP = 2: 3
$\therefore$ Co-ordinates of P are

$$\left(\frac{2\times(-3)+3\times 7}{2+3}, \frac{2\times 4+3\times(-1)}{2+3}\right)$$

$$= \left(\frac{-6+21}{5}, \frac{8-3}{5}\right)$$

$$= \left(\frac{15}{5}, \frac{5}{5}\right)$$

$$= (3, 1)$$

Slope of BP = $\frac{1-1}{3-4} = 0$

Required equation of the line passing through points B and P is

$y - y_1 = m(x - x_1)$
$y - 1 = 0(x - 3)$
$y = 1$

Exercise 14D

Question 1.
Find the slope and y-intercept of the line:
(i) y = 4
(ii) ax − by = 0
(iii) 3x − 4y = 5

Solution:

(i) y = 4
Comparing this equation with y = mx + c, we have:
Slope = m = 0
y-intercept = c = 4

(ii) $ax - by = 0 \Rightarrow by = ax \Rightarrow y = \frac{a}{b}x$

Comparing this equation with y = mx + c, we have:

Slope = m = $\frac{a}{b}$

y-intercept = c = 0

(iii) $3x - 4y = 5 \Rightarrow 4y = 3x - 5 \Rightarrow y = \frac{3}{4}x - \frac{5}{4}$

Comparing this equation with y = mx + c, we have:

Slope = m = $\frac{3}{4}$

y-intercept = c = $-\frac{5}{4}$

Question 2.
The equation of a line x – y = 4. Find its slope and y-intercept. Also, find its inclination.

Solution:

Given equation of a line is x - y = 4

$\Rightarrow$ y = x - 4

Comparing this equation with y = mx + c. We have:

Slope = m = 1

y-intercept = c = -4

Let the inclination be θ.

Slope = 1 = tan θ = tan 45°

$\therefore \theta = 45^\circ$

Question 3.
(i) Is the line 3x + 4y + 7 = 0 perpendicular to the line 28x – 21y + 50 = 0?
(ii) Is the line x – 3y = 4 perpendicular to the line 3x – y = 7?
(iii) Is the line 3x + 2y = 5 parallel to the line x + 2y = 1?
(iv) Determine x so that the slope of the line through (1, 4) and (x, 2) is 2.

Solution:

(i) $3x + 4y + 7 = 0$

$\Rightarrow 4y = -3x - 7$

$\Rightarrow y = -\frac{3}{4}x - \frac{7}{4}$

Slope of this line = $\frac{-3}{4}$

$28x - 21y + 50 = 0$

$\Rightarrow 21y = 28x + 50$

$\Rightarrow y = \frac{28}{21}x + \frac{50}{21}$

$\Rightarrow y = \frac{4}{3}x + \frac{50}{21}$

Slope of this line = $\frac{4}{3}$

Since, product of slopes of the two lines = -1, the lines are perpendicular to each other.

(ii) $x - 3y = 4$

$3y = x - 4$

$y = \frac{1}{3}x - \frac{4}{3}$

Slope of this line = $\frac{1}{3}$

$3x - y = 7$

$y = 3x - 7$

Slope of this line = 3

Product of slopes of the two lines = $1 \neq -1$

So, the lines are not perpendicular to each other.

(iii) $3x + 2y = 5$

$2y = -3x + 5$

$y = \frac{-3}{2}x + \frac{5}{2}$

Slope of this line = $\frac{-3}{2}$

$x + 2y = 1$

$2y = -x + 1$

$y = \frac{-1}{2}x + \frac{1}{2}$

Slope of this line = $\frac{-1}{2}$

Product of slopes of the two lines = 3 ≠ -1
So, the lines are not perpendicular to each other.
(iv) Given, the slope of the line through (1, 4) and (x, 2) is 2.

$$\therefore \frac{2-4}{x-1} = 2$$

$$\frac{-2}{x-1} = 2$$

$$\frac{-1}{x-1} = 1$$

$$-1 = x - 1$$

$$x = 0$$

Question 4.

Find the slope of the line which is parallel to:

(i) x + 2y + 3 = 0 (ii) $\frac{x}{2} - \frac{y}{3} - 1 = 0$

Solution:

(i) x + 2y + 3 = 0
2y = -x - 3

$$y = \frac{-1}{2}x - \frac{3}{2}$$

Slope of this line = $\frac{-1}{2}$

Slope of the line which is parallel to the given line = Slope of the given line = $\frac{-1}{2}$

(ii) $\frac{x}{2} - \frac{y}{3} - 1 = 0$

$$\frac{y}{3} = \frac{x}{2} - 1$$

$$y = \frac{3}{2}x - 3$$

Slope of this line = $\frac{3}{2}$

Slope of the line which is parallel to the given line = Slope of the given line = $\frac{3}{2}$

Question 5.

Find the slope of the line which is perpendicular to:

(i) $x - \frac{y}{2} + 3 = 0$ (ii) $\frac{x}{3} - 2y = 4$

Solution:

(i) $x - \frac{y}{2} + 3 = 0$

$\frac{y}{2} = x + 3$

$y = 2x + 6$

Slope of this line = 2

Slope of the line which is perpendicular to the given line = $\frac{-1}{\text{Slope of the given line}} = \frac{-1}{2}$

(ii) $\frac{x}{3} - 2y = 4$

$2y = \frac{x}{3} - 4$

$y = \frac{x}{6} - 2$

Slope of this line = $\frac{1}{6}$

Slope of the line which is perpendicular to the given line = $\frac{-1}{\text{Slope of this line}} = \frac{-1}{\frac{1}{6}} = -6$

Question 6.

(i) Lines $2x - by + 3 = 0$ and $ax + 3y = 2$ are parallel to each other. Find the relation connecting a and b.

(ii) Lines $mx + 3y + 7 = 0$ and $5x - ny - 3 = 0$ are perpendicular to each other. Find the relation connecting m and n.

Solution:

(i) $2x - by + 3 = 0$

$by = 2x + 3$

$y = \frac{2}{b}x + \frac{3}{b}$

Slope of this line = $\frac{2}{b}$

$ax + 3y = 2$

$3y = -ax + 2$

$y = \frac{-a}{3}x + \frac{2}{3}$

Slope of this line = $\frac{-a}{3}$

Since, the lines are parallel, so the slopes of the two lines are equal.

$\therefore \frac{2}{b} = \frac{-a}{3}$

$ab = -6$

(ii) $mx + 3y + 7 = 0$

$3y = -mx - 7$

$y = \frac{-m}{3}x - \frac{7}{3}$

Slope of this line = $\frac{-m}{3}$

$5x - ny - 3 = 0$

$ny = 5x - 3$

$y = \frac{5}{n}x - \frac{3}{n}$

Slope of this line = $\frac{5}{n}$

Since, the lines are perpendicular; the product of their slopes is -1.

$\therefore \left(\frac{-m}{3}\right)\left(\frac{5}{n}\right) = -1$

$5m = 3n$

Question 7.

Find the value of p if the lines, whose equations are $2x - y + 5 = 0$ and $px + 3y = 4$ are perpendicular to each other.

Solution:

$2x - y + 5 = 0$

$y = 2x + 5$

Slope of this line = 2

$px + 3y = 4$

$3y = -px + 4$

$y = \frac{-p}{3}x + \frac{4}{3}$

Slope of this line = $\frac{-p}{3}$

Since, the lines are perpendicular to each other, the product of the slopes is -1.

$\therefore (2)\left(\frac{-p}{3}\right) = -1$

$\frac{2p}{3} = 1$

$p = \frac{3}{2}$

Question 8.
The equation of a line AB is $2x - 2y + 3 = 0$.
(i) Find the slope of the line AB.
(ii) Calculate the angle that the line AB makes with the positive direction of the x-axis.

Solution:

(i) $2x - 2y + 3 = 0$

$2y = 2x + 3$

$y = x + \frac{3}{2}$

Slope of the line AB = 1

(ii) Required angle = θ

Slope = $\tan\theta = 1 = \tan 45^\circ$

$\theta = 45^\circ$

Question 9.
The lines represented by $4x + 3y = 9$ and $px - 6y + 3 = 0$ are parallel. Find the value of p.

Solution:

$4x + 3y = 9$
$3y = -4x + 9$
$y = \frac{-4}{3}x + 3$
Slope of this line = $\frac{-4}{3}$
$px - 6y + 3 = 0$
$6y = px + 3$
$y = \frac{p}{6}x + \frac{1}{2}$
Slope of this line = $\frac{p}{6}$
Since, the lines are parallel, their slopes will be equal.
$\therefore \frac{-4}{3} = \frac{p}{6}$
$-4 = \frac{p}{2}$
$p = -8$

Question 10.
If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, find the value of p.

Solution:

$y = 3x + 7$
Slope of this line = 3
$2y + px = 3$
$2y = -px + 3$
$y = -\frac{p}{2}x + \frac{3}{2}$
Slope of this line = $-\frac{p}{2}$

$y = 3x + 7$
Slope of this line = 3
$2y + px = 3$
$2y = -px + 3$

$$y = -\frac{p}{2}x + \frac{3}{2}$$

Slope of this line = $-\frac{p}{2}$

Since, the lines are perpendicular to each other, the product of their slopes is -1.

$$\therefore (3)\left(-\frac{p}{2}\right) = -1$$

$$\frac{3p}{2} = 1$$

$$p = \frac{2}{3}$$

Question 11.
The line through A(-2,3) and B(4,b) is perpendicular to the line 2x – 4y =5. Find the value of b.

Solution:

The slope of the line passing through two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line passing through two given points $A(-2, 3)$ and $B(4, b)$ is

$$\text{Slope of AB} = \frac{b - 3}{4 - (-2)} = \frac{b - 3}{4 + 2} = \frac{b - 3}{6}$$

Equation of the given line is $2x - 4y = 5$

$\Rightarrow$ Equation is $4y = 2x - 5$

$\Rightarrow$ Equation is $y = \frac{1}{4}(2x - 5)$

$\Rightarrow$ Equation is $y = \frac{x}{2} - \frac{5}{4}$

Comparing this equation with the general equation,

$y = mx + c$, we have $m = \frac{1}{2}$

Since the given line and AB are perpendicular to each other, the product of their slopes is -1

$\therefore \left(\frac{b-3}{6}\right) \times \frac{1}{2} = -1$

$\Rightarrow b - 3 = -12$

$\Rightarrow b = 3 - 12$

$\Rightarrow b = -9$

Question 12.
Find the equation of the line through (-5, 7) and parallel to:
(i) x-axis (ii) y-axis

Solution:

(i) The slope of the line parallel to x-axis is 0.

$(x_1, y_1) = (-5, 7)$

Required equation of the line is

$y - y_1 = m(x - x_1)$

$y - 7 = 0(x + 5)$

$y = 7$

(ii) The slope of the line parallel to y-axis is not defined.

That is slope of the line is $\tan 90^0$ and hence the given line is parallel to y-axis.

$(x_1, y_1) = (-5, 7)$

Required equation of the line is

$x - x_1 = 0$

$\Rightarrow x + 5 = 0$

Question 13.
(i) Find the equation of the line passing through (5, -3) and parallel to x – 3y = 4.
(ii) Find the equation of the line parallel to the line 3x + 2y = 8 and passing through the point (0, 1).

Solution:

(i) $x - 3y = 4$

$\Rightarrow 3y = x - 4$

$\Rightarrow y = \frac{1}{3}x - \frac{4}{3}$

Slope of this line = $\frac{1}{3}$

Slope of a line parallel to this line = $\frac{1}{3}$

Required equation of the line passing through (5, -3) is

$y - y_1 = m(x - x_1)$

$y + 3 = \frac{1}{3}(x - 5)$

$3y + 9 = x - 5$

$x - 3y - 14 = 0$

(ii) $2y = -3x + 8$

Or $y = -\frac{3}{2}x + \frac{8}{2}$

$\therefore$ Slope of given line = $-\frac{3}{2}$

Since the required line is parallel to given straight line.

$\therefore$ Slope of required line (m) = $-\frac{3}{2}$

Now the equation of the required line is given by:

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 1 = -\frac{3}{2}(x - 0)$

$\Rightarrow 2y - 2 = -3x$

$\Rightarrow 3x + 2y = 2$

Question 14.

Find the equation of the line passing through (-2, 1) and perpendicular to 4x + 5y = 6.

Solution:

4x + 5y = 6
5y = -4x + 6

$y = \frac{-4}{5}x + \frac{6}{5}$

Slope of this line = $\frac{-4}{5}$

The required line is perpendicular to the line 4x + 5y = 6.

$\therefore$ Slope of the required line = $\frac{-1}{\text{Slope of the given line}} = \frac{-1}{\frac{-4}{5}} = \frac{5}{4}$

The required equation of the line is given by

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{5}{4}(x + 2)$

4y - 4 = 5x + 10
5x - 4y + 14 = 0

Question 15.
Find the equation of the perpendicular bisector of the line segment obtained on joining the points (6, -3) and (0, 3).

Solution:

Let A = (6, -3) and B = (0, 3).
We know the perpendicular bisector of a line is perpendicular to the line and it bisects the line, that it, it passes through the mid-point of the line.
Co-ordinates of the mid-point of AB are

$\left(\frac{6+0}{2}, \frac{-3+3}{2}\right) = (3, 0)$

Thus, the required line passes through (3, 0).

Slope of AB = $\frac{3+3}{0-6} = \frac{6}{-6} = -1$

$\therefore$ Slope of the required line = $\frac{-1}{\text{Slope of AB}} = 1$

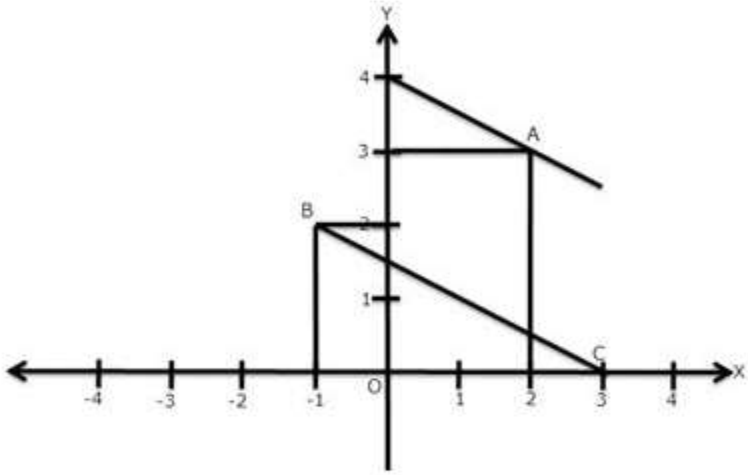
Thus, the equation of the required line is given by:

$y - y_1 = m(x - x_1)$

y - 0 = 1(x - 3)
y = x - 3

Question 16.

In the following diagram, write down:
(i) the co-ordinates of the points A, B and C.
(ii) the equation of the line through A and parallel to BC.



Solution:

(i) The co-ordinates of points A, B and C are (2, 3), (-1, 2) and (3, 0) respectively.

(ii) Slope of BC = $\frac{0-2}{3+1} = \frac{-2}{4} = \frac{-1}{2}$

Slope of a line parallel to BC = Slope of BC = $\frac{-1}{2}$

Required equation of a line passing through A and parallel to BC is given by

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-1}{2}(x - 2)$

$2y - 6 = -x + 2$

$X + 2y = 8$

Question 17.

B (-5, 6) and D (1, 4) are the vertices of rhombus ABCD. Find the equation of diagonal BD and of diagonal AC.

Solution:

We know that in a rhombus, diagonals bisect each other at right angle.

Let O be the point of intersection of the diagonals AC and BD.

Co-ordinates of O are

$\left(\frac{-5+1}{2}, \frac{6+4}{2}\right) = (-2, 5)$

Slope of BD = $\frac{4-6}{1+5} = \frac{-2}{6} = \frac{-1}{3}$

For line BD:

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{-1}{3}(x + 5)$

$3y - 18 = -x - 5$

$x + 3y = 13$

For line AC:

Slope = m = $\frac{-1}{\text{Slope of BD}} = 3$, $(x_1, y_1) = (-2, 5)$

Equation of the line AC is

$y - y_1 = m(x - x_1)$

$y - 5 = 3(x + 2)$

$y - 5 = 3x + 6$

$y = 3x + 11$

Question 18.

A = (7, -2) and C = (-1, -6) are the vertices of square ABCD. Find the equations of diagonal BD and of diagonal AC.

Solution:

We know that in a square, diagonals bisect each other at right angle.

Let O be the point of intersection of the diagonals AC and BD.

Co-ordinates of O are

$\left(\frac{7-1}{2}, \frac{-2-6}{2}\right) = (3, -4)$

Slope of AC = $\frac{-6+2}{-1-7} = \frac{-4}{-8} = \frac{1}{2}$

For line AC:

Slope = m = $\frac{1}{2}$, $(x_1, y_1) = (7, -2)$

Equation of the line AC is

$y - y_1 = m(x - x_1)$

$y + 2 = \frac{1}{2}(x - 7)$

$2y + 4 = x - 7$

$2y = x - 11$

For line BD:

Slope = m = $\frac{-1}{\text{Slope of AC}} = \frac{-1}{\frac{1}{2}} = -2$, $(x_1, y_1) = (3, -4)$

Equation of the line BD is

$y - y_1 = m(x - x_1)$

$y + 4 = -2(x - 3)$

$y + 4 = -2x + 6$

$2x + y = 2$

Question 19.
A (1, -5), B (2, 2) and C (-2, 4) are the vertices of triangle ABC, find the equation of:
(i) the median of the triangle through A.
(ii) the altitude of the triangle through B.
(iii) the line through C and parallel to AB.

Solution:

(i) We know the median through A will pass through the mid-point of BC. Let AD be the median through A.
Co-ordinates of the mid-point of BC, i.e., D are

$$\left(\frac{2-2}{2}, \frac{2+4}{2}\right) = (0, 3)$$

Slope of AD = $\frac{3+5}{0-1} = -8$

Equation of the median AD is
y - 3 = -8(x - 0)
8x + y = 3
(ii) Let BE be the altitude of the triangle through B.

Slope of AC = $\frac{4+5}{-2-1} = \frac{9}{-3} = -3$

$\therefore$ Slope of BE = $\frac{-1}{\text{Slope of AC}} = \frac{1}{3}$

Equation of altitude BE is

y - 2 = $\frac{1}{3}$(x - 2)

3y - 6 = x - 2
3y = x + 4

(iii) Slope of AB = $\frac{2+5}{2-1} = 7$

Slope of the line parallel to AB = Slope of AB = 7
So, the equation of the line passing through C and parallel to AB is
y - 4 = 7(x + 2)
y - 4 = 7x + 14
y = 7x + 18

Question 20.
(i) Write down the equation of the line AB, through (3, 2) and perpendicular to the line 2y = 3x + 5.
(ii) AB meets the x-axis at A and the y-axis at B. Write down the co-ordinates of A and B. Calculate the area of triangle OAB, where O is the origin.

Solution:

(i) $2y = 3x + 5$

$\Rightarrow y = \frac{3}{2}x + \frac{5}{2}$

Slope of this line = $\frac{3}{2}$

Slope of the line AB = $\frac{-1}{\frac{3}{2}} = \frac{-2}{3}$

$(x_1, y_1) = (3, 2)$

The required equation of the line AB is

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-2}{3}(x - 3)$

$3y - 6 = -2x + 6$

$2x + 3y = 12$

(ii) For the point A (the point on x-axis), the value of y = 0.

$\therefore 2x + 3y = 12 \Rightarrow 2x = 12 \Rightarrow x = 6$

Co-ordinates of point A are (6, 0).

For the point B (the point on y-axis), the value of x = 0.

$\therefore 2x + 3y = 12 \Rightarrow 3y = 12 \Rightarrow y = 4$

Co-ordinates of point B are (0, 4).

Area of Δ OAB = $\frac{1}{2} \times OA \times OB = \frac{1}{2} \times 6 \times 4 = 12$ sq units

Question 21.

The line $4x - 3y + 12 = 0$ meets the x-axis at A. Write the co-ordinates of A. Determine the equation of the line through A and perpendicular to $4x - 3y + 12 = 0$.

Solution:

For the point A (the point on x-axis), the value of y = 0.

$\therefore 4x - 3y + 12 = 0 \Rightarrow 4x = -12 \Rightarrow x = -3$

Co-ordinates of point A are (-3, 0).

Here, $(x_1, y_1) = (-3, 0)$

The given line is $4x - 3y + 12 = 0$

$3y = 4x + 12$

$y = \frac{4}{3}x + 4$

Slope of this line = $\frac{4}{3}$

$\therefore$ Slope of a line perpendicular to the given line = $\frac{-1}{\frac{4}{3}} = \frac{-3}{4}$

Required equation of the line passing through A is

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-3}{4}(x + 3)$

$4y = -3x - 9$

$3x + 4y + 9 = 0$

Question 22.

The point P is the foot of perpendicular from A (-5, 7) to the line whose equation is 2x – 3y + 18 = 0. Determine:

(i) the equation of the line AP

(ii) the co-ordinates of P

Solution:

(i) The given equation is

$2x - 3y + 18 = 0$

$3y = 2x + 18$

$y = \frac{2}{3}x + 6$

Slope of this line = $\frac{2}{3}$

Slope of a line perpendicular to this line = $\frac{-1}{\frac{2}{3}} = \frac{-3}{2}$

$(x_1, y_1) = (-5, 7)$

The required equation of the line AP is given by

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{-3}{2}(x + 5)$

$2y - 14 = -3x - 15$

$3x + 2y + 1 = 0$

(ii) P is the foot of perpendicular from point A.

So P is the point of intersection of the lines $2x - 3y + 18 = 0$ and $3x + 2y + 1 = 0$.

$2x - 3y + 18 = 0 \Rightarrow 4x - 6y + 36 = 0$

$3x + 2y + 1 = 0 \Rightarrow 9x + 6y + 3 = 0$

Adding the two equations, we get,

$13x + 39 = 0$

$x = -3$

$\therefore 3y = 2x + 18 = -6 + 18 = 12$

$y = 4$

Thus, the co-ordinates of the point P are (-3, 4).

Question 23.
The points A, B and C are (4, 0), (2, 2) and (0, 6) respectively. Find the equations of AB and BC.
If AB cuts the y-axis at P and BC cuts the x-axis at Q, find the co-ordinates of P and Q.

Solution:

For the line AB:

$$Slope\ of\ AB = m = \frac{2-0}{2-4} = \frac{2}{-2} = -1$$

$(x_1, y_1) = (4, 0)$

Equation of the line AB is
$y - y_1 = m(x - x_1)$
$y - 0 = -1(x - 4)$
$y = -x + 4$
$x + y = 4$ ....(1)

For the line BC:

$$Slope\ of\ BC = m = \frac{6-2}{0-2} = \frac{4}{-2} = -2$$

$(x_1, y_1) = (2, 2)$

Equation of the line BC is
$y - y_1 = m(x - x_1)$
$y - 2 = -2(x - 2)$
$y - 2 = -2x + 4$
$2x + y = 6$ ....(2)

Given that AB cuts the y-axis at P. So, the abscissa of point P is 0.
Putting x = 0 in (1), we get,
$y = 4$

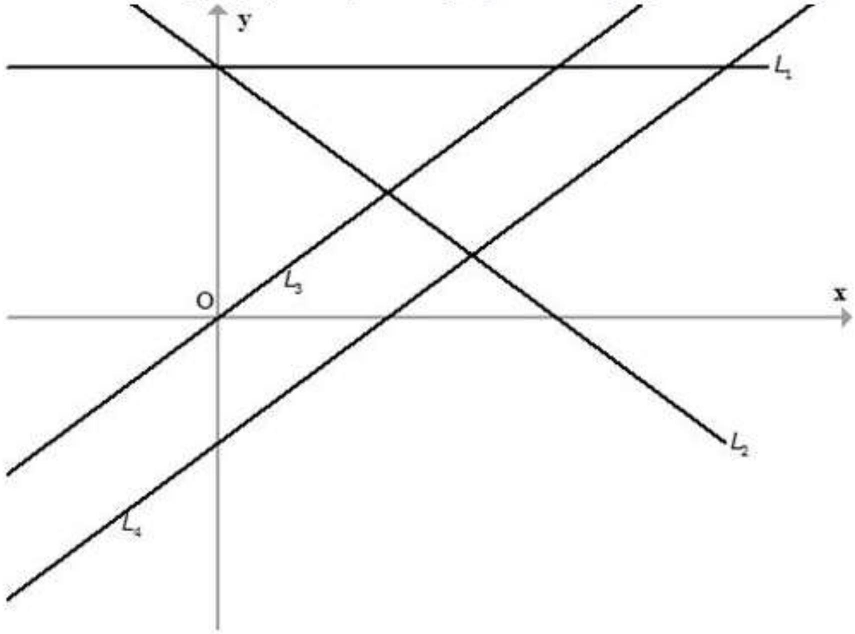
Thus, the co-ordinates of point P are (0, 4).

Given that BC cuts the x-axis at Q. So, the ordinate of point Q is 0.
Putting y = 0 in (2), we get,
$2x = 6 \Rightarrow x = 3$

Thus, the co-ordinates of point Q are (3, 0).

Question 24.

Match the equations A, B, C and D with lines L_1, L_2, L_3 and L_4, whose graphs are roughly drawn in the given diagram.



$A \equiv y = 2x$; $B \equiv y - 2x + 2 = 0$;
$C \equiv 3x + 2y = 6$; $D \equiv y = 2$

Solution:

Putting x = 0 and y = 0 in the equation y = 2x, we have:
LHS = 0 and RHS = 0
Thus, the line y = 2x passes through the origin.
Hence, $A = L_3$
Putting x = 0 in y - 2x + 2 = 0, we get, y = -2
Putting y = 0 in y - 2x + 2 = 0, we get, x = 1
So, x-intercept = 1 and y-intercept = -2
So, x-intercept is positive and y-intercept is negative.
Hence, $B = L_4$
Putting x = 0 in 3x + 2y = 6, we get, y = 3
Putting y = 0 in 3x + 2y = 6, we get, x = 2
So, both x-intercept and y-intercept are positive.
Hence, $C = L_2$
The slope of the line y = 2 is 0.
So, the line y = 2 is parallel to x-axis.
Hence, $D = L_1$

Question 25.
Find the value of a for which the points A(a, 3), B(2, 1) and C(5, a) are collinear. Hence, find the equation of the line.

Solution:

If 3 points are collinear, the slope between any 2 points is the same.
Thus, for A(a, 3), B(2, 1) and C(5, a) to be collinear, the slope between A and B and between B and C should be the same.

$$\Rightarrow \frac{1-3}{2-a} = \frac{a-1}{5-2}$$
$$\Rightarrow \frac{-2}{2-a} = \frac{a-1}{3}$$
$$\Rightarrow \frac{2}{a-2} = \frac{a-1}{3}$$
$$\Rightarrow 6 = (a-2)(a-1)$$
$$\Rightarrow a^2 - 3a + 2 = 6$$
$$\Rightarrow a^2 - 3a - 4 = 0$$
$$\Rightarrow a = -1 \text{ or } 4$$

Thus, slope can be :

$$\frac{2}{a-2} = \frac{2}{-1-2} = -\frac{2}{3} \quad \text{OR} \quad \frac{2}{a-2} = \frac{2}{4-2} = 1$$

Thus, the equation of the line can be :

$$\frac{y-1}{x-2} = -\frac{2}{3}$$
$$\Rightarrow 3y + 2x = 5$$

or

$$\frac{y-1}{x-2} = 1$$
$$\Rightarrow y - x = -1$$
$$\Rightarrow x - y = 1$$

Exercise 14E

Question 1.
Point P divides the line segment joining the points A (8, 0) and B (16, -8) in the ratio 3: 5.
Find its co-ordinates of point P.
Also, find the equation of the line through P and parallel to 3x + 5y = 7.

Solution:

Using section formula, the co-ordinates of the point P are

$$\left(\frac{3\times 16+5\times 8}{3+5},\frac{3\times(-8)+5\times 0}{3+5}\right)$$

$$=(11,-3)=(x_1,y_1)$$

$3x + 5y = 7$

$$\Rightarrow y=\frac{-3}{5}x+\frac{7}{5}$$

Slope of this line = $\frac{-3}{5}$

As the required line is parallel to the line 3x + 5y = 7,

Slope of the required line = Slope of the given line = $\frac{-3}{5}$

Thus, the equation of the required line is

$y - y_1 = m(x - x_1)$

$y + 3 = \frac{-3}{5}(x - 11)$

$5y + 15 = -3x + 33$

$3x + 5y = 18$

Question 2.

The line segment joining the points A(3, -4) and B (-2, 1) is divided in the ratio 1: 3 at point P in it. Find the co-ordinates of P. Also, find the equation of the line through P and perpendicular to the line 5x – 3y + 4 = 0.

Solution:

Using section formula, the co-ordinates of the point P are

$$\left(\frac{1\times(-2)+3\times 3}{1+3},\frac{1\times 1+3\times(-4)}{1+3}\right)$$

$$=\left(\frac{7}{4},\frac{-11}{4}\right)=(x_1,y_1)$$

The equation of the given line is

$5x - 3y + 4 = 0$

$$\Rightarrow y=\frac{5}{3}x+\frac{4}{3}$$

Slope of this line = $\frac{5}{3}$

Since, the required line is perpendicular to the given line, Slope of the required line = $\dfrac{-1}{\frac{5}{3}} = \dfrac{-3}{5}$

Thus, the equation of the required line is

y - y1 = m(x - x1)

$y + \dfrac{11}{4} = \dfrac{-3}{5}\left(x - \dfrac{7}{4}\right)$

$\dfrac{4y + 11}{4} = \dfrac{-3}{5}\left(\dfrac{4x - 7}{4}\right)$

$20y + 55 = -12x + 21$

$12x + 20y + 34 = 0$

$6x + 10y + 17 = 0$

Point P lies on y-axis, so putting x = 0 in the equation 5x + 3y + 15 = 0, we get, y = -5

Thus, the co-ordinates of the point P are (0, -5).

$x - 3y + 4 = 0 \Rightarrow y = \dfrac{1}{3}x + \dfrac{4}{3}$

Slope of this line = $\dfrac{1}{3}$

The required equation is perpendicular to given equation x - 3y + 4 = 0.

$\therefore$ Slope of the required line = $\dfrac{-1}{\frac{1}{3}} = -3$

(x1, y1) = (0, -5)

Thus, the required equation of the line is

y - y1 = m(x - x1)

y + 5 = -3(x - 0)

3x + y + 5 = 0

kx - 5y + 4 = 0

$\Rightarrow 5y = kx + 4$

$\Rightarrow y = \dfrac{k}{5}x + \dfrac{4}{5}$

Slope of this line = m1 = $\dfrac{k}{5}$

5x - 2y + 5 = 0

$\Rightarrow 2y = 5x + 5$

$\Rightarrow y = \dfrac{5}{2}x + \dfrac{5}{2}$

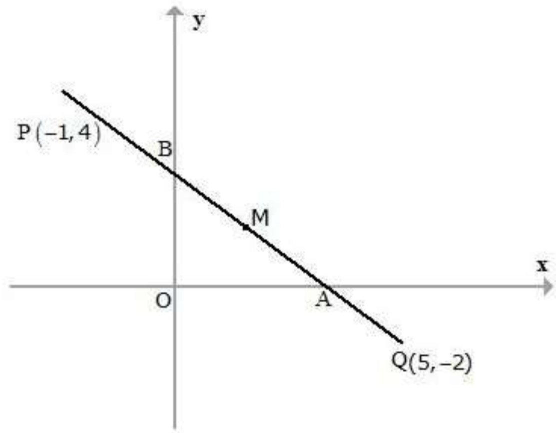
Slope of this line = m2 = $\dfrac{5}{2}$

Since, the lines are perpendicular, m1m2 = -1

$\Rightarrow \dfrac{k}{5} \times \dfrac{5}{2} = -1$

$\Rightarrow k = -2$

A straight line passes through the points P (-1, 4) and Q (5, -2). It intersects the co-ordinate axes at points A and B. M is the mid-point of the segment AB. Find:
(i) the equation of the line.
(ii) the co-ordinates of A and B.
(iii) the co-ordinates of M.



(i) Slope of PQ = $\frac{-2-4}{5+1} = \frac{-6}{6} = -1$

Equation of the line PQ is given by

$y - y1 = m(x - x1)$

$y - 4 = -1(x + 1)$

$y - 4 = -x - 1$

$x + y = 3$

(ii) For point A (on x-axis), y = 0.

Putting y = 0 in the equation of PQ, we get,

x = 3

Thus, the co-ordinates of point A are (3, 0).

For point B (on y-axis), x = 0.

Putting x = 0 in the equation of PQ, we get,

y = 3

Thus, the co-ordinates of point B are (0, 3).

(iii) M is the mid-point of AB.

So, the co-ordinates of point M are

$$\left(\frac{3+0}{2}, \frac{0+3}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$$

A = (1, 5) and C = (-3, -1)

We know that in a rhombus, diagonals bisect each other at right angle.

Let O be the point of intersection of the diagonals AC and BD.

Co-ordinates of O are

$$\left(\frac{1-3}{2}, \frac{5-1}{2}\right) = (-1, 2)$$

Slope of AC = $\frac{-1-5}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$

For line AC:

Slope = m = $\frac{3}{2}$, (x1, y1) = (1, 5)

Equation of the line AC is

y - y1 = m(x - x1)

$y - 5 = \frac{3}{2}(x - 1)$

2y - 10 = 3x - 3

3x - 2y + 7 = 0

For line BD:

Slope = m = $\frac{-1}{\text{Slope of AC}} = \frac{-2}{3}$, (x1, y1) = (-1, 2)

Equation of the line BD is

y - y1 = m(x - x1)

$y - 2 = \frac{-2}{3}(x + 1)$

3y - 6 = -2x - 2

2x + 3y = 4

$Using\ distance\ formula, we\ have:$

$AB = \sqrt{(6-3)^2 + (-2-2)^2} = \sqrt{9+16} = 5$

$BC = \sqrt{(2-6)^2 + (-5+2)^2} = \sqrt{16+9} = 5$

$Thus, AC = BC$

$Also, Slope\ of\ AB = \frac{-2-2}{6-3} = \frac{-4}{3}$

$Slope\ of\ BC = \frac{-5+2}{2-6} = \frac{-3}{-4} = \frac{3}{4}$

$Slope\ of\ AB \times Slope\ of\ BC = -1$

$Thus, AB \perp BC$

$Hence, A, B, C\ can\ be\ the\ vertices\ of\ a\ square..$

$(i)\ \ Slope\ of\ AB = \frac{-2-2}{6-3} = = Slope\ of\ CD$

$Equation\ of\ the\ line\ CD\ is$

$y - y_1 = m(x - x_1)$

$\Rightarrow y + 5 = \frac{-4}{3}(x-2)$

$\Rightarrow 3y + 15 = -4x + 8$

$\Rightarrow 4x + 3y = -7....(1)$

$Slope\ of\ BC = \frac{-5+2}{2-6} = \frac{-3}{-4} = \frac{3}{4} = Slope\ of\ AD$

$Equation\ of\ the\ line\ AD\ is$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = \frac{3}{4}(x-3)$

$\Rightarrow 4y - 8 = 3x - 9$

$\Rightarrow 3x - 4y = 1...(2)$

Now, D is the point of intersection of CD and AD.

$(1) \Rightarrow 16x + 12y = -28$

$(2) \Rightarrow 9x - 12y = 3$

Adding the above two equations we get,

$25x = -25$

$\Rightarrow x = -1$

So, $4y = 3x - 1 = -3 - 1 = -4$

$\Rightarrow y = -1$

Thus, the co-ordinates of point D are $(-1, -1)$.

(ii)

The equation of line AD is found in part (i)

It is $3x - 4y = 1$ *or* $4y = 3x - 1$.

Slope of BD $= \frac{-1+2}{-1-6} = \frac{1}{-7} = \frac{-1}{7}$

The equation of diagonal BD is

$y - y_1 = m(x - x_1)$

$\Rightarrow y + 1 = \frac{-1}{7}(x + 1)$

$\Rightarrow 7y + 7 = -x - 1$

$\Rightarrow x + 7y + 8 = 0$

The given line is

$x = 3y + 2$ ...(1)

$3y = x - 2$

$y = \frac{1}{3}x - \frac{2}{3}$

Slope of this line is $\frac{1}{3}$.

The required line intersects the given line at right angle.

$\therefore$ Slope of the required line = $\frac{-1}{\frac{1}{3}} = -3$

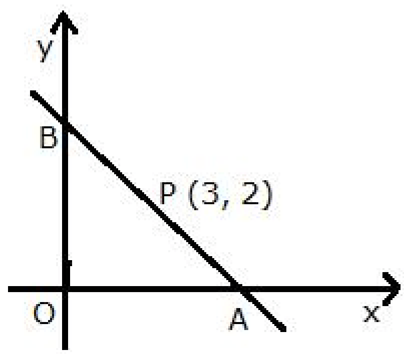
The required line passes through (0, 0) = (x1, y1)
The equation of the required line is
y - y1 = m(x - x1)
y - 0 = -3(x - 0)
3x + y = 0 ...(2)

Point X is the intersection of the lines (1) and (2).
Using (1) in (2), we get,
9y + 6 + y = 0

$$y = \frac{-6}{10} = \frac{-3}{5}$$

$$\therefore\ x = 3y + 2 = \frac{-9}{5} + 2 = \frac{1}{5}$$

Thus, the co-ordinates of the point X are $\left(\frac{1}{5}, \frac{-3}{5}\right)$.



Let the line intersect the x-axis at point A (x, 0) and y-axis at point B (0, y).
Since, P is the mid-point of AB, we have:

$$\left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (3, 2)$$

$$\left(\frac{x}{2}, \frac{y}{2}\right) = (3, 2)$$

$x = 6,\ y = 4$

Thus, A = (6, 0) and B = (0, 4)

Slope of line AB = $\frac{4-0}{0-6} = \frac{4}{-6} = \frac{-2}{3}$

Let (x1, y1) = (6, 0)
The required equation of the line AB is given by
y - y1 = m(x - x1)

$$y - 0 = \frac{-2}{3}(x - 6)$$

3y = -2x + 12
2x + 3y = 12

Question 3.
A line $5x + 3y + 15 = 0$ meets y-axis at point P. Find the co-ordinates of point P. Find the equation of a line through P and perpendicular to $x - 3y + 4 = 0$.

Question 4.
Find the value of k for which the lines $kx - 5y + 4 = 0$ and $5x - 2y + 5 = 0$ are perpendicular to each other.

Question 5.
(1, 5) and (-3, -1) are the co-ordinates of vertices A and C respectively of rhombus ABCD. Find the equations of the diagonals AC and BD.

Question 7.
Show that A (3, 2), B (6, -2) and C (2, -5) can be the vertices of a square.
(i) Find the co-ordinates of its fourth vertex D, if ABCD is a square.
(ii) Without using the co-ordinates of vertex D, find the equation of side AD of the square and also the equation of diagonal BD.

Question 8.
A line through origin meets the line $x = 3y + 2$ at right angles at point X. Find the co-ordinates of X.

Question 9.
A straight line passes through the point (3, 2) and the portion of this line, intercepted between the positive axes, is bisected at this point. Find the equation of the line.

Question 10.
Find the equation of the line passing through the point of intersection of $7x + 6y = 71$ and $5x - 8y = -23$; and perpendicular to the line $4x - 2y = 1$.

Solution:

$7x + 6y = 71 \Rightarrow 28x + 24 = 284$ ...(1)
$5x - 8y = -23 \Rightarrow 15x - 24y = -69$ ...(2)
Adding (1) and (2), we get,
$43x = 215$
$x = 5$

From (2), $8y = 5x + 23 = 25 + 23 = 48 \Rightarrow y = 6$

Thus, the required line passes through the point (5, 6).

$4x - 2y = 1$
$2y = 4x - 1$
$y = 2x - \frac{1}{2}$
Slope of this line = 2
Slope of the required line = $\frac{-1}{2}$
The required equation of the line is
$y - y1 = m(x - x1)$
$y - 6 = \frac{-1}{2}(x - 5)$
$2y - 12 = -x + 5$
$x + 2y = 17$

Question 11.

Find the equation of the line which is perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ at the point where this line meets y-axis.

Solution:

The given line is
$\frac{x}{a} - \frac{y}{b} = 1 \Rightarrow \frac{y}{b} = \frac{x}{a} - 1 \Rightarrow y = \frac{b}{a}x - b$
Slope of this line = $\frac{b}{a}$
Slope of the required line = $\frac{-1}{\frac{b}{a}} = \frac{-a}{b}$
Let the required line passes through the point P (0, y).
Putting x = 0 in the equation $\frac{x}{a} - \frac{y}{b} = 1$, we get,
$0 - \frac{y}{b} = 1$
$\Rightarrow y = -b$
Thus, P = (0, -b) = (x1, y1)
The equation of the required line is
$y - y1 = m(x - x1)$
$y + b = \frac{-a}{b}(x - 0)$
$by + b2 = -ax$
$ax + by + b2 = 0$

Question 12.
O (0, 0), A (3, 5) and B (-5, -3) are the vertices of triangle OAB. Find:

(i) the equation of median of triangle OAB through vertex O.
(ii) the equation of altitude of triangle OAB through vertex B.

Solution:

(i) Let the median through O meets AB at D. So, D is the mid-point of AB.
Co-ordinates of point D are

$$\left(\frac{3-5}{2}, \frac{5-3}{2}\right) = (-1, 1)$$

Slope of OD = $\frac{1-0}{-1-0} = -1$

(x1, y1) = (0, 0)

The equation of the median OD is

y - y1 = m(x - x1)

y - 0 = -1(x - 0)

x + y = 0

(ii) The altitude through vertex B is perpendicular to OA.

Slope of OA = $\frac{5-0}{3-0} = \frac{5}{3}$

Slope of the required altitude = $\frac{-1}{\frac{5}{3}} = \frac{-3}{5}$

The equation of the required altitude through B is

y - y1 = m(x - x1)

y + 3 = $\frac{-3}{5}(x+5)$

5y + 15 = -3x - 15

3x + 5y + 30 = 0

Question 13.
Determine whether the line through points (-2, 3) and (4, 1) is perpendicular to the line 3x = y + 1.

Does the line 3x = y + 1 bisect the line segment joining the two given points?

Solution:

Let A = (-2, 3) and B = (4, 1)

Slope of AB = m1 = $\frac{1-3}{4+2} = \frac{-2}{6} = \frac{-1}{3}$

Equation of line AB is

y - y1 = m1(x - x1)

y - 3 = $\frac{-1}{3}$(x + 2)

3y - 9 = -x - 2

x + 3y = 7 ...(1)

Slope of the given line 3x = y + 1 is 3 = m2.

$\therefore m_1 \times m_2 = -1$

Hence, the line through points A and B is perpendicular to the given line.

Given line is 3x = y + 1 ...(2)

Solving (1) and (2), we get,

x = 1 and y = 2

So, the two lines intersect at point P = (1, 2).

The co-ordinates of the mid-point of AB are

$\left(\frac{-2+4}{2}, \frac{3+1}{2}\right) = (1,2) = P$

Hence, the line 3x = y + 1 bisects the line segment joining the points A and B.

Question 14.

Given a straight line x cos 30^o + y sin 30^o = 2. Determine the equation of the other line which is parallel to it and passes through (4, 3).

Solution:

x cos 30^o + y sin 30^o = 2

$\Rightarrow x\frac{\sqrt{3}}{2} + y\frac{1}{2} = 2$

$\Rightarrow \sqrt{3}x + y = 4$

$\Rightarrow y = -\sqrt{3}x + 4$

Slope of this line = $-\sqrt{3}$

Slope of a line which is parallel to this given line = $-\sqrt{3}$

Let (4, 3) = (x1, y1)

Thus, the equation of the required line is given by:

$y - y1 = m1(x - x1)$

$y - 3 = -\sqrt{3}(x - 4)$

$\sqrt{3}x + y = 4\sqrt{3} + 3$

Question 15.

Find the value of k such that the line (k − 2)x + (k + 3)y − 5 = 0 is:

(i) perpendicular to the line 2x − y + 7 = 0

(ii) parallel to it.

Solution:

$(k - 2)x + (k + 3)y - 5 = 0$ ....(1)

$(k + 3)y = -(k - 2)x + 5$

$$y = \left(\frac{2-k}{k+3}\right)x + \frac{5}{k+3}$$

Slope of this line = $m_1 = \dfrac{2-k}{k+3}$

(i) $2x - y + 7 = 0$

$y = 2x + 7 = 0$

Slope of this line = $m_2 = 2$

Line (1) is perpendicular to $2x - y + 7 = 0$

$\therefore m_1 m_2 = -1$

$$\Rightarrow \left(\frac{2-k}{k+3}\right)(2) = -1$$

$\Rightarrow 4 - 2k = -k - 3$

$\Rightarrow k = 7$

(ii) Line (1) is parallel to $2x - y + 7 = 0$

$\therefore m_1 = m_2$

$$\Rightarrow \frac{2-k}{k+3} = 2$$

$\Rightarrow 2 - k = 2k + 6$

$\Rightarrow 3k = -4$

$$\Rightarrow k = -\frac{4}{3}$$

Question 16.
The vertices of a triangle ABC are A (0, 5), B (-1, -2) and C (11, 7). Write down the equation of BC. Find:
(i) the equation of line through A and perpendicular to BC.
(ii) the co-ordinates of the point, where the perpendicular through A, as obtained in (i), meets BC.

Solution:

Slope of BC = $\frac{7+2}{11+1} = \frac{9}{12} = \frac{3}{4}$
Equation of the line BC is given by
$y - y1 = m1(x - x1)$
$y + 2 = \frac{3}{4}(x + 1)$
$4y + 8 = 3x + 3$
$3x - 4y = 5$....(1)

(i) Slope of line perpendicular to BC = $\frac{-1}{\frac{3}{4}} = \frac{-4}{3}$
Required equation of the line through A (0, 5) and perpendicular to BC is
$y - y1 = m1(x - x1)$
$y - 5 = \frac{-4}{3}(x - 0)$
$3y - 15 = -4x$
$4x + 3y = 15$ ....(2)

(ii) The required point will be the point of intersection of lines (1) and (2).

(1) $\Rightarrow 9x - 12y = 15$
(2) $\Rightarrow 16x + 12y = 60$

Adding the above two equations, we get,
$25x = 75$
$x = 3$

So, $4y = 3x - 5 = 9 - 5 = 4$
$y = 1$

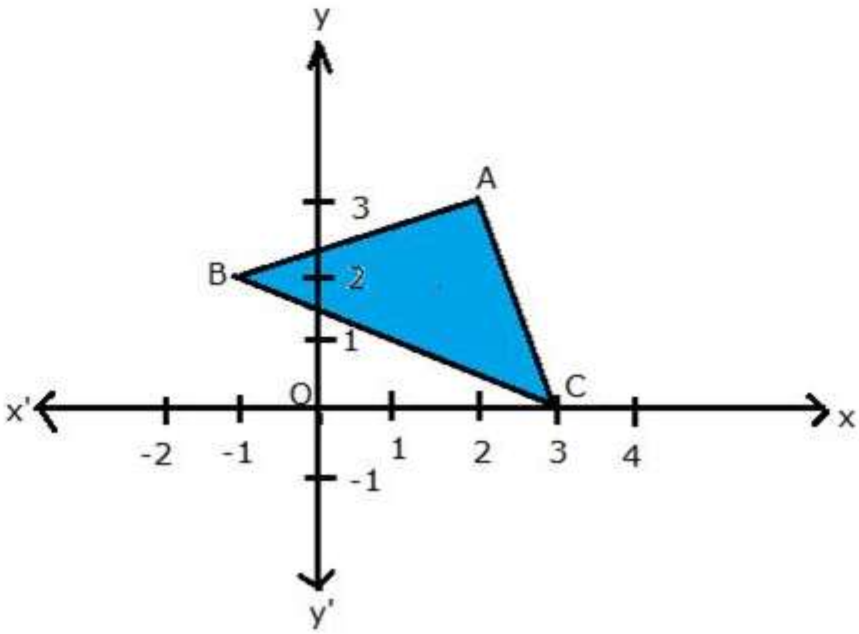
Thus, the co-ordinates of the required point is (3, 1).

Question 17.

From the given figure, find:

(i) the co-ordinates of A, B and C.

(ii) the equation of the line through A and parallel to BC.



Solution:

(i) A = (2, 3), B = (-1, 2), C = (3, 0)

(ii) Slope of BC = $\frac{0-2}{3+1} = -\frac{2}{4} = -\frac{1}{2}$

Slope of required line which is parallel to BC = Slope of BC = $-\frac{1}{2}$

$(x1, y1) = (2, 3)$

The required equation of the line through A and parallel to BC is given by:

$y - y1 = m1(x - x1)$

$y - 3 = -\frac{1}{2}(x - 2)$

$2y - 6 = -x + 2$

$x + 2y = 8$

Question 18.

P (3, 4), Q (7, -2) and R (-2, -1) are the vertices of triangle PQR. Write down the equation of the median of the triangle through R.

Solution:

The median (say RX) through R will bisect the line PQ.
The co-ordinates of point X are

$$\left(\frac{3+7}{2}, \frac{4-2}{2}\right) = (5, 1)$$

$$\text{Slope of RX} = \frac{1+1}{5+2} = \frac{2}{7} = m$$

(x1, y1) = (-2, -1)
The required equation of the median RX is given by:
y - y1 = m1(x - x1)

$$y + 1 = \frac{2}{7}(x+2)$$

7y + 7 = 2x + 4
7y = 2x - 3

Question 19.
A (8, -6), B (-4, 2) and C (0, -10) are vertices of a triangle ABC. If P is the mid-point of AB and Q is the mid-point of AC, use co-ordinate geometry to show that PQ is parallel to BC. Give a special name of quadrilateral PBCQ.

Solution:
P is the mid-point of AB. So, the co-ordinate of point P are

$$\left(\frac{8-4}{2}, \frac{-6+2}{2}\right) = (2, -2)$$

Q is the mid-point of AC. So, the co-ordinate of point Q are

$$\left(\frac{8+0}{2}, \frac{-6-10}{2}\right) = (4, -8)$$

$$\text{Slope of PQ} = \frac{-8+2}{4-2} = \frac{-6}{2} = -3$$

$$\text{Slope of BC} = \frac{-10-2}{0+4} = \frac{-12}{4} = -3$$

Since, slope of PQ = Slope of BC,
∴ PQ || BC
Also, we have:

$$\text{Slope of PB} = \frac{-2-2}{2+4} = \frac{-2}{3}$$

$$\text{Slope of QC} = \frac{-8+10}{4-0} = \frac{1}{2}$$

Thus, PB is not parallel to QC.
Hence, PBCQ is a trapezium.

Question 20.
A line AB meets the x-axis at point A and y-axis at point B. The point P (-4, -2) divides the line segment AB internally such that AP : PB = 1 : 2. Find:
(i) the co-ordinates of A and B.
(ii) the equation of line through P and perpendicular to AB.

Solution:

(i) Let the co-ordinates of point A (lying on x-axis) be (x, 0) and the co-ordinates of point B (lying y-axis) be (0, y).
Given, P = (-4, -2) and AP: PB = 1:2
Using section formula, we have:

$$(-4,-2) = \left(\frac{1\times 0 + 2\times x}{1+2}, \frac{1\times y + 2\times 0}{1+2}\right)$$

$$(-4,-2) = \left(\frac{2x}{3}, \frac{y}{3}\right)$$

$$\Rightarrow -4 = \frac{2x}{3} \qquad -2 = \frac{y}{3}$$

$$\Rightarrow x = -6 \qquad y = -6$$

Thus, the co-ordinates of A and B are (-6, 0) and (0, -6).

(ii) Slope of AB = $\frac{-6-0}{0+6} = \frac{-6}{6} = -1$

Slope of the required line perpendicular to AB = $\frac{-1}{-1} = 1$

(x1, y1) = (-4, -2)
Required equation of the line passing through P and perpendicular to AB is given by
y - y1 = m(x - x1)
y + 2 = 1(x + 4)
y + 2 = x + 4
y = x + 2

Question 21.
A line intersects x-axis at point (-2, 0) and cuts off an intercept of 3 units from the positive side of y-axis. Find the equation of the line.

Solution:

The required line intersects x-axis at point A (-2, 0).
Also, y-intercept = 3
So, the line also passes through B (0, 3).

Slope of line AB = $\frac{3-0}{0+2} = \frac{3}{2} = m$

(x1, y1) = (-2, 0)

Required equation of the line AB is given by
y - y1 = m(x - x1)

$y - 0 = \frac{3}{2}(x + 2)$

2y = 3x + 6

Question 22.
Find the equation of a line passing through the point (2, 3) and having the x-intercept of 4 units.

Solution:

The required line passes through A (2, 3).
Also, x-intercept = 4
So, the required line passes through B (4, 0).

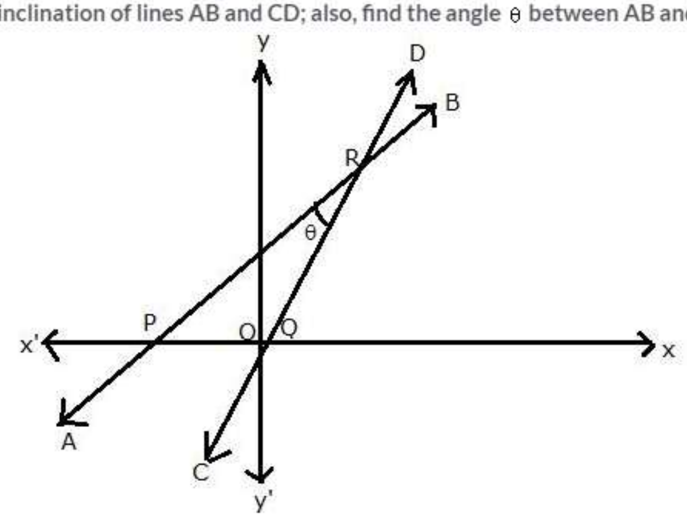
Slope of AB = $\frac{0-3}{4-2} = \frac{-3}{2} = m$

(x1, y1) = (4, 0)
Required equation of the line AB is given by
y - y1 = m(x - x1)

$y - 0 = \frac{-3}{2}(x - 4)$

2y = -3x + 12
3x + 2y = 12

Question 23.
The given figure (not drawn to scale) shows two straight lines AB and CD. If equation of the line AB is: y = x + 1 and equation of line CD is: $y = \sqrt{3}x - 1$. Write down the inclination of lines AB and CD; also, find the angle θ between AB and CD.



Solution:

Equation of the line AB is y = x + 1

Slope of AB = 1

Inclination of line AB = $45°$ (Since, tan 45o = 1)

$\Rightarrow \angle RPQ = 45^0$

Equation of line CD is y = $\sqrt{3}$ x - 1

Slope of CD = $\sqrt{3}$

Inclination of line CD = 60o (Since, tan 60o = $\sqrt{3}$)

$\Rightarrow \angle DQX = 60°$

$\therefore \angle DQP = 180° - 60° = 120°$

Using angle sum property in Δ PQR,

$\theta = 180° - 45° - 120° = 15°$

Question 24.

Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the lin

6) and B (3, -4) in the ratio 2: 3.

Solution:

Given, P divides the line segment joining A (-2, 6) and B (3, -4) in the ratio 2: 3.

Co-ordinates of point P are

$$\left(\frac{2 \times 3 + 3 \times (-2)}{2 + 3}, \frac{2 \times (-4) + 3 \times 6}{2 + 3}\right)$$

$$= \left(\frac{6 - 6}{5}, \frac{-8 + 18}{5}\right)$$

$$= (0, 2) = (x_1, y_1)$$

Slope of the required line = m = $\frac{3}{2}$

The required equation of the line is given by

y - y1 = m(x - x1)

y - 2 = $\frac{3}{2}$ (x - 0)

2y - 4 = 3x

2y = 3x + 4

Question 25.
The ordinate of a point lying on the line joining the points (6, 4) and (7, -5) is -23. Find the co-ordinates of that point.

Solution:

Let A = (6, 4) and B = (7, -5)

Slope of the line AB = $\frac{-5-4}{7-6} = -9$

$(x_1, y_1) = (6, 4)$

The equation of the line AB is given by

$y - y_1 = m(x - x_1)$

$y - 4 = -9(x - 6)$

$y - 4 = -9x + 54$

$9x + y = 58$ ...(1)

Now, given that the ordinate of the required point is -23.

Putting y = -23 in (1), we get,

$9x - 23 = 58$

$9x = 81$

$x = 9$

Thus, the co-ordinates of the required point is (9, -23).

Question 26.
Points A and B have coordinates (7, -3) and (1, 9) respectively. Find:
(i) the slope of AB.
(ii) the equation of the perpendicular bisector of the line segment AB.
(iii) the value of 'p' if (-2, p) lies on it.

Solution:

Given points are A(7, -3) and B(1, 9).

(i) Slope of AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{9+3}{1-7} = \frac{12}{-6} = -2$

(ii) Slope of perpendicular bisector = $\frac{-1}{-2} = \frac{1}{2}$

Mid-point of AB = $\left(\frac{7+1}{2}, \frac{-3+9}{2}\right) = (4, 3)$

$\therefore$ Equation of perpendicular bisector is:

$y - 3 = \frac{1}{2}(x - 4)$

$2y - 6 = x - 4$

$x - 2y + 2 = 0$

(iii) Point (-2, p) lies on $x - 2y + 2 = 0$.

$\therefore -2 - 2p + 2 = 0$

$\Rightarrow 2p = 0$

$\Rightarrow p = 0$

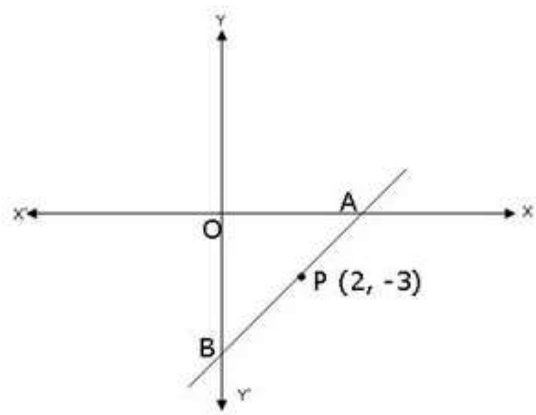
Question 27.

A and B are two points on the x-axis and y-axis respectively. P (2, -3) is the mid-point of AB. Find the

(i) coordinates of A and B

(ii) slope of line AB

(iii) equation of line AB.



Solution:

(i) Let the co-ordinates be A(x, 0) and B(0, y).

Mid-point of A and B is given by $\left(\frac{x+0}{2}, \frac{y+0}{2}\right) = \left(\frac{x}{2}, \frac{y}{2}\right)$

$\Rightarrow \quad (2, -3) = \left(\frac{x}{2}, \frac{y}{2}\right)$

$\Rightarrow \quad \frac{x}{2} = 2 \quad \text{and} \quad \frac{y}{2} = -3$

$\Rightarrow \quad x = 4$ and $y = -6$

$\therefore \quad A = (4, 0)$ and $B = (0, -6)$

(ii) Slope of line AB, m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{0 - 4} = \frac{3}{2} = 1\frac{1}{2}$

(iii) Equation of line AB, using A(4, 0)

$y - 0 = \frac{3}{2}(x - 4)$

2y = 3x - 12

Question 28.
The equation of a line 3x + 4y − 7 = 0. Find:
(i) the slope of the line.
(ii) the equation of a line perpendicular to the given line and passing through the intersection of the lines x − y + 2 = 0 and 3x + y − 10 = 0.

Solution:

3x + 4y - 7 = 0 ...(1)
4y = -3x + 7

$y = \frac{-3}{4}x + \frac{7}{4}$

(i) Slope of the line = m = $-\frac{3}{4}$

(ii) Slope of the line perpendicular to the given line = $\frac{-1}{\frac{-3}{4}} = \frac{4}{3}$

Solving the equations x - y + 2 = 0 and 3x + y - 10 = 0, we get x = 2 and y = 4.
So, the point of intersection of the two given lines is (2, 4).

Given that a line with slope $\frac{4}{3}$ passes through point (2, 4).

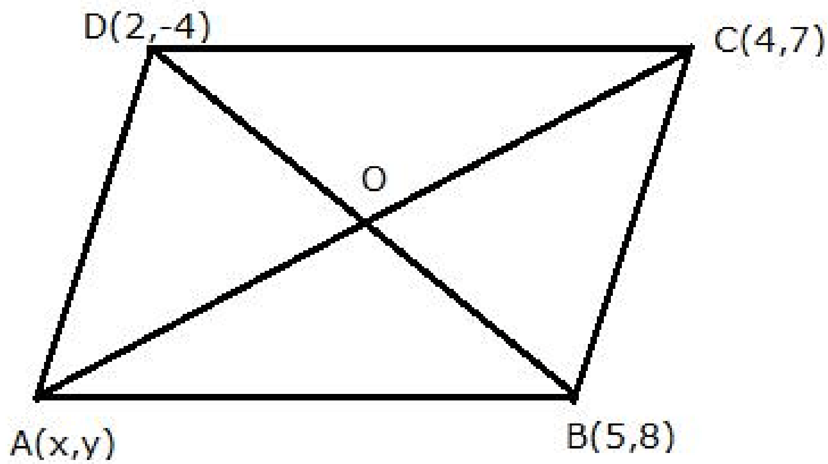
Thus, the required equation of the line is

$y - 4 = \frac{4}{3}(x - 2)$

3y - 12 = 4x - 8
4x - 3y + 4 = 0

Question 29.
ABCD is a parallelogram where A(x, y), B(5, 8), C(4, 7) and D(2, -4). Find:
(i) Co-ordinates of A
(ii) Equation of diagonal BD

Solution:



In parallelogram ABCD, A(x, y), B(5, 8), C(4, 7) and D(2, -4).
The diagonals of the parallelogram bisect each other.
O is the point of intersection of AC and BD
Since O is the midpoint of BD, its coordinates will be

$$\left(\frac{2+5}{2}, \frac{-4+8}{2}\right) \text{ or } \left(\frac{7}{2}, \frac{4}{2}\right) \text{ or } \left(\frac{7}{2}, 2\right)$$

(i)
Since O is the midpoint of AC also,

$$\frac{x+4}{2} = \frac{7}{2} \Rightarrow x + 4 = 7 \Rightarrow x = 7 - 4 = 3$$

$$\text{and } \frac{y+7}{2} = 2 \Rightarrow y + 7 = 14 \Rightarrow y = 14 - 7 = 7$$

Thus, coordinates of A are $(3,7)$

(ii)
Equation of BD will be

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)} \times (x - x_1)$$

$$\Rightarrow y + 4 = \frac{8+4}{5-2} \times (x - 2)$$

$$\Rightarrow y + 4 = \frac{12}{3} \times (x - 2)$$

$$\Rightarrow y + 4 = 4(x - 2)$$

$$\Rightarrow y + 4 = 4x - 8$$

$$\Rightarrow 4x - y = 12$$

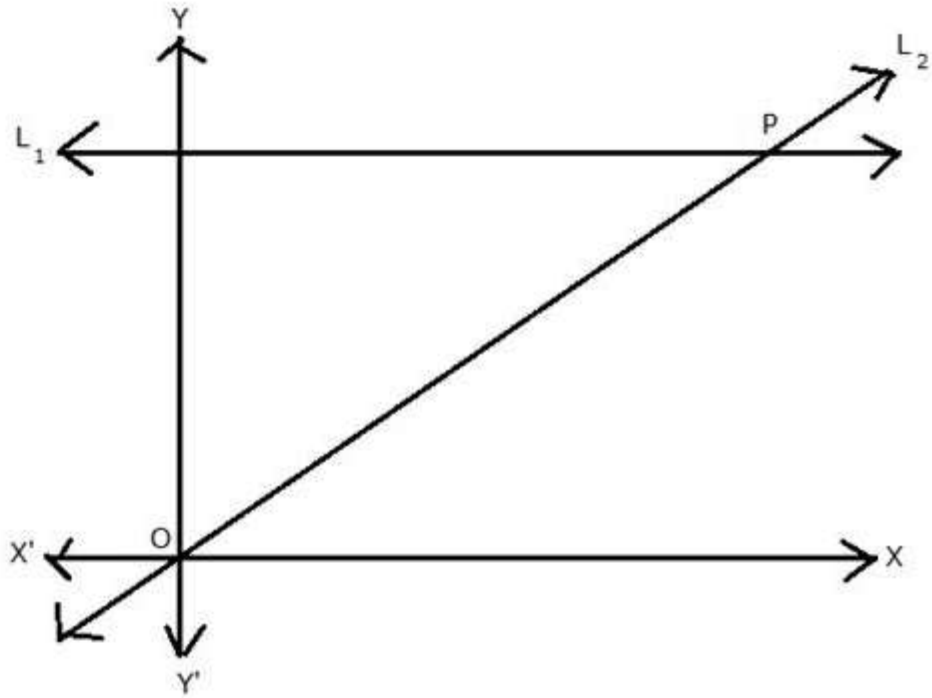
Question 30.

Given equation of the line L_1 is y = 4.

(i)Write the slope of the line L_2 if L_2 is the bisector of angle O

(ii)Write the coordinates of point P

(iii)Find the equation of L_2

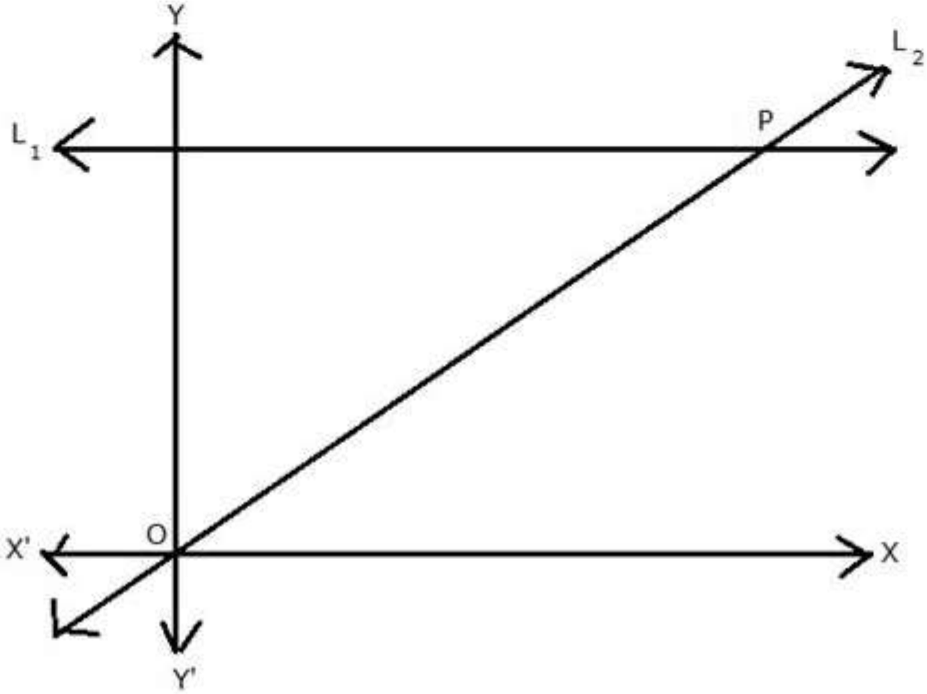


Solution:

(i)

Equation of line L_1 is y=4

$\because L_2$is the bisector of $\angle O$



$\therefore \angle POX = 45^\circ$

Slope = tan 45° = 1

Let coordinates of P be (x, y)

$\because$ P lies on L_1

(ii)

$\therefore$ Slope of $L_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$1 = \dfrac{4-0}{x-0} \Rightarrow 1 = \dfrac{4}{x}$

$\Rightarrow x = 4$

$\therefore$ coordinates of P are $(4,4)$

(iii)

Equation of L_2 is

$y - y_1 = m(x - x_1)$

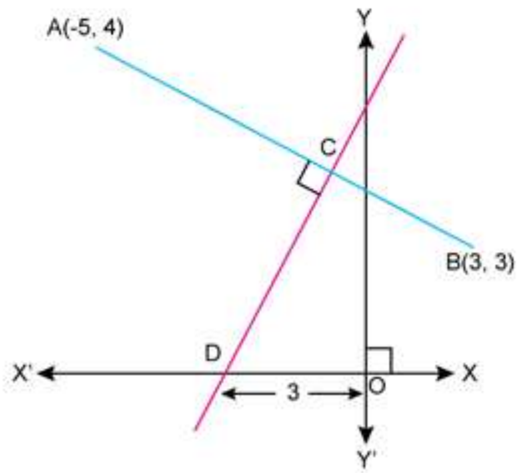
$\Rightarrow y - 4 = 1(x - 4)$

$\Rightarrow y - 4 = x - 4$

$\Rightarrow x = y$

Question 31.

(i) equation of AB
(ii) equation of CD



Solution:

(i) Slope of AB = $\frac{3-4}{3-(-5)} = \frac{-1}{8}$

$\therefore$ Equation of AB is given by

$y - 4 = -\frac{1}{8}(x - (-5))$

$8y - 32 = -(x + 5)$

$8y - 32 = -x - 5$

$x + 8y = 27$

(ii) AB and CD are perpendicular to each other.

Thus, product of their slopes = −1

Slope of AB x Slope of CD = −1

$\Rightarrow \frac{-1}{8} \times$ Slope of CD = −1

$\Rightarrow$ Slope of CD = 8

Now, from graph we have coordinates of D = (−3, 0)

$\therefore$ Equation of line CD is given by

$y - 0 = 8(x + 3)$

$y = 8x + 24$

Question 32.
Find the equation of the line that has x-intercept = -3 and is perpendicular to 3x + 5y = 1.

Solution:

Slope of $3x + 5y = 1$ is given by $-\frac{3}{5} =$

$\Rightarrow$ Slope of line perpendicular to $3x + 5y = 1$:$- \frac{1}{\text{Slope of } 3x + 5y = 1} = -\frac{1}{-3/5} = \frac{5}{3}$

Now, x-intercept $= -3$

$y = mx + c$

$\Rightarrow 0 = \frac{5}{3} \times (-3) + c$

$\Rightarrow c = 5$

$\therefore$ Equation of required line is given by $y = \frac{5}{3}x + 5$

i.e. $3y = 5x + 15$

i.e. $5x - 3y + 15 = 0$

Question 33.
A straight line passes through the points P(-1, 4) and Q(5, -2). It intersects x-axis at point A and y-axis at point B. M is the mid- t point of the line segment AB. Find:

(i) the equation of the line.
(ii) the co-ordinates of points A and B.
(iii) the co-ordinates of point M

Solution:

(i) The equation of the line passing through the points P(−1, 4) and Q(5, − 2) is

$y - 4 = \frac{-2-4}{5-(-1)}[x-(-1)]$

i.e. $y - 4 = \frac{-6}{6}(x+1)$

i.e. $y - 4 = -1(x+1)$

i.e. $y - 4 = -x - 1$

i.e. $x + y = 3$

(ii) The line x + y = 3 cuts x-axis at point A. Hence, its y-coordinate is 0.

And, x-coordinate is given by

$x + 0 = 3 \Rightarrow x = 3$

So, the coordinates of A are (3,0).

The line x + y = 3 cuts y-axis at point B. Hence, its x-coordinate is 0.

And, y-coordinate is given by

$0 + y = 3 \Rightarrow y = 3$

So, the coordinates of B are (0,3).

(iii) Since M is the mid-point of line segment AB,

So, coordinates of $M = \left(\frac{3+0}{2}, \frac{0+3}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$

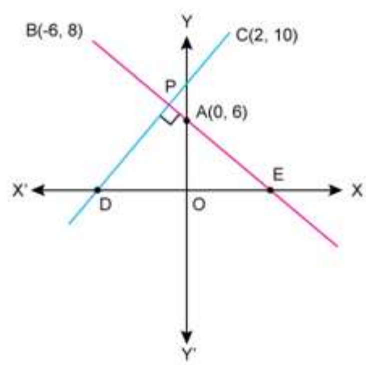
Question 34.

In the given figure. line AB meets y-axis at point A. Line through C(2, 10) and D intersects line AB at right angle at point R Find:

(i) equation of line AB

(ii) equation of line CD

(iii) co-ordinates of points E and D



Solution:

(i) Slope of line AB $= m = \frac{8-6}{-6-0} = \frac{2}{-6} = -\frac{1}{3}$

The y-intercept of the line AB is 6.

Thus, the equation of the given line is given by the slope-intercept form

$y = mx + c$

i.e. $y = -\frac{1}{3}x + 6$

i.e. $3y = -x + 18$

i.e. $x + 3y = 18$, which is the required equation.

(ii) Since AB and CD intersect at right angles,

$\text{Slope}_{AB} \times \text{Slope}_{CD} = -1$

$\Rightarrow -\frac{1}{3} \times \text{Slope}_{CD} = -1$

$\Rightarrow \text{Slope}_{CD} = 3$

Using the slope-point form, the equation of CD is given by

$y - y_1 = m(x - x_1)$

i.e. $y - 10 = 3(x - 2)$

i.e. $y - 10 = 3x - 6$

i.e. $3x - y + 4 = 0$, which is the required equation of line CD.

(iii) Since point E satisfies the equation of AB, and the y-coordinate of E is 0, we can find the x-coordinate of E.

$x + 3(0) = 18$

$\Rightarrow x = 18$

So, the coordinates of E are (18, 0).

Now, since point D satisfies the equation of CD, and the y-coordinate of D is 0, we can find the x-coordinate of D.

$3x - (0) + 4 = 0$

$\Rightarrow 3x = -4$

$\Rightarrow x = -\frac{4}{3}$

So, the coordinates of D are $\left(-\frac{4}{3}, 0\right)$.

Question 35.

A line through point P(4, 3) meets x-axis at point A and the y-axis at point B. If BP is double of PA, find the equation of AB.

Solution:

Since a line through point P meets x-axis at point A and y-axis at point B,

Co-ordinates of A are (x, 0) and co-ordinates of B are (0, y).

Now, BP = 2PA

$\Rightarrow \frac{BP}{PA} = \frac{2}{1}$

$\Rightarrow$ P divides AB in the ratio 2 : 1.

So, the coordinates of P are $\left(\frac{2 \times x + 1 \times 0}{2 + 1}, \frac{2 \times 0 + 1 \times y}{2 + 1}\right) = \left(\frac{2x}{3}, \frac{y}{3}\right)$

But, coordinates of P are (4, 3).

$\Rightarrow \frac{2x}{3} = 4 \Rightarrow 2x = 12 \Rightarrow x = 6$ and $\frac{y}{3} = 3 \Rightarrow y = 9$

$\Rightarrow$ Co-ordinates of A are (6, 0) and coordinates of B are (0, 9).

$\therefore$ Slope of line AB $= \frac{9-0}{0-6} = \frac{9}{-6} = -\frac{3}{2}$

Thus, the equation of line AB is given by

$y - 0 = -\frac{3}{2}(x-6)$

i.e. $2y = -3x + 18$

i.e. $3x + 2y = 18$

Question 36.
Find the equation of line through the intersection of lines 2x − y = 1 and 3x + 2y = -9 and making an angle of 30° with positive direction of x-axis.

Solution:

Since the line passing through the x-axis makes an angle of 30° with the positive direction of the x-axis, the slope of the line is given by $\tan 30° = \frac{1}{\sqrt{3}}$.

The intersection of the lines $2x - y = 1$ and $3x + 2y = -9$ is given by solving the equations simultaneously.

So, multiplying equation $2x - y = 1$ by 2, we get.

$4x - 2y = 2$

Now add this resultant to the second equation $3x + 2y = -9$.

$\Rightarrow 7x = -7 \Rightarrow x = -1$

Substituting the value of x in $2x - y = 1$, we get $y = -3$.

Thus, the intersection of the lines is $(-1, -3)$.

To find the equation of the required line, we use the slope-point form, so we get

$y - (-3) = \frac{1}{\sqrt{3}}[x - (-1)]$

i.e. $y + 3 = \frac{1}{\sqrt{3}}(x+1)$

i.e. $y = \frac{x}{\sqrt{3}} + \frac{1}{\sqrt{3}} - 3$

Question 37.
Find the equation of the line through the Points A(-1, 3) and B(0, 2). Hence, show that the points A, B and C(1, 1) are collinear.

Solution:

Slope of line AB = $m = \dfrac{2-3}{0-(-1)} = \dfrac{-1}{1} = -1$

Using the slope-point form, the equation of line AB is given by

$y - y_1 = m(x - x_1)$

i.e. $y - 3 = -1[x - (-1)]$

i.e. $y - 3 = -1(x + 1)$

i.e. $y - 3 = -x - 1$

i.e. $x + y = 2$

Now, slope of line BC = $\dfrac{1-2}{1-0} = \dfrac{-1}{1} = -1$

Since, Slope of line AB = Slope of line BC, points A, B and C are collinear.

Question 38.
Three vertices of a parallelogram ABCD taken in order are A(3, 6), B(5, 10) and C(3, 2), find :
(i) the co-ordinates of the fourth vertex D.
(ii) length of diagonal BD.
(iii) equation of side AB of the parallelogram ABCD.

Solution:

(i) Let (x, y) be the co-ordinates of D.

We know that the diagonals of a parallelogram bisect each other.

$\therefore$ Mid-point of diagonal AC = Mid-point of diagonal BD

$\Rightarrow \left(\dfrac{3+3}{2}, \dfrac{6+2}{2}\right) = \left(\dfrac{5+x}{2}, \dfrac{10+y}{2}\right)$

$\Rightarrow (3, 4) = \left(\dfrac{5+x}{2}, \dfrac{10+y}{2}\right)$

$\Rightarrow \dfrac{5+x}{2} = 3 \Rightarrow 5 + x = 6 \Rightarrow x = 1$ and $\dfrac{10+y}{2} = 4 \Rightarrow 10 + y = 8 \Rightarrow y = -2$

$\therefore$ Co-ordinates of D are $(1, -2)$.

(ii) Length of diagonal BD $= \sqrt{(1-5)^2 + (-2-10)^2}$

$= \sqrt{(-4)^2 + (-12)^2}$

$= \sqrt{16 + 144}$

$= \sqrt{160}$

$= 4\sqrt{10}$ units

(iii) Slope of side AB = $m = \frac{10-6}{5-3} = \frac{4}{2} = 2$

Thus, the equation of side AB is given by

$y - 6 = 2(x - 3)$

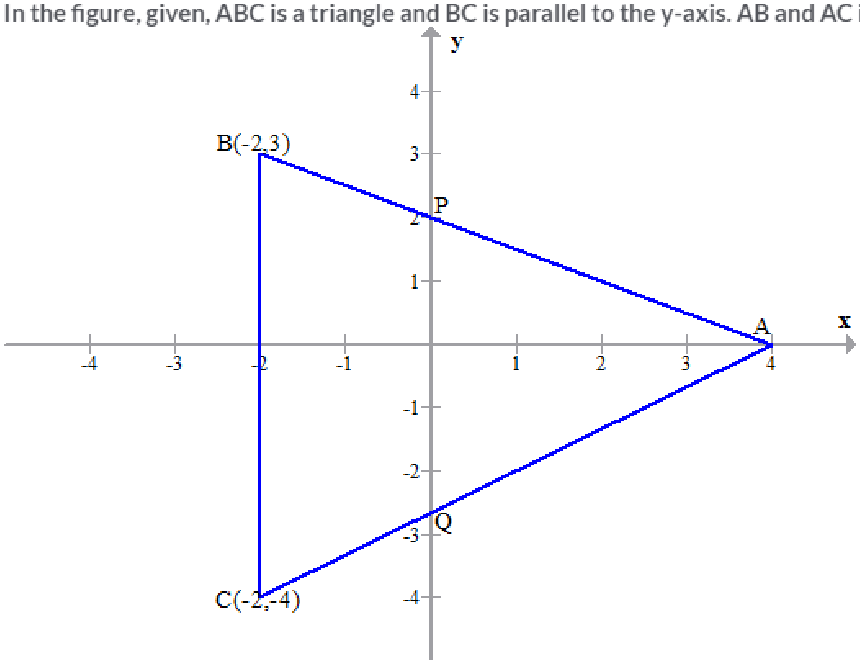
i.e. $y - 6 = 2x - 6$

i.e. $2x - y = 0$

i.e. $y = 2x$

Question 39.

In the figure, given, ABC is a triangle and BC is parallel to the y-axis. AB and AC intersect the y-axis at P and Q respectively.



(i) Write the co-ordinates of A.
(ii) Find the length of AB and AC.
(iii) Find the radio in which Q divides AC.
(iv) Find the equation of the line AC.

Solution:

(i) The line intersects the x-axis where y = 0.
Hence, the co-ordinates of A are (4, 0).

(ii) Length of AB = $\sqrt{(4-(-2))^2 + (0-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$ units

Length of AC = $\sqrt{(4-(-2))^2 + (0+4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

(iii) Let k be the required ratio which divides the line segment joining the co-ordinates A(4, 0) and C(−2, −4).

Let the co-ordinates of Q be x and y.

$\therefore\ x = \dfrac{k(-2)+1(4)}{k+1}$ and $y = \dfrac{k(-4)+0}{k+1}$

Q lies on the y-axis where x = 0.

$\Rightarrow \dfrac{-2k+4}{k+1} = 0$

$\Rightarrow -2k + 4 = 0$

$\Rightarrow 2k = 4$

$\Rightarrow k = \dfrac{4}{2} = \dfrac{2}{1}$

Thus, the required ratio is 2 : 1.

(iv) Slope of line AC = m = $\dfrac{-4-0}{-2-4} = \dfrac{-4}{-6} = \dfrac{2}{3}$

Thus, the equation of the line AC is given by

$y - 0 = \dfrac{2}{3}(x-4)$

i.e. $3y = 2x - 8$

i.e. $2x - 3y = 8$

Question 40.

(i) Slope of PQ = $\frac{3-k}{1-3k-6}$

$\Rightarrow \frac{1}{2} = \frac{3-k}{-3k-5}$

$\Rightarrow -3k-5 = 2(3-k)$

$\Rightarrow -3k-5 = 6-2k$

$\Rightarrow k = -11$

(ii) Substituting k in P and Q, we get

$P(6,k) = P(6,-11)$ and $Q(1-3k,3) = Q(34,3)$

$\therefore$ Mid-point of PQ $= \left(\frac{6+34}{2}, \frac{-11+3}{2}\right) = \left(\frac{40}{2}, \frac{-8}{2}\right) = (20,-4)$

Question 41.

i. Since A lies on the X-axis, let the co-ordinates of A be (x, 0).
Since B lies on the Y-axis, let the co-ordinates of B be (0, y).
Let m = 1 and n = 2
Using Section formula,

Coordinates of P $= \left(\frac{1(0)+2(x)}{1+2}, \frac{1y+2(0)}{1+2}\right)$

$\Rightarrow (4,-1) = \left(\frac{2x}{3}, \frac{y}{3}\right)$

$\Rightarrow \frac{2x}{3} = 4$ and $\frac{y}{3} = -1$

$\Rightarrow$ x = 6 and y = -3

So, the co-ordinates of A are (6, 0) and that of B are (0, -3).

ii. Slope of AB $= \frac{-3-0}{0-6} = \frac{-3}{-6} = \frac{1}{2}$

$\Rightarrow$ Slope of line perpendicular to AB = m = -2
P = (4, -1)
Thus, the required equation is
$y - y_1 = m(x - x_1)$
$\Rightarrow y - (-1) = -2(x - 4)$
$\Rightarrow y + 1 = -2x + 8$
$\Rightarrow 2x + y = 7$