JEE Advanced

Single Correct Answer Type

1. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the center of the sphere is
   a. zero
   b. 10 V
   c. same as at a point 5 cm away from the surface
   d. same as at a point 25 cm away from the surface

(IIT-JEE 1983)

2. Two equal negative charges $-q$ are fixed at points $(0, \pm a)$ on the y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will
   a. execute simple harmonic motion about the origin
   b. move to the origin and remain at rest
   c. move to infinity
   d. execute oscillatory but not simple harmonic motion

(IIT-JEE 1984)

3. A charge Q is placed at the center of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to
   a. $-Q/2$ **b.** $-Q/4$
   c. $+Q/4$ **d.** $-Q/2$

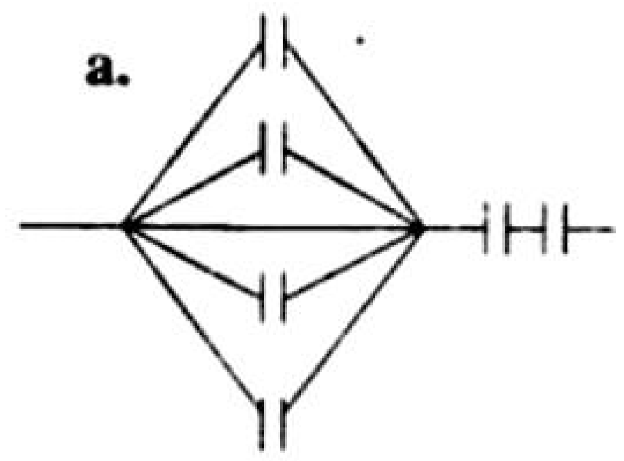
(IIT-JEE 1987)

4. A solid conducting sphere having charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is
   a. V **b.** $2V$
   c. $4V$ **d.** $-2V$

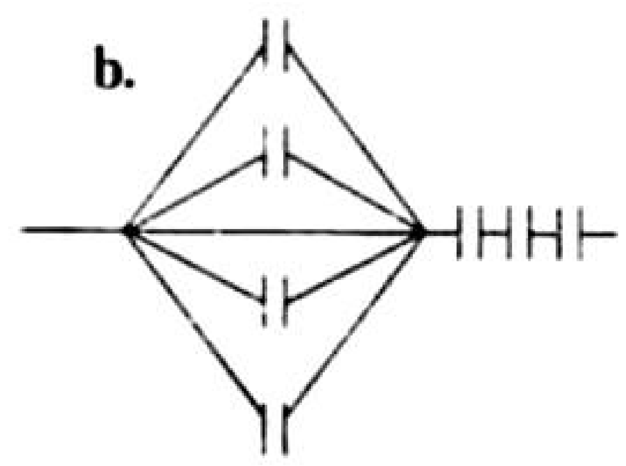
(IIT-JEE 1989)

5. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of 10/11 μF. Which of the combination (s) shown in the given graphs will achieve the desired result?

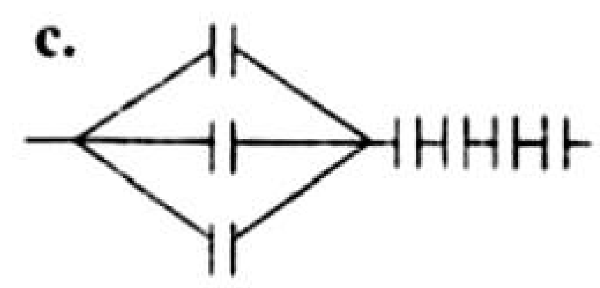
a.



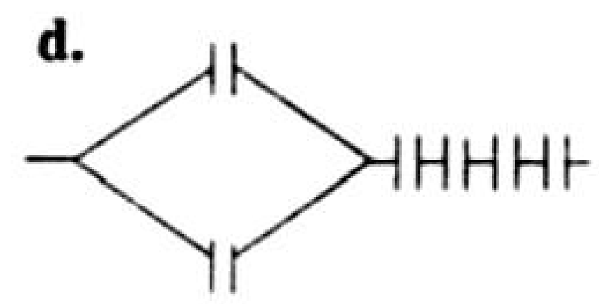
b.



c.



d.



(IIT-JEE 1990)

6. Two identical thin rings, each of radius R meters, are coaxially placed a distance r meters apart. If Q_1 coulomb and Q_2 coulomb are, respectively, the charges uniformly spread on the two rings, the work done in moving a charge q from the center of one ring to that of the other is
   a. zero
   b. $\dfrac{q(Q_1 - Q_2)(\sqrt{2}-1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$
   c. $\dfrac{q\sqrt{2}(Q_1 + Q_2)}{(4\pi\varepsilon_0 R)}$
   d. $\dfrac{q(Q_1 + Q_2)(\sqrt{2}+1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

(IIT-JEE 1992)

7. Two point charges $+q$ and $-q$ are held fixed at $(-d, 0)$ and $(d, 0)$, respectively, of a xy coordinate system. Then
   a. electric field E at all points on the x-axis has the same direction
   b. electric field at all points on the y-axis is along the x-axis
   c. work has to be done in bringing a test charge from ∞ to the origin
   d. the dipole moment is $2qd$ along the x-axis

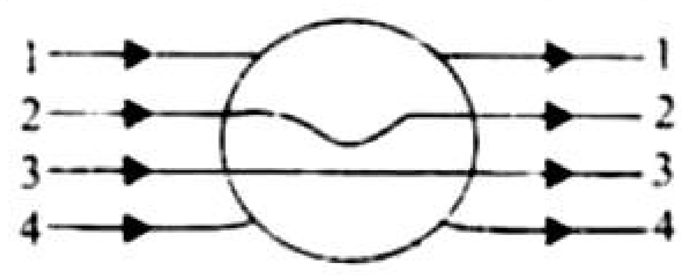
(IIT-JEE 1995)

8. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the combination is
   a. zero **b.** $\frac{3}{2}CV^2$ **c.** $\frac{25}{6}CV^2$ **d.** $\frac{9}{2}CV^2$

(IIT-JEE 1995)

9. The magnitude of electric field $\vec{E}$ in the annular region of a charged cylindrical capacitor
a. is same throughout
b. is higher near the outer cylinder than near the inner cylinder
c. varies as $1/r$, where r is the distance from axis
d. varies as $1/r^2$ where r is the distance from the axis
(IIT-JEE 1996)

10. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in the figure as



a. 1 **b.** 2
c. 3 **d.** 4 **(IIT-JEE 1996)**

11. An electron of mass m_e initially at rest moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p also initially at rest takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to

a. 1 **b.** $(m_p/m_e)^{1/2}$ **c.** $(m_e/m_p)^{1/2}$ **d.** 1836
(IIT-JEE 1997)

12. A nonconducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed nonuniformly on its circumference producing an electric field E everywhere in space. The value of the integral $\int_{l=\infty}^{l=0} -\vec{E}.d\vec{l}$ ($l = 0$ being center of the ring) in volts is
a. +2 **b.** −1 **c.** −2 **d.** zero
(IIT-JEE 1997)

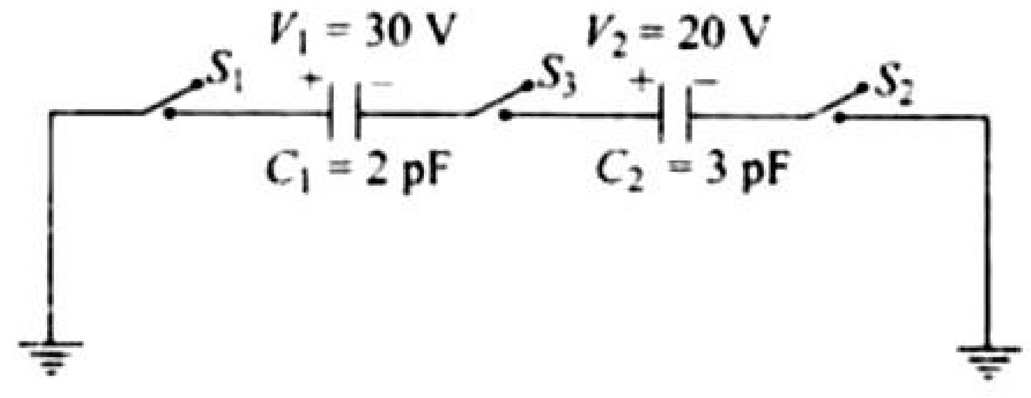
13. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$, and so on, on the x-axis, and a charge $-q$ is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0$, and so on. Here, x_0 is a positive constant. Take the electric potential at a point due to charge Q at a distance r from it to be $Q/4\pi\varepsilon_0 r$. Then, the potential at the origin due to the above system of charges is

a. 0 **b.** $\dfrac{q}{8\pi\varepsilon_0 x_0 \ln 2}$ **c.** ∞ **d.** $\dfrac{q \ln 2}{4\pi\varepsilon_0 x_0}$

(IIT-JEE 1998)

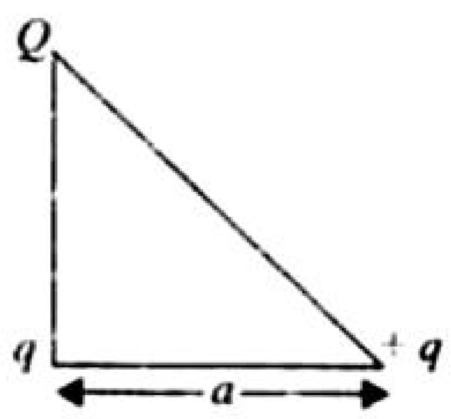
14. Two identical metal plates are given positive charges Q_1 and $Q_2 (< Q_1)$, respectively. If they are brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them will be
a. $(Q_1 + Q_2)/2C$ **b.** $(Q_1 + Q_2)/C$
c. $(Q_1 - Q_2)/C$ **d.** $(Q_1 - Q_2)/2C$
(IIT-JEE 1999)

15. For the circuit shown in the figure, which of the following statements is true?



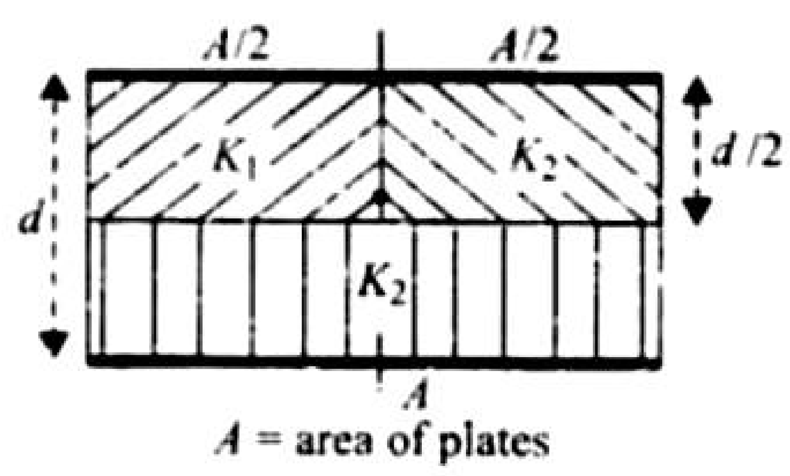
a. With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V
b. With S_3 closed, $V_1 = V_2 = 25$ V
c. With S_1 and S_2 closed, $V_1 = V_2 = 0$
d. With S_1 and S_2 closed, $V_1 = 30$ V, $V_2 = 20$ V
(IIT-JEE 1999)

16. Three charges q, $+q$, and $+q$ are placed at the vertices of a right-angled isosceles triangle as shown in the figure. The net electrostatic energy of the configuration is zero if Q is equal to



a. $\dfrac{-q}{1+\sqrt{2}}$ **b.** $\dfrac{-2q}{2+\sqrt{2}}$
c. $-2q$ **d.** $+q$
(IIT-JEE 2000)

17. A parallel plate capacitor of area A, plate separation d, and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2, and k_3 as shown in the figure. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant k is given by

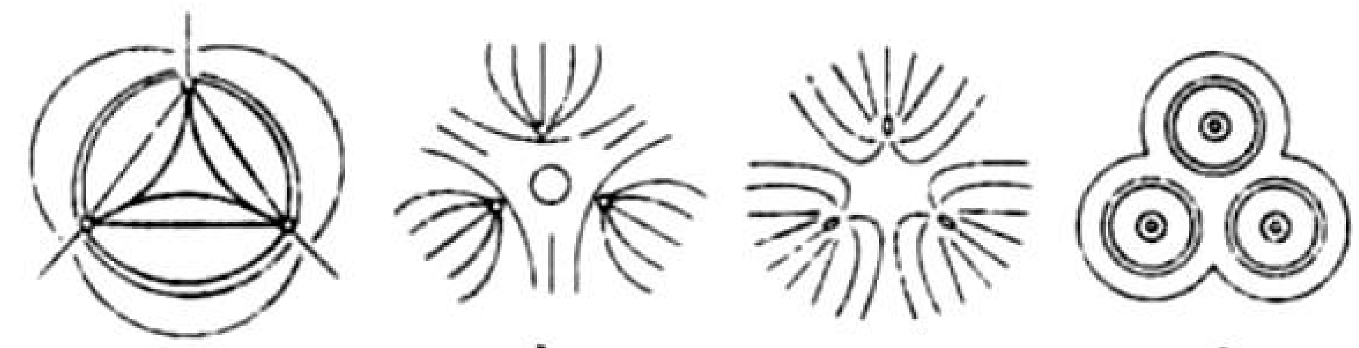


(IIT-JEE 2000)

a. $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \dfrac{1}{2k_3}$ **b.** $\dfrac{1}{k} = \dfrac{1}{k_1 + k_2} + \dfrac{1}{2k_3}$

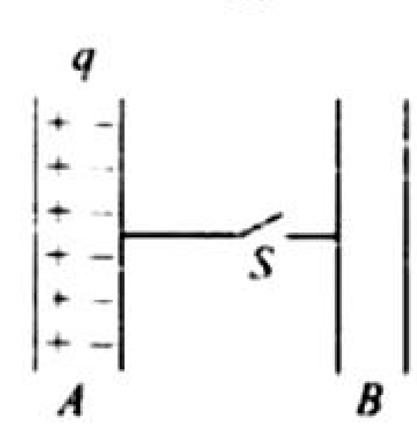
c. $k = \dfrac{k_1 k_2}{k_1 + k_2} + 2k_3$ **d.** $k = k_1 + k_2 + 2k_3$

18. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in **(IIT-JEE 2001)**



a. **b.** **c.** **d.**

19. Consider the situation shown in the figure. The capacitor A has a charge q on it, whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



a. zero b. $q/2$ c. q d. $2q$

(IIT-JEE 2001)

20. A uniform electric field pointing in the positive x-direction exists in a region. Let A be the origin. B be the point on the x-axis at $x = +1$ cm, and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B, and C satisfy

a. $V_A < V_B$ b. $V_A > V_B$ c. $V_A < V_C$ d. $V_A > V_C$

(IIT-JEE 2001)

21. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to

a. x b. x^2 c. x^3 d. $1/x$

(IIT-JEE 2002)

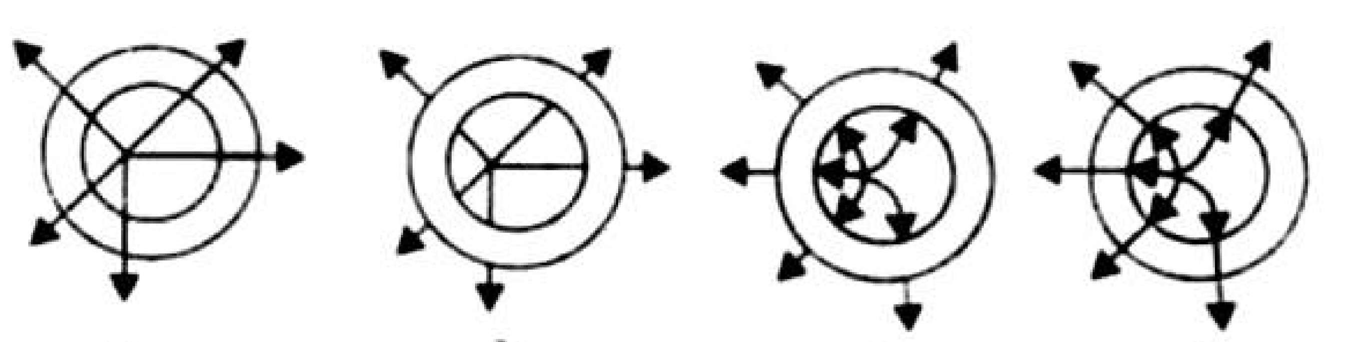
22. Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

a. $C(V_1^2 - V_2^2)/4$ b. $C(V_1^2 + V_2^2)/4$

c. $C(V_1 - V_2)^2/4$ d. $C(V_1 + V_2)^2/4$

(IIT-JEE 2002)

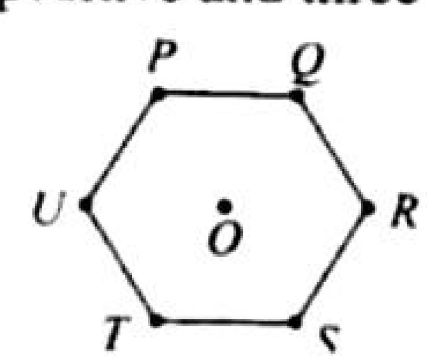
23. A metallic shell has a point charge q kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of force?



a. b. c. d.

(IIT-JEE 2003)

24. Six charges of equal magnitude, three positive and three negative, are to be placed on $PQRSTU$ corners of a regular hexagon, such that field at the center is double that of what it would have been if only one positive charge is placed at R.

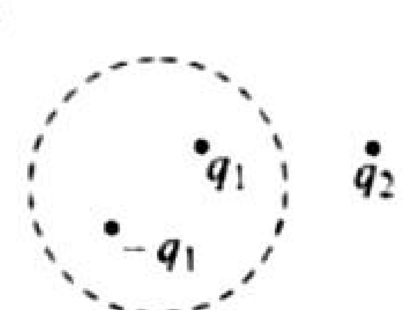


a. +,+,+,−,−,− b. −,+,+,+,−,−

c. −,+,+,−,+,− d. +,−,+,−,+,−

(IIT-JEE 2004)

25. A Gaussian surface in the figure is shown by dotted line. The electric field on the surface will be



a. due to q_1 and q_2 only

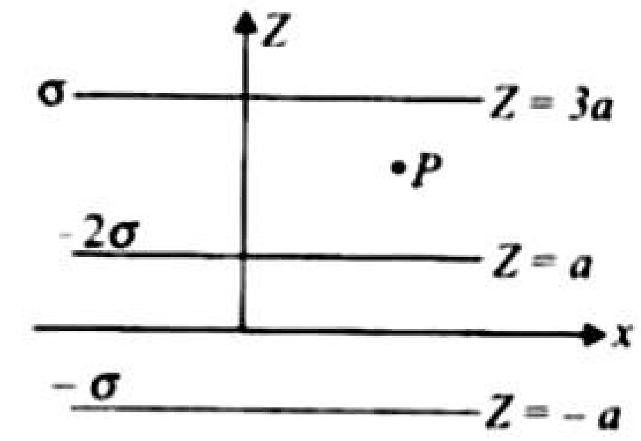
b. due to q_2 only

c. zero

d. due to all

(IIT-JEE 2004)

26. Three infinitely long charge sheets are placed as shown in the figure. The electric field at point P is



a. $2\sigma\,\varepsilon_0\,\hat{k}$ b. $4\sigma/\varepsilon_0\,\hat{k}$

c. $-2\sigma\,\varepsilon_0\,\hat{k}$ d. $-4\sigma/\varepsilon_0\,\hat{k}$

(IIT-JEE 2005)

27. A long hollow conducting cylinder is kept coaxially inside another long hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. Then

a. a potential difference appears between the two cylinders when a charge density is given to the inner cylinder

b. a potential difference appears between the two cylinders when a charge density is given to the outer cylinder

c. no potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders

d. no potential difference appears between the two cylinders when same charge density is given to both the cylinders

(IIT-JEE 2007)

28. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then

a. negative and distributed uniformly over the surface of the sphere

b. negative and appears only at the point on the sphere closest to the point charge

c. negative and distributed nonuniformly over the entire surface of the sphere

d. zero

(IIT-JEE 2007)

29. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the empty space is

a. zero everywhere

b. nonzero and uniform

c. nonuniform

d. zero only at its center

(IIT-JEE 2007)

30. Positive and negative point charges of equal magnitude are kept at $(0, 0, a/2)$ and $(0, 0, -a/2)$, respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is

a. positive

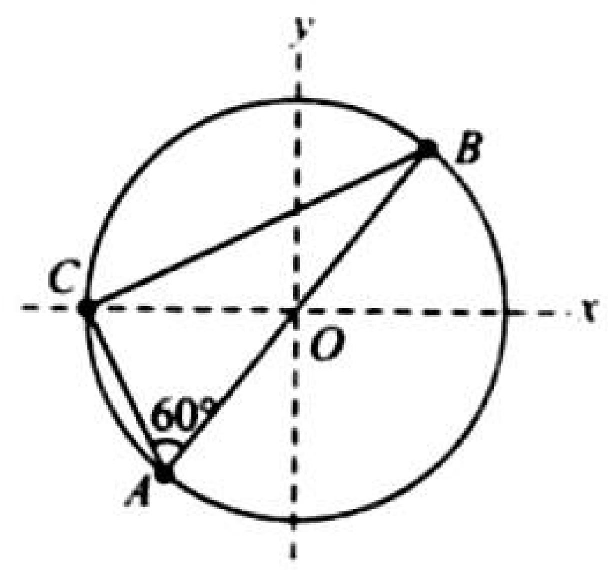
b. negative

c. zero

d. depends on the path connecting the initial and final positions

(IIT-JEE 2007)

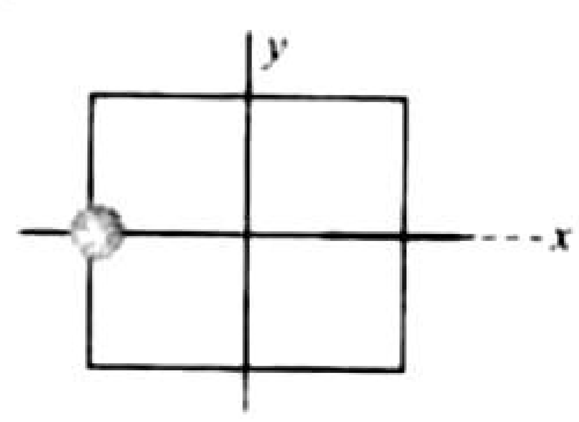
31. Consider a system of three charges $q/3$, $q/3$, and $-2q/3$ placed at points A, B, and C, respectively, as shown in the figure. Take O to be the center of the circle of radius R and angle $CAB = 60°$. Then



a. the electric field at point O is $q/8\pi\varepsilon_0 R^2$ directed along the negative x-axis
b. the potential energy of the system is zero
c. the magnitude of the force between the charges at C and B is $q^2/54\pi\varepsilon_0 R^2$
d. the potential at point O is $q/12\pi\varepsilon_0 R$

(IIT-JEE 2008)

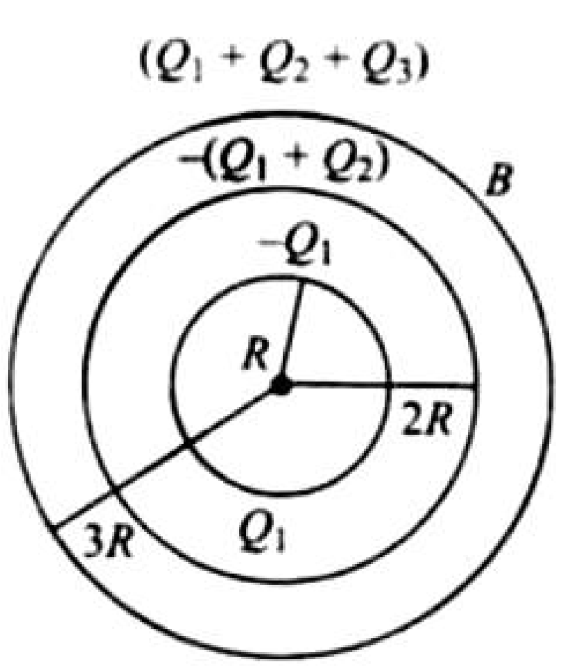
32. A disk of radius $a/4$ having a uniformly distributed charge 6 C is placed in the xy plane with its center at $(-a/2, 0, 0)$. A rod of length carrying a uniformly distributed charge 8 C is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges -7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$. The electric flux through this cubical surface is



a. $-2C/\varepsilon_0$ b. $2C/\varepsilon_0$
c. $10C/\varepsilon_0$ d. $12C/\varepsilon_0$

(IIT-JEE 2009)

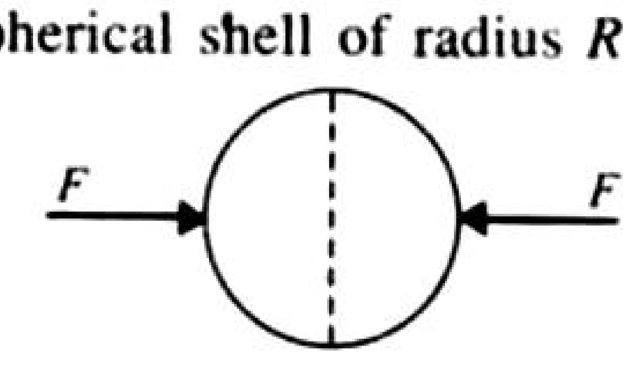
33. Three concentric metallic spherical shells of radii R, $2R$, and $3R$ are given charges Q_1, Q_2, and Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio $Q_1 : Q_2 : Q_3$ of the charges given to the shells is



a. 1:2:3 b. 1:3:5 c. 1:4:9 d. 1:8:18

(IIT-JEE 2009)

34. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F. F is proportional to



a. $\frac{1}{\varepsilon_0}\sigma^2 R^2$ b. $\frac{1}{\varepsilon_0}\sigma^2 R$ c. $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ d. $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

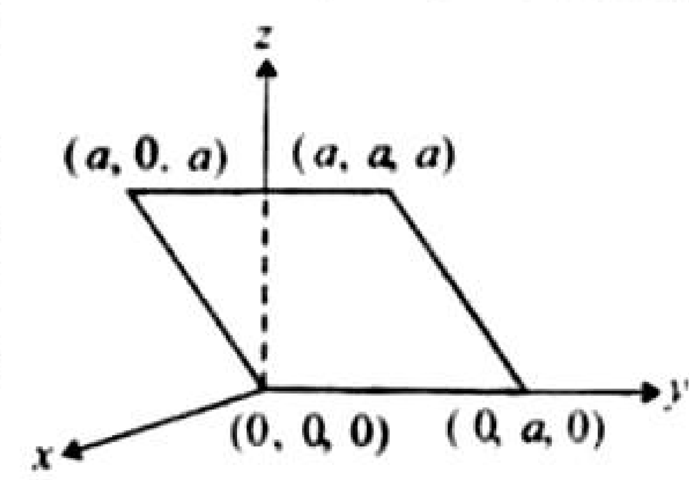
(IIT-JEE 2010)

35. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $(81\pi/7) \times 105$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given $g = 9.8$ ms^{-2}, viscosity of the air is 1.8×10^{-5} Ns m^{-2}, and the density of oil is 900 kg m^{-3}, the magnitude of q is.

a. 1.6×10^{-19} C b. 3.2×10^{-19} C
c. 4.8×10^{-19} C d. 8.0×10^{-10} C

(IIT-JEE 2010)

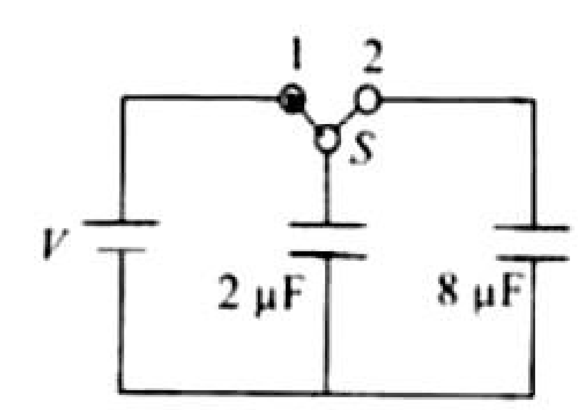
36. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



a. $2E_0a^2$ b. $\sqrt{2}E_0a^2$
c. E_0a^2 d. $\frac{E_0a^2}{\sqrt{2}}$

(IIT-JEE 2011)

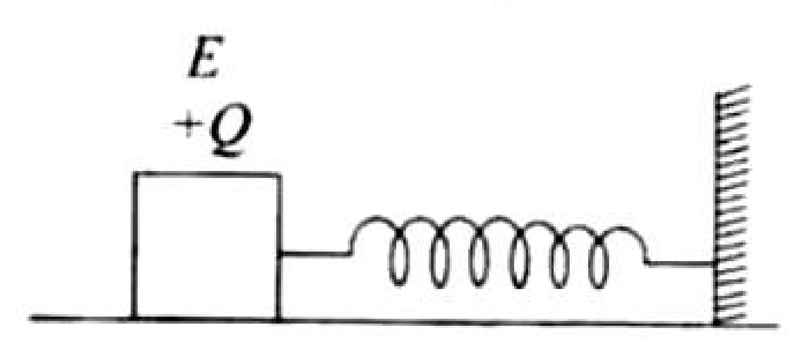
37. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after switch S is turned to position 2 is



a. 0% b. 20%
c. 75% d. 80%

(IIT-JEE 2011)

38. A wooden block performs SHM on a frictionless surface with frequency v_0. The block carries a charge $+Q$ on its surface. If a uniform electric field E is switched on as shown, then the SHM of the block will be



a. of the same frequency and with shifted mean position
b. of the same frequency and with the same mean position
c. of changed frequency and with shifted mean position
d. of changed frequency and with the same mean position

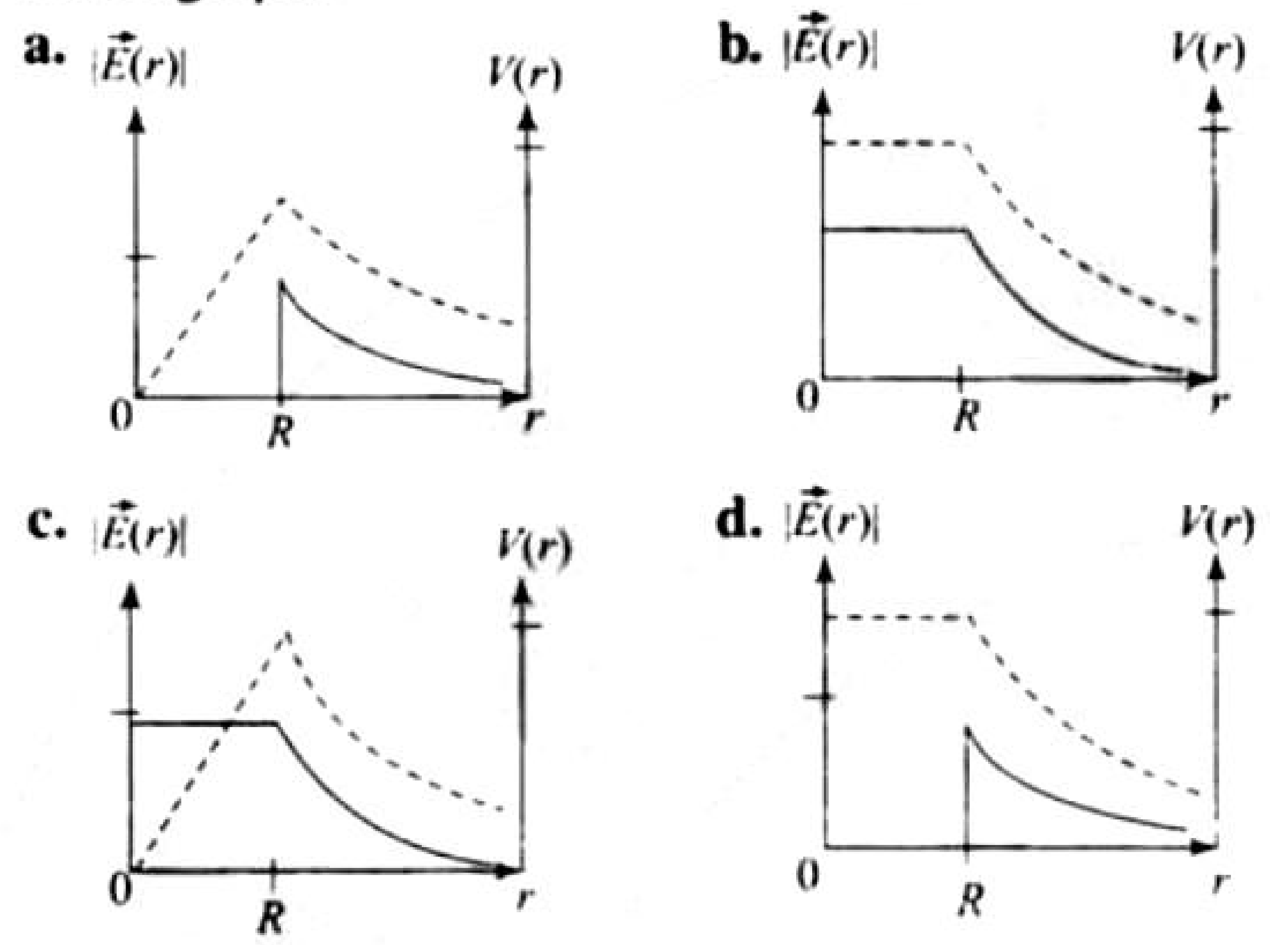
(IIT-JEE 2011)

39. Which of the following statement(s) is/are correct?
a. If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then Gauss's law will still be valid.
b. Gauss's law can be used to calculate the field distribution around an electric dipole.

c. If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

d. The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$. **(IIT-JEE 2011)**

40. Consider a thin spherical shell of radius R with its center at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the center, is best represented by which graph?



(IIT-JEE 2012)

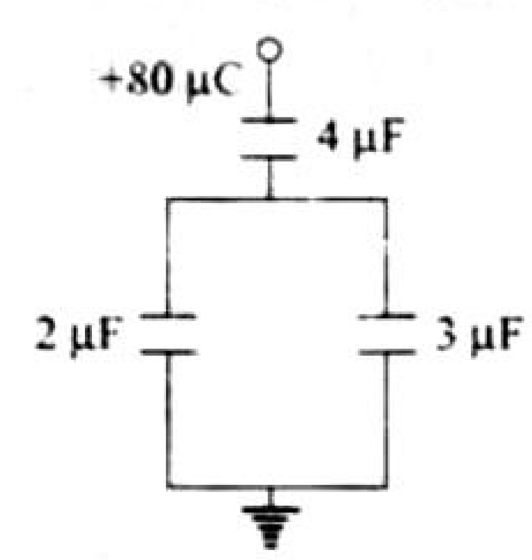
41. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly

a. 1×10^{-5} V **b.** 1×10^{-7} V

c. 1×10^{-9} V **d.** 1×10^{-10} V

(IIT-JEE 2012)

42. In the given circuit, a charge of $+80\ \mu C$ is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is

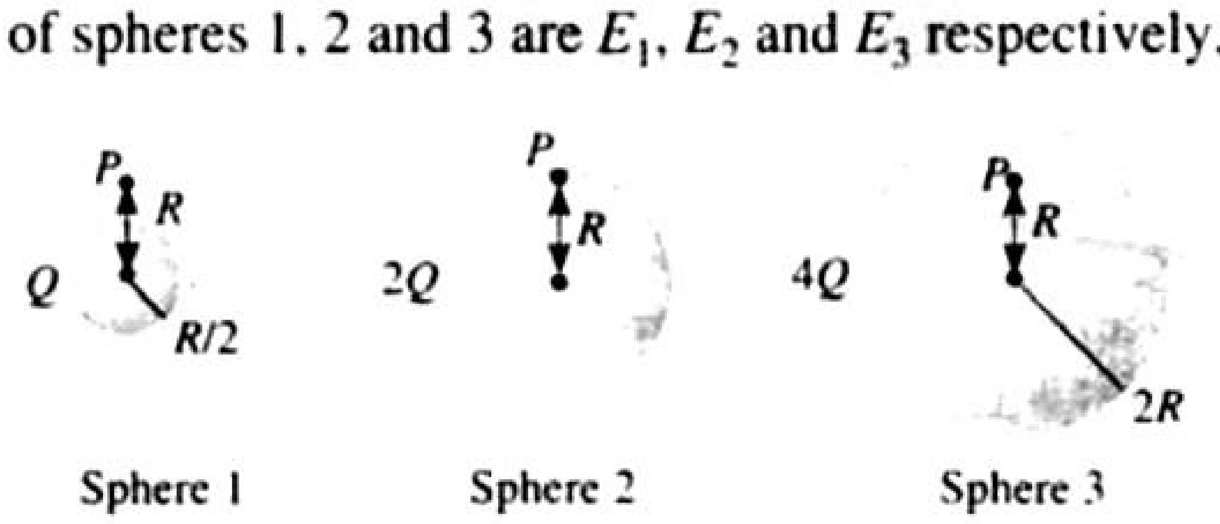


a. +32 μC **b.** +40 μC

c. +48 μC **d.** +80 μC

(IIT-JEE 2012)

43. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then

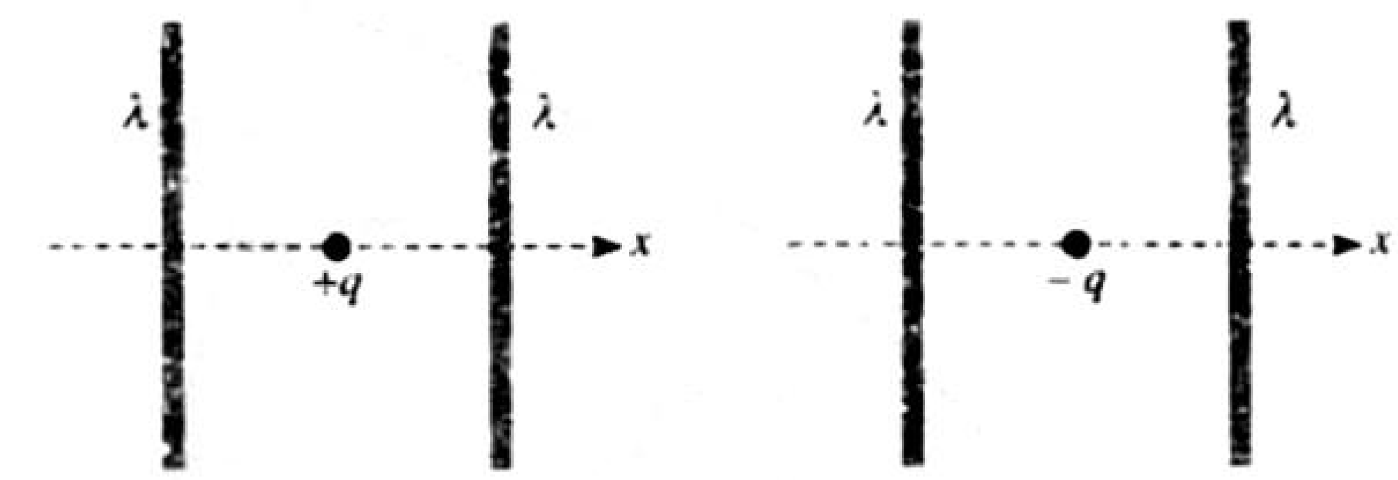


a. $E_1 > E_2 > E_3$ **b.** $E_3 > E_1 > E_2$

c. $E_2 > E_1 > E_3$ **d.** $E_3 > E_2 > E_1$

(JEE Advanced 2014)

44. The figure below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x-direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is (are):



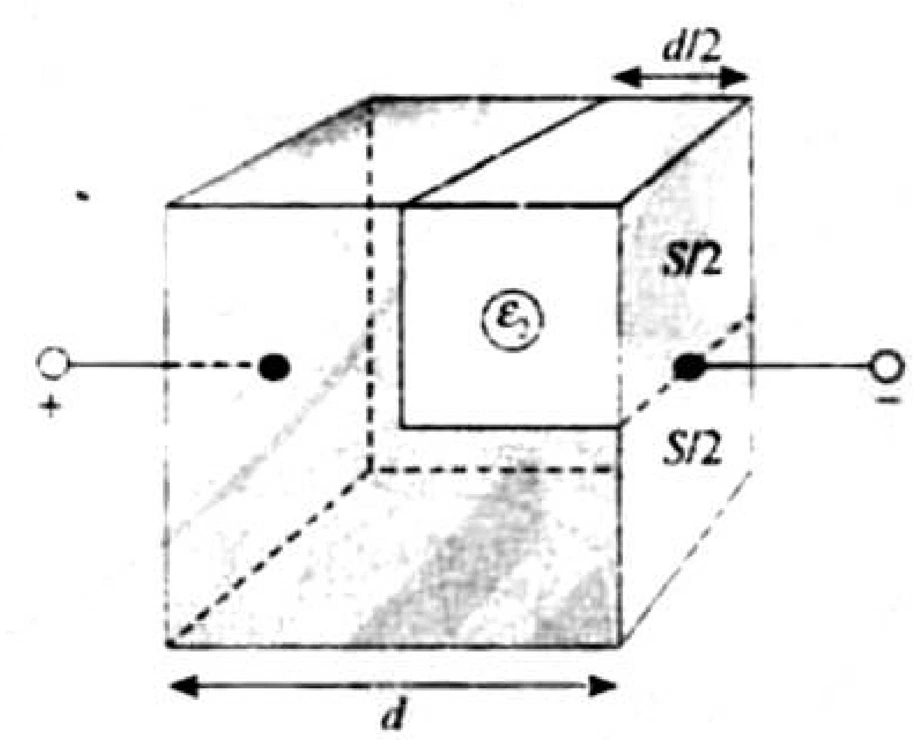
a. Both charges execute simple harmonic motion.

b. Both charges will continue moving in the direction of their displacement.

c. Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.

d. Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement. **(JEE Advanced 2015)**

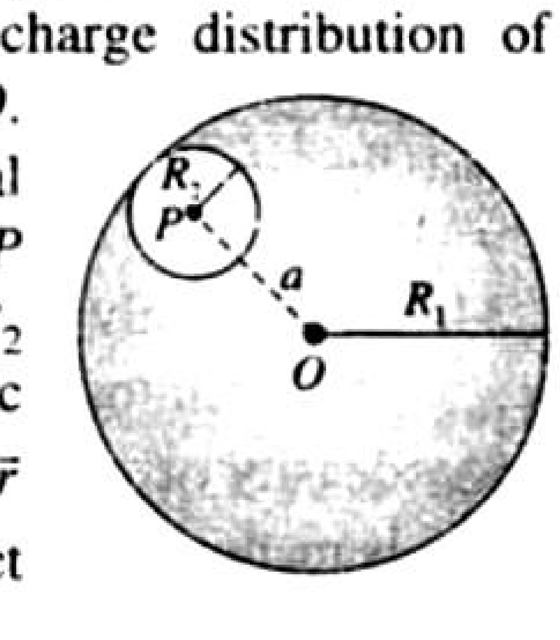
45. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is



a. 6/5 **b.** 5/3 **c.** 7/5 **d.** 7/3

(JEE Advanced 2015)

46. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)



a. $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

b. $\vec{E}$ is uniform, its magnitude depends of R_2 and its direction depends on $\vec{r}$

c. $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

d. $\vec{E}$ is uniform, and both its magnitude and direction depends on $\vec{a}$ **(JEE Advanced 2015)**

Multiple Correct Answers Type

1. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with the capacitor are given by Q_0, V_0, E_0, and U_0, respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities now given by Q, V, E, and U are related to the previous one as

   a. $Q > Q_0$ b. $V > V_0$ c. $E > E_0$ d. $U > U_0$

   (IIT-JEE 1985)

2. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles

   a. the charge on the capacitor increases
   b. the voltage across the plates increases
   c. the capacitance increases
   d. the electrostatic energy stored in the capacitor increases

   (IIT-JEE 1987)

3. A parallel plate capacitor of plate area a and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E, and W denote, respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and work done on the system, in question, in the process of inserting the slab, then

   a. $Q = \frac{\varepsilon_0 AV}{d}$ b. $Q = \frac{\varepsilon_0 KAV}{d}$

   c. $E = \frac{V}{Kd}$ d. $W = \frac{\varepsilon_0 AV^2}{2d}\left[1 - \frac{1}{K}\right]$

   (IIT-JEE 1991)

4. A dielectric slab of thickness d is inserted in a parallel-plate capacitor whose negative plate is at $x = 0$ and positive plate is $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$,

   a. the magnitude of the electric field remains the same
   b. the direction of the electric field remains the same
   c. the electric potential increases continuously
   d. the electric potential increases at first, then decreases and again increases **(IIT-JEE 1998)**

5. A positively charged thin metal ring of radius R is fixed in the xy plane with its center at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$, where $z_0 > 0$. Then the motion of P is

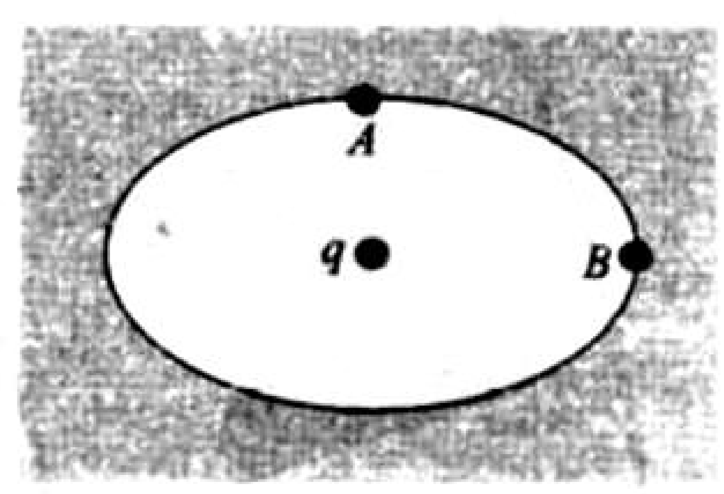
   a. periodic, for all values of z_0 satisfying $0 < z_0 < \infty$
   b. simple harmonic, for all values of z_0 satisfying $0 < z_0 \le R$
   c. approximately simple harmonic, provided $z_0 << R$
   d. such that P crosses O and continues to move along the negative z-axis toward $z = -\infty$.

   (IIT-JEE 1998)

6. A nonconducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its center

   a. increases as r increases, for $r < R$
   b. decreases as r increases, for $0 < r < \infty$
   c. decreases as r increases, for $R < r < \infty$
   d. is discontinuous at $r = R$ **(IIT-JEE 1998)**

7. An elliptical cavity is carved within a perfect conductor (see figure). A positive charge q is placed at the center of the cavity. The points A and B are on the cavity surface as shown in the figure. Then

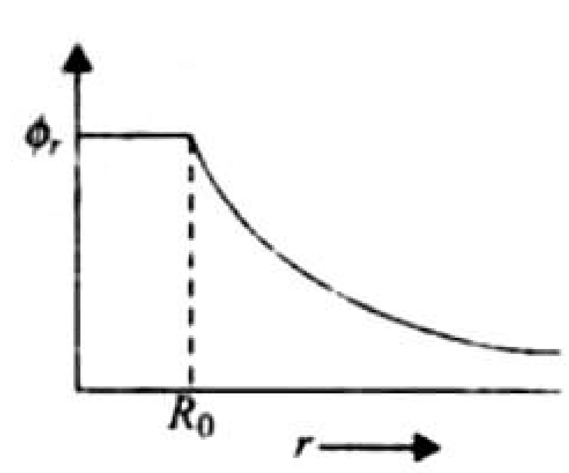


   a. electric field near A in the cavity = electric field near B in the cavity
   b. charge density at A = charge density at B
   c. potential at A = potential at B
   d. total electric field flux through the surface of the cavity is q/ε_0 **(IIT-JEE 1999)**

8. The electrostatic potential (ϕ_r) of a spherical symmetric system, kept at origin, is shown in the figure, and given as

$$\phi_r = \frac{q}{4\pi\varepsilon_0 r}\ (r \ge R_0),\ \phi_r = \frac{q}{4\pi\varepsilon_0 R_0}\ (r \le R_0)$$

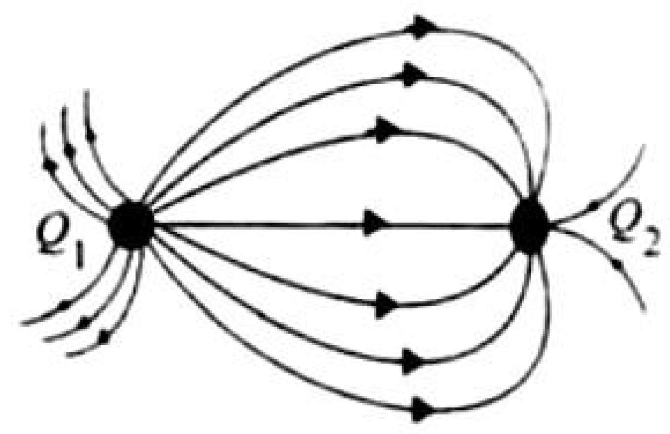
   Which of the following option(s) is/are correct?



   a. For spherical region $r \le R_0$, total electrostatic energy stored is zero.
   b. Within $r = 2R_0$, total charge is Q.
   c. There will be no charge anywhere except at $r = R_0$.
   d. Electric field is discontinuous at $r = R_0$.

   (IIT-JEE 2006)

9. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



a. $|Q_1| > |Q_2|$

b. $|Q_1| < |Q_2|$

c. at a finite distance to the left of Q_1, the electric field is zero

d. at a finite distance to the right of Q_2, the electric field is zero

(IIT-JEE 2010)

10. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($<R_A$) are kept far apart and each is given charge Q. Now they are connected by a thin metal wire. Then

a. $E_A^{\text{inside}} = 0$

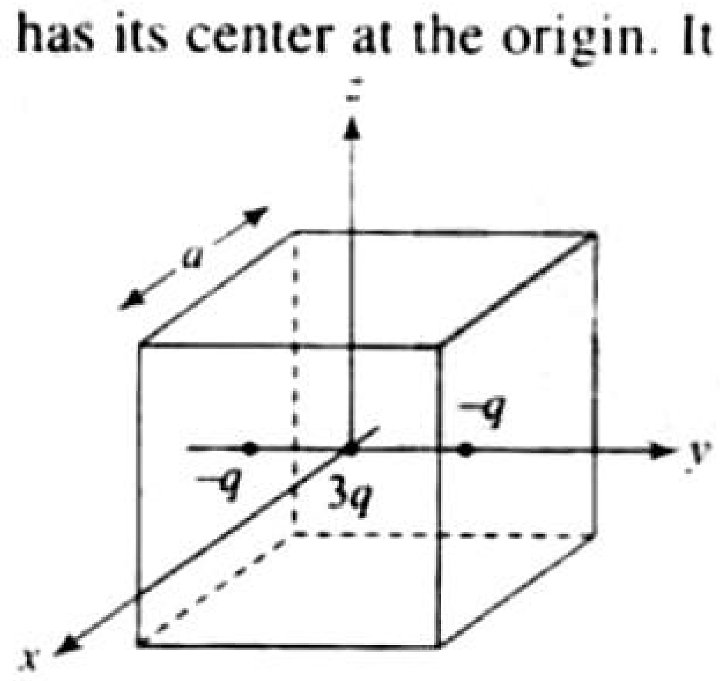
b. $Q_A > Q_B$

c. $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$

d. $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

(IIT-JEE 2011)

11. A cubical region of side a has its center at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$, and $-q$ at $(0, +a/4, 0)$. Choose the correct option(s).



a. The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$.

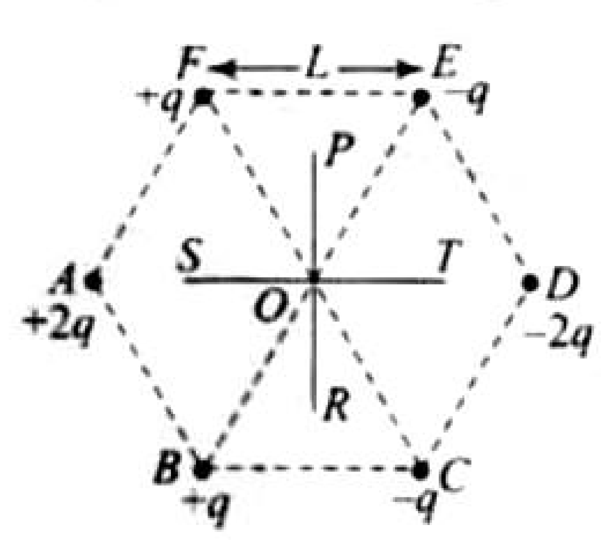
b. The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$.

c. The net electric flux crossing the entire origin is q/ε_0.

d. The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$.

(IIT-JEE 2012)

12. Six point charges are kept at the vertices of a regular hexagon of side L and center O, as shown in figure. Given that $K = q/4\pi\varepsilon_0 L^2$, which of the following statement(s) is/are correct?



a. The electric field at O is $6K$ along OD.

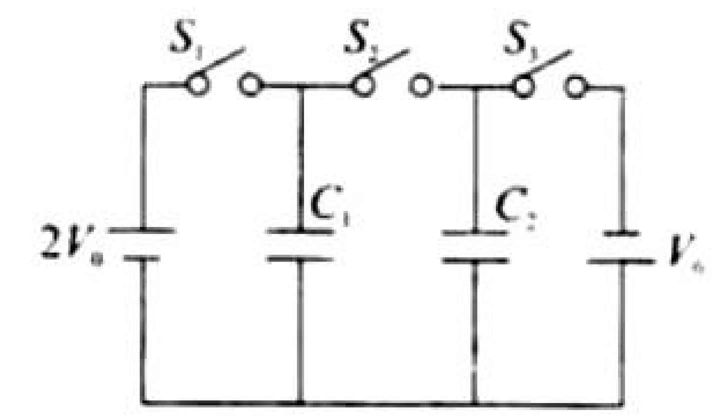
b. The potential at O is zero.

c. The potential at all points on the line PR is same.

d. The potential at all points on the line ST is same.

(IIT-JEE 2012)

13. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S is pressed. After some time,



a. the charge on the upper plate of C_1 is $2CV_0$

b. the charge on the upper plate of C_1 is CV_0

c. the charge on the upper plate of C_2 is 0

d. the charge on the upper plate of C_2 is $-CV_0$

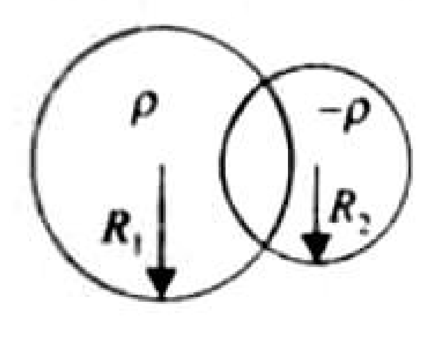
(JEE Advanced 2013)

14. Two nonconducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2, respectively, touch each other. The net electric field at a distance $2R$ from the center of the smaller sphere, along the line joining the center of the spheres is zero. The ratio ρ_1/ρ_2 can be

a. -4 b. $-32/25$ c. $32/25$ d. 4

(JEE Advanced 2013)

15. Two nonconducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



a. the electrostatic field is zero

b. the electrostatic potential is constant

c. the electrostatic field is constant in magnitude

d. the electrostatic field has same direction

(JEE Advanced 2013)

16. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

a. $Q = 4\sigma\pi r_0^2$

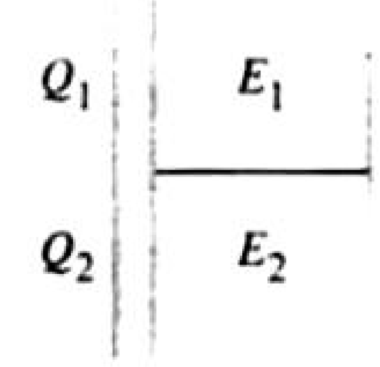
b. $r_0 = \dfrac{\lambda}{2\pi\sigma}$

c. $E_1\left(\dfrac{r_0}{2}\right) = 2E_2\left(\dfrac{r_0}{2}\right)$

d. $E_2\left(\dfrac{r_0}{2}\right) = 4E_3\left(\dfrac{r_0}{2}\right)$

(JEE Advanced 2014)

17. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric



field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.

a. $\frac{E_1}{E_2} = 1$ b. $\frac{E_1}{E_2} = \frac{1}{K}$

c. $\frac{Q_1}{Q_2} = \frac{3}{K}$ d. $\frac{C}{C_1} = \frac{2+K}{K}$

(JEE Advanced 2014)

Linked Comprehension Type

For Problems 1 and 2

A positive point charge q is fixed at origin. A dipole with a dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along the positive x-axis.

(IIT-JEE 2003)

1. Find the kinetic energy of the dipole when it reaches a distance d from the origin.

a. $\frac{3}{4\pi\varepsilon_0}\frac{pq}{d^2}$ b. $\frac{3}{2\pi\varepsilon_0}\frac{pq}{d^2}$ c. $\frac{1}{2\pi\varepsilon_0}\frac{pq}{d^2}$ d. $\frac{1}{4\pi\varepsilon_0}\frac{pq}{d^2}$

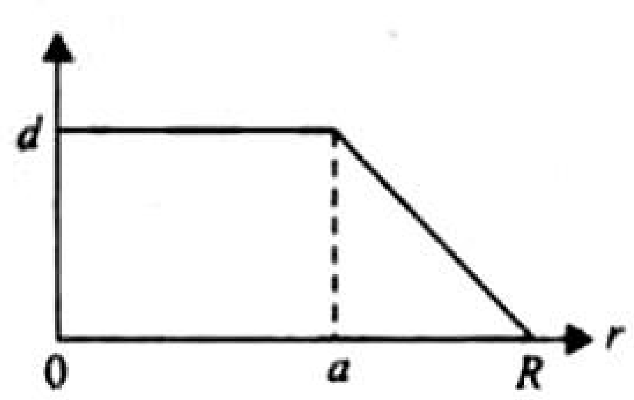
2. Find the force experienced by the charge q at this moment.

a. $\frac{1}{4\pi\varepsilon_0}\frac{pq}{d^2}$ b. $\frac{1}{2\pi\varepsilon_0}\frac{pq}{d^2}$

c. $\frac{1}{2\sqrt{2}\pi\varepsilon_0}\frac{pq}{d^2}$ d. $\frac{\sqrt{2}}{\pi\varepsilon_0}\frac{pq}{d^2}$

For Problems 3–5

The nuclear charge (Z_e) is nonuniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ (charge per unit volume) is dependent only on the radial distance r from the center of the nucleus as shown in the figure. The electric field is only along the radial direction.



(IIT-JEE 2008)

3. The electric field at $r = R$ is
   a. independent of a
   b. directly proportional to a
   c. directly proportional to a^2
   d. inversely proportional to a

4. For $a = 0$, the value of d (maximum value of ρ as shown in Fig. A1.173) is

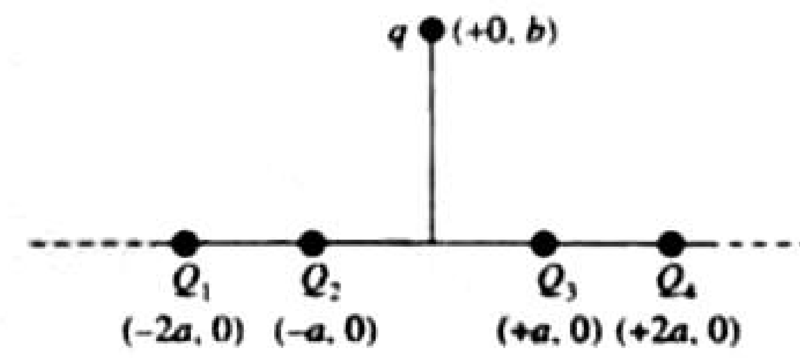
a. $\frac{3Ze}{4\pi R^3}$ b. $\frac{3Ze}{\pi R^3}$ c. $\frac{4Ze}{3\pi R^3}$ d. $\frac{Ze}{3\pi R^3}$

5. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

a. $a = 0$ b. $a = \frac{R}{2}$ c. $a = R$ d. $a = \frac{2R}{3}$

Matching Column Type

1. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists.



List I	List II
P. Q_1, Q_2, Q_3, Q_4 all positive	1. $+x$
Q. Q_1, Q_2 positive; Q_3, Q_4 negative	2. $-x$
R. Q_1, Q_4 positive ; Q_2, Q_3 negative	3. $+y$
S. Q_1, Q_3 positive; Q_2, Q_4 negative	4. $-y$

Code:

a. P-3, Q-1, R-4, S-2 b. P-4, Q-2, R-3, S-1

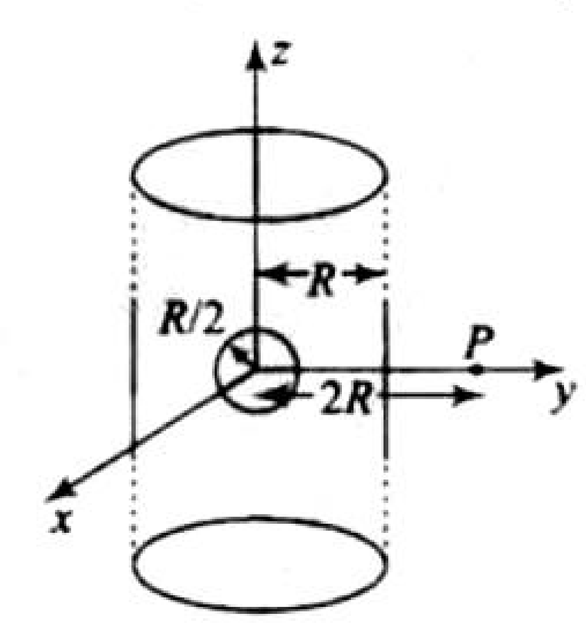
c. P-3, Q-1, R-2, S-4 d. P-4, Q-2, R-1, S-3

(JEE Advanced 2014)

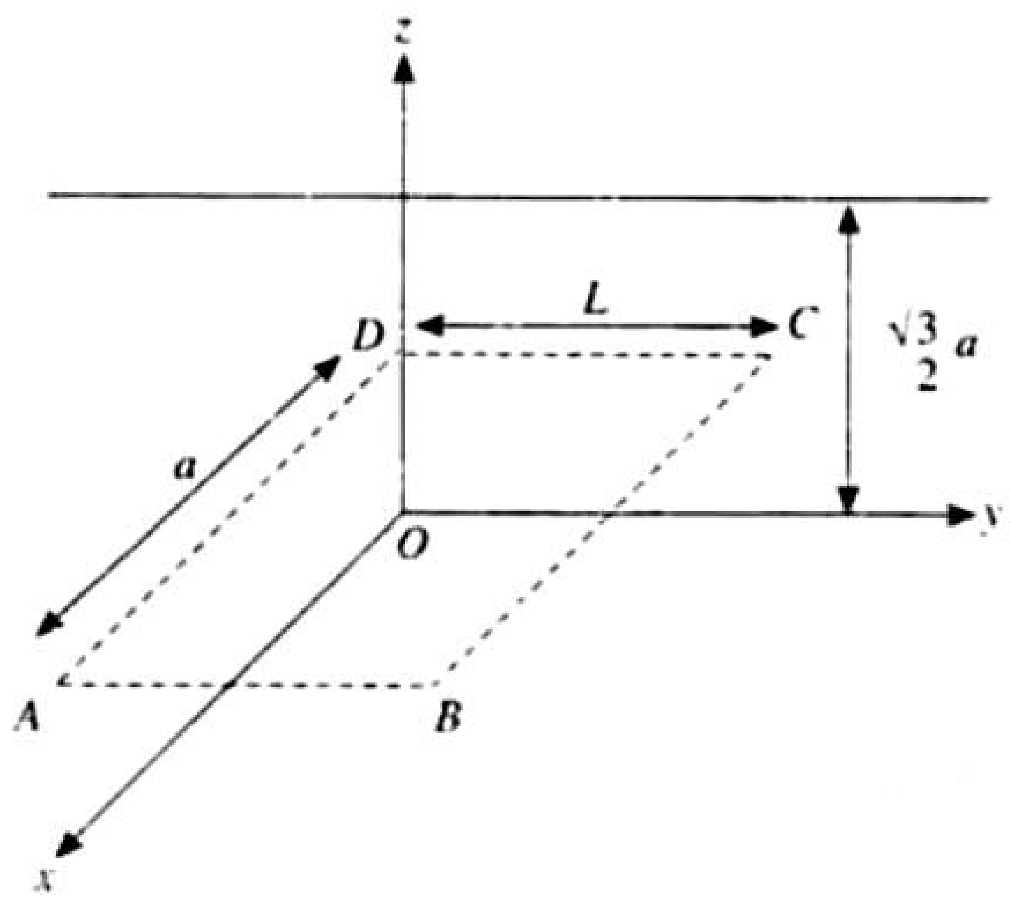
Integer Answer Type

1. A solid sphere of radius r has a charge Q distributed in its volume with a charge density $\rho = kr^a$ where k and a are constants and r is the distance from its centre. If the electric field at $r = R/2$ is 1/8 times that at $r = R$, find the value of a. **(IIT-JEE 2009)**

2. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its center on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $23\rho R/16k\varepsilon_0$. The value of k is **(IIT-JEE 2012)**



3. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field though the rectangular surface $ABCD$ lying in the $x-y$ plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is _________. **(JEE Advanced 2015)**



Assertion-Reasoning Type

a. Statement I is True, Statement II is True; Statement II is the correct explanation for Statement I.

b. Statement I is True, Statement II is True; Statement II is **NOT** the correct explanation for Statement I.

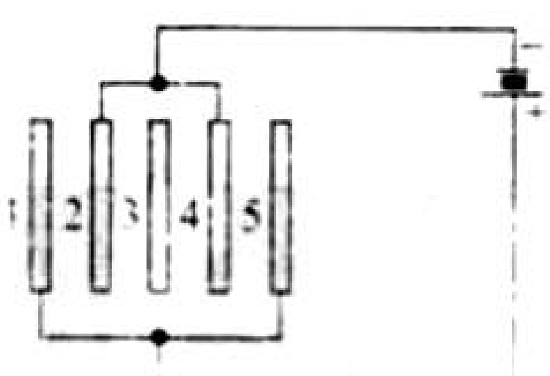
c. Statement I is True but Statement II is False.

d. Statement I is False but Statement II is True.

1. **Statement I:** For practical purposes, the earth is used as a reference at zero potential in electrical circuits.
Statement II: The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $Q/4\pi\varepsilon_0 R$.

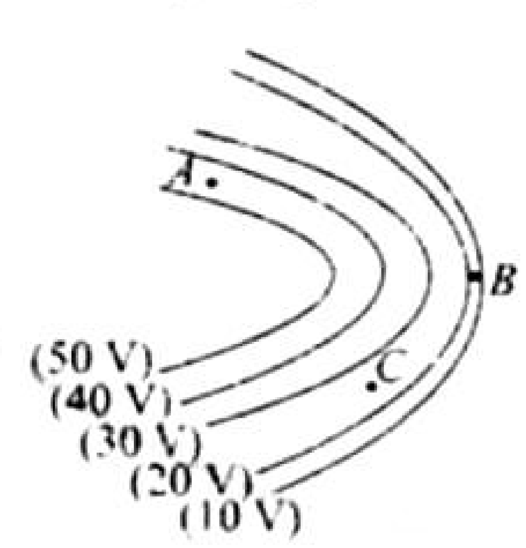
Fill in the Blanks Type

1. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at distance d apart. The plates are connected to a source of emf V as shown in the figure.



(IIT-JEE 1984)

2. Figure shows line of constant potentials in a region in which an electric field is present. The values of the potentials are written in brackets. Of the points A, B, and C, the magnitude of the electric field is greater at the point ________.

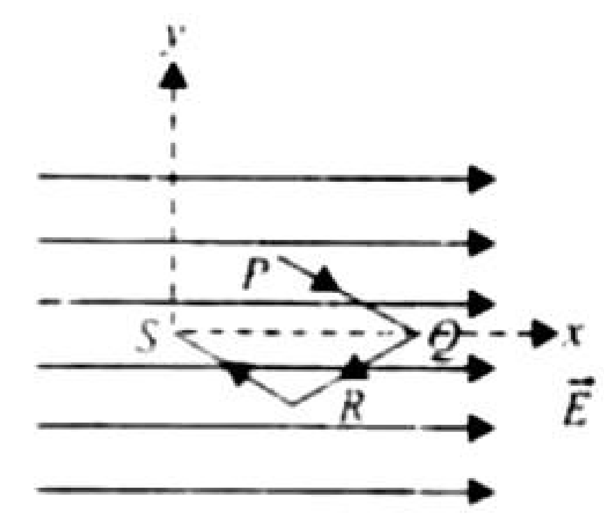


(IIT-JEE 1984)

3. Two small balls, having equal positive charge Q (coulomb) on each, are suspended by two insulating strings of equal length L (meter) from a hook fixed to a stand. The whole setup is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the two strings is ________ and the tension in each string is ________ newton. **(IIT-JEE 1986)**

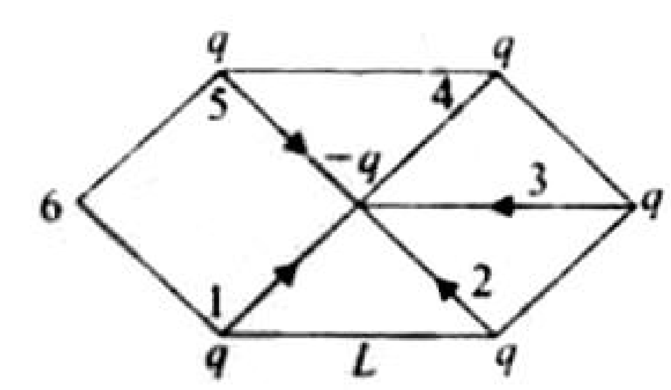
4. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to potential difference V. The battery is then disconnected, and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes ________. **(IIT-JEE 1988)**

5. A point charge q moves from point P to point S along the path $PQRS$ in a uniform electric field E pointing parallel to the positive direction of the x-axis (See figure). The coordinates of the points P, Q, R, and S are $(a, b, 0)$ $(2a, 0, 0)$, $(a, -b, 0)$, and $(0, 0, 0)$, respectively. The work done by the field in the above process is given by the expression ________. **(IIT-JEE 1989)**



6. The electric potential V at any point x, y, z (all in meters) in space is given by $V = 4x^2$ V. The electric field at the point (1 m, 0, 2 m) is ________ Vm^{-1}. **(IIT-JEE 1992)**

7. Five point charges, each of value $+q$ coulomb, are placed on five vertices of a regular hexagon of side L meters. The magnitude of the force on the point charge of value $-q$ coulomb placed at the center of the hexagon is ________ newton. **(IIT-JEE 1992)**



True/False Type

1. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried. **(IIT-JEE 1981)**

2. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulomb and the other an equal negative charge. Their masses after charging are different. **(IIT-JEE 1983)**

3. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If a high-energy X-ray beam falls on the ball, then the ball will be deflected in the direction of the field. **(IIT-JEE 1983)**

4. Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference V as shown in the figure. The forces on the two protons are identical. **(IIT-JEE 1986)**

5. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the center of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring. **(IIT-JEE 1988)**

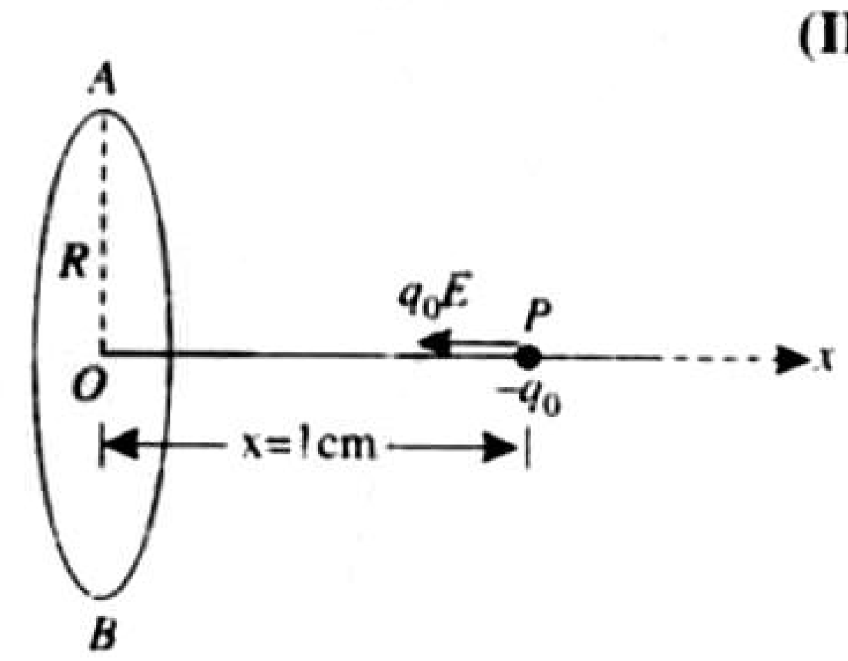
Subjective Type

1. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} coulomb is at rest in a horizontal uniform electric field of 20,000 volt/metre. Find the tension in the thread of the pendulum and the angle it makes with the vertical. **(IIT-JEE 1979)**

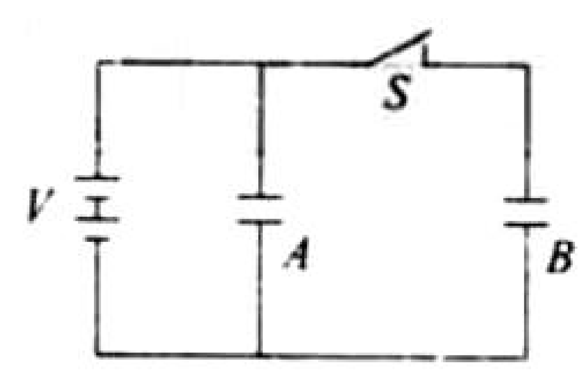
2. A charged particle is free to move in an electric field. Will it always move along an electric line of force? **(IIT-JEE 1979)**

3. A charge 'Q' is distributed over two concentric hollow spheres of radii 'r' and 'R' ($>r$) such that the surface densities are equal. Find the potential at the common centre. **(IIT-JEE 1981)**

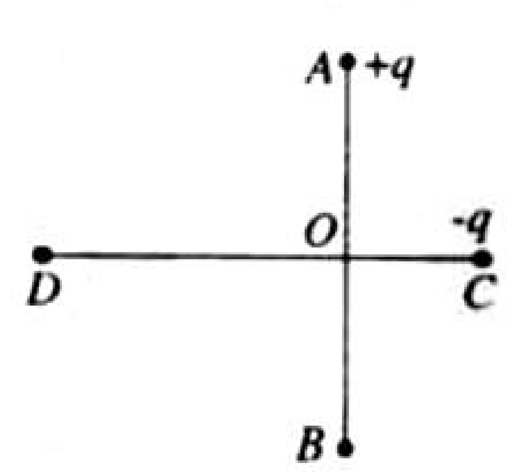
4. A thin fixed ring of radius 1m has a positive charge 1×10^{-5} coulomb uniformly distributed over it. A particle of mass 0.9g and having a negative charge of 1×10^{-6} coulomb is placed on the axis at a distance of 1 cm from the centre of the ring. Show that the motion of the negatively charged particle is Simple Harmonic Motion. Calculate the time period of oscillation. **(IIT-JEE 1982)**



5. Figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened, and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) $K = 3$. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **(IIT-JEE 1983)**



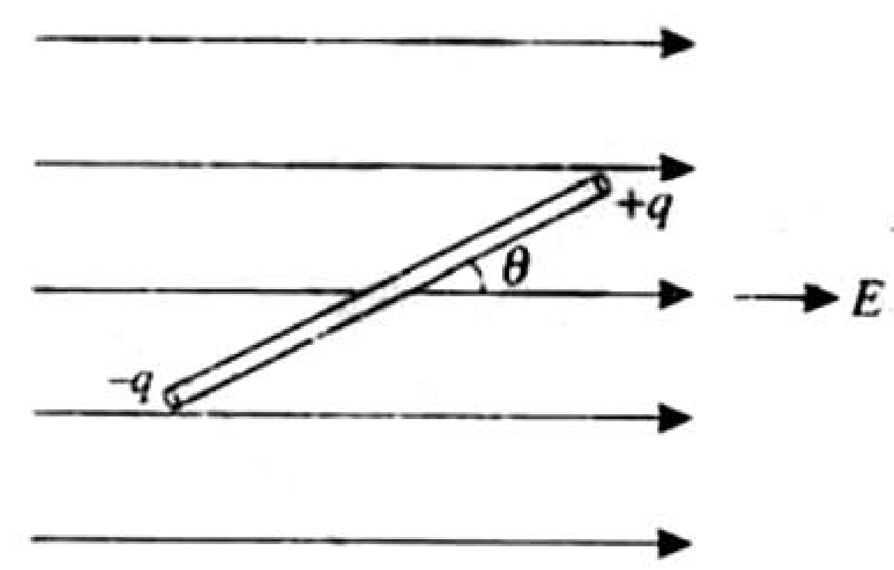
6. Two fixed, equal, positive charges, each of magnitude 5×10^{-5} coulomb are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when it reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 joules. Calculate the distances of the farthest point D which the negative charge will reach before returning towards C. **(IIT-JEE 1985)**



7. Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the position where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges. **(IIT-JEE 1987)**

8. Three particles, each of mass 1 gm and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. **(IIT-JEE 1988)**

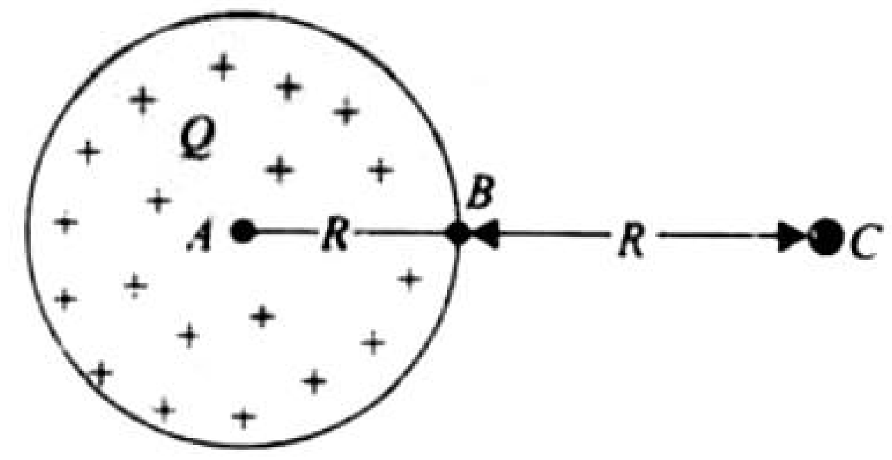
9. A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$ respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small time needed for the rod to become parallel to the field after it is set free. **(IIT-JEE 1989)**



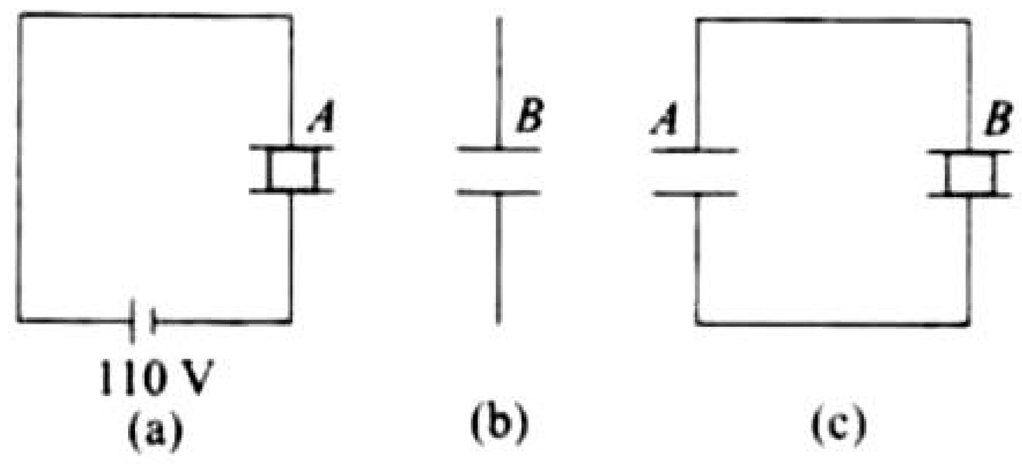
10. Three concentric spherical metallic shells A, B and C of radii a, b and c ($a < b < c$) have surface charge densities σ, $-\sigma$ and σ respectively. **(IIT-JEE 1990)**
    i. Find the potential of the three shells A, B and C.
    ii. If the shells A and C are at the same potential, obtain the relation between the radii a, b and c

11. Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$ respectively in the x-y plane. **(IIT-JEE 1991)**
    a. Show that all points in the x-y plane where the electric potential due to the two charges is zero lie on a circle. Find its radius and the location of its centre.
    b. Give the expression $V(x)$ at a general point on the x-axis and sketch the function $V(x)$ on the whole x-axis.
    c. If a particle of charge $+q$ starts form rest at the centre of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so.

12. Find the electric work done in bringing a charge q from A to B in a sphere of charge Q distributed uniformly throughout its volume. **(IIT-JEE 1992)**



13. Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate areas of A and B are 0.04 m^2 and 0.02 m^2, respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B.



i. The dielectric slab is placed inside A as shown in figure (a). A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.

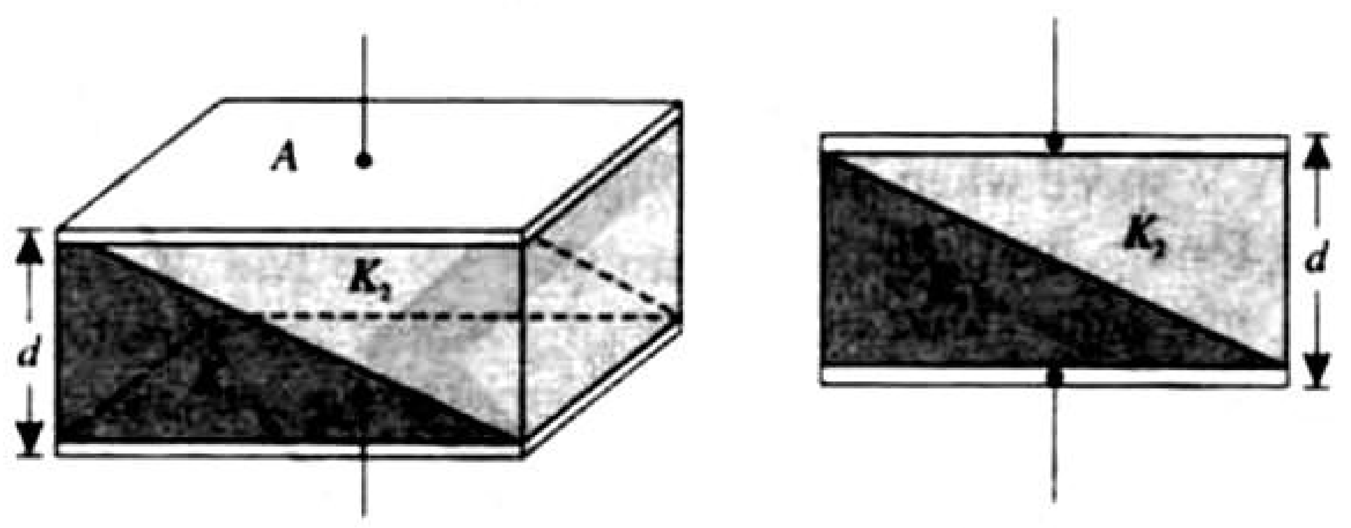
ii. The battery is disconnected, and then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A.

iii. The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure (c). Calculate the energy stored in the system. **(IIT-JEE 1993)**

14. A circular ring of radius R with uniform positive charge density λ per unit length is located in the yz plane with its center at the origin O. A particle of mass m and positive charge q is projected from the point $p(-\sqrt{3}R, 0, 0)$ on the negative x-axis directly toward O, with initial speed v. Find the smallest (nonzero) value of the speed such that the particle does not return to P? **(IIT-JEE 1993)**

15. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 ms^{-1}. Calculate the current drawn from the battery during the process. (Dielectric constant of oil = 11, ε_0 = 8.85 × $10^{-12} C^2 N^{-1} m^{-1}$) **(IIT-JEE 1994)**

16. The capacitance of a parallel plate capacitor with plate area A and separation d is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2, respectively. Find the capacitance of the resulting capacitor. **(IIT-JEE 1996)**

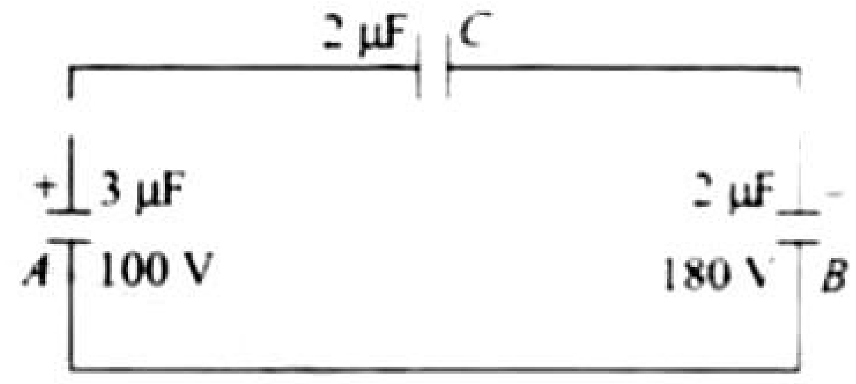


17. Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. Find the new charge density on the bigger sphere. **(IIT-JEE 1996)**

18. Two capacitors A and B with capacities 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V, respectively. The plates of the capacitors are connected as shown in the figure with one wire free from each capacitor. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit.

i. Calculate the final charge on the three capacitors.

ii. Find the amount of electrostatic energy stored in the system before and after the completion of the circuit.



(IIT-JEE 1997)

19. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged.

S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q, and it is again brought into contact with S_2 and removed. This procedure is repeated n times.

i. Find the electrostatic energy of S_2 after n such contacts with S_1.

ii. What is the limiting value of this energy as $n \to \infty$? **(IIT-JEE 1998)**

20. A nonconducting disk of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disk, from a height H with zero initial velocity. The particle has $q/m = 4\varepsilon_0 g/\sigma$.

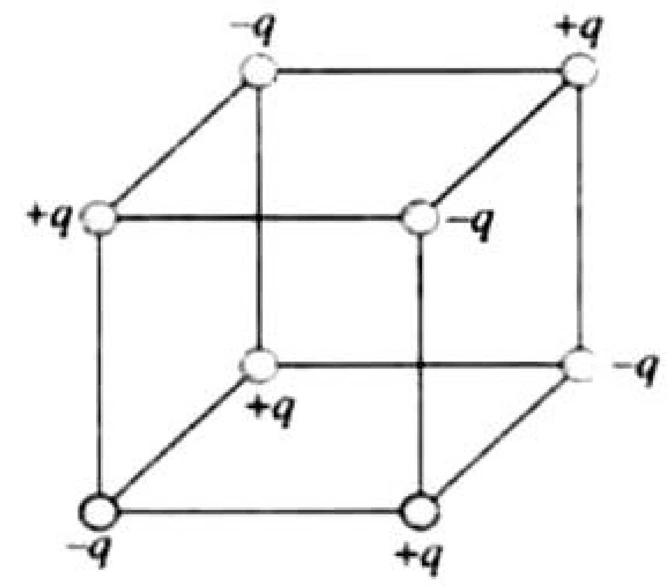
i. Find the value of H if the particle just reaches the disk.

ii. Sketch the potential energy of the particle as a function of its height and find its equilibrium position. **(IIT-JEE 1999)**

21. Four point charges +8 mC, −1 mC, −1 mC and +8 mC are fixed at the points $-\sqrt{\frac{27}{2}}\,m$, $-\sqrt{\frac{3}{2}}\,m$, $+\sqrt{\frac{3}{2}}\,m$ and $+\sqrt{\frac{27}{2}}\,m$ respectively on the y-axis. A particle of mass 6×10^{-4} kg and charge +0.1 μC moves along the x-direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$. **(IIT-JEE 2000)**

22. A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball, so that it can make complete revolution. **(IIT-JEE 2001)**

23. Charges $+q$ and $-q$ are located at the corners of a cube of side as show in the figure. Find the work done to separate the charges to infinite distance. **(IIT-JEE 2003)**



24. A charge $+Q$ is fixed at the origin of the co-ordinate system while a small electric dipole of dipole moment $\bar{p}$ pointing away from the charge along the x-axis is set free from a point far away from the origin.
   a. Calculate the K.E. of the dipole when it reaches to a point $(d, 0)$.
   b. Calculate the force on the charge $+Q$ at this moment. **(IIT-JEE 2003)**

25. Two uniformly charged large plane sheets S_1 and S_2 having charge densities σ_1 and $\sigma_2 (\sigma_1 > \sigma_2)$ are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length $a (a < d)$ at an angle 45° with the normal to S_1. Calculate the work done by the electric field. **(IIT-JEE 2004)**

26. A conducting liquid bubble of radius a and thickness $t (t << a)$ is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet. **(IIT-JEE 2005)**

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. b.	**2.** d.	**3.** b.	**4.** a.	**5.** a.
6. b.	**7.** b.	**8.** b.	**9.** c.	**10.** d.
11. b.	**12.** a.	**13.** d.	**14.** d.	**15.** b.
16. b.	**17.** b.	**18.** c.	**19.** a.	**20.** b.
21. b.	**22.** c.	**23.** c.	**24.** c.	**25.** d.
26. c.	**27.** a.	**28.** d.	**29.** b.	**30.** c.
31. c.	**32.** a.	**33.** b.	**34.** a.	**35.** b.
36. c.	**37.** d.	**38.** a.	**39.** c.	**40.** d.
41. c.	**42.** c.	**43.** c.	**44.** c.	**45.** d.
46. d.				

Multiple Correct Answers Type

1. a., d.	**2.** b., d.	**3.** a., c., d.
4. b., c.	**5.** a., c.	**6.** a., c.
7. c., d.	**8.** a., b., c., d.	**9.** a., d.
10. a., b., c., d.	**11.** a., c., d.	**12.** a., b., c.
13. b. d.	**14.** b., d.	**15.** c., d.
16. a., b., c.	**17.** a., d.	

Linked Comprehension Type

1. d. **2.** b. **3.** a. **4.** b. **5.** c.

Matching Column Type

1. a. (P) – (3), (Q) – (1), (R) – (4), (S) – (2)

Integer Answer Type

1. (2) **2.** (6) **3.** (6)

Assertion–Reasoning Type

1. b.

Fill in the Blanks Type

1. $\dfrac{\varepsilon_0 AV}{d}, -\dfrac{2\varepsilon_0 AV}{d}$ **2.** B **3.** $180°, \dfrac{Q^2}{16\pi\varepsilon_0 L^2}$

4. $\left(\dfrac{3}{K+2}\right)V$ **5.** $-qEa$ **6.** $-8\hat{i}$

7. $9 \times 10^9 \dfrac{q^2}{L^2}$

True/False Type

2. True **4.** True **5.** False

Subjective Type

1. $T = 9 \times 10^{-4}$ N and $\theta = 27°$

2. No.

3. $\dfrac{1}{4\pi\varepsilon_0} \dfrac{Q(R+r)}{R^2 + r^2}$

4. $T = \dfrac{\pi}{5}$ sec

5. $\dfrac{3}{5}$

6. Maximum distance from $O = 8.48$ m

7. (a) Charge q should be at a distance of 3 cm from $2q$
(b) Electric field = 0

8. 3.17×10^{-9} C

9. Time for the dipole to align along the direction of electric field will be $\frac{T}{2} = \frac{2\pi}{4}\sqrt{\frac{ML}{2qE}} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$

10. (i) $V_A = \frac{\sigma}{\varepsilon_0}[a - b + c]$

$$V_B = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} + b + c\right] \text{ and } V_C = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$$

(ii) $c = a + b$

11. (a) radius = $4a$. Center = $(5a, 0)$

(b) $V(x) = \frac{1}{4\pi\varepsilon_0}Q\left[\frac{1}{x - 3a} - \frac{2}{x + 3a}\right]$ For $|x| > 3a$

$V(x) = \frac{1}{4\pi\varepsilon_0}Q\left[\frac{1}{3a - x} - \frac{2}{3a + x}\right]$ For $|x| \le 3a$

(c) $v = \sqrt{\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{2ma}\right)}$

12. (a) $U = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 R}$ (b) $U = -\frac{3}{5}\frac{GM^2}{R}$ $E = 1.5 \times 10^{32}$ J

(c) $U = \frac{Q^2}{8\pi\varepsilon_0 R}$

13. (i) $C_A = 2 \times 10^{-9}$ F, $U_A = 1.21 \times 10^{-5}$ J
(ii) $W = 4.84 \times 10^{-5}$ J (iii) $U = 1.1 \times 10^{-5}$ J

14. $v_{min} = \sqrt{\frac{q\lambda}{2\varepsilon_0 m}}$

15. $I = 4.425 \times 10^{-9}$ amp

16. $C = \frac{\varepsilon_0 K_1 K_2 A}{d(K_1 - K_2)}\log\frac{K_1}{K_2}$ where $C = \frac{\varepsilon_0 A}{d}$

17. $\frac{5}{6}\sigma$

18. (i) $q_1 = 90$ μC, $q_2 = 150$ μC, $q_3 = 210$ μC
(ii) $U_i = 47.4$ mJ and $U_f = 18$ mJ

19. (i) $U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$

(ii) $U_\infty = \frac{Q^2 R}{8\pi\varepsilon_0 R}$. Hence, $q_n = \frac{QR}{r}\left[1 - \left(\frac{R}{R + r}\right)^n\right]$

20. (i) $H = \frac{4}{3}a$ (ii) $H = \frac{a}{\sqrt{3}}$

21. $v_0 = 3$ m/s, 3×10^{-4} J

22. 5.86 m/s.

23. $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}.\frac{4}{\sqrt{6}}[3\sqrt{3} - 3\sqrt{6} - \sqrt{2}]$

24. (a) KE $= \frac{1}{4\pi\varepsilon_0}\frac{pQ}{d^2}$ (b) $F = -\frac{1}{4\pi\varepsilon_0}\frac{2pQ}{d^3}$

25. $W = q_0 E\left(\frac{a}{\sqrt{2}}\right)$

26. $V_{droplet} = V\left[\frac{a}{3t}\right]^{1/3}$

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. b. The potential at the surface of a sphere is the same as the potential at the center of the sphere.

2. d. Let us consider the positive charge Q at any instant of time t at a distance x from the origin. It is under the influence of two forces, F_1 (= I) and F_2(= F). On resolving these two forces, we find that $F \sin\theta$ cancels out. The resultant force is

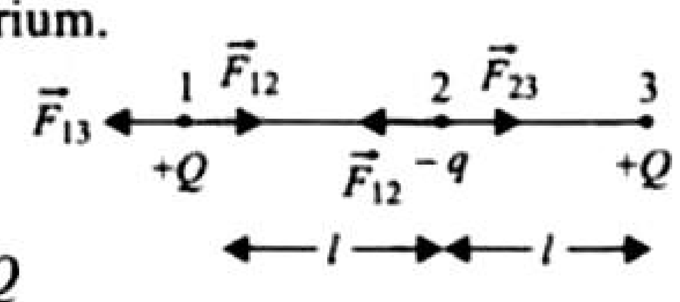
$$F_R = 2F\cos\theta = 2\times\frac{kQq}{\sqrt{x^2+a^2}}\times\frac{x}{\sqrt{x^2+a^2}}$$

$$= \frac{2kQqx}{(x^2+a^2)^{3/2}}$$

Since F_R is not proportional to x, the motion is not simple harmonic. The charge Q will accelerate till the origin and gain velocity. At the origin, the net force is zero but due to momentum it will cross the origin and move toward left. As it comes on the negative x-axis, the force is again toward the origin.

3. b. q has to be negative for equilibrium.

Considering equilibrium of 1,

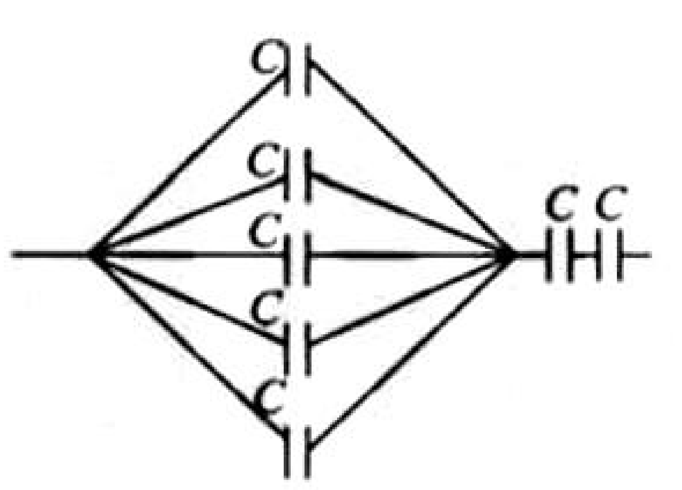


$$F_{13} = F_{12}$$

or $$\frac{KQ\times Q}{(2l)^2} = \frac{kQq}{l^2} \text{ or } q = \frac{Q}{4}$$

4. a. The potential inside the shell will be the same everywhere as on its surface. As we add $3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore, the potential difference remains the same.

5. a. The equivalent capacitance (see figure) is



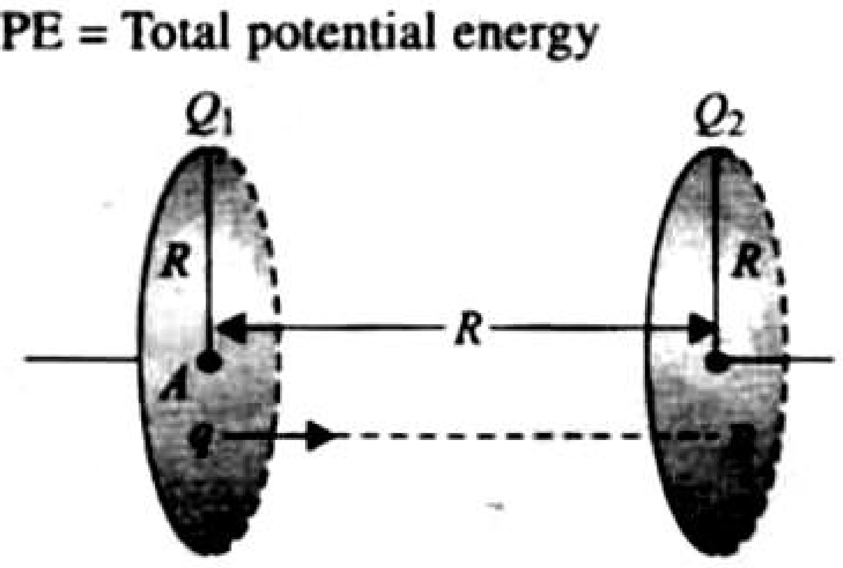
$$\frac{1}{C_{eq}} = \frac{1}{2}+\frac{1}{2}+\frac{1}{2\times 5} = \frac{11}{10}$$

or $$C_{eq} = \frac{11}{10}\lambda F$$

6. b. The work done in moving a charge from A to B is

$$W = (\text{TPE})_A - (\text{TPE})_B$$

TPE = Total potential energy



$(\text{TPE})_A$ = [PE due to Q_1 + PE due to Q_2]

$$= \left[\left(\frac{Q_1}{4\pi\varepsilon_0 R}\right)\times q + \left(\frac{Q_2}{4\pi\varepsilon_0\sqrt{R^2+R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}}\right]$$

$(TPE)_B$ = [PE due to Q_2 + PE due to Q_1]

$$= \left[\left(\frac{Q_2}{4\pi\varepsilon_0 R}\right)q + \left(\frac{Q_1}{4\pi\varepsilon_0\sqrt{R^2+R^2}}\right)q\right]$$

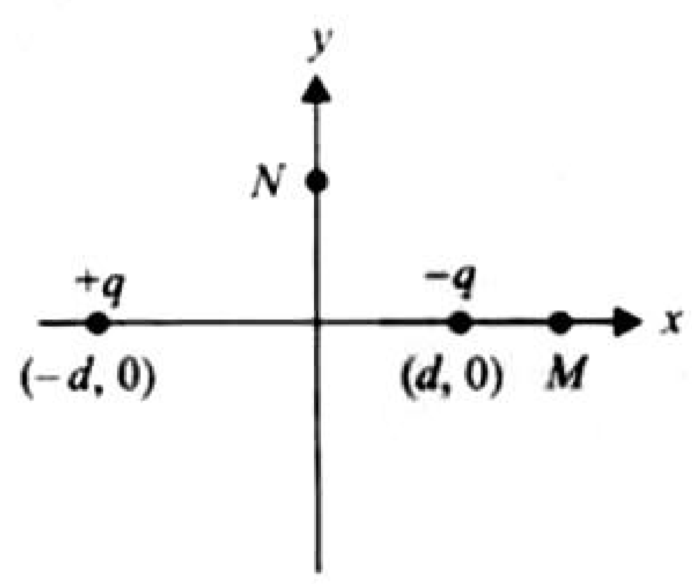
$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_2 + \frac{Q_1}{\sqrt{2}}\right]$$

$$\therefore \quad W = \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1\left(1-\frac{1}{\sqrt{2}}\right) - Q_2\left(1-\frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{q(Q_1-Q_2)}{4\pi\varepsilon_0 R}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

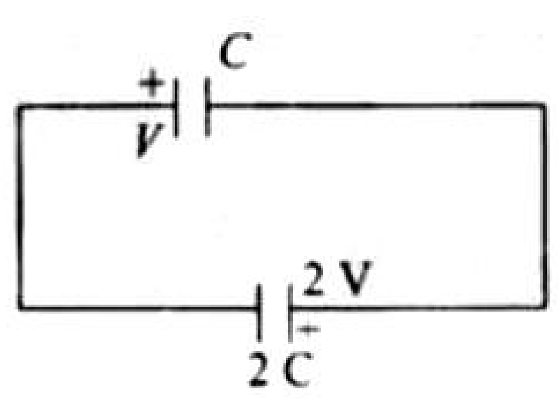
7. b. If we take a point M on the x-axis as shown in the figure, then the net electric field is in the x-direction. Therefore, option (a) is incorrect. If we take a point N on y-axis, we find net electric field along $+x$-direction. The same will be true for any point on Y-axis. Option (c) is incorrect. The direction of dipole moment is from negative to positive. Therefore, (d) is incorrect.



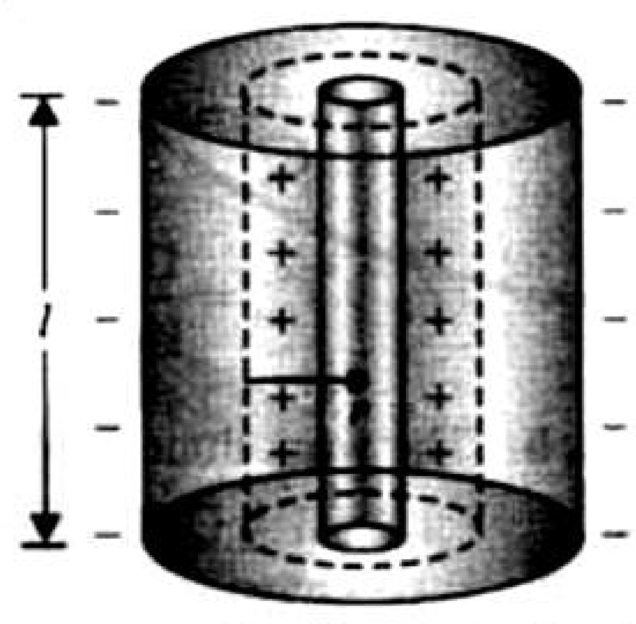
8. b. Energy stored,

$$U = \frac{1}{2}C_{eq}V_{net}^2 = \frac{1}{2}(3C)V^2 \quad \text{(see figure)}$$

$$= \frac{3}{2}CV^2$$



9. c. Let λ be the charge per unit length. Let us consider a Gaussian surface (dotted cylinder) (see figure).



Applying Gauss's law, we get

$$\phi = \oint \vec{E}\cdot\vec{ds} = \frac{\lambda l}{\varepsilon_0}$$

For the flat portions of the Gaussian surface, the angle between electric field and surface is 90°. Hence, flux through flat portions is zero. By symmetry, the electric field on the curved surface is the same throughout. The angle between $\vec{E}$ and $\vec{ds}$ is 0° (for curved surface)

$$E\int ds = \frac{\lambda l}{\varepsilon_0} \text{ or } E\times 2\pi rl = \frac{\lambda l}{\varepsilon_0} \text{ or } E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

or $\quad E \propto \frac{1}{r}$

10. d. The electric lines of force cannot enter the metallic sphere as electric field inside the solid metallic sphere is zero. Also, the origination and termination of the electric lines of force from the metallic surface is normal (directed toward the center).

11. b. Let the distance to be traveled be x. Let the strength of uniform electric field be E. For the electron.

$$u = 0,\ S = x,\ a = \frac{eE}{m_e},\ t = t_1$$

$$S = ut + \frac{1}{2}at^2 \text{ or } x = \frac{1}{2}\frac{eE}{m_e}\times t_1^2 \qquad \text{(i)}$$

For the proton,

$$u = 0,\ s = x,\ a = \frac{eE}{mp},\ t = t_1$$

$$S = ut + \frac{1}{2}at^2 \text{ or } x = \frac{1}{2}\frac{eE}{m_p}\times t_2^2 \qquad \text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{t_2^2}{t_1^2} = \frac{m_p}{m_e} \text{ or } \frac{t_2}{t_1} = \left[\frac{m_p}{m_e}\right]^{1/2}$$

12. a. $V_0 = K\dfrac{q}{R},\ V_\infty = 0$

$$\int_{t=a}^{t=0} -\vec{E}\cdot\vec{dl} = V_0 - V_\infty = \frac{Kq}{r} - 0 = \frac{9\times10^9\times1.11\times10^{-10}}{0.5} = 2\text{ V}$$

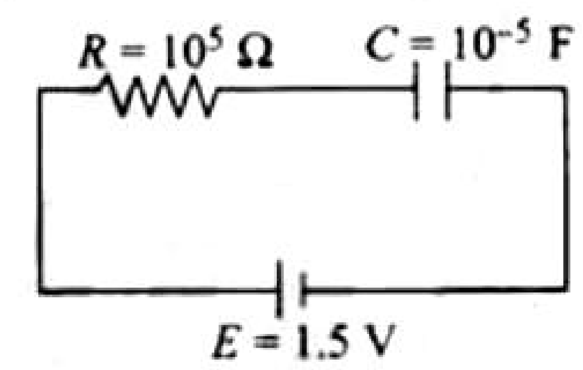
13. d. Potential at origin will be given by

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + \cdots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right]$$

$$= \frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

14. d. For the capacitor to get charged upto 0.75 V, the charge on the plates should be



$$q = CV = 10^{-5}\times 0.75$$
$$= 0.75\times10^{-5}\text{ C}$$

Using the equation of charging of capacitor

$q = CE\,[1 - e^{-t/RC}]$, we get

$$0.75\times10^{-5} = 10^{-5}\times1.5\left[1 - e^{\frac{t}{10^5\times10^{-5}}}\right]$$

or $\quad \frac{1}{2} = [1 - e^{-t}]$ or $e^{-t} = \frac{1}{2}$

Taking log on both sides, we get

$$-t = \ln 2$$

or $\quad t = 0.693$ s

15. b. With the closing of switch S_3, the potential across C_1 and C_2 would become identical to the average of V_1 and V_2, i.e., (30 V + 20 V)/2 = 25 V

16. b. Here, we have $= \dfrac{Qq}{a} + \dfrac{q^2}{a} + \dfrac{Qq}{a\sqrt{2}} = 0$

or $\quad Q\sqrt{2} + q\sqrt{2} + Q = 0$ or $Q(\sqrt{2}+1) = -q\sqrt{2}$

or $\quad Q = -\dfrac{q\sqrt{2}}{\sqrt{2}+1} = -\dfrac{2q}{2+\sqrt{2}}$

17. b. The effective capacitance is given by

$$\frac{\varepsilon_0 A}{d}\left[\frac{1}{(k_1+k_2)}+\frac{1}{2k_3}\right]^{-1}$$

The capacitance of a single capacitor will be $(\varepsilon_0 A/d)k$.

$$k = \left[\frac{1}{(k_1+k_2)}+\frac{1}{2k_3}\right]^{-1} \text{ or } \frac{1}{k} = \frac{1}{(k_1+k_2)}+\frac{1}{2k_3}$$

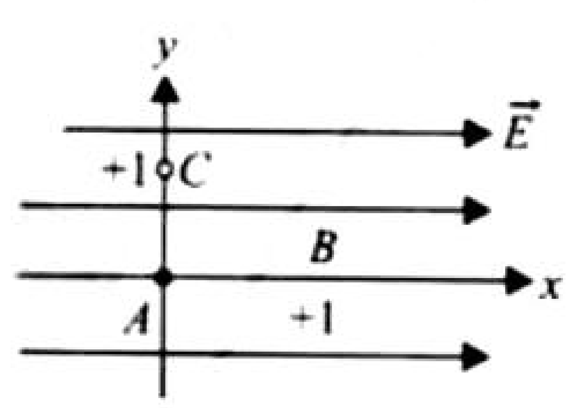
18. c. Option (a) is not possible because all the three charges are positive and the electric lines of force will expand laterally and not contract longitudinally.

Option (b) is not possible as electric lines of force are continuous lines but there are three lines that end up abruptly somewhere in between the electric field, which is not possible.

Option (d) makes no sense.

19. a. When S is closed, there will be no shifting of negative charge from plate A to B as the charge $-q$ is held by the charge $+q$. Neither there will be any shifting of charge from B to A.

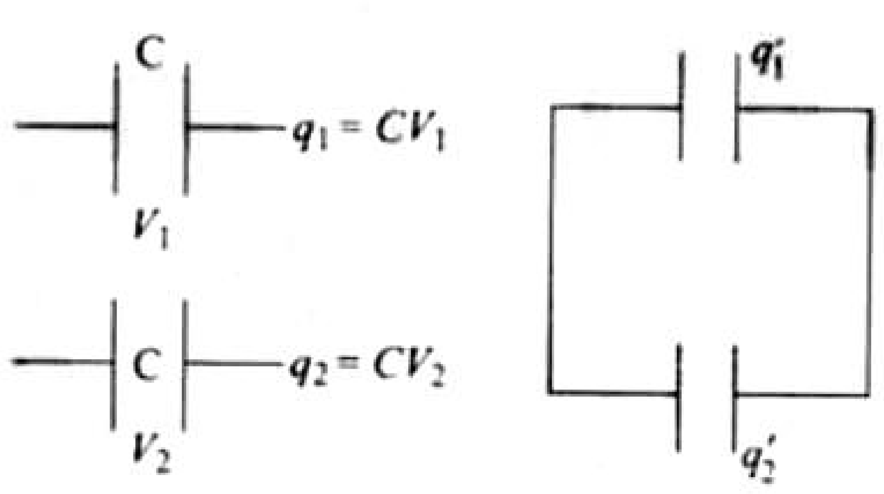
20. b. As we move along the direction of electric field, potential decreases. So $V_A < V_B$.



21. b. $U_i = \dfrac{2Qq}{4\pi\varepsilon_0(a)}$; $U_f = \dfrac{Qq}{4\pi\varepsilon_0}\left[\dfrac{1}{a+x}+\dfrac{1}{a-x}\right]$

$U_i - U_f = \dfrac{(Qqx^2)}{2\pi\varepsilon_0 a^3}$ [for $x << a$, x^2 can be neglected in comparison to a^2 in denominator]

22. c. Initially,



Initial energy $= \dfrac{1}{2}C(V_1^2+V_2^2)q_1' + q_2' = CV_1 + CV_2$

$$\frac{q_1'}{C} = \frac{q_2'}{C} \text{ or } q_1' = q_2' \text{ (charge conservation)}$$

$$q_1' = \frac{C(V_1+V_2)}{2}$$

Final energy $= \dfrac{C(V_1+V_2)^2}{4}$

Therefore, change in energy = Initial energy − Final energy

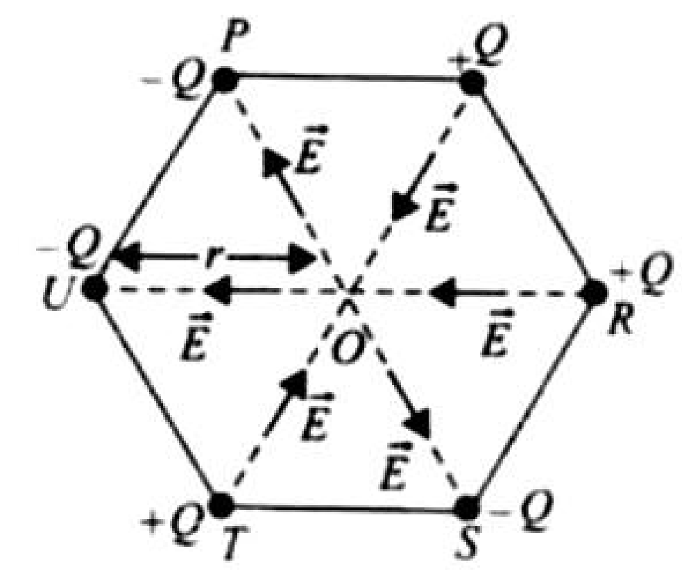
$$= \frac{1}{2}C(V_1^2+V_2^2) - \frac{C}{4}(V_1^2+V_2^2+2V_1V_2)$$

$$= \frac{C}{4}[2V_1^2+2V_2^2-V_1^2-V_2^2-2V_1V_2] = \frac{C}{4}(V_1-V_2)^2$$

23. c. Electric field is perpendicular to the equipotential surfaces and is zero everywhere inside the metal.

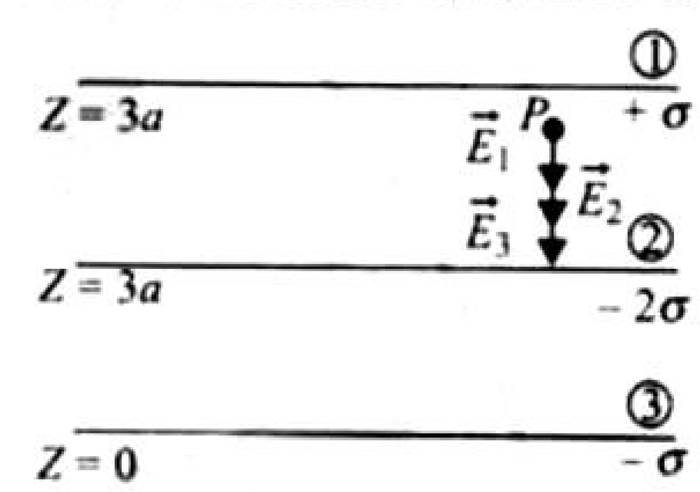
24. c. $|\vec{E}| = \dfrac{kq}{r^2}$

Electric field due to P on O is canceled by electric field due to S on O (figure). Similarly, electric field due to Q to O is canceled by electric field due to T and O. The electric field due to R on O in the same direction as that of U and O. Therefore, the net electric field is $2\vec{E}$.



25. d. The flux through the Gaussian surface is due to the charges inside the Gaussian surface. But the electric field on the Gaussian surface will be due to the charges present in the Gaussian surface and outside it. It will be due to all the charges.

26. c. Figure shows the electric fields due to the sheets 1, 2, and 3 at point P. The direction of electric fields is according to the charge on the sheets (away from positively charged sheet and toward the negatively charged sheet and perpendicular).



The total electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= E_1(-\hat{k}) + E_2(-\hat{k}) + E_3(-\hat{k})$$

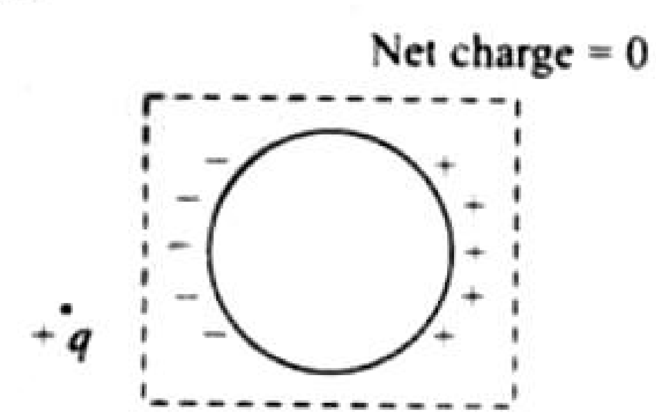
$$= \left[\frac{\sigma}{2\varepsilon_0}+\frac{2\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}\right](-\hat{k})$$

$$= -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

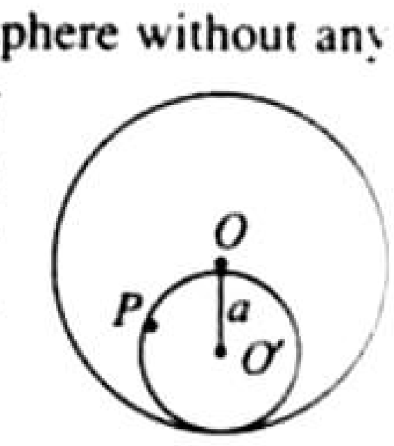
27. a. When a charge density is given to the inner cylinder, the potential developed at its surface is different from that on the outer cylinder. This is because the potential decreases with distance from a charged conducting cylinder when the point of consideration is outside the cylinder.

But when a charge density is given to the outer cylinder, it will change its potential by the same amount as that of the inner cylinder. Therefore, no potential difference will be produced between the cylinders in this case.

28. d. When a positive point charge is placed outside a conducting sphere, a rearrangement of charge takes place on the surface. But the total charge on the sphere is zero as no charge has left or entered the sphere.



29. b. Let us consider a uniformly charged solid sphere without any cavity. Let the charge per unit volume be σ and O be the center of the sphere. Let us consider a uniformly charged sphere of negative charged density σ having its center at O'. Also, let OO' be equal to a (figure).



Let us consider an arbitrary point P in the small sphere. The electric field due to charge on big sphere $\vec{E} = (\sigma/3\varepsilon_0)\overline{OP}$.

Also, the electric field due to small sphere is

$$\vec{E}_2 = \frac{\sigma}{3\varepsilon_0}\overline{PO}$$

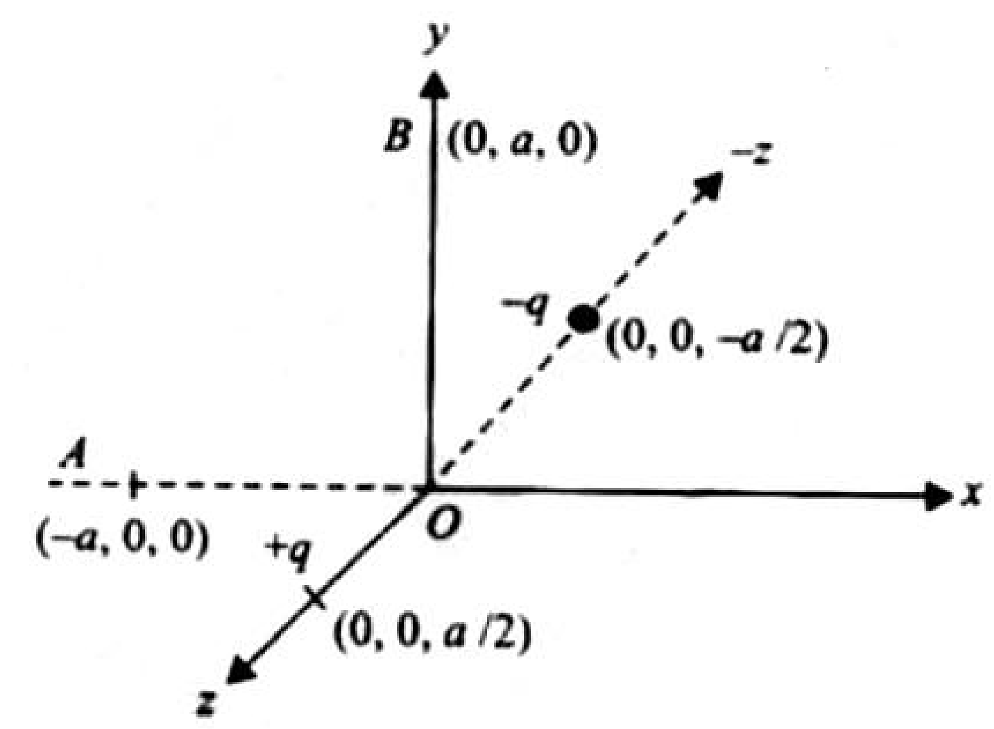
Therefore, the total electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{3\varepsilon_0}[\overline{OP} + \overline{PO}] = \frac{\sigma}{3\varepsilon_0}\overline{OO'}$$

This will have a finite value, which will be uniform.

30. c. The charges make an electric dipole. Points A and B lie on the equatorial plane of the dipole. Therefore, potential at A = potential at $B = 0$

$$W = q(V_A - V_B) = q \times 0 = 0$$



31. c. $F_{BC} = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{(R/\sqrt{3})^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}$

32. a. Total charge enclosed by cube is $-2C$. Hence, electric flux through the cube is $-2C/\varepsilon_0$.

33. b. $\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$

or $\quad Q_1 : Q_2 : Q_3 :: 1 : 3 : 5$

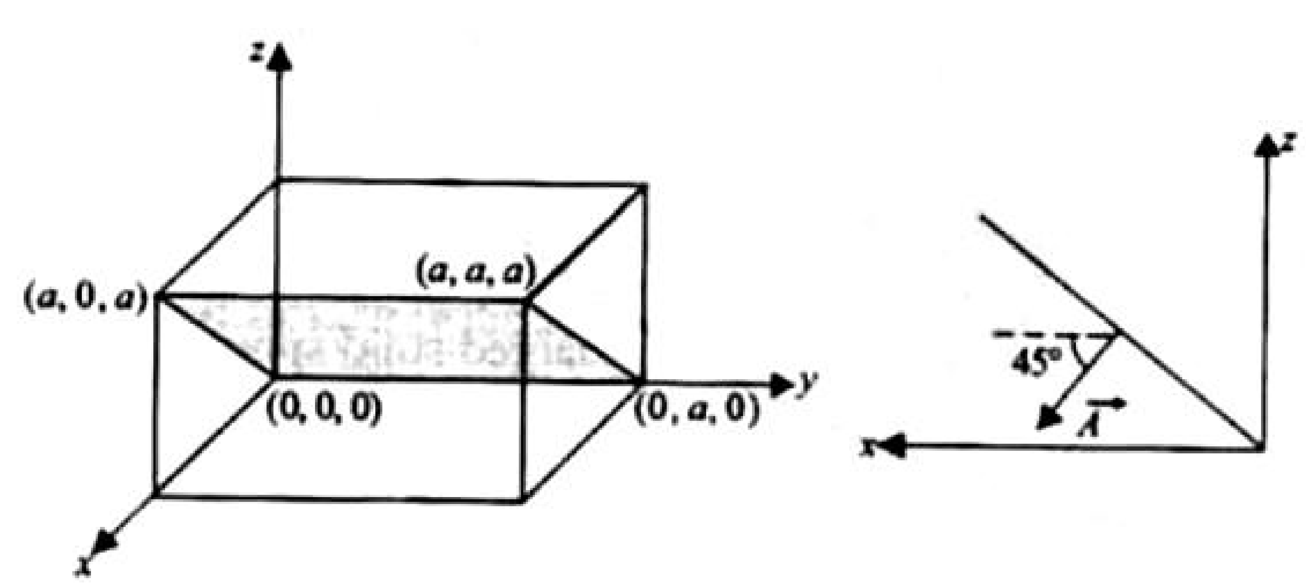
34. a. Pressure $= \frac{\sigma^2}{2\varepsilon_0}$ and force $= \frac{\sigma^2}{2\varepsilon_0} \times \pi R^2$

35. b. $\frac{4}{5}\pi R^3 \rho g = qE = 6\rho\eta R v_T$

$\therefore q = 8.0 \times 10^{-19}$ C

36. c. $\vec{A} = A\cos 45°\hat{i} - A\sin 45°\hat{k}$

$$= \sqrt{2}a^2\frac{1}{\sqrt{2}}\hat{i} - \sqrt{2}a^2\frac{1}{\sqrt{2}}\hat{k} = a^2\hat{i} - a^2\hat{k}$$



$$\phi = \vec{E}\cdot\vec{A} = E_0\hat{i}\cdot(a^2\hat{i} - a^2\hat{k}) = E_0 a^2$$

37. d. $U_i = \frac{1}{2}(2)V^2 = V^2$.

After connecting S to 2.

$$V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{2V + 8 \times 0}{2 + 8} = \frac{V}{5}$$

$$U_f = \frac{1}{2}(2 + 8)\left(\frac{V}{5}\right)^2 = \frac{V^2}{5}$$

Percent energy dissipated $= \frac{U_i - U_f}{U_i} \times 100$

$$= \frac{V^2 - \frac{V^2}{5}}{V^2} \times 100 = 80\%$$

38. a. The frequency will be the same $\nu = \sqrt{k/m}/2\pi$. Frequency is independent of any external constant force. But due to the constant force qE toward right, the equilibrium position gets shifted by qE/K in forward direction.

39. c.

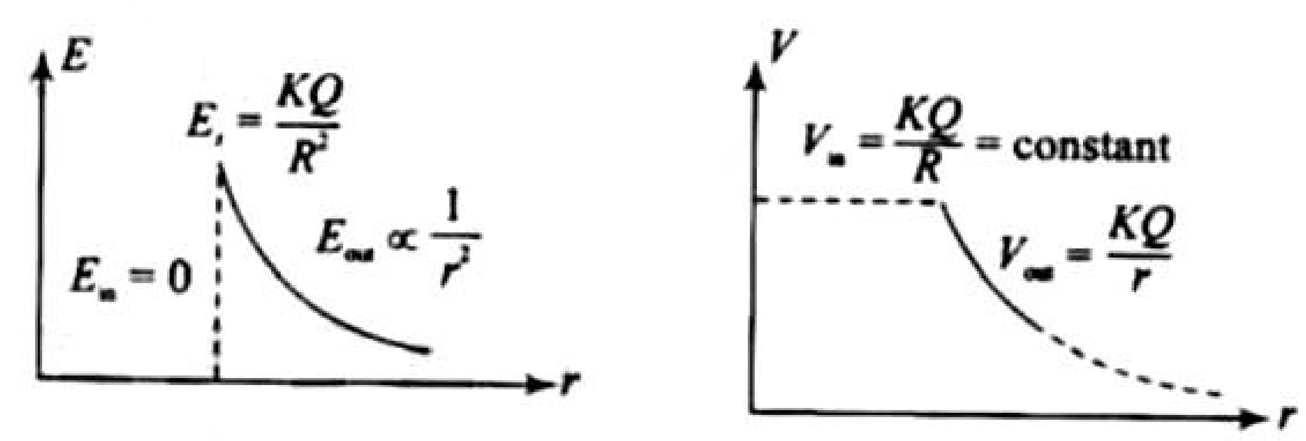
a. This option is wrong, because flux due a charge through a Gaussian surface (in which the charge is present) will depend upon the shape and size of the surface.

b. It is almost impossible to calculate the field distribution around an electric dipole using Gauss's law because proper symmetry will not be formed.

c. This is obvious.

d. This option is not correct because it is not given that charge is moving slowly.

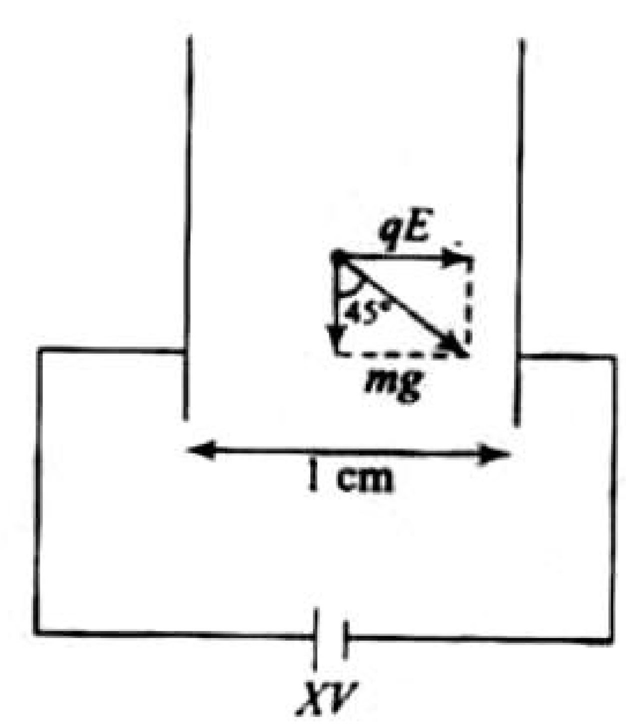
40. d.



41. c. $\quad mg = qE$

$$1.67 \times 10^{-27} \times 10 = 1.6 \times 10^{-19} \times \frac{X}{0.01}$$

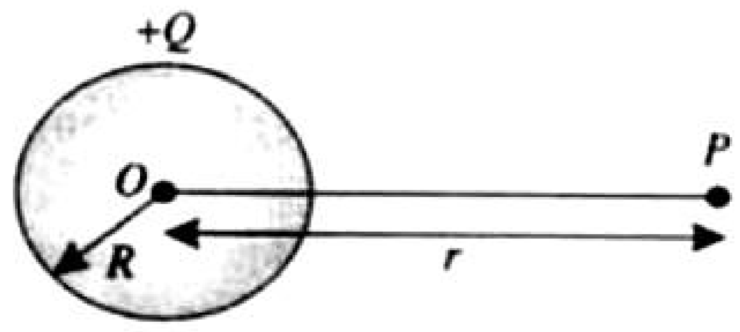
or $\quad X = \frac{1.67}{1.6} \times 10^{-9}\text{ V} = 1 \times 10^{-9}\text{ V}$



42. c. $q_3 = \frac{C_3}{C_2 + C_3}Q$

$$q_3 = \frac{3}{3 + 2} \times 80 = \frac{3}{5} \times 80 = 48\ \mu\text{C}$$

43. c. Electric field due to uniformly charged dielectric solid sphere of radius R at a point P is given by $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^3}r,\ r\le R$



$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2},\ r>R$$

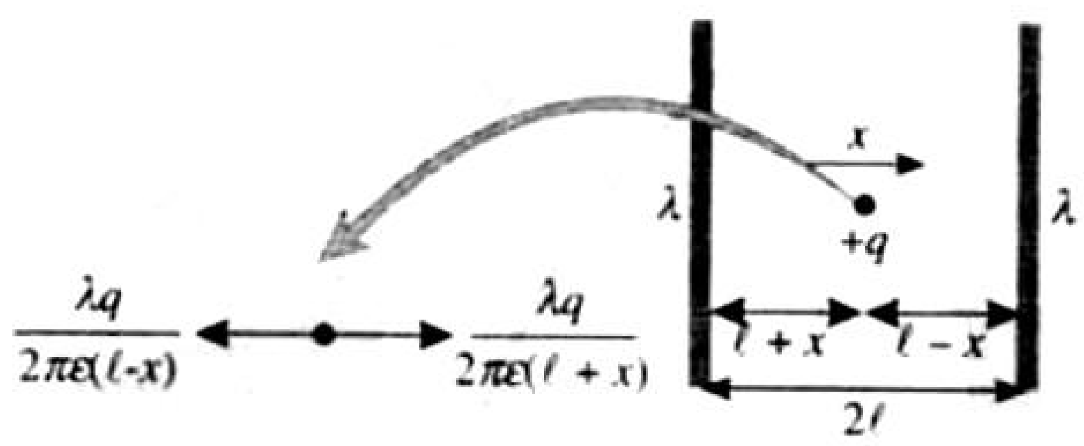
For solid sphere 1, $E_1=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

For solid sphere 2, $E_2=\frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}=2E_1$

For solid sphere 3, $E_3=\frac{1}{4\pi\varepsilon_0}\frac{(4Q)R}{(2R)^3}=\frac{\frac{1}{4\pi\varepsilon_0}}{2R^2}=\frac{E_1}{2}$

$\therefore\quad E_1 : E_2 : E_3$

44. c. For positive charge: If the charge displaced towards right by a distance x



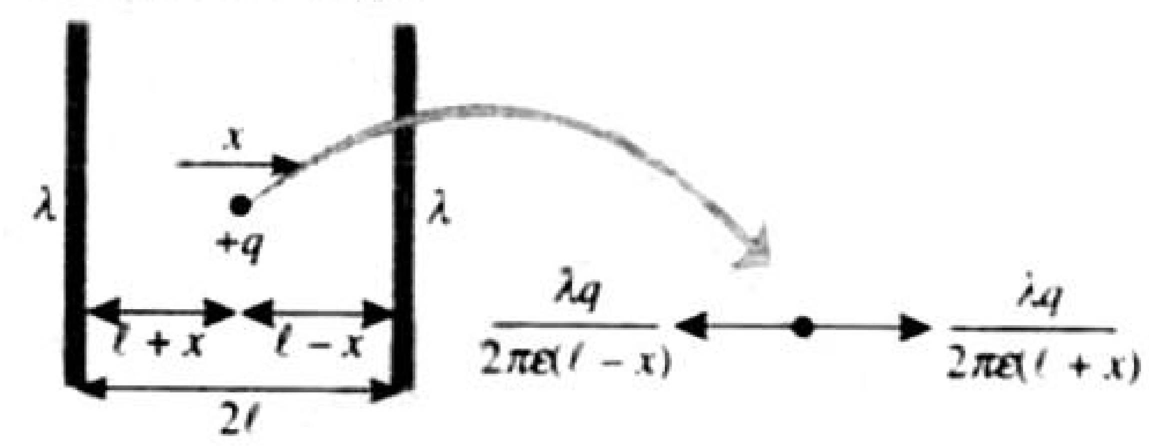
Hence restoring force on the charge

$$F_{rest}=\frac{\lambda q}{2\pi\epsilon_0(l-x)}-\frac{\lambda q}{2\pi\epsilon_0(l+x)}$$
$$=\frac{\lambda q(2x)}{2\pi\epsilon_0(l^2-x^2)}$$

$\Rightarrow\quad F_{rest}=\frac{\lambda q x}{\pi\epsilon_0 l^2}\Rightarrow F_{rest}\propto x$

$\Rightarrow$ It executes SHM.

For negative charge:

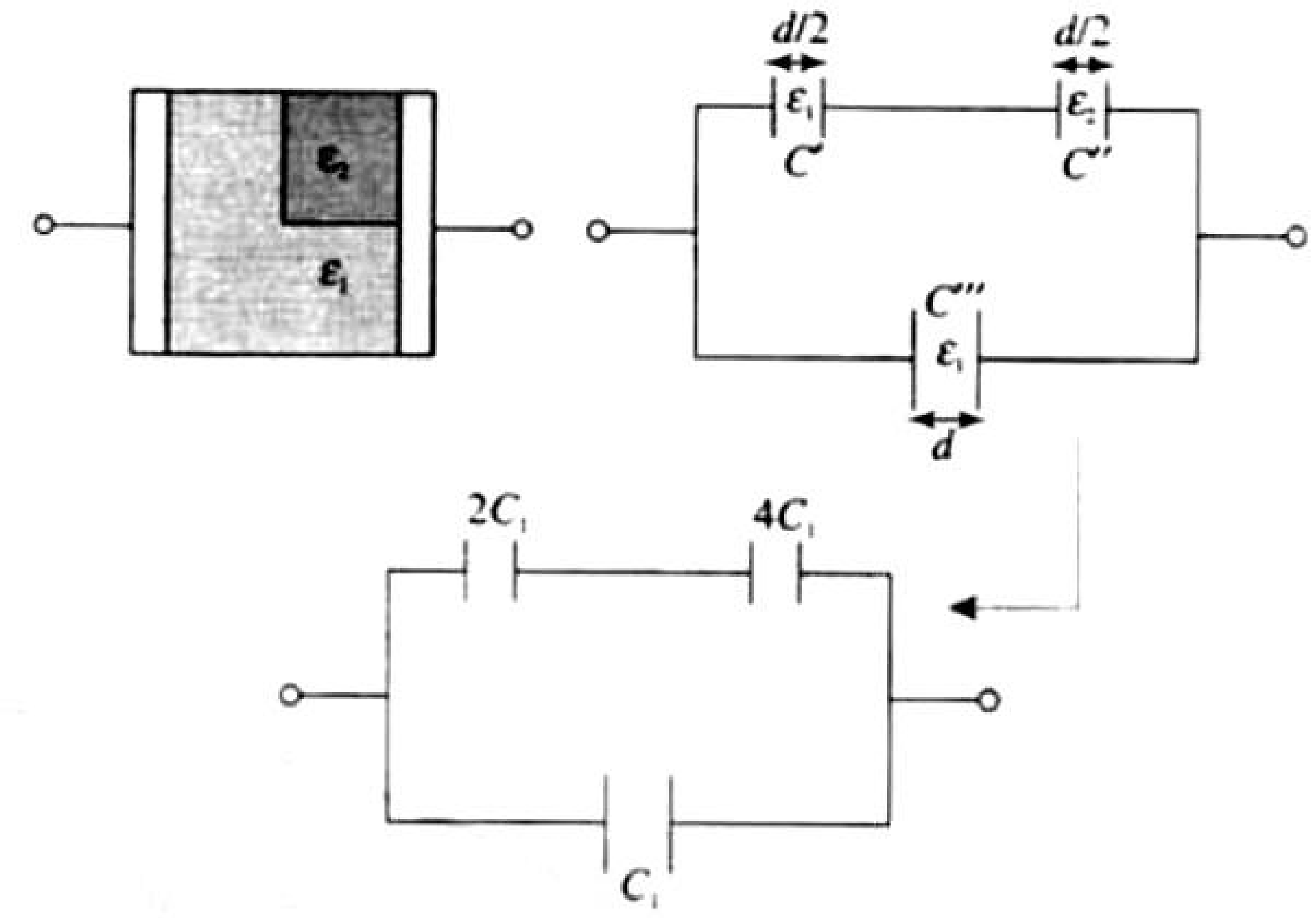


If we displace the charge towards right. The force in the direction of displacement increases. Hence the net force on the charge particle will be towards right hence the particle keeps going towards right.

45. d. A capacitance of the capacitor without dielectric.

$$C_1=\frac{\varepsilon_0 A}{d}$$

Now consider the case with filled dielectric. The given capacitor can be divided into three capacitors as shown.



$$C'=\frac{2\varepsilon_0\frac{S}{2}}{d/2}=\frac{2\varepsilon_0 S}{d}=2C_1$$

$$C''=\frac{4\varepsilon_0\frac{S}{2}}{d/2}=\frac{4\varepsilon_0 S}{d}=4C_1$$

$$C'''=\frac{2\varepsilon_0\frac{S}{2}}{d}=\frac{\varepsilon_0 S}{d}=C_1$$

$$C_2=\frac{C'C''}{C'+C''}+C'''=\frac{4}{3}\frac{\varepsilon_0 S}{d}+\frac{\varepsilon_0 S}{d}=\frac{7}{3}\frac{\varepsilon_0 S}{d}$$

This gives : $\frac{C_2}{C_1}=\frac{7}{3}$

46. d. Since the sphere carries a uniform charge density it will be a non-conducting sphere. Hence, no concept of conducting sphere/cavity should be used.

For a spherical cavity, in a uniformly charged sphere, we use the standard result that:

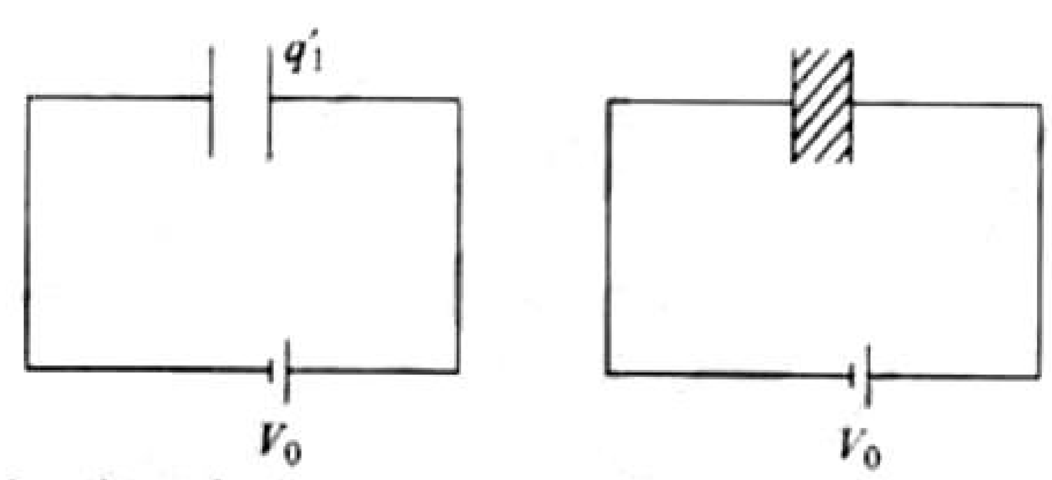
$$\vec{E}=\frac{\rho\vec{a}}{3\epsilon_0}$$

Where $\vec{a}$ = position vector of centre of cavity WRT centre of sphere.

Hence (d).

Multiple Correct Answer Type

1. a., d.



Before introducing dielectric slab:

Potential difference = V_0

Capacitance = C

Charge, $q_0=CV_0$

$$\text{PE, } U_0=\frac{1}{2}CV_0^2$$

After introduction of dielectric slab:

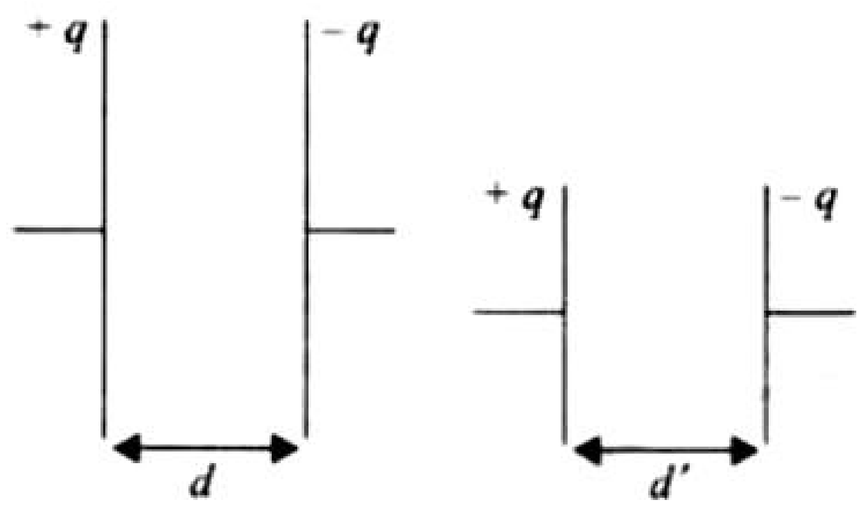
Potential difference = V_0

Capacitance = KC [K is the dielectric constant of slab; $K > 1$]

New charge, $Q = KCV_0$

New PE, $U = \frac{1}{2}KCV_0^2$

2. b., d.



From figure, charge on plate is q

$$C = \frac{\varepsilon_0 A}{d};\quad C' = \frac{\varepsilon_0 A}{d'} \text{ or } C' < C$$

$$q = CV \text{ or } V = \frac{q}{C};\quad V' = \frac{q}{C'} \text{ or } V' > V$$

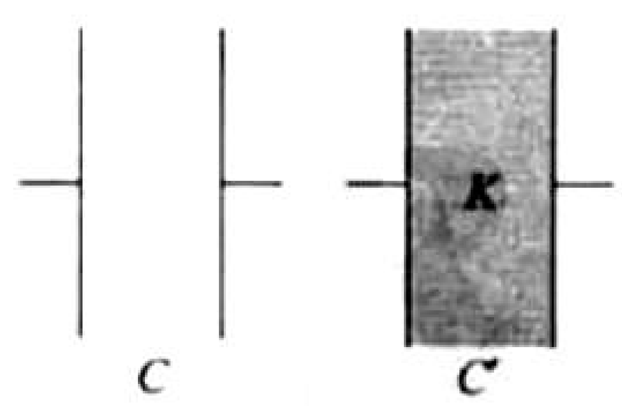
$$U = \frac{1}{2}q \times V;\quad U' = \frac{1}{2}qV' \text{ or } U' > U$$

3. a., c., d.

$$C = \frac{\varepsilon_0}{d}A,\quad C' = \frac{K\varepsilon_0 A}{d}$$

$$Q = CV = \frac{\varepsilon_0 A}{d} \times V = \frac{\varepsilon_0 AV}{d}$$

[Q will remain the same as no charge is leaving or entering the plates during the process of slab insertion.]



$$Q = C'V' = C'E'd$$

or $$E' = \frac{Q}{C'd} = \frac{(\varepsilon_0 AV/d)}{(K\varepsilon_0 A/d)} \times d = \frac{V}{Kd}$$

Work done is the change in energy stored, i.e.,

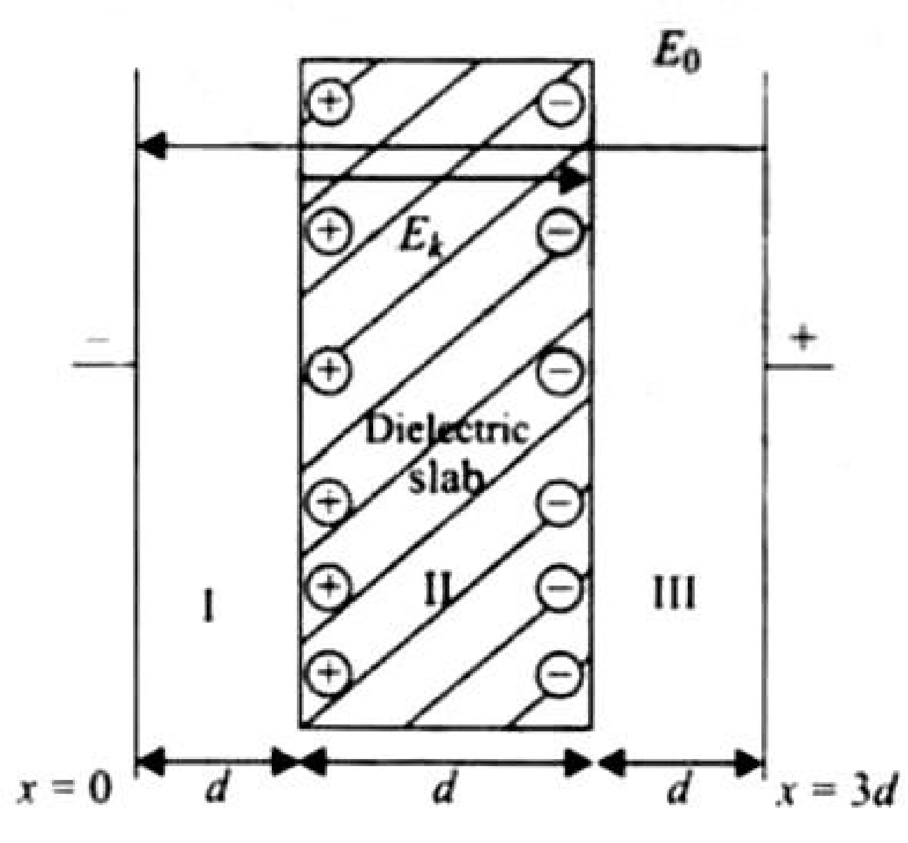
$$W = \frac{1}{2}C'V'^2 - \frac{1}{2}CV^2$$

$$= \frac{1}{2}\frac{K\varepsilon_0 A}{d} \times \frac{V^2}{K^2} - \frac{1}{2}\frac{\varepsilon_0 A}{d} \times V^2 \quad \left[\because V' = E'd = \frac{V}{K}\right]$$

$$= \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2\left[\frac{1}{K} - 1\right]$$

4. b., c.

In regions I and III, there will be electric field $\vec{E}_0$ directed from positive to negative. In region II, due to orientation of dipoles, there is an electric field $\vec{E}_0$ present in the opposite direction of $\vec{E}_0$. But since $\vec{E}_0$ is also present, the net electric field is $E_0 - E_k$ in the direction of $\vec{E}_0$ as shown in the figure ($\because E_0 > E_k$).



Also, please note that when one moves opposite to the direction of electric field, the potential always increases. The stronger the electric field, the more is the increase in potential. Since in region II, the electric field is less as compared to regions I and III, the increase in potential will be less but there has to be an increase in potential in all the regions from $x = 0$ to $x = 3d$.

5. a., c.

Let Q be the charge on the ring; the negative charge $-q$ is released from point $P(0, 0, Z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{QZ_0}{(R^2 + Z_0^2 - 4pt)^{3/2}}$$

Therefore, force on charge P will be toward the center as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(R^2 + Z_0^2)^{3/2}}Z_0 \qquad \text{(i)}$$

Similarly, when it crosses the origin, the force is again toward center O. Thus, the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞. Secondly, if $Z_0 \ll R, (R^2 + Z_0^2)^{3/2} \rightarrow R^3$

$$F_e = \frac{1}{4p\varepsilon_0} \times \frac{Qq}{R^3} \times Z_0 \text{ [from Eq. (i)]}$$

That is, the restoring force $F_e \propto -Z_0$. Hence, the motion of the particle will be simple harmonic. (Here, negative sign implies that the force is toward its mean position.)

6. a., c.

The expressions of the electrical field are

Inside the sphere $(r < R)$, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$

Outside the sphere $(R < r < \infty)$, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

Hence, E increases for $r < R$ and decreases for $R < r < \infty$.

7. c., d.

When two points are connected with a conducting path in electrostatic condition, then the potential of the two points is equal. Therefore, option (c) is the correct option. Option (d) follows from Gauss's law. Options (a) and (b) are dependent on the curvature, which are different at points A and B.

8. a., b., c., d.

For $\quad r > R_0,\ E = \frac{d\phi}{dr} = \frac{Q}{4\pi\varepsilon_0 r^2}$

Therefore, charge enclosed by concentric spherical surface of r is

$$2R_0 = \varepsilon_0\phi_E 4\pi r^2 = \varepsilon_0\frac{Q}{4\pi\varepsilon_0 r^2}4\pi r^2 = Q$$

For $\quad r < R_0,\ E = -\frac{dV}{dr} = 0$

and for $r > R_0$ $\ E = -\frac{dV}{dr} = 4\pi\varepsilon_0 r^2$ (Here, $V = \phi$)

9. a., d.

A number of electric field lines of forces emerging from Q_1 are larger than terminating at Q_2.

10. a., b., c., d.

From conservation of charge:

$$Q_A + Q_B = 2Q \quad \text{(i)}$$

Potential of both should be the same:

$$\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B} \quad \text{(ii)}$$

From Eqs. (i) and (ii)

$$\frac{Q_A}{Q_B} = \frac{R_A}{R_B} \quad \text{(iii)}$$

Solving Eqs. (i) and (iii), we get

$$Q_A = \frac{2QR_A}{R_A + R_B} \text{ and } Q_B = \frac{2QR_B}{R_A + R_B} \text{ or } Q_A > Q_B$$

$$\frac{\sigma_A}{\sigma_B} = \frac{Q_A / 4\pi R_A^2}{Q_B / 4\pi R_B^2} = \frac{R_B}{R_A} \quad \text{using Eq. (iii)}$$

$$\sigma_B > \sigma_A \quad (\text{as } R_A > R_B)$$

On surface, $E_A = \frac{\sigma_A}{\varepsilon_0}$ and $E_B = \frac{\sigma_B}{\varepsilon_0}$

or $E_A < E_B \quad (\because \sigma_A < \sigma_B)$

11. a., c., d.

Position of all the charges are symmetric about the planes $x = +a/2$ and $x = -a/2$. So net electric flux through them will be same. Similarly, flux through $y = +a/2$ is equal to flux through $y = -a/2$.

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{3q - q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

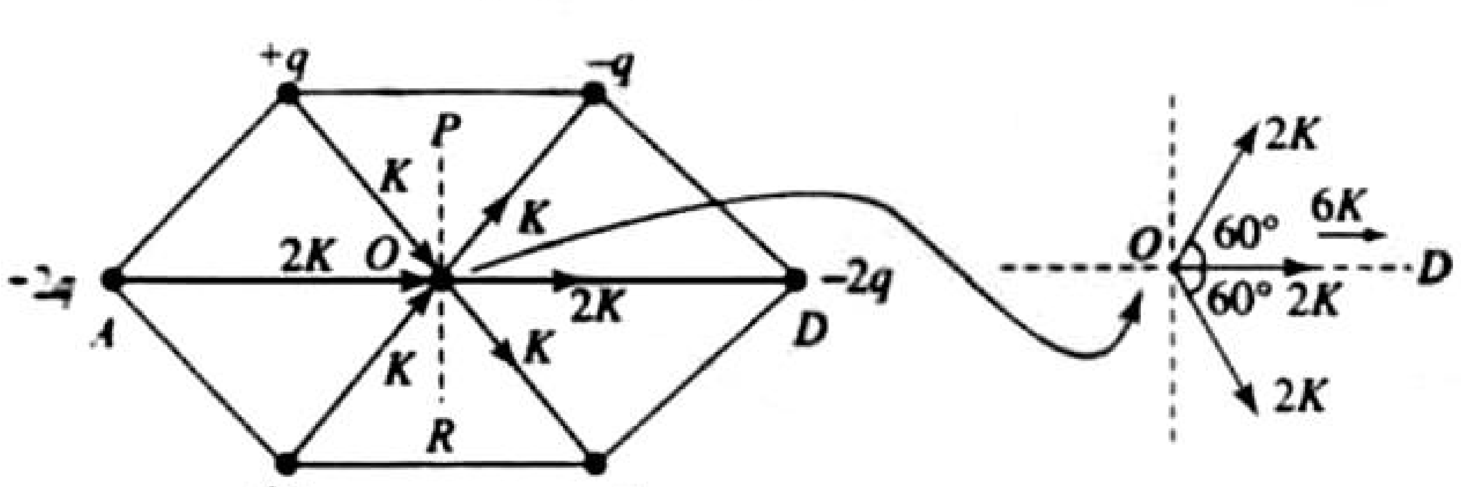
By symmetry flux through $z = +q/2$ is equal to flux through $x = +q/2$.

12. a., b., c.

Electric field at 0

$$E_0 = 2K + 2 \times 2K \cos 60°$$
$$= 2K + 4K$$

Hence, $E_0 = 6K$ (along OD)



$V_0 = 0$

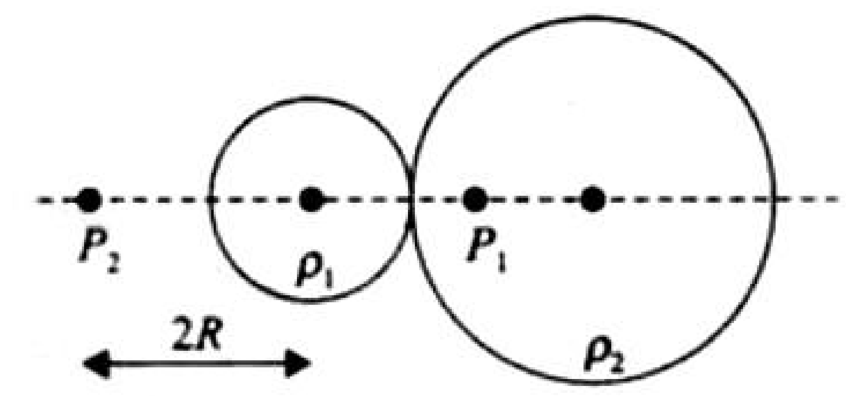
Potential on line PR is zero.

13. b, d.

After switch S_1 is closed, C_1 is charged by $2CV_0$; when switch S_2 is closed, C_1 and C_2 both have upper plate charge CV_0. When S_3 is closed, then upper plate of C_2 becomes charged by $-CV_0$ and lower plate by $+CV_0$.

14. b., d.

At point $P_1 = \frac{1}{4\pi\varepsilon_0}\frac{\rho_1(4/3)\pi R^3}{4R^2} = \frac{\rho_2 R}{3\varepsilon_0}$



$$\frac{\rho_1 R}{12} = \frac{\rho_2 R}{3} \text{ or } \frac{\rho_1}{\rho_2} = 4$$

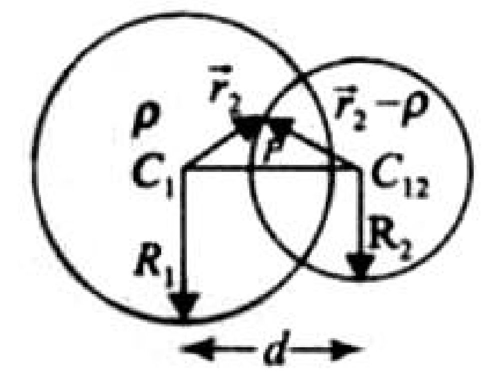
At point P_2

$$\frac{\rho_1(4/3)\pi R^3}{(2R)^2} + \frac{\rho_2(4/3)\pi 8R^3}{(5R)^2} = 0$$

or $\quad \frac{\rho_1}{\rho_2} = -\frac{32}{25}$

15. c., d.

In triangle PC_1C_2, $\vec{r_2} = \vec{d} + \vec{r_1}$



The electrostatic field at point P is

$$\vec{E} = \frac{K\left(\rho\frac{4}{3}\pi R_1^3\right)\vec{r_2}}{R_1^3} + \frac{K\left(\rho\frac{4}{3}\pi R_2^3\right)(\vec{r_1})}{R_2^3}$$

$$= K\rho\frac{4}{3}\pi(\vec{r_2} - \vec{r_1}) = \frac{\rho}{3\varepsilon_0}\vec{d}$$

16. a., b., c.

We have $E_1(r_0) = E_2(r_0) = E_3(r_0)$ (given)

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$$

Then $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2} = \frac{\sigma}{2\varepsilon_0}$

or $\quad Q = 2\sigma\pi r_0^2$

Hence, option (a) is correct.

Now, $\frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$ or $r_0 = \frac{\lambda}{\pi\sigma}$

Hence option (b) is correct.

At $\quad r = \frac{r_0}{2}$

$$E_1\left(\frac{r_0}{2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(r_0/2)^2} = \frac{4}{4\pi\varepsilon_0}\frac{Q}{r_0^2} = 4E_1(r_0)$$

or $\quad E_1(r_0) = \frac{1}{4}E_1\left(\frac{r_0}{2}\right)$

$$E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{2\pi\varepsilon_0(r_0/2)} = \frac{2\lambda}{2\pi\varepsilon_0 r_0} = 2E_2(r_0)$$

or $\quad E_2(r_0) = \frac{1}{2}E_2\left(\frac{r_0}{2}\right)$

$\because \quad E_1(r_0) = E_2(r_0)$

$\therefore \quad \frac{1}{4}E_1 = \frac{1}{2}E_2\left(\frac{r_0}{2}\right)$

$E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right)$

Hence, option (c) is correct.

$E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\varepsilon_0} = E_3(r_0)$

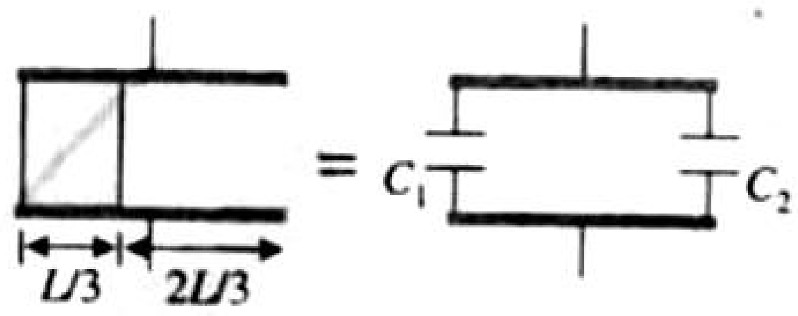
$\because \quad E_2(r_0) = E_3(r_0)$

$\therefore \quad \frac{1}{2}E_2\left(\frac{r_0}{2}\right) = E_3\left(\frac{r_0}{2}\right)$

or $\quad E_2\left(\frac{r_0}{2}\right) = 2E_3\left(\frac{r_0}{2}\right)$

Hence, option (d) is incorrect.

17. a., d.



If $C = \frac{\varepsilon_0 A}{3d}$ then $C_1 = kc \qquad C_2 = 2c$

$C_1 + C_2 = C \Rightarrow (k+2)c = C \Rightarrow c = \frac{C}{k+2}$

$\therefore \quad C_1 = \frac{kc}{k+2}, C_2 = \frac{2c}{k+z} \Rightarrow \frac{C}{C_1} = \frac{2+k}{k}$

If charging voltage is V: charges will be in the ratio of capacities and as potential difference is same. Electric field should be equal.

Linked Comprehension Type

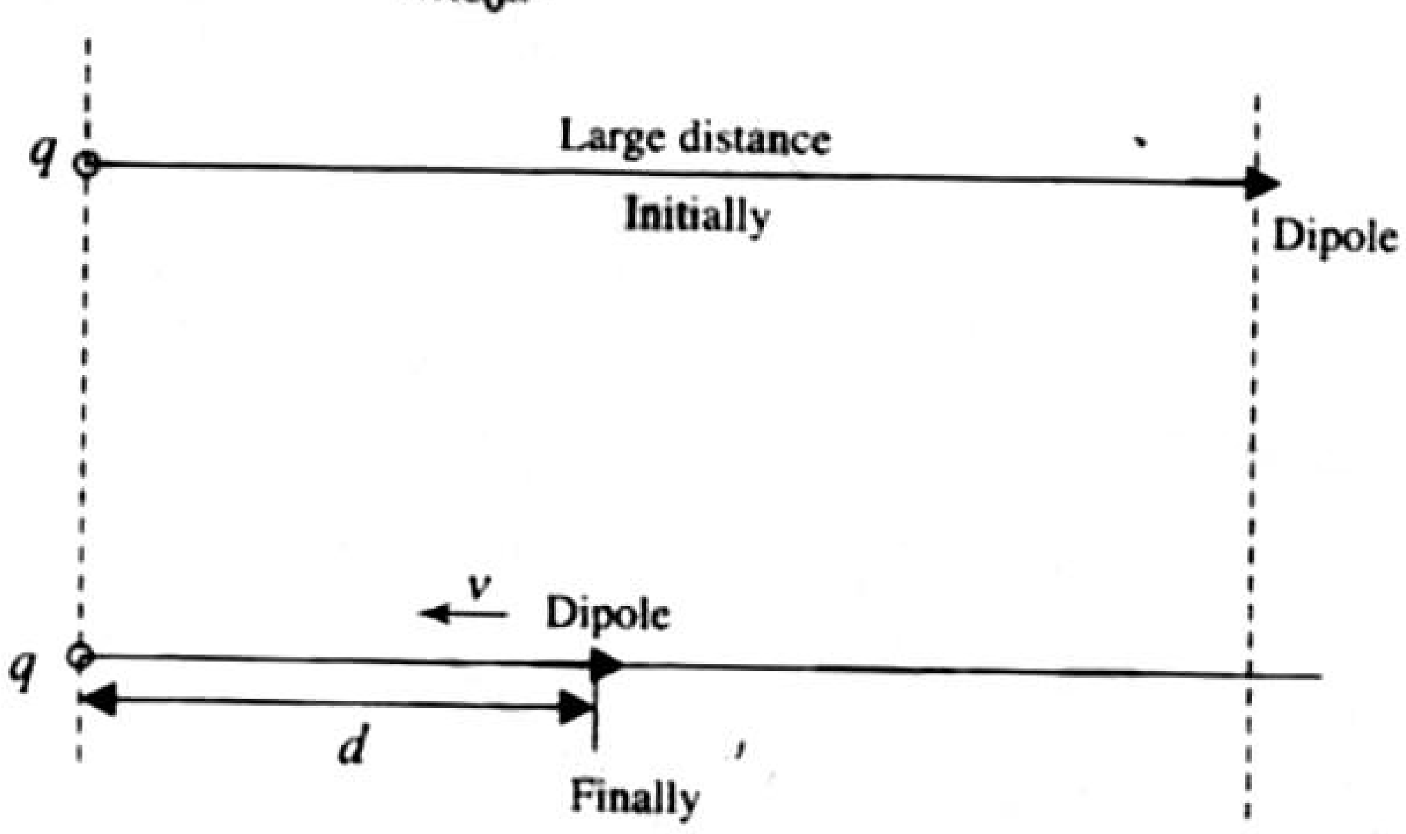
1. d., 2. b.

Conserving energy (see figure)

$(-pE)_{\text{initial}} + (\text{KE})_{\text{initial}} = \text{constant}$

or $\quad 0 + 0 = -p \times \frac{1}{4\pi\varepsilon_0}\frac{q}{q^2} + \text{KE}$

or $\quad \text{KE} = \frac{pq}{4\pi\varepsilon_0 d^2}$



$|F| = \left|\frac{dU}{dx}\right| = \frac{pq}{4\pi\varepsilon_0}\frac{d}{dx}\left(\frac{1}{x^2}\right)$

$= \frac{pq}{4\pi\varepsilon_0 0}(-2x^{-3}) = \frac{pq}{2\pi\varepsilon_0}\frac{1}{x^3}$

Force on dipole $|F| = \frac{1}{2\pi\varepsilon_0}\frac{pq}{d^3}$

3. a., 4. b., 5. c.

Net charge within $r < R$ is constant; hence, electric field is independen of a.

$q = \int_0^R \frac{d}{R}(R - x)4\pi x^2 dx = Ze$

$d = \frac{3Ze}{\pi R^3}$

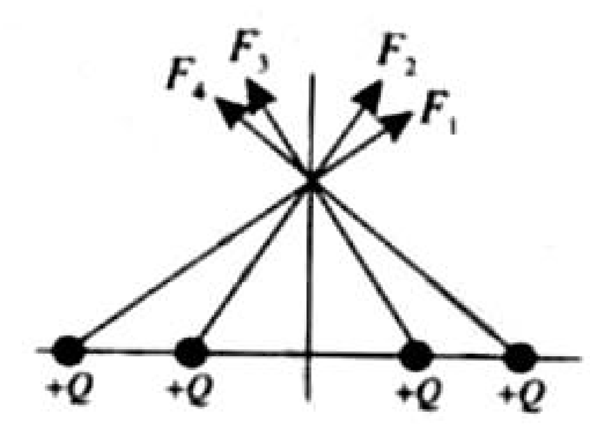
If within a sphere ρ is constant, then $E \propto r$.

Matching Column Type

1. a.

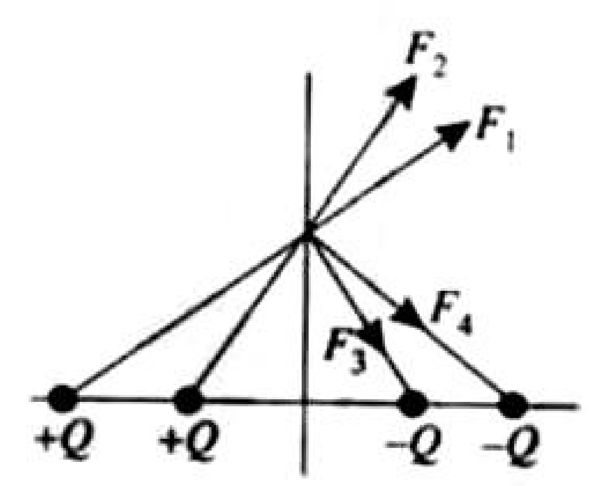
(P) – (3), (Q) – (1), (R) – (4), (S) – (2)

(P) – (3)



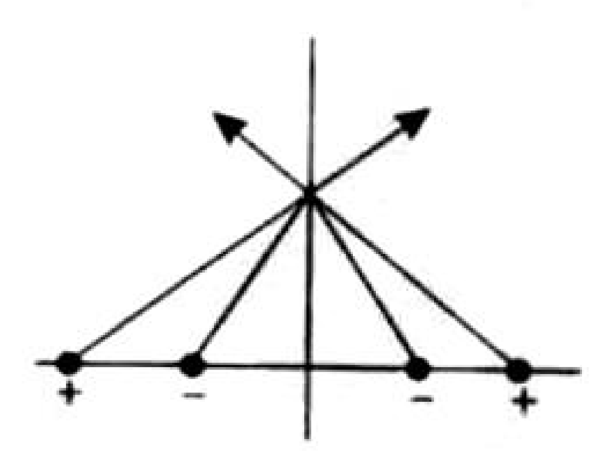
All x components of force will be cancelled only y will exist, i.e., along $+y$ direction.

(Q) – (1)



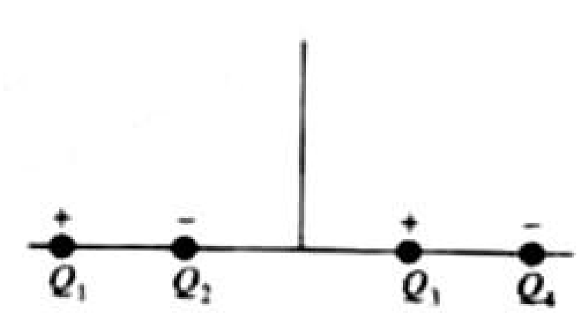
All y components of force will be cancelled and only +ve x will exist.

(R) – (4)



Each pair will cancel x of each other and y of +ve pair will be smaller than –ve so net will be along –ve y.

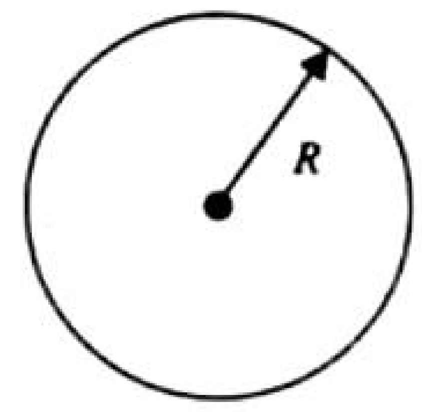
(S) – (2)



Each dipole pair will cancel y of each other and x of $Q_2 - Q_3$ pair along –ve x is stronger than that of $Q_1 - Q_4$ along +ve x.

Integer Answer Type

1. (2) $\rho = kr^a$



$$E\left(r-\frac{R}{2}\right)=\frac{1}{8}E(r-R)$$

$$\frac{q_{enclosed}}{4\pi\varepsilon_0(R/2)^2}=\frac{1}{8}\frac{Q}{4\pi\varepsilon_0 R^2}$$

or $\quad 32q_{enclosed} = Q$

$\quad q_{enclosed} = Q$

or $\quad q_{enclosed} = \int_0^{R/2} kr^a\, 4\pi r^2 dr = \dfrac{4\pi k}{(a+3)}\left(\dfrac{R}{2}\right)^{(a+3)}$

or $\quad Q = \dfrac{4\pi k}{(a+3)}R^{(a+3)}$

or $\quad \dfrac{Q}{q_{enclosed}} = 2^{(a+3)}$

or $\quad 2^{a+3} = 32$

or $\quad a = 2$

2. (6) $\quad E_1 = \dfrac{\lambda}{2\pi\varepsilon_0 2R} = \dfrac{\rho\pi R^2}{4\pi\varepsilon_0 R} = \dfrac{\rho R}{4\varepsilon_0}$

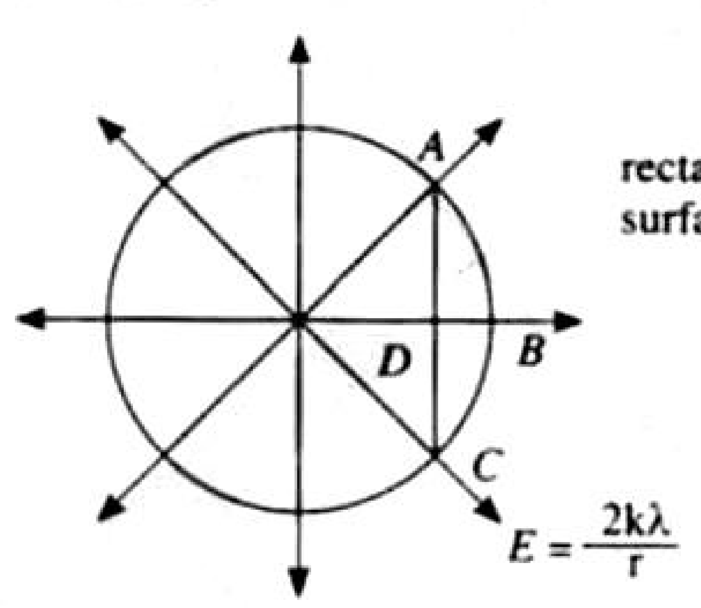
$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\rho\cdot\frac{4}{3}\pi\cdot\frac{R^3}{8}}{(2R)^2}$$

$$E_1 - E_2 = \frac{\rho R}{4\varepsilon_0} - \frac{\rho R}{\varepsilon_0\cdot 24\times 4} = \frac{\rho R}{4\varepsilon_0}\left[1-\frac{1}{24}\right]$$

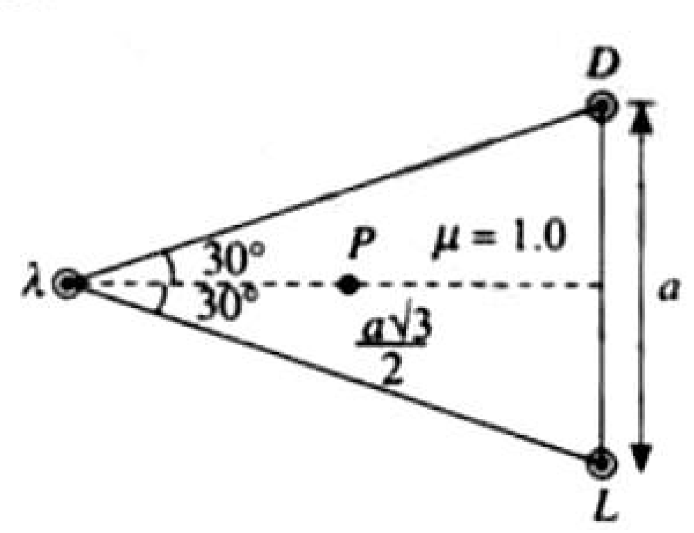
$$= \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{16k\varepsilon_0} \qquad \therefore k = 6$$

3. (6) Suppose that an infinite, charged wire is placed parallel to a rectangular area. Then, number of field lines passing through the rectangular area (denoted by ADC in figure) is equal to number of field lines passing through portion ABC of cylindrical surface.

Hence flux through both of them will be the same.



Let us redraw the figure from the point of view of an observer sitting y-axis.



Consider a cylindrical surface of length L, centred on the charged wire, having a radius $r = \sqrt{\left(\dfrac{a}{2}\right)^2 + \left(\dfrac{a\sqrt{3}}{2}\right)^2}$.

From the figure, it is clear that the flux passing through the rectangular area is equal to $\dfrac{1}{6}$ of the flux passing through this cylindrical surface.

So, $\quad \phi = \dfrac{1}{6}\cdot\dfrac{Q_{enc}}{\varepsilon_0}$

$\Rightarrow \quad = \dfrac{1}{6}\cdot\dfrac{\lambda L}{\varepsilon_0}$

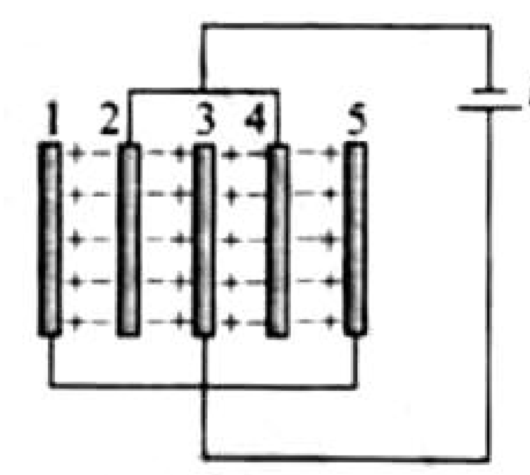
$\therefore \quad n = 6$

Assertion-Reasoning Type

1. b. Both are basic facts and have no relation with each other

Fill in the Blanks Type

1. For plate 1 (see figure):



$$q = CV = \frac{\varepsilon_0 A}{d}\times V$$

For plate 2:

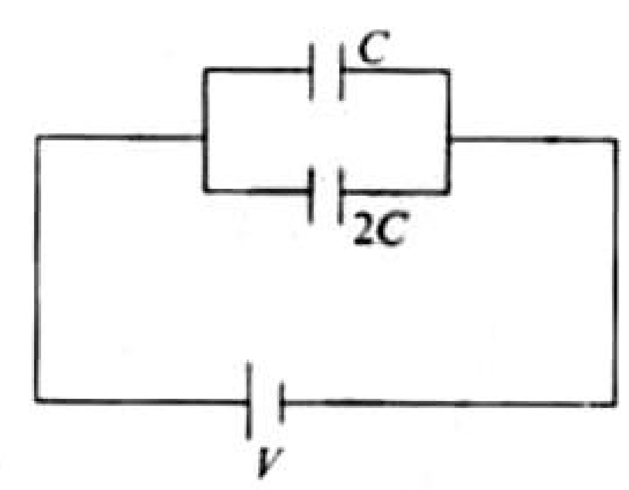
$$2q = \frac{2\,\varepsilon_0 A}{d}\times V$$

2. It is greatest at point B since at B, the equipotential surfaces are closest.

3. Where there is no gravitational force, then in this case only electrostatic force of repulsion is acting which will take the two balls as far as possible. The angle between the two strings will be 180°. The tension in the string will be equal to the electrostatic force of repulsion, i.e.,

$$T = \frac{1}{4\pi\varepsilon_0}\times\frac{Q\times Q}{(2L)^2} = \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{4L^2}$$

4. Initially, charge on capacitance C is $q_1 = CV$.



Charge on capacitance $2C$ is $q_2 = 2CV$.

Finally, charge on capacitance C is $q_1' = KCV'$.

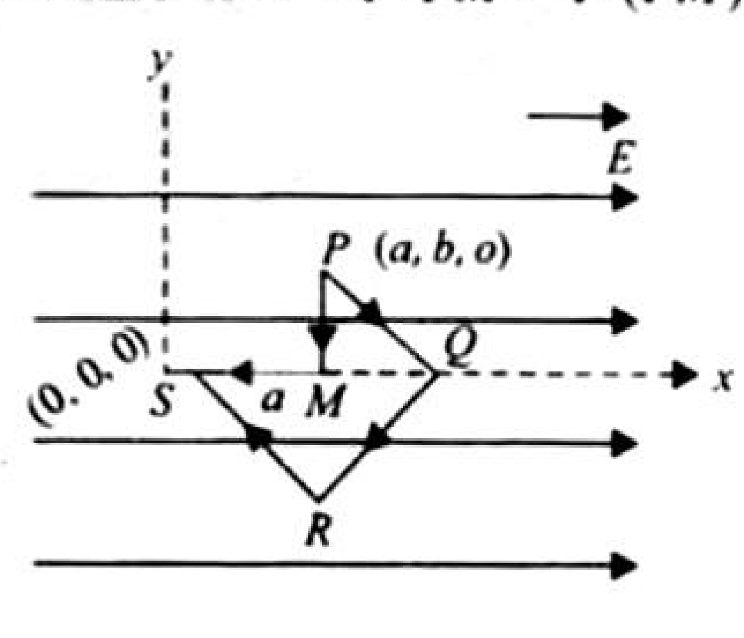
Charge on capacitance $2C$ is $q_2' = 2CV'$.

Since charge will not change, we get

$$CV + 2CV = KCV' + 2CV' \text{ or } V' = 3V/(k+2) - 4pt$$

5. Since electric field is conservative in nature, the work done by the field along $PQRS$ will be the same as along P to M to S (see figure).

Work done from P to M $= \vec{F}\cdot\overline{PM} = F\,(PM)\cos 90° = 0$



Work done from M to $S = \vec{F}\cdot\overline{MS} = F(MS)\cos 180° = -qEa$

6. $V = 4x^2$ V

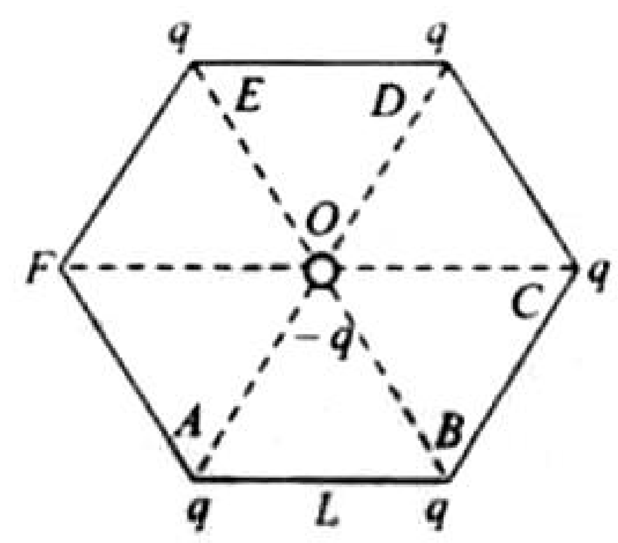
The electric potential changes only along x-axis. We know that

$$E_x = \frac{-dV}{dx} = -\frac{d}{dx}(4x^2) = -8x$$

The electric field at point (1, 0, 2) will be (here $x = 1$)

$$E_x = -8 \text{ V}$$

7. If we place a charge q at the sixth vertex of the regular hexagon, then the net force on the charge $-q$ placed at the center of hexagon will be zero due to symmetry.



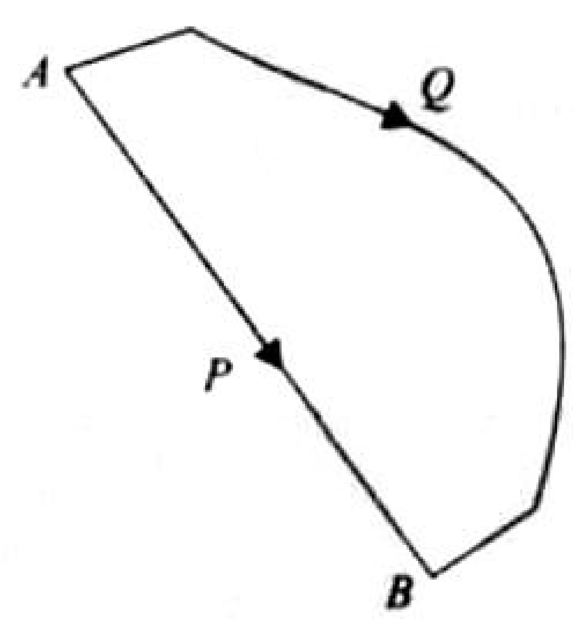
The force on charge $-q$ due to the charge q placed on the sixth vertex balances the net force on charge $-q$ due to the other five charges placed at the five vertices. The force on charge $-q$ due to charge q placed at the sixth vertex will be

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{L^2}$$

where l is the distance of the center of hexagon from any vertex (directed from O to C). The magnitude of force on the point charge of value $-q$ coulomb placed at the center of the hexagon is $(1/4\pi\varepsilon_0)(q^2/L^2)$ directed from O to F.

True/False Type

1. Let us consider two points A and B in an electric field. Let the potentials at A and B be V_A and V_B, respectively. Now, by the definition of potential difference, the potential difference between two points B and A is the amount of work done in carrying a unit positive charge from A to B.



Mathematically, for path $A \to P \to B$,

$$\frac{W_{APB}}{q} = V_B - V_A$$

$$W_{APB} = q(V_B - V_A) \qquad \text{(i)}$$

For path $A \to Q \to B$,

$$\frac{W_{AQB}}{q} = V_B - V_A$$

or $\quad W_{AQB} = q(V_B - V_A) \qquad$ (ii)

Since the RHS of Eqs. (i) and (ii) is the same,

$$W_{APB} = W_{AQB} \qquad \text{(iii)}$$

2. The statement is true. The metallic sphere that gets negatively charged gains electrons and hence its mass increases. The metallic sphere which gets positively charged loses electrons and hence its mass decreases.

3. When a high-energy X-ray beam falls, it will knock out electrons from the small metal ball making it positively charged. Therefore, the ball will be deflected in the direction of electric field.

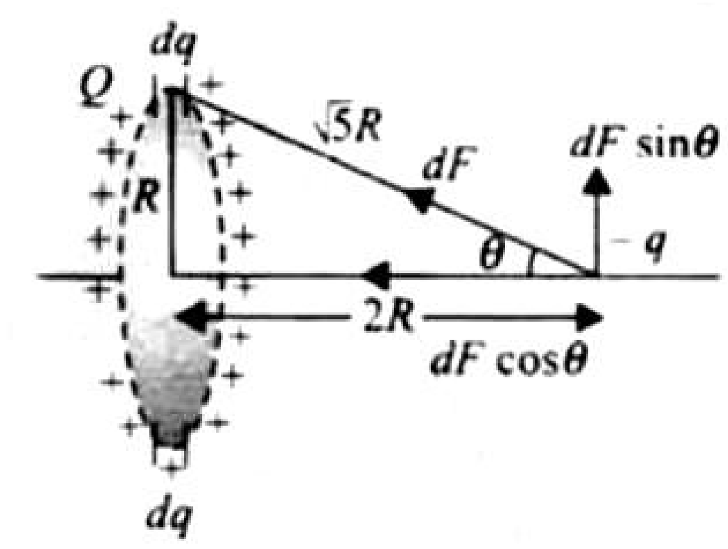
4. The electric field produced between the parallel plate capacitor is uniform. The force acing on a charged particle placed in an electric field is given by $F = qE$. In the case of two protons, q and E are equal and hence force will be equal. The statement is true.

5. Force on charge $-q$ due to small charge dq situated at length dl is

$$dF = k\frac{qdq}{5R^2}$$

Resolving this force into two parts $dF\cos\theta$ and $dF\sin\theta$ as shown in the figure.

If we take another diametrically opposite length dl, the charge on it being dq, then the force on charge $-q$ by this small charge dq will be



$$dF = k\frac{qdq}{5R^2}$$

Again, resolving this force, we find $dF\sin\theta$ cancels out with $dF\sin\theta$ of the previous force and $dF\cos\theta$ components add up.

$$F = \int_0^{2\pi R} dF\cos\theta = \int_0^{2\pi R}\frac{kqdq}{5R^2}\times\frac{2R}{\sqrt{5}R}$$

Charge on length $2\pi R = Q$

Charge on length $dl = \frac{Qdl}{2\pi R} = dq$

$$F = \int_0^{2\pi R}\frac{2kq}{5\sqrt{5}R^2}\times\frac{Qdl}{2\pi R}$$

$$= \frac{2kQq}{5\sqrt{5}\times 2\pi R^3}\times 2\pi R = \frac{2kQq}{5\sqrt{5}R^2}$$

This is not an equation of simple harmonic motion. Therefore, the statement is false.

Subjective Type

1. a. We are given 80×10^{-3} g $= 80 \times 10^{-6}$ kg, $q = 2 - 10^{-8}$ C; $E = 20{,}000$ V/m

For equilibrium of bob.

$T\sin\theta = mg \qquad$ (i)

$T\cos\theta = mg \qquad$ (ii)

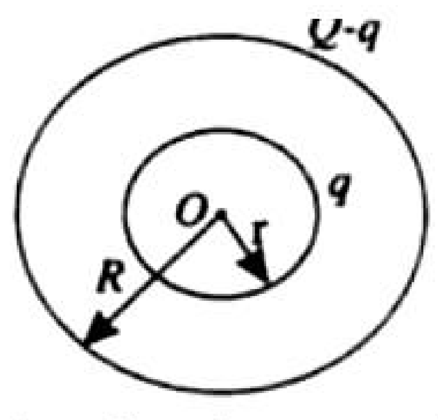
$$\tan\theta = \frac{qE}{mg} \Rightarrow E = \frac{2\times 10^{-8}\times 2\times 10^4}{80\times 10^{-6}\times 10} = 0.5$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Squaring and adding equation (i) and (ii)

$\Rightarrow \quad T = 9 \times 10^{-4}$ N

2. No. When a positively charged particle starts from rest in a uniform electric field, it will move along the direction of electric field. But if the electric field is variable the charged particle will move under the influence of instantaneous velocity and the force of electric field.

3. 

Let the charge on the inner surface on outer sphere. Q and $(Q-q)$

As surface charge densities are equal.

$$\frac{q}{4\pi r^2} = \frac{Q-q}{4\pi R^2}$$

$$qR^2 = (Q-q)r^2$$

$$qR^2 = Qr^2 - qr^2$$

$$q = \frac{Qr^2}{R^2+r^2}$$

Potential at common center O due to inner sphere

$$V_{\text{inner}} = k\frac{q}{r} = \frac{k}{r}\left(\frac{Qr^2}{R^2+r^2}\right)$$

$$V_{\text{inner}} = k\left(\frac{Qr}{R^2-r^2}\right) \quad \text{(i)}$$

Potential at O due to outer sphere

$$V_0 = k\frac{(Q-q)}{R} = \frac{k}{R}\left[Q - \frac{Qr^2}{R^2+r^2}\right]$$

$$= \frac{k}{R}\frac{[QR^2+Qr^2-Qr^2]}{(R^2+r^2)}$$

$$\Rightarrow \quad V_{\text{outer}} = \frac{kQR}{R^2+r^2}$$

The total potential $V = V_{\text{inner}} + V_{\text{outer}}$

$$= \frac{kQr}{R^2+r^2} + \frac{kQR}{R^2+r^2} = \frac{kQ(R+r)}{R^2+r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{R^2+r^2}$$

4. The electric field intensity due to the charged ring R at a distance x from its centre

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q\vec{x}}{(R^2+x^2)^{3/2}}$$

$\therefore$ The force on the negative charge $(-q_0)$ placed at axis at a distance x from centre

$$\vec{F} = -q_0\vec{E}$$

$\therefore$ Then $\vec{F} = -\frac{1}{4\pi\varepsilon_0}\frac{qq_0x}{(R^2+x^2)^{3/2}}$

As $x << R$, hence $x^2 << R^2$, we can write

$$\vec{F} = -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{R^3}\vec{x} \quad \text{(i)}$$

If $\vec{a}$ be the its acceleration of the charged particle, then

Then $m\vec{a} = -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{R^3}\vec{x}$

$$\vec{a} = -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{mR^3}\vec{x} \quad \text{(ii)}$$

i.e. $\quad \vec{a} \propto -\vec{x}$

It is necessary and sufficient condition for simple harmonic motion. It means motion of negatively charged particle on the axis of a charged ring under the condition $x << R$ is S.H.M.
The standard equation of S.H.M.

$$\vec{a} = -\omega^2\vec{x} \quad \text{(iii)}$$

Comparing (ii) and (iii) we get $\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{mR^3}$

i.e., angular frequency, $\omega = \frac{2\pi}{T} = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{mR^3}\right)}$

or $\quad$ Time period $T = 2\pi\sqrt{\left(\frac{4\pi\varepsilon_0 mR^3}{qq_0}\right)} \quad \text{(iv)}$

Now substituting the given
Mass of particle $m = 0.9\text{g} = 0.9 \times 10^{-3}$ kg, charge on particle, $q_0 = 1 \times 10^{-6}$ C,
Radius of ring, $R = 1$ m
and Charge on ring, $q = 1 \times 10^{-5}$ coulomb
Substituting the values in (iv) we get $T = \frac{\pi}{5}$ sec

5. Initially, when the switch is closed, both the capacitors A and B are in parallel and, therefore, the energy stored in the capacitors is

$$U_i = 2\times\frac{1}{2}CV^2 = CV^2 \quad \text{(i)}$$

When switch S is opened, B gets disconnected from the battery. The capacitor B is now isolated, and the charge on an isolated capacitor remains constant, often referred to as bound charge. On the other hand, A remains connected to the battery. Hence, potential V remains constant on it.
When the capacitors are filled with dielectric, their capacitance increases to KC. Therefore, energy stored in B changes to $Q^2/2KC$, where $Q = CV$ is the charge on B, which remains constant, and energy stored in A changes to $1/2\ KCV^2$, where V is the potential on A, which remains constant. Thus, the final total energy stored in the capacitors is

$$U_f = \frac{1}{2}\frac{(CV)^2}{KC} + \frac{1}{2}KCV^2 = \frac{1}{2}CV^2\left(K+\frac{1}{K}\right) \quad \text{(ii)}$$

From Eqs. (i) and (ii), we find

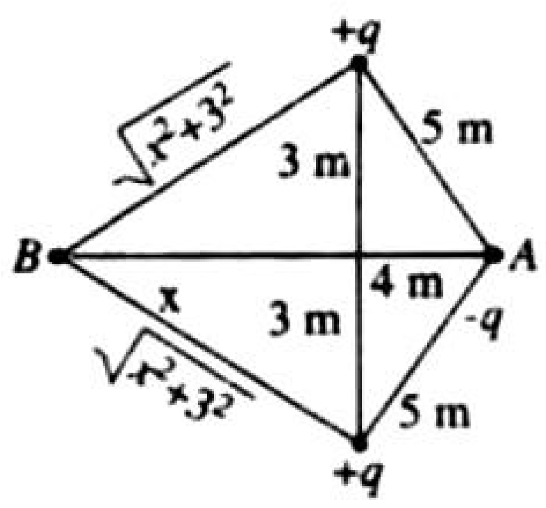
$$\frac{U_i}{U_f} = \frac{2K}{K^2+1}$$

It is given that $K = 3$. Therefore, we have

$$\frac{U_i}{U_f} = \frac{3}{5}$$

6. The electric field of the charged particles at A and B moves the negative charge along the line COD. Total energy of the system of three charges will be same of potential and kinetic energy. When the charge $-q$ is at A, = $P.E. + K.E.$

$$= \left[\frac{Kq\times q}{6} + \frac{K(q)(-q)}{5} + \frac{K(q)(-q)}{5}\right] + 4 \quad \text{(i)}$$



Final energy of the system of three charges when $-q$ is at B and momentarily at rest

$$E_D = \left[\frac{Kq\times q}{6} + \frac{Kq(-q)}{\sqrt{x^2+3^2}} + \frac{Kq(-q)}{\sqrt{x^2+3^2}}\right]$$

$$= \frac{Kq\times q}{6} + \frac{2Kq(-q)}{\sqrt{x^2+3^2}} \qquad \text{(ii)}$$

Using principle of conservation of energy from (i) and (ii)

$$\frac{Kq\times q}{6} + \frac{2Kq(-q)}{5} + 4 = \frac{Kq\times q}{6} + \frac{2Kq(-q)}{\sqrt{x^2+3^2}}$$

$$2 = kq^2\left[\frac{1}{5} - \frac{1}{\sqrt{x^2+3^2}}\right]$$

We are given $q = 5\times 10^{-5}$ C $\quad\therefore\quad x = 8.48$ m

7. For the potential energy to be minimum the bigger charges should be farthest. Therefore the combination would be as shown in the figure.

The potential energy of the system is

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{8q\times 2q}{0.09} + \frac{8q\times q}{x} + \frac{q(2q)}{(0.09-x)}\right]$$

For potential energy to be minimum using $\frac{dU}{dx} = 0$

$$\left[\frac{-8q\times q}{x^2} - \frac{q\times 2q(-1)}{(0.09\times x^2)}\right] = 0$$

$$\frac{4}{x^2} = \frac{1}{(0.09-x)^2}$$

$$2(0.09-x) = x$$

$$2\times 0.09 - 2x = x$$

$$x = \frac{2\times 0.09}{3} = 2\times 0.03 = 0.06 \text{ m}$$

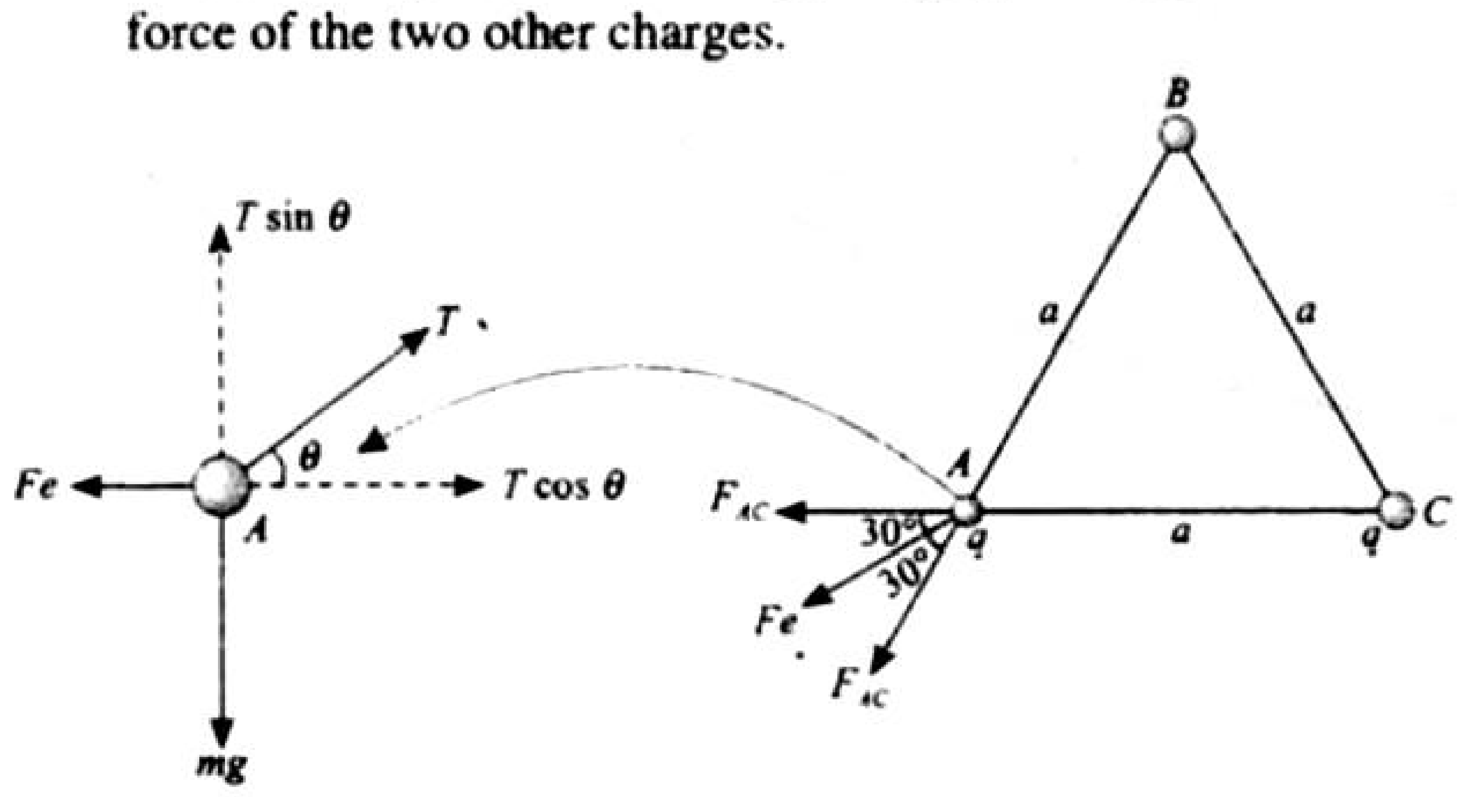
Electric field at q due to the two charges

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E = \frac{k\times 8q}{x^2} - \frac{k(2q)}{(0.09-x)^2}$$

$$= \frac{9\times 10^9\times 8q}{(0.06)^2} - \frac{9\times 10^9\times 2q}{(0.03)^2} = 0$$

8. Each charge will be in equilibrium under the action of three forces namely tension of string, weight, resultant electrostatic force of the two other charges.



F is the resultant of electrostatic forces on A due to charges at B and C. Let θ be the angle of string with horizontal in equilibrium.

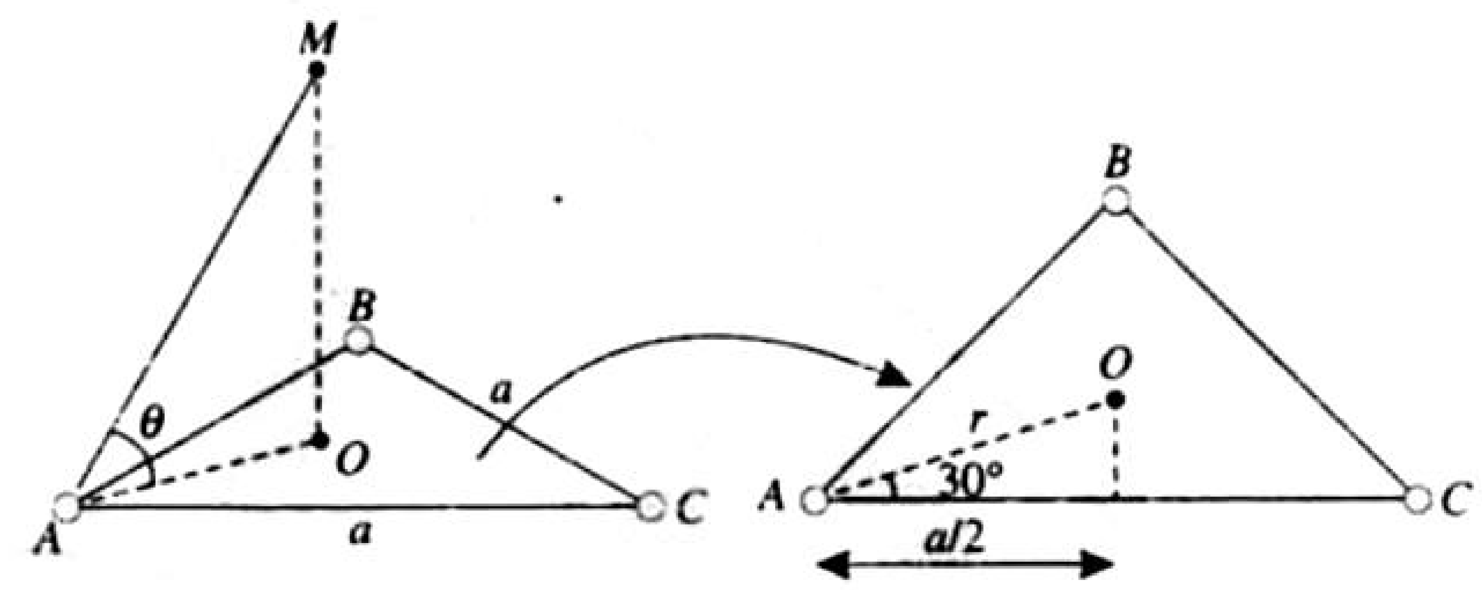
From figure $|\vec{F}_{AC}| = |\vec{F}_{AB}|$

$$\text{Fe} = 2F_{AC}\cos 30°$$

$$= 2\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a^2}\right]\frac{\sqrt{3}}{2}$$

$$= \frac{2\times 10^9\times 9\times q^2\times\sqrt{3}}{(3\times 10^{-2})^2\times 2}$$

$$= \sqrt{3}\times 10^{13}q^2$$



$$\cos\theta = \frac{r}{l} = \frac{(a/2)\sec 30°}{l}$$

$$= \frac{a}{\sqrt{3}l} = \frac{3}{100\sqrt{3}}$$

$\therefore\quad \theta \approx 89°$

Now, the particle is in equilibrium under three concurrent forces, F, T and mg. Therefore, applying Lami's theorem:

$$\frac{F}{\sin(90°+\theta)} = \frac{mg}{\sin(180°-\theta)}$$

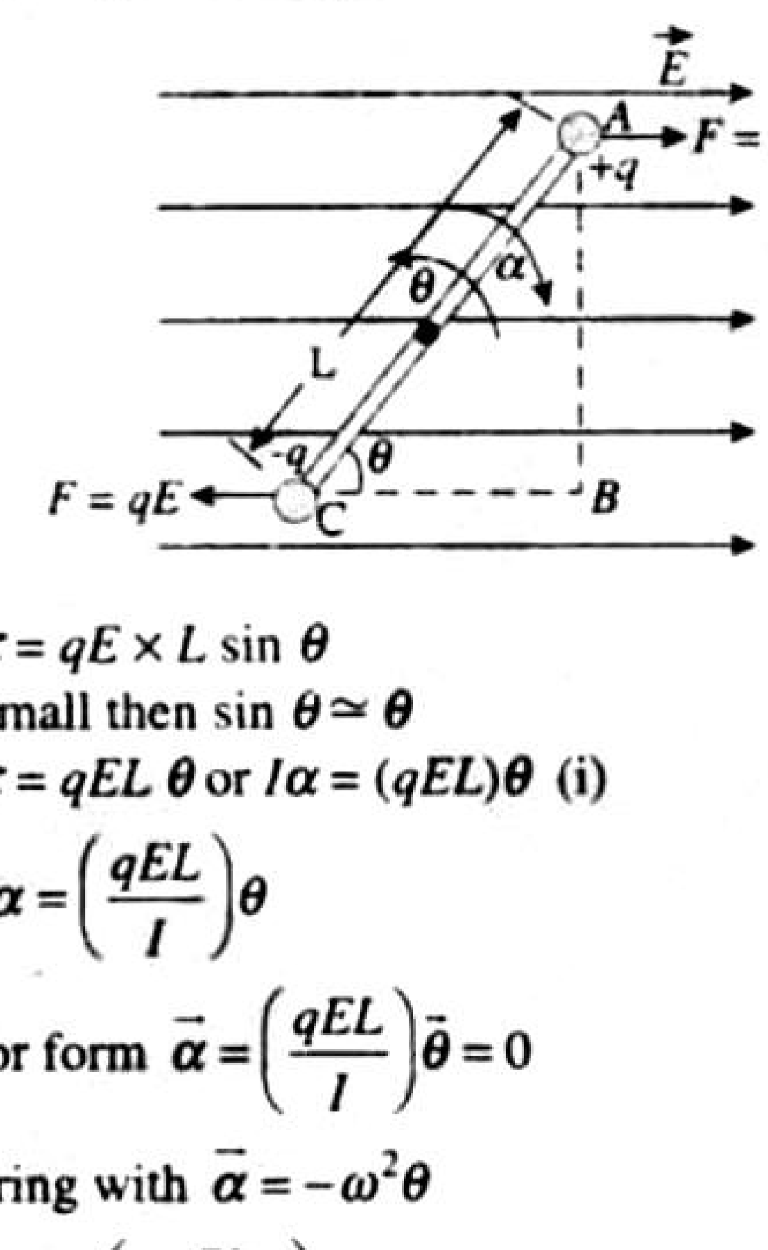
or $\quad \sqrt{3}\times 10^{13}q^2 = (1\times 10^{-3})(10)\cot 89°$

solving this equation, we get

$$q = 0.317\times 10^{-8}\text{ C}$$

or $\quad q = 3.17\times 10^{-9}$ C

9. Torque acting on the dipole



$$\tau = qE\times L\sin\theta$$

If θ is small then $\sin\theta \simeq \theta$

$\therefore\quad \tau = qEL\,\theta$ or $I\alpha = (qEL)\theta$ (i)

or $\quad \alpha = \left(\frac{qEL}{I}\right)\theta$

In vector form $\vec{\alpha} = \left(\frac{qEL}{I}\right)\vec{\theta} = 0$

Comparing with $\vec{\alpha} = -\omega^2\theta \qquad$ (ii)

$$\omega^2 = \left(\frac{qEL}{ML^2/2}\right)$$

Now, $I = \frac{ML^2}{4} + \frac{ML^2}{4} = \frac{ML^2}{2}$

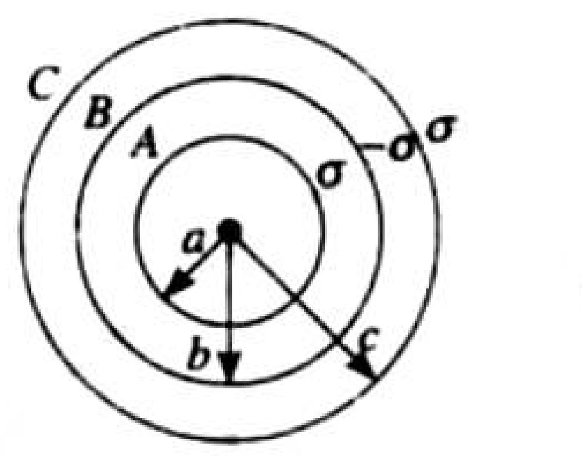
From (i) and (ii): $\frac{ML^2}{2}w^2 = qEL \Rightarrow w = \sqrt{\frac{2qE}{ML}}$

$$\Rightarrow \quad \frac{2\pi}{T} = \sqrt{\frac{2qE}{ML}} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{ML}{2qE}}$$

This is the time period of complete oscillation

$\therefore$ Time for the dipole to align along the direction of electric field will be $\frac{T}{2} = \frac{2\pi}{4}\sqrt{\frac{ML}{2qE}} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$

10. Charge on Shell $A = q_A = \sigma(4\pi a^2)$



Charge on Shell $B = q_B = -\sigma(4\pi b^2)$

Charge on Shell $C = q_C = -\sigma(4\pi c^2)$

The potential of shell A

$$V_A = \frac{kq_A}{a} - \frac{kq_B}{b} + \frac{kq_C}{c}$$

$$= \frac{k\sigma(4\pi a^2)}{a} - \frac{k\sigma(4\pi b^2)}{b} + \frac{k\sigma(4\pi c^2)}{c}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \sigma \times \frac{4\pi a^2}{a} - \frac{1}{4\pi\varepsilon_0}\frac{\sigma(4\pi b^2)}{b} + \frac{1}{4\pi\varepsilon_0}\frac{\sigma(4\pi c^2)}{c}$$

$$= \frac{\sigma}{\varepsilon_0}[a - b + c]$$

Similarly, $V_B = \frac{kq_A}{b} + \frac{kq_B}{b} + \frac{kq_C}{c}$

and $\quad V_C = \frac{kq_A}{c} + \frac{kq_B}{c} + \frac{kq_C}{c}$

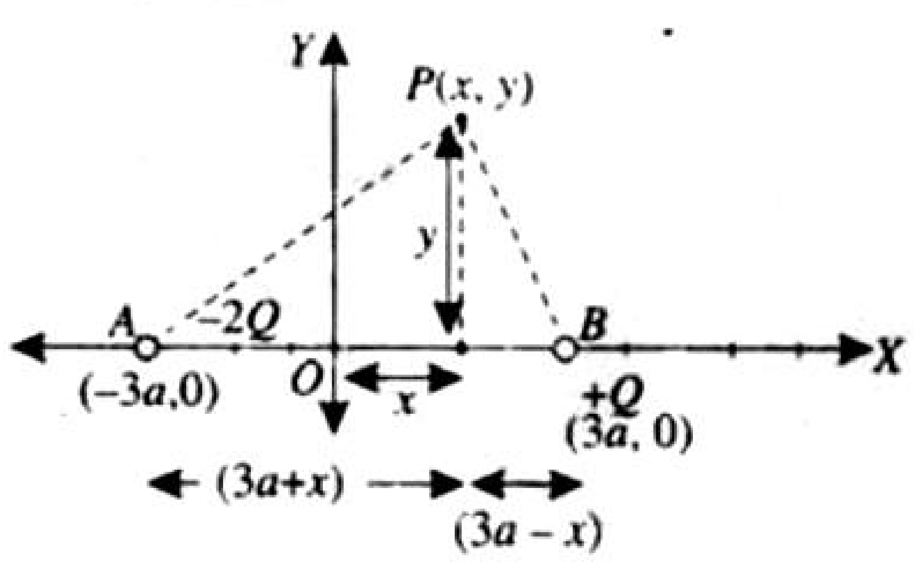
$$V_B = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} + b + c\right] \text{ and } V_C = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$$

Given that $V_A = V_C$

$$\frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$$

$$ac - bc + c^2 = a^2 - b^2 + c^2 \Rightarrow c = a + b$$

11. a. Let P be a point in the X-Y plane with co-ordinates (x, y) at which the potential due to charges $-2Q$ and $+Q$ placed at A and B respectively be zero.



$$\frac{1}{4\pi\varepsilon_0}\frac{(-2Q)}{\sqrt{(3a+x)^2 + y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{(+Q)}{\sqrt{(3a-x)^2 + y^2}}$$

$$2\sqrt{(3a-x)^2 + y^2} = \sqrt{(3a+x)^2 + y^2}$$

$$\Rightarrow \quad 4[(3a - x)^2 + y^2] = [(3a + x)^2 + y^2]$$

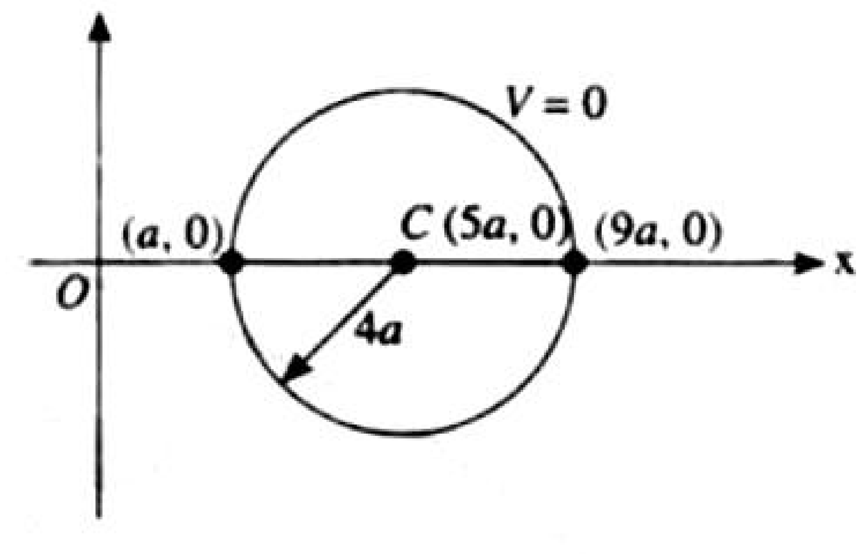
$$4[6a^2 + x^2 - 12ax + y^2] = [6a^2 + x^2 + 12ax + y^2]$$

$$3x^2 + 3y^2 - 30ax + 27a^2 = 0$$

$$x^2 + y^2 - 10ax + 9a^2 = 0$$

$$(x - 5a)^2 + (y - 0)^2 = (4a)^2$$

This is the equation of a circle with centre at $(5a, 0)$ and radius $4a$. Thus $c\ (5a, 0)$ is the centre of the circle.



b. For $x > 3a$.

To find $V(x)$ at any point X-axis let us consider a point (arbitrary) M at a distance x from the origin.

The potential at M will be

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{(-2Q)}{x + 3a} + \frac{1}{4\pi\varepsilon_0}\frac{(+Q)}{(x - 3a)}$$

$$V(x) = \frac{1}{4\pi\varepsilon_0}Q\left[\frac{1}{x - 3a} - \frac{2}{x + 3a}\right] \text{For } |x| > 3a$$

Similarly, for $0 < |x| < 3a$

$$V(x) = \frac{1}{4\pi\varepsilon_0}Q\left[\frac{1}{3a - x} - \frac{2}{3a + x}\right]$$

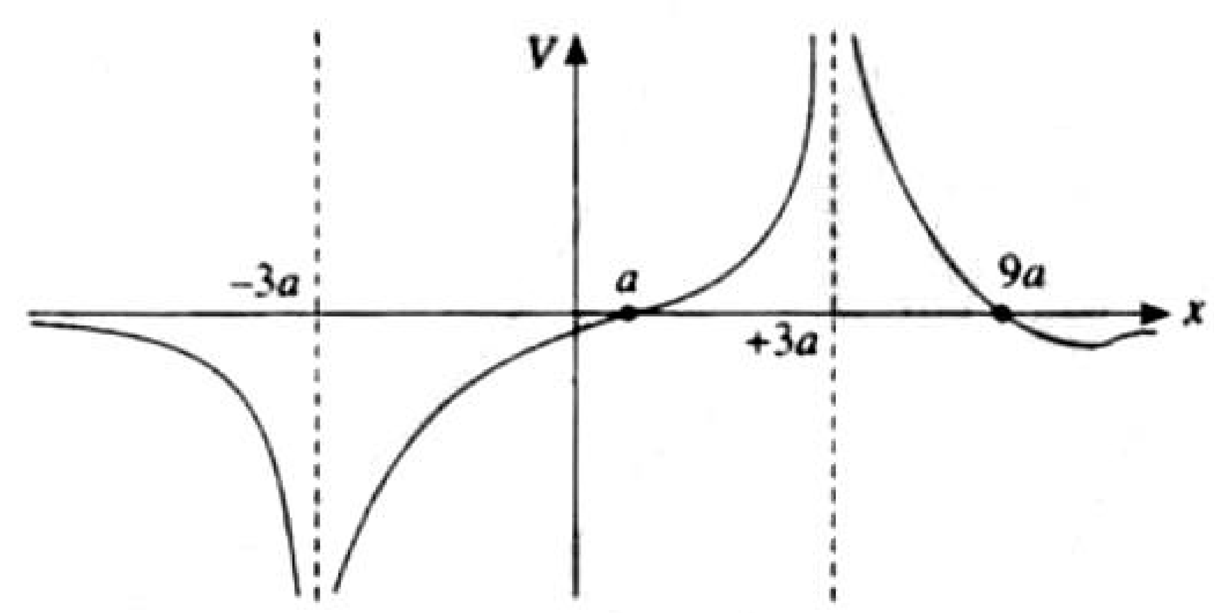
Since circle of zero potential cuts the x-axis at $(a, 0)$ and $(9a, 0)$ hence $V(x) = 0$ at $x = a$ at $x = 9a$.

From the above expressions

$V(x) \to \infty$ at $x \to 3a$ and $V(x) \to -\infty$ and $x \to -3a$

$V(x) \to 0$ as $x \to \pm\infty$

$V(x)$ varies as $\frac{1}{x}$ in general.



c. Applying energy conservation

$$(\text{K.E.} + \text{P.E.})_{\text{centre}} = (\text{K.E.} + \text{P.E.})_{\text{circumference}}$$

$$0 + \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{2a} - \frac{2Qq}{8a}\right] = \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{6a} - \frac{2Qq}{12a}\right]$$

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{4a} \Rightarrow v = \sqrt{\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{2ma}\right)}$$

12. The work done by electric force in bringing charge q from A to B is $W_{ele} = q(V_A - V_B)$ where

$$V_A = \frac{3Q}{8\pi\varepsilon_0 R},$$

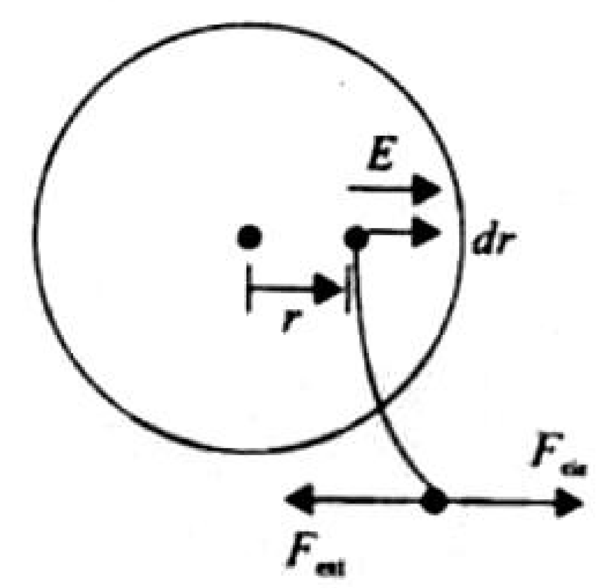
$$V_B = \frac{Q}{4\pi\varepsilon_0 R}$$

Hence,

$$W_{elc} = q\left(\frac{3Q}{8\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 R}\right) = \frac{Qq}{8\pi\varepsilon_0 R}$$

Alternative method:

The work done from A to B can also be given by $W_{elc} = q\int E\,dr$,



where

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3};\ r \le R$$

or $$W = q\int_0^R\left(\frac{Q}{4\pi\varepsilon_0 R^3}r\right)dr = \frac{Qq}{8\pi\varepsilon_0 R}$$

13. i. Capacitor A with a dielectric can be regarded as two capacitors in parallel, one having a dielectric and the other having no dielectric state. Such a capacitor has an area of $A/2$. So the combined capacitance is

$$C = C_1 + C_2 = \frac{(A/2)\varepsilon_0}{d} + \frac{(A/2)\varepsilon_0 K}{d} = \frac{A}{2}\frac{\varepsilon_0}{d}(1+K)$$

$$= \frac{0.04\times 8.85\times 10^{-12}}{2\times 8.85\times 10^{-4}}(1+9) = 2\times 10^{-9}\ \text{F}$$

Thus, energy stored is

$$\frac{1}{2}CV^2 = \frac{1}{2}\times 2\times 10^{-9}\times(110)^2 = 1.21\times 10^{-5}\ \text{J}$$

ii. Work done in removing the dielectric state = (Energy stored in capacitor without dielectric) – (Energy stored in capacitor with dielectric).

It may be noted that on taking out the dielectric, the charge on the capacitor plate remains the same.

$$W = \frac{q^2}{2C'} - \frac{q^2}{2C}$$

Here,

$$C = 2\times 10^{-9}\ \text{F},$$

$$C' = \frac{A\varepsilon_0}{d} = \frac{0.04\times 8.85\times 10^{-12}}{8.85\times 10^{-4}} = 0.4\times 10^{-9}\ \text{F}$$

$$q = CV = 2\times 10^{-9}\times 110 = 2.2\times 10^{-7}\ \text{C}$$

$$\therefore\quad W = \frac{(2.2\times 10^{-7})^2}{2}\left[\frac{1}{0.4\times 10^{-9}} - \frac{1}{2\times 10^{-9}}\right]$$

$$= 4.84\times 10^{-5}\ \text{J}$$

iii. The capacitance of B is $\varepsilon_0 K A_B/d$. So

$$C_B = 1.8\times 10^{-9}\ \text{F}$$

The charge on A, $q_A = 2.2\times 10^{-7}$ C, gets distributed into two parts q_1 and q_2. So.

$$q_1 + q_2 = 2.2\times 10^{-7}\ \text{C}$$

Also, the potential difference across A is equal to the potential difference across B. Thus,

$$\frac{q_1}{C_A} = \frac{q_2}{C_B} \text{ or } q_1 = \frac{C_A}{C_B}q_2 = \frac{0.4\times 10^{-9}}{1.8\times 10^{-9}}q_2 = 0.22q_2$$

or $$0.22q_2 + q_2 = 2.2\times 10^{-7}$$

or $$q_2 = \frac{2.2}{1.22}\times 10^{-7} = 1.8\times 10^{-7}\ \text{C}$$

and $q_1 = 0.4\times 10^{-7}$ C

Total energy stored is

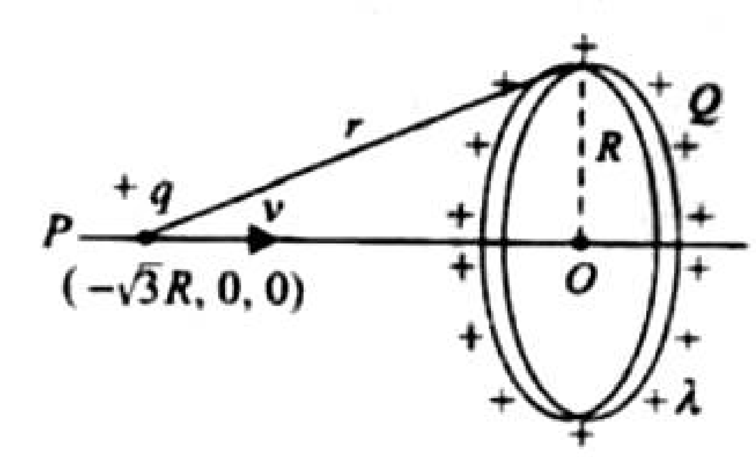
$$\frac{q_1^2}{2C_A} + \frac{q_2^2}{2C_B} = 0.2\times 10^{-5} + 0.9\times 10^{-5} = 1.1\times 10^{-5}\ \text{J}$$

Alternatively, the combined capacitance of the two capacitors can be found. The total charge on the two capacitors is known. The energy can be found using the formula $Q^2/2C_{eq}$.

14. As the electric field at the center of a ring is zero, the particle will not come back due to repulsion if it crosses the center (see figure), i.e.,

$$\frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} > \frac{1}{4\pi\varepsilon_0}\frac{qQ}{R}$$

But here, $Q = 2\pi R\lambda$ and $r = \sqrt{(\sqrt{3}R)^2 + R^2} = 2R$

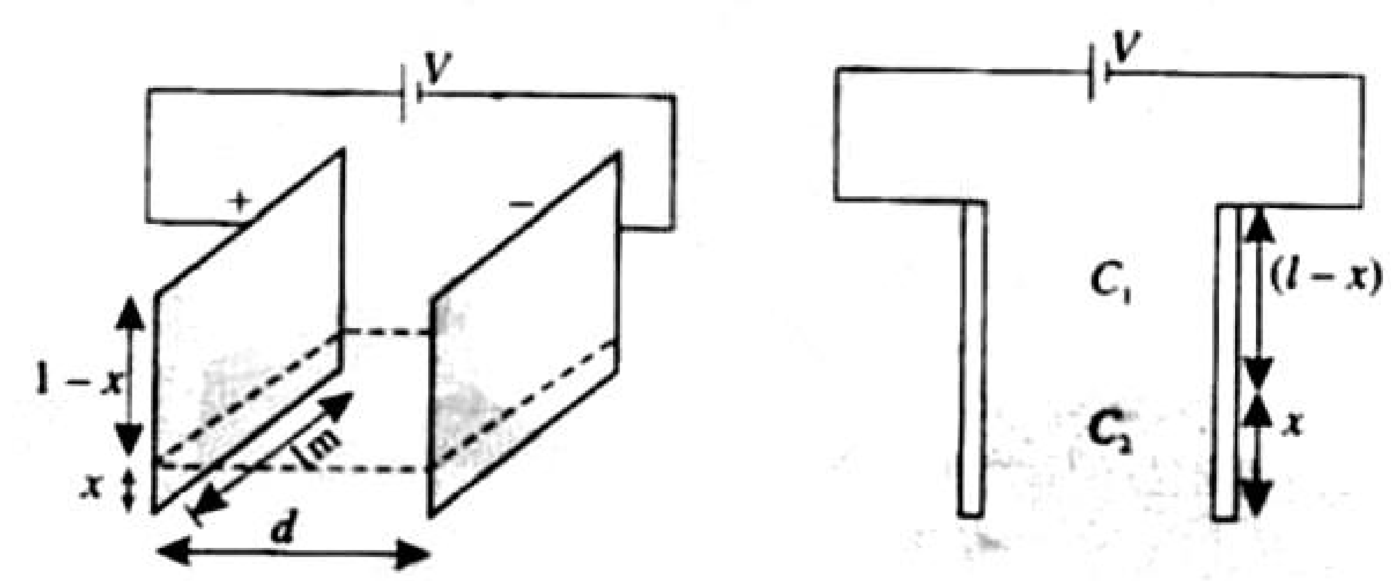


So $$\frac{1}{2}mv^2 > \frac{1}{4\pi\varepsilon_0}\frac{2\pi R\lambda q}{R}\left[1 - \frac{1}{2}\right]$$

or $$v > \sqrt{\frac{\lambda q}{2\varepsilon_0 m}},$$

Therefore, $$v_{min} = \sqrt{\frac{\lambda q}{2\varepsilon_0 m}}$$

15. The adjacent figure is a case of parallel plate capacitor. The combined capacitance will be



$$C = C_1 + C_2$$

$$= \frac{k\varepsilon_0(x\times 1)}{d} + \frac{\varepsilon_0[(1-x)\times 1]}{d}$$

$$C = \frac{\varepsilon_0}{d}[kx + 1 - x] \qquad \text{(i)}$$

Charge in capacitor $q = CV$

Hence current drown from battery

$$\frac{dq}{dt} = V\frac{dC}{dt}$$

$$\Rightarrow \quad I = V\frac{\varepsilon_0}{d}(k-1)\left(\frac{dx}{dt}\right) = \frac{\varepsilon_0 V}{d}(k-1)v \qquad \text{(ii)}$$

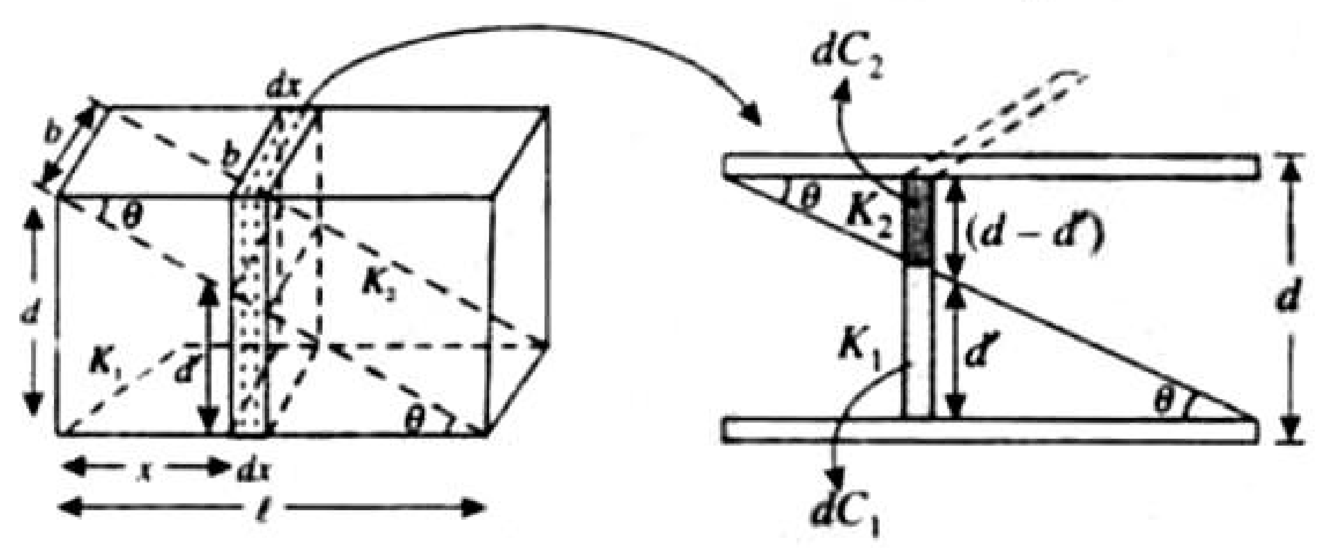
After substituting the given values we get

$$I = 4.425 \times 10^{-9} \text{ amp}$$

16. Case (i) When no dielectric :

Given $C = \frac{\varepsilon_0 A}{d}$

Case (ii) When dielectric is filled: A small dotted element is considered of thickness dx as shown in the figure.



The small capacitance of the dotted portion

$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2}$ where dC_1 = capacitance of capacitor with dielectric k_1

dC_2 = capacitance of capacitor with dielectric k_2.

Let l, b the length and breadth of the capacitor plate. Therefore $l \times b = A$

$$dC_1 = \frac{\varepsilon_0 K_1 (bdx)}{d'}$$

$$d' = d - x\tan\theta = d - x\left(\frac{d}{l}\right) = d\left[1 - \frac{x}{l}\right]$$

$$dC_1 = \frac{\varepsilon_0 K_1 b\,dx}{d\left[1 - \frac{x}{l}\right]} = \frac{\varepsilon_0 K_1 bldx}{d(l-x)} = \frac{\varepsilon_0 K_1 Adx}{d(l-x)}$$

Similarly, $dC_2 = \frac{\varepsilon_0 K_2 (bdx)}{d-d'} = \frac{\varepsilon_0 K_2 bdx}{d - d + \frac{xd}{l}} = \frac{\varepsilon_0 K_2 Adx}{xd}$

$$\frac{1}{dC} = \frac{d(l-x)}{\varepsilon_0 K_1 Adx} + \frac{xd}{\varepsilon_0 K_2 Adx} = \frac{1}{\varepsilon_0 Adx}\left[\frac{ld - xd}{K_1} + \frac{xd}{K_2}\right]$$

$$\frac{1}{dC} = \frac{1}{\varepsilon_0 Adx}\left[\frac{K_2 ld - K_2 xd + K_1 xd}{K_1 K_2}\right]$$

$$\frac{1}{dC} = \frac{1}{\varepsilon_0 Adx} \times \frac{1}{K_1 K_2}[K_2 ld + d(K_1 - K_2)x]$$

$$\frac{\varepsilon_0 K_1 K_2 A\,dx}{K_2 ld + d(K_1 - K_2)x} = dC$$

To find the capacitance of the whole capacitor, we integrate the above equation.

$$C = \int_0^l \frac{\varepsilon_0 K_1 K_2 Adx}{K_2 ld + d(K_1 - K_2)x}$$

$$= \varepsilon_0 K_1 K_2 A \int_0^l \frac{dx}{K_2 ld + d(K_1 - K_2)x}$$

$$= \varepsilon_0 K_1 K_2 A\left[\frac{\log[K_2 ld + d(K_1 - K_2)]x}{d(K_1 - K_2)}\right]_0^l$$

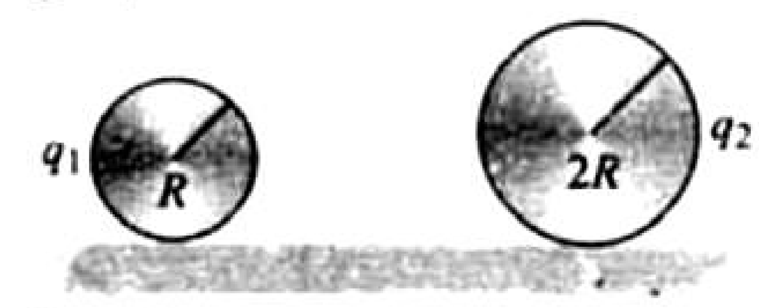
$$= \frac{\varepsilon_0 K_1 K_2 A}{d(K_1 - K_2)}[\log\{K_2 ld + d(K_1 - K_2)l\} - \log K_2 ld]$$

$$= \frac{\varepsilon_0 K_1 K_2 A}{d(K_1 - K_2)}[\log\{K_2 ld + dK_1 l - dK_2 l\} - \log K_2 ld]$$

$$C = \frac{\varepsilon_0 K_1 K_2 A}{d(K_1 - K_2)}\log\frac{K_1}{K_2}$$

17. For sphere of radius R (see figure).

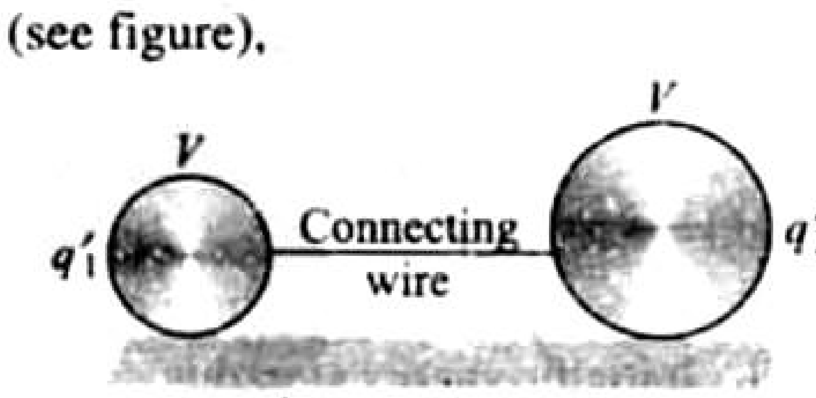
$$\sigma = \frac{q_1}{4\pi R^2}$$



or $\quad q_1 = \sigma \times 4\pi R^2$

For sphere of radius $2R$ (see figure),

$$\sigma = \frac{q_2}{4\pi(2R)^2}$$



or $\quad q_2 = \sigma \times 16\pi R^2$

When the two spheres are connected, the potential on the two spheres will be the same. There will be a rearrangement of charge for this to happen.

Let q_1' and q_2' be the new charges on the two spheres. Since the total charge remains the same, we get

$$q_1' + q_2' = q_1 + q_2 = \sigma \times 20\pi R^2 \qquad \text{(i)}$$

Also, since $V_1 = V_2$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1'}{R} = \frac{1}{4\pi\varepsilon_0}\frac{q_2'}{2R}$$

$$q_1' = \frac{q_2'}{2} \qquad \text{(ii)}$$

Substituting the value of q_1' from Eq. (ii) in Eq. (i)

$$\frac{q_2'}{2} + q_2' = \sigma \times 20\pi R^2$$

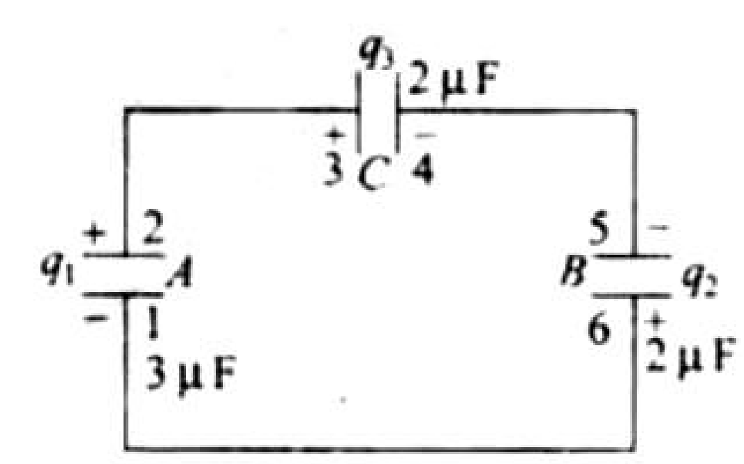
or $\quad \frac{3q_2'}{2} = \sigma \times 20\pi R^2$

or $\quad \frac{q_2'}{4\pi(2R)^2} = \frac{\sigma}{3} \times \frac{5}{2}$

New charge density on the bigger sphere is

$$\frac{q_2'}{4\pi(2R)^2} = \frac{5\sigma}{6}$$

18. i. Method 1: Initial charge on capacitor A is $q_A = 3 \times 100 = 300\ \mu C$. Initial charge on capacitor B is $q_B = 2 \times 180 = 360\ \mu C$. After completing the circuit, let the charge on capacitor A be q_1, on B be q_2, and on C be q_3 with polarities as shown in the figure. Applying conservation of charge for plates 2 and 3, we get



$$q_1 + q_3 = 300 + 0 = 300 \qquad \text{(i)}$$

Applying conservation of charge for plates 4 and 5, we get

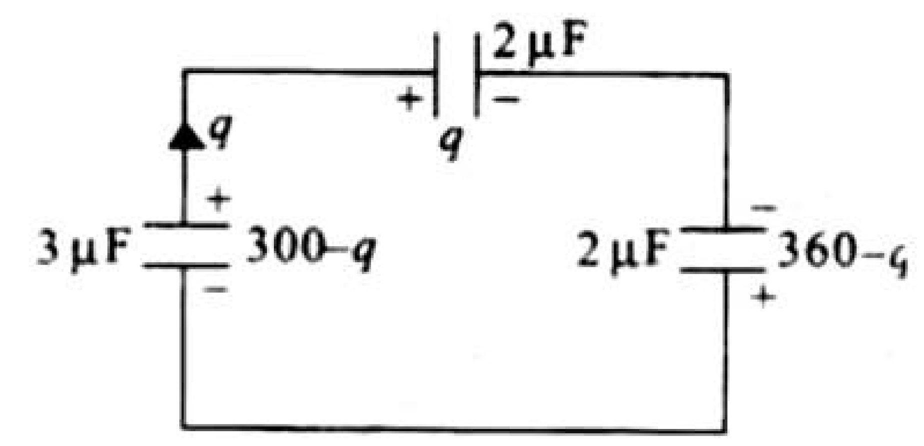
$$-q_2 - q_3 = -360 + 0 \text{ or } q_2 + q_3 = 360 \quad \text{(ii)}$$

Applying Kirchhoff's law in the whole circuit, we get

$$\frac{q_1}{3} - \frac{q_3}{2} + \frac{q_2}{2} = 0 \quad \text{(iii)}$$

On solving, we get $q_1 = 90\ \mu C$, $q_2 = 150\ \mu C$, and $q_3 = 210\ \mu C$.

Method 2: After completing the circuit, let the charge q flow in the circuit in the clockwise direction. Then the final charges on the capacitors will be as shown in the figure.



Applying Kirchhoff's law, we get

$$\frac{300-q}{3} - \frac{q}{2} + \frac{360-q}{2} = 0 \text{ or } q = 210\ \mu C$$

Now charges on all three capacitors can be found.

ii. Initially, energy stored is

$$U_i = \frac{1}{2}\times 3\times 10^{-6}\times(100)^2 + \frac{1}{2}\times 2\times 10^{-6}(180)^2$$
$$= 4.74\times 10^{-2}\ J$$

Finally, the energy stored is

$$U_f = \frac{(90\times 10^{-6})^2}{2\times 3\times 10^{-6}} + \frac{(150\times 10^{-6})^2}{2\times 2\times 10^{-6}} + \frac{(210\times 10^{-6})^2}{2\times 2\times 10^{-6}}$$
$$= 1.8\times 10^{-2}\ J$$

19 i. When S_1 and S_2 come in contact, there is transfer of charges till the potentials of the two spheres become equal. During first contact,

$$V_1 = V_2 \quad (q_1 \text{ charge shifts from } S_1 \text{ to } S_2)$$

$$\Rightarrow \frac{K(Q-q_1)}{r} = \frac{Kq_1}{R} \text{ or } q_1 = Q\left(\frac{R}{R+r}\right)$$

During second contact, again

$$V_1 = V_2$$

$$\Rightarrow \frac{K[Q-(q_2-q_1)]}{r} = \frac{Kq_2}{R}$$

$[(q_2 - q_1)$ charge shifts from S_1 to $S_2]$

$$\therefore \quad q_2 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

On third contact, again

$$V_1 = V_2$$

$$\Rightarrow \frac{K[Q-(q_3-q_2)]}{r} = \frac{Kq_3}{R}$$

$[(q_3 - q_2)$ charge shifts from S_1 to $S_2]$

$$\therefore \quad q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

On nth contact, by symmetry

$$V_1 = V_2$$

$$\Rightarrow \frac{K[Q-(q_n-q_{n-1})]}{r} = \frac{Kq_n}{R}$$

$[(q_n - q_{n-1})$ charge shift from S_1 to $S_2]$

$$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \cdots + \left(\frac{R}{R+r}\right)^n\right]$$

$$= \frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]$$

The electrostatic energy of S_2 after n contacts is

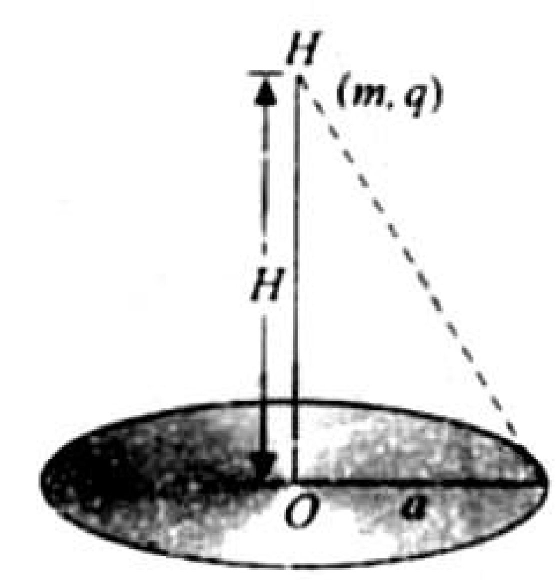
$$U_n = \frac{1}{2}\frac{q_n^2}{C} = \frac{1}{2}\times\frac{1}{4\pi\varepsilon_0 R}\times\left\{\frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]\right\}^2$$

ii. The limiting value is

$$\underset{n\to\infty}{\text{Lt}}\ U_n = \underset{n\to\infty}{\text{Lt}}\left[\frac{1}{2}\times\frac{1}{4\pi\varepsilon_0 R}\left\{\frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]\right\}^2\right]$$

$$= \frac{Q^2 R}{2(4\pi\varepsilon_0)r^2}$$

20. i. Given that a is the radius of disk, σ is the surface charge density and $q/m = 4\varepsilon_0 g/\sigma$. The kinetic energy of the particle, when it reaches the disk, can be taken as zero. Potential due to a charged disk at any axial point situated at a distance x from O is



$$V(x) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+x^2} - x\right]$$

Hence,

$$V(H) = \frac{\sigma}{2\varepsilon_0}[\sqrt{a^2+H^2} - H]$$

and $$V(O) = \frac{\sigma a}{2\varepsilon_0}$$

According to the law of conservation of energy, we have the loss of gravitation potential energy is equal to the gain in electric potential energy.

$$mg\,H = q\Delta V = q[V(0) - V(H)]$$

$$= q[a - \{\sqrt{(a^2+H^2)} - H\}]\frac{\sigma}{2\varepsilon_0} \quad \text{(i)}$$

We are given

$$\frac{\sigma}{2\varepsilon_0} = 2\,mg$$

Putting this in Eq. (i), we get

$$mg\,H = 2mg[a - \{\sqrt{(a^2+H^2)} - H\}]$$

or $$H = 2[a + H - \sqrt{(a^2+H^2)}]$$

$$= 2a + 2H - 2\sqrt{(a^2+H^2)}$$

or $$2\sqrt{(a^2+H^2)} = H + 2a$$

or $$4a^2 + 4H^2 = H^2 + 4a^2 + 4aH$$

or $$3H^2 = 4aH$$

or $$H = \frac{4a}{3} \quad \text{(Since } H = 0 \text{ is not valid)}$$

ii. The total potential energy of the particle at height x is

$$U(x) = mgx + qV(x) = mgx + \frac{q\sigma}{2\varepsilon_0}(\sqrt{a^2+x^2} - x)$$

$$= mgx + 2mg + \left[\sqrt{(a^2+x^2)} - x\right]$$

$$= mg\,[2\sqrt{(a^2+x^2)} - x] \qquad \text{(ii)}$$

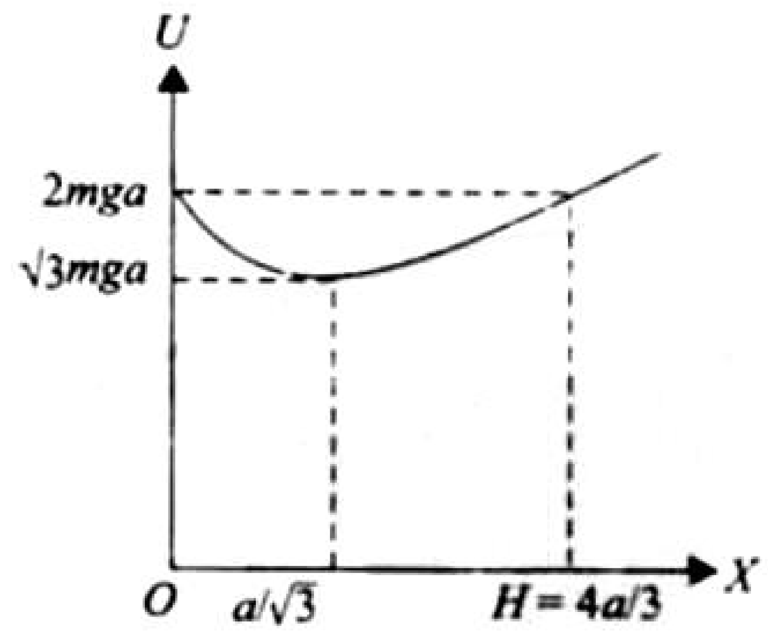
For equilibrium

$$\frac{dU}{dx} = 0$$

This gives

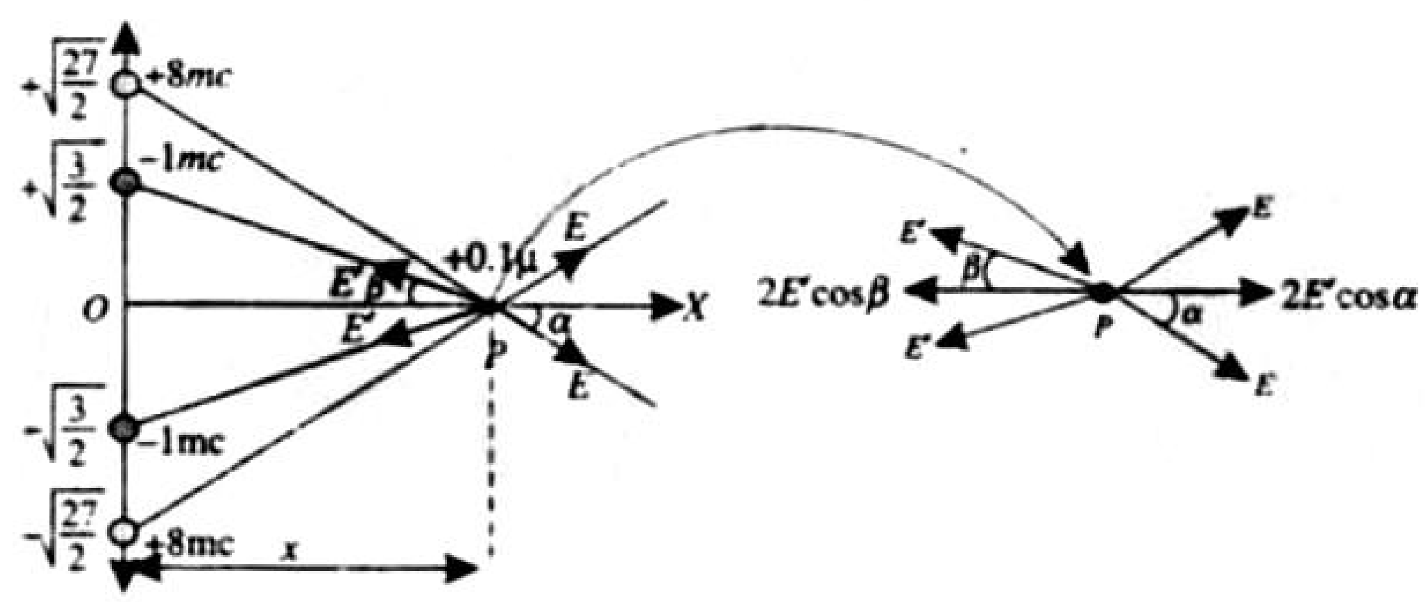
$$x = a/\sqrt{3}$$

From Eq. (ii), graph between $U(x)$ and x can be plotted as shown in the figure.



21. If we throw the charge particle upto neutral point. Thereafter the attractive forces will make the charge is move to origin.

Let the particle be, at some instant, at a point P distant x from the origin. As shown in the figure, electric field due to two charges of $+8mC$ will be $2E\cos\alpha$ towards right.



Similarly the electric field due to two charges of $-1\ mC$ will be $2E'\cos\beta$ towards left.

At neutral point

$$2E\cos\alpha = 2E'\cos\beta$$

$$\frac{K\times 8\times 10^{-6}}{\sqrt{x^2+\frac{27}{2}}}\cdot\frac{x}{\sqrt{x^2+\frac{27}{2}}} = \frac{K\times 1\times 10^{-6}}{\sqrt{x^2+\frac{3}{2}}}\cdot\frac{x}{\sqrt{x^2+\frac{3}{2}}}$$

$$\frac{8}{\left[x^2+\frac{27}{2}\right]^{3/2}} = \frac{1}{\left[x^2+\frac{3}{2}\right]^{3/2}}$$

Which gives $x = \pm\sqrt{\frac{5}{2}}$

This means that we need to move the charge from $-\infty$ to $\sqrt{\frac{5}{2}}$.

Thereafter the attractive forces will make the charge move to origin.

The electric potential of the force charges at $x = \sqrt{\frac{5}{2}}$ is

$$V = \frac{2\times 9\times 10^9\times 8\times 10^{-6}}{\sqrt{\frac{5}{2}+\frac{27}{2}}} - \frac{2\times 9\times 10^9\times 10^{-6}}{\sqrt{\frac{5}{2}+\frac{3}{2}}}$$

$$= 2\times 9\times 10^9\times 10^{-6}\left[\frac{8}{4}-\frac{1}{2}\right] = 2.7\times 10^4\ \text{V}$$

Kinetic energy is required to overcome the force of repulsion from ∞ to $x = \sqrt{\frac{5}{2}}$.

The work done in this process is $W = q(V)$ where V = p.d. between ∞ and $x = \sqrt{\frac{5}{2}}$.

$\therefore \quad W = 0.1\times 10^{-6}\times 2.7\times 10^4 = 2.7\times 10^{-3}$ J

By energy conservation $\frac{1}{2}mv_0^2 = 2.7\times 10^{-3}$

$\Rightarrow \quad \frac{1}{2}\times 6\times 10^{-4}v_0^2 = 2.7\times 10^{-3}$

$\Rightarrow \quad v_0 = 3$ m/s

K.E. at the origin

Again by energy conservation

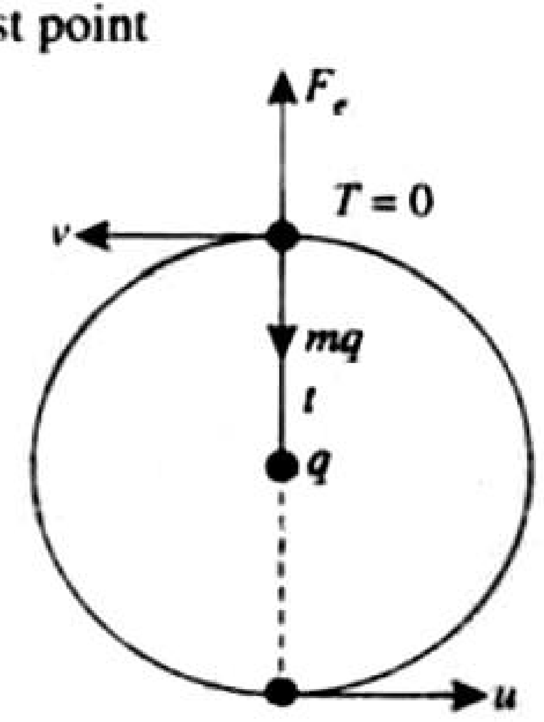
$$K.E. = q\left[V_{x=\frac{\sqrt{5}}{2}} - V_{x=0}\right]$$

$$V_{x=0} = \frac{2\times 9\times 10^9\times 8\times 10^{-6}}{\sqrt{\frac{27}{2}}} - \frac{2\times 9\times 10^9\times 10^{-6}}{\sqrt{\frac{3}{2}}}$$

$$= 2.4\times 10^4$$

$$\text{K.E.} = 0.1\times 10^{-6}\,[2.7\times 10^4 - 2.4\times 10]$$
$$= 0.1\times 10^{-6}\times 0.3\times 10^4 = 3\times 10^{-4}\ \text{J}$$

22. Let u be the speed of the particle at its lowest point and v its speed at highest point



At highest point three forces are acting on the particle.

a. Electrostatic repulsion $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}$ (outwards)

b. weight $W = mg$ (inwards)

c. Tension T (inwards)

$T = 0$, if the particle has just to complete the circle and the necessary centripetal force is provided by $W - F_e$ i.e.

$$\frac{mv^2}{l} = W - F_e$$

or $$v^2 = \frac{l}{m}\left(mg - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\right)$$

Given $q = 1\ \mu C = 10^{-6}\ C$

$m = 2\times 10^{-3}$ kg and $l = 0.8$ m

$$v^2 = \frac{0.8}{2\times10^3}\left(2\times10^{-3}\times10 - \frac{9.0\times10^9\times(10^{-6})^2}{(0.8)^2}\right)m^2/s^2$$

or $\quad v^2 = 2.4\ m^2/s^2$ (i)

Now, the electrostatic potential energy at the lowest and highest point are equal. Hence, from conservation of mechanical energy.

Increase in gravitational potential energy = Decrease in kinetic energy.

or $\quad mg(2l) = \frac{1}{2}m(u^2 - v^2)$

or $\quad u^2 = v^2 + 4gl$

Substituting the values of v^2 from equation (i), we get

$u^2 = 2.4 + 4(10)(0.8) = 34.4\ m^2/g^2$

$\therefore \quad u = 5.86$ m/s

Therefore, minimum horizontal velocity imparted lower ball, so that it can make complete revolution, is 5.86 m/s.

23. $W_{external} = \Delta PE = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[-\frac{3}{1}+\frac{3}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right]\times 4$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\cdot\frac{4}{\sqrt{6}}[3\sqrt{3} - 3\sqrt{6} - \sqrt{2}]$$

24. a. Initial potential energy of the dipole-charge system

$U_i = 0$ (Since the charge is far away)

Final potential energy $U_f = -Q\times\frac{1}{4\pi\varepsilon_0}\frac{p}{d^2}$

$$K.E. = |U_f - U_i| = \frac{1}{4\pi\varepsilon_0}\frac{pQ}{d^2}$$

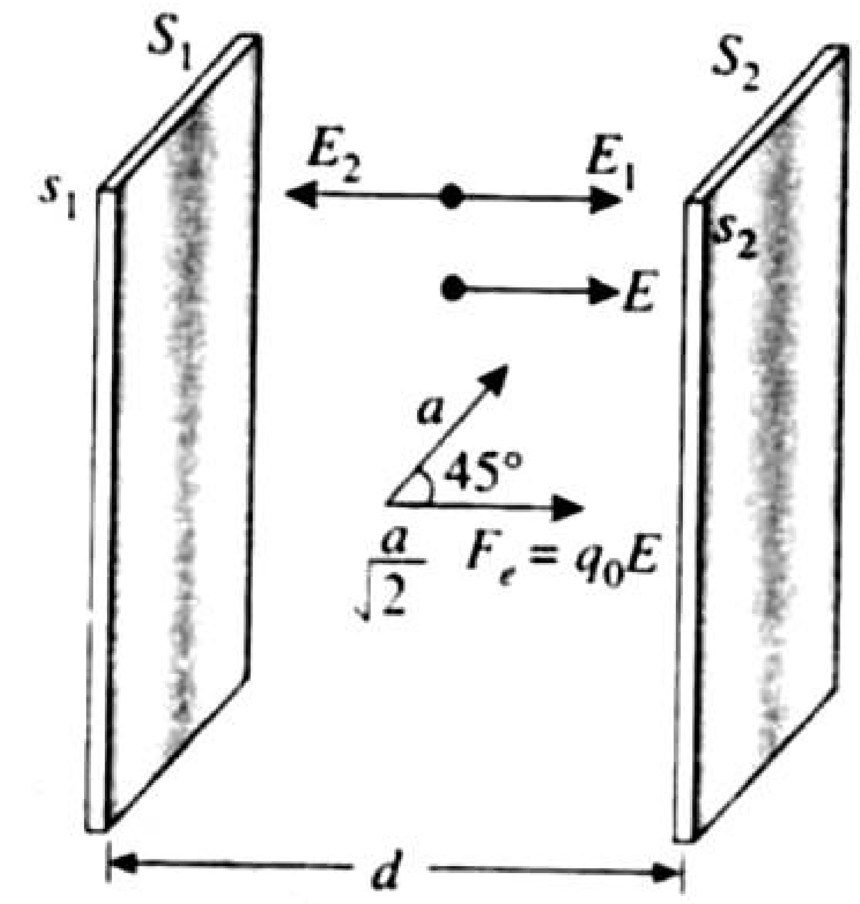
b. Forces on charge

$$F = -\frac{dU}{d(d)} = -\frac{d}{d(d)}\left[-\frac{1}{4\pi\varepsilon_0}\frac{pQ}{d^2}\right]$$

$$F = -\frac{1}{4\pi\varepsilon_0}p\times Q\left[\frac{-2}{d^3}\right] = -\frac{1}{4\pi\varepsilon_0}\frac{2pQ}{d^3}$$

25. Electric field due to left plane $E_1 = \frac{\sigma_1}{2\varepsilon_0}$

$$E_1 = \frac{\sigma_1}{2\varepsilon_0}$$



Net electric field $E = E_1 - E_2 = \frac{\sigma_1 - \sigma_2}{2\varepsilon_0}$

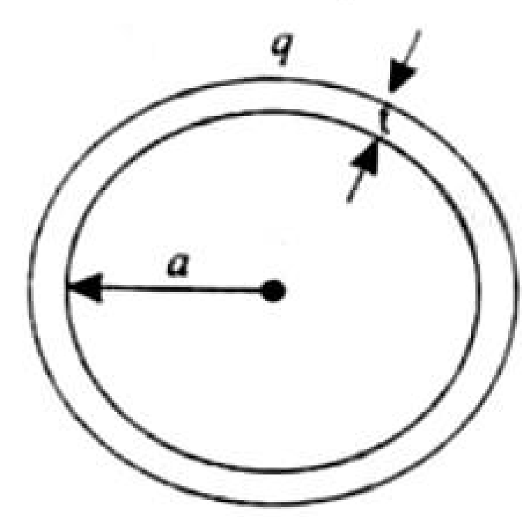
Work done by electric field

$W = F_e \cos 45° \quad \Rightarrow \quad W = q_0E\left(\frac{a}{\sqrt{2}}\right)$

26. The potential of the liquid bubble is V.

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} \quad \Rightarrow \quad q = 4\pi\varepsilon_0 aV$ (i)

where q is the charge on the liquid bubble.



The volume of liquid droplet = Volume (of the liquid) in liquid bubble.

$\frac{4}{3}\pi r^3 = 4\pi a^2 t$

$r^3 = 3a^2t$

$r = [3a^2t]^{1/3}$ (ii)

By charge conservation we can conclude that charge on liquid bubble is equal to charge on liquid droplet

$\Rightarrow$ Charge on liquid droplet is q.

$\therefore$ Potential on liquid droplet $V_{droplet} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$$V_{droplet} = \frac{1}{4\pi\varepsilon_0}\times\frac{(4\pi\varepsilon_0 qVa)}{[3a^2t]^{1/3}} \Rightarrow V_{droplet} = V\left[\frac{a}{3t}\right]^{1/3}$$