STREAMS

Normally by speed of the boat or swimmer we mean the speed of the boat (or swimmer in still water). If the boat (or the swimmer) moves against the stream then it is called *upstream* and if it moves with the stream, it is called *downstream.*

If the speed of the boat (or the swimmer) is x and if the speed of the stream is y then, while upstream the effective speed of the boat = $x - y$ and while downstream the effective speed of the boat = $x + y$.

Theorem : If x km per hour be the man's rate in still water, and y km per hour the rate of the current. Then,

$x + y$ = man's rate with current

$x - y$ = man's rate against current.

Adding and subtracting and then dividing by 2

x = 1/2 (man's rate with current + his rate against current)

y = 1/2 (man's rate with current – his rate against current)

Hence, we have the following two facts :

(*i*) A man's rate in still water is half the sum of his rates with and against the current.

(*ii*) The rate of the current is half the difference between the rates of the man with and against the current.

Tricks For Boats and Streams Related Problems

Suppose,

x = Speed of a boat or man in still water.

y = Speed of the stream or the current or the river.

Then,

Type I :

1. Speed of boat or man with the stream (down-stream)
   = Down rate = $x + y$
2. Speed of boat or man against the stream (up stream)
   = Up rate = $x - y$

Type II :

1. x = 1/2 (Down rate + Up rate)
2. y = 1/2 (Down rate – Up rate)

Special Hints :

1. If a body covers a distance at the rate of x kmph and another equal distance at the rate of y kmph. Then,

$$\textbf{Average Speed} = \frac{2xy}{x+y}$$

2. If a man changes his speed in the ratio of $U : V$, then the ratio of the time taken to cover the same distance is $V : U$.

Precautions :

1. All given quantities (speed or distance) should be in the same manner *i.e.*, either in kilometre (distance) and kilometre per hour (speed) or in metre (distance) and in metre per second (speed).
2. Sometimes we have to change the unit of given quantity according to the need of the question. It must be remembered.

EXERCISE

1. The current of stream runs at 1 kmph. A motorboat goes 35 km upstream and back again to the starting point in 12 hours. The speed of the motorboat in still water is :
(*a*) 6 km/hr (*b*) 7 km/hr
(*c*) 8 km/hr (*d*) 8.5 km/hr
(*e*) None of these

2. A man can row 5 kmph in still water. If the river is running at 1 kmph, it takes him 75 minutes to row to a place and back. How far is the place?
(*a*) 3 km (*b*) 2.5 km
(*c*) 4 km (*d*) 8.5 km
(*e*) None of these

3. A man can row upstream at 7 kmph and downstream at 10 kmph. Find man's rate in still water and the rate of current.
(*a*) 2.5 , 1.5 km/hr (*b*) 8.5, 1.5 km/hr
(*c*) 3, 2.5 km/hr (*d*) 3.5, 2.5 km/hr
(*e*) None of these

4. A man can row 9 1/3 km/hr in still water and he takes thrice as much time to row up than as to row down the same distance in river. The speed of the current is :
(*a*) 5 km/hr (*b*) $4\frac{2}{3}$ km/hr
(*c*) $5\frac{1}{4}$ km/hr (*d*) $4\frac{1}{4}$ km/hr
(*e*) None of these

5. A boat travels upstream from *B* to *A* and downstream from *A* to *B* in 3 hours. If the speed of the boat in still water is 9 km/hr and the speed of the current is 3 km/hr, the distance between *A* and *B* is :
(*a*) 10 km (*b*) 12 km
(*c*) 11 km (*d*) 13 km
(*e*) None of these

6. A man can row upstream at 8 kmph and downstream at 13 kmph. The speed of the stream is :
(*a*) 5 km/hr (*b*) 2.5 km/hr
(*c*) 10.5 km/hr (*d*) 4.2 km/hr
(*e*) None of these

7. A man rows 13 km upstream in 5 hours and also 28 km downstream in 5 hours. The speed of the stream is :
(*a*) 1.5 km/hr (*b*) 2 km/hr
(*c*) 2.5 km/hr (*d*) 3 km/hr
(*e*) None of these

8. A man can row a boat at 10 kmph in still water. If the speed of the stream is 6 kmph the time taken to row a distance of 80 km down the stream is :
(*a*) 8 hours (*b*) 5 hours
(*c*) 10 hours (*d*) 20 hours
(*e*) None of these

9. If a man rows at 6 kmph in still water and 4.5 kmph against the current, then his rate along the current is :
(*a*) 7.5 km/hr (*b*) 6 km/hr
(*c*) 8 km/hr (*d*) 9 km/hr
(*e*) None of these

EXPLANATORY ANSWERS

1. (*a*) : Let the speed of motorboat in still water be x kmph.
Then, speed upstream = $(x-1)$ kmph
speed downstream = $(x+1)$ kmph

$$\because \quad \frac{35}{x-1}+\frac{35}{x+1}=12$$

$$\Rightarrow \quad \frac{35x+35+35x-35}{x^2-1}=12$$

$$\Rightarrow \quad 6x^2-35x-6=0$$

$$\Rightarrow (x-6)(6x+1)=0 \Rightarrow x=6$$

Hence, the speed of motor boat in still water = 6 kmph.

2. (*a*) : Speed downstream = (5 + 1) km/hr = 6 km/hr
Speed upstream = (5 – 1) km/hr = 4 km/hr
Let the required distance be x km.

Then, $\frac{x}{6}+\frac{x}{4}=\frac{75}{60}$

$$\Rightarrow \quad \frac{2x+3x}{12}=\frac{5}{4}$$

$$\Rightarrow \quad 2x+3x=15$$

$$\Rightarrow \quad x=3$$

So, required distance = 3 km.

3. (*b*) : Rate of man in still water = 1/2 (10 + 7) km/hr = 8.5 km/hr

Rate of current = 1/2 (10 – 7) km/hr
= 1.5 km/hr

4. (*b*) : Let the speed of current = x km/hr
Then, $28/3 + x = 3\ (28/3 - x)$

$$\Rightarrow \quad 4x = 28 - \frac{28}{3}$$

$$\Rightarrow \quad x = \frac{14}{3} = 4\frac{2}{3} \text{ km/hr.}$$

5. (*b*) : Let the distance be x km.
Now, upstream speed = 9 – 3 = 6 km/hr.
and downstream speed = 9 + 3 = 12 km/hr
Total time taken in upstream and downstream journey

$$\frac{x}{6} + \frac{x}{12} = 3 \quad \Rightarrow \quad \frac{3x}{12} = 3$$

$$\Rightarrow \quad x = 12 \text{ km}$$

Quicker Maths (Direct formula) :

Distance = Total time $\times$

$$\left\{\frac{(\text{speed of boat in still water})^2 - (\text{speed of current})^2}{2 \times \text{speed of boat in still water}}\right\}$$

$$= 3 \times \left\{\frac{(9)^2 - (3)^2}{2 \times 9}\right\} = \frac{3 \times 72}{18} = 12 \text{ km}$$

6. (*b*) : Speed of stream = $\frac{1}{2}(13 - 8)$ kmph
= 2.5 kmph

7. (*a*) : Speed upstream = 13/5 kmph
Speed downstream = 28/5 kmph

$$\text{Speed of stream} = \frac{1}{2}\left(\frac{28}{5} - \frac{13}{5}\right) \text{kmph}$$

= 1.5 kmph

8. (*b*) : Speed downstream = (10 + 6) km/hr
= 16 km/hr
Time taken to cover 80 km downstream
= (80/16) hrs = 5 hrs

9. (*a*) : Let the rate of the stream be x kmph.
Then, rate against the current = $(6 - x)$ kmph.

$$\Rightarrow \quad 6 - x = 4.5$$

$$\Rightarrow \quad x = 1.5$$

So, rate of current = 1.5 kmph
Rate along the current = 6 + 1.5 = 7.5 kmph

MISCELLANEOUS EXERCISE

1. A monkey ascends a greased pole 12 metres high. He ascends 2 metres in first minute and slips down 1 metre in the alternate minute. In which minute, he reaches the top?
(*a*) 10th (*b*) 11th
(*c*) 12th (*d*) 13th
(*e*) None of these

2. Two trains start at the same time from Aligarh and Delhi and proceed towards each other at the rate of 16 km/hr and 21 km/hr respectively. When they meet, it is found that one train has travelled 60 km more than the other. The distance between the two stations is :
(*a*) 445 km (*b*) 444 km
(*c*) 440 km (*d*) 450 km
(*e*) None of these

3. A train 100 metres long, moving at a speed of 50 km per hour, crosses a train 120 metres long coming from opposite direction in 6 seconds. What is the speed of the second train?
(*a*) 132 kmph (*b*) 82 kmph
(*c*) 60 kmph (*d*) 50 kmph
(*e*) None of these

4. If I walk at 4 kmph, I miss the bus by 10 minutes. If I walk at 5 kmph, I reach 5 minutes before the arrival of the bus. How far I walked to reach the bus stand?
(*a*) 5 km (*b*) 10 km
(*c*) 7 km (*d*) 4 km
(*e*) None of these

5. Two buses travel to a place at speeds of 45 kmph and 60 kmph respectively. If the second bus takes $5\frac{1}{2}$ hours less than the first for the journey, the length of the journey is :
(*a*) 900 km (*b*) 945 km
(*c*) 990 km (*d*) 1350 km
(*e*) None of these

6. A man leaves a point P at 6 a.m. and reaches the point Q at 10 a.m. Another man leaves the point Q at 8 a.m. and reaches the point P at 12 noon. At what time do they meet?
(*a*) 9 a.m. (*b*) 10 a.m.
(*c*) 8 a.m. (*d*) 7 a.m.
(*e*) None of these

7. A man travels 360 km in 4 hrs, partly by air and partly by train. If he had travelled all the way by air, he would have saved 4/5 of the time he was in train and would have arrived at his destination 2 hours early. Find the distance he travelled by air and train.
(*a*) 260 km and 80 km (*b*) 270 km and 90 km
(*c*) 260 km and 70 km (*d*) 270 km and 95 km
(*e*) None of these

8. A person covers a distance in 40 mintues if he runs at a speed of 45 km per hour on an average. Find the speed at which he must run to reduce the time of journey to 30 minutes.
(*a*) 70 km/hr (*b*) 75 km/hr
(*c*) 60 km/hr (*d*) 65 km/hr
(*e*) None of these

9. A boat moves downstream at the rate of 1 km in 6 minutes and upstream at the rate of 1 km in 10 minutes. The speed of the current is :
(*a*) 2 km/hr (*b*) 3 km/hr
(*c*) 1 km/hr (*d*) 4 km/hr
(*e*) None of these

10. Two trains travel in the same direction at 90 km/hr and 72 km/hr respectively and the faster train passes a man in the slower train in 23 seconds. Find the length of the train that runs faster.
(*a*) 135 m (*b*) 120 m
(*c*) 115 m (*d*) 150 m
(*e*) None of these

11. A man rows upstream 13 km and downstream 28 km taking 5 hours each time. What is the speed of the current?
(*a*) $2\frac{1}{2}$ km (*b*) 1 km
(*c*) $1\frac{1}{2}$ km (*d*) 2 km
(*e*) None of these

12. By walking at 3/4 of his usual speed, a man reaches office 20 minutes later than usual time. What is his usual time?
(*a*) 65 min (*b*) 60 min
(*c*) 70 min (*d*) 64 min
(*e*) None of these

13. Ravi runs 15.6 km per hour. How many metres does he run in two minutes?
(*a*) 400 metres (*b*) 520 metres
(*c*) 200 metres (*b*) 450 metres
(*e*) None of these

14. A monkey tries to ascend a greased pole 14 metres high. He ascends 2 metres in first minute and slips down 1 metre in the alternate minute. If he continues to ascend in this fashion, how long does he take to reach the top?
(*a*) 25 min. (*b*) 28 min.
(*c*) 20 min. (*d*) 30 min.
(*e*) None of these

15. Two runners cover the same distance at the rate of 15 km and 16 km per hour respectively. Find the distance travelled when one takes 16 minutes longer than the other.
(*a*) 60 km (*b*) 70 km
(*c*) 64 km (*d*) 80 km
(*e*) None of these

16. Two cars run to a place at the speeds of 45 km/hr and 60 km/hr respectively. If the second car takes 5 hours less than the first for the journey. Find the length of the journey.
(*a*) 1000 km (*b*) 900 km
(*c*) 850 km (*d*) 1200 km
(*e*) None of these

17. A cyclist travels for 10 hours, the first half at 21 km per hour and the other half at 24 kmph. Find the distance travelled.
(*a*) 235 km (*b*) 224 km
(*c*) 255 km (*d*) 275 km
(*e*) None of these

18. On a tour, a man travels at the rate of 64 km an hour for the first 160 km, then travels the next 160 km at the rate of 80 km an hour. What is the average speed in km per hour for the first 320 km of the tour?
(*a*) 75 kmph (*b*) 85 kmph
(*c*) 71.1 kmph (*d*) 75.12 kmph
(*e*) None of these

19. A man can row 4.5 km/hr in still water and he finds that it takes him twice as long to row upstream as to row down the river. Find the rate of stream.
(*a*) 4 km/hr (*b*) 7 km/hr
(*c*) 1.5 km/hr (*d*) 2 km/hr
(*e*) None of these

20. A man can row 5 km per hour in still water. If the river is flowing at 1 km per hour, it takes him 75 minutes to row to a place and back. How far is the place?
(*a*) 3 km (*b*) 2.5 km
(*c*) 4 km (*d*) 5 km
(*e*) None of these

21. A train passes through the stationary man standing on the platform in 7 seconds and passes through the platform completely in 28 seconds. If the length of the platform is 330 metres, what is the length of the train?
(*a*) 82.5 m (*b*) 220 m
(*c*) 110 m (*d*) 100 m
(*e*) None of these

22. A man covers a certain distance between his house and office on scooter. Having an average speed of 30 km/hr, he is late by 10 minutes. However, with a speed of 40 km/hr, he reaches his office 5 minutes earlier. Find the distance between his house and office.
(*a*) 30 km (*b*) 35 km
(*c*) 40 km (*d*) 45 km
(*e*) None of these

23. The distance between two stations, Delhi and Amritsar, is 450 km. A train starts at 4 p.m. from Delhi and moves towards Amritsar at an average speed of 60 km/hrs. Another train starts from Amritsar at 3:20 p.m. and moves towards Delhi at an average speed of 80 km/hrs. How far from Delhi will the two trains meet and at what time?
(*a*) 70 km and 4 p.m.
(*b*) 140 km and 3 p.m.
(*c*) Data is inadequate
(*d*) 170 km and 6:50 p.m.
(*e*) None of these

EXPLANATORY ANSWERS

1. (*e*) : The monkey climbs 1 metre in every 2 minutes, so it will take 20 minutes to climb 10 metres. It will take 1 minute to climb the rest 2 metres and will reach the top. Therefore, it will reach the top in 21st minute.

2. (*b*) : Suppose, they meet after x hours
So, $21x - 16x = 60$;
So, $x = 12$
So, distance $= 16 \times 12 + 21 \times 12 = 444$ km.

3. (*b*) : Suppose, requisite speed = x km/hrs.
Now, $22 \times 60 \times 60 = 6 \times 100\ (50 + x)$
So, $x = 82$ km/hrs.

4. (*a*) : **Trick :** 10 + 5 = 15 min = 1/4 hour.
Requisite distance $= \frac{1}{4} \times \frac{4 \times 5}{5 - 4} = 5$ km

5. (*c*) : **Trick :**
Distance $= \frac{11}{2} \times \frac{60 \times 45}{60 - 45} = 990$ km

6. (*a*) : Let the distance $PQ = A$ km.
And they meet x hrs after the first man starts.
Average speed of first man
$$= \frac{A}{10 - 6} = \frac{A}{4} \text{ km/hr.}$$
Average speed of second man
$$= \frac{A}{12 - 8} = \frac{A}{4} \text{ km/hr}$$
Distance travelled by first man $= \frac{Ax}{4}$ km
They meet x hrs after the first man starts. The second man, as he starts 2 hrs late, meets after $(x - 2)$ hrs from his start. Therefore, the distance travelled by the second man
$$= \frac{A(x-2)\ km}{4}$$
Now, $\frac{Ax}{4} + \frac{A(x-2)\ km}{4} = A$
$\Rightarrow \quad 2x - 2 = 4$
$\Rightarrow \quad x = 3$ hrs.
So, they meet at 6 a.m. + 3 hrs = 9 a.m.

Quicker Approach : Since both the persons take equal time of 4 hrs to cover the distance, their meeting time will be exactly in the middle of 6 a.m. and 12 noon, *i.e.*, at 9 a.m.
But what happens when they take different times? In that case, the following formula works good. They will meet at
$$= \text{First's starting time} + \frac{(\text{Time taken by first})\ (\text{2nd's arrival time} - \text{1st's starting time})}{\text{Sum of time taken by both}}$$
$$= 6 \text{ a.m.} + \frac{(10 - 6)\ (12 - 6)}{(10 - 6) + (12 - 8)}$$
$$= 6 \text{ a.m.} + \frac{4 \times 6}{4 + 4} = 9 \text{ a.m.}$$

7. (*b*) : 4/5 of total time in train = 2 hours.

So, total time in train = $2 \times \frac{5}{4} = 5/2$ hrs

So, total time spent in air = $4 - 5/2 = 3/2$ hrs

By the given hypothesis, if 360 km is covered by air, then time taken is $(4 - 2) = 2$ hrs

So, when 3/2 hrs is spent in air, distance covered

$$= \frac{360}{2} \times \frac{3}{2} = 270 \text{ km}$$

So, distance covered by train = $360 - 270$

$= 90$ km

8. (c) : **Theorem :** Speed and time taken are inversely proportional.

Therefore, $S_1 T_1 = S_2 T_2 = S_3 T_3 \ldots$

Where $S_1, S_2, S_3, \ldots$ are the speeds and $T_1, T_2, T_3, \ldots$ are the time taken to travel the same distance. Thus in this case :

$45 \times 40 = S_2 \times 30$

$$\therefore \quad S_2 = \frac{45 \times 40}{30} = 60 \text{ km/hrs.}$$

9. (*a*) : Speed in downstream $= \left(\frac{1}{6} \times 60\right)$ km/hr

$= 10$ km/hr

Speed in upstream $= \left(\frac{1}{10} \times 60\right)$ km/hr

$= 6$ km/hr

So, speed of the current $= \frac{1}{2}(10 - 6)$

$= 2$ km/hrs

10. (*c*) : **Trick :** Length of faster train

$$= 23 \times \left(18 \times \frac{5}{18}\right) = 115 \text{ m}$$

11. (*c*) : Speed in upstream = 13/5 = 2.6 km/hr

Speed in downstream = 28/5 = 5.6 km/hr

So, speed of current $= \frac{1}{2}(5.6 - 2.6)$

$$= 1\frac{1}{2} \text{ km}$$

12. (*b*) : Let usual speed and time are x min and t sec respectively

$$xt = \frac{3x}{4}(t + 20 \times 60)$$

$4t = 3t + 3600$

$t = 3600$ sec or $\frac{3600}{60} = 60$ min

13. (*b*) : Requisite distance $= \frac{15600}{60} \times 2$

$= 520$ metres.

14. (*a*) : In every 2 minutes, he is able to ascend $2 - 1 = 1$ metre. This way he ascends up to 12 metres because when he reaches at the top, he does not slip down. Thus, up to 12 metres, he takes $12 \times 2 = 24$ minutes and for the last 2 metres, he takes 1 minute. Therefore, he takes $24 + 1 = 25$ minutes to reach the top.

15. (*c*) : Let the distance be x km.

Time taken by the first runner $= \frac{x}{15}$ hrs

Time taken by the second runner $= \frac{x}{16}$ hrs

Now,

$$\frac{x}{15} - \frac{x}{16} = \frac{16}{60}$$

$$\Rightarrow \quad \frac{x(16-15)}{15 \times 16} = \frac{16}{60}$$

$$\Rightarrow \quad x = \frac{16}{60} \times 15 \times 16 = 64 \text{ km}$$

Direct Formula : Distance

$$= \frac{\text{Products of speeds}}{\text{Difference of speeds}} \times \text{Difference in time to cover the distance}$$

$$= \frac{15 \times 16}{(16 - 15)} \times \frac{16}{60} = 64 \text{ km}$$

16. (*b*) : "One takes 5 hrs less than the other" means the second takes 5 hrs more than the first to reach the destination. So, the above direct formula works in this case also.

So, distance $= \frac{45 \times 60}{60 - 45} \times 5 = 900$ km

17. (*b*) : Let the total distance be $2x$ km.
According to question,

$$\frac{x}{21}+\frac{x}{24} = 10$$
$$24x + 21x = 21 \times 24 \times 10$$
$$45x = 5040$$
$$x = 112$$

Hence, total distance travelled $= 2 \times 112$
$= 224$ km

18. (*c*) : Average speed $= \dfrac{2 \times 64 \times 80}{64 + 80}$
$= 71.11$ km/hour.

19. (*c*) : Let man's rate with current $= x$ km/hr.

So, $\dfrac{1}{2}[x + 2x] = 4.5$

So, $x = 3$ km/hr
i.e., With current = 3 km/hr. and against current = 6 km/hr.
So, rate of current

$= \dfrac{1}{2}[6 - 3] = 1.5$ km/hr.

20. (*a*) : Suppose, distance $= x$ km

Then, $\dfrac{x}{6} + \dfrac{x}{4} = \dfrac{5}{4}$

$\Rightarrow x = 3$ km

21. (*c*) : Suppose train's length $= x$ metres

So, speed $= \dfrac{x}{7}$ metres/sec.

So, distance covered in 28 seconds

$$= \frac{x}{7} \times 28 = 4x \text{ metres}$$

Now, $4x - x = 330$
So, $x = 110$ metres.

22. (*a*) : Let the distance be x km
Time taken to cover x km at 30 km/hr

$= \dfrac{x}{30}$ hrs.

Time taken to cover x km at 40 km/hr

$= \dfrac{x}{40}$ hrs.

Difference between the time taken = 15 min = 1/4 hr

So, $\dfrac{x}{30} - \dfrac{x}{40} = \dfrac{1}{4}$ or $4x - 3x = 30$
or $x = 30$
Hence, the required distance is 30 km.

Direct Formula :

Required distance
$= \dfrac{\text{Product of two speeds}}{\text{Difference of two speeds}}$
$\times$ Difference between arrival times
Thus in this case, the required distance

$= \dfrac{30 \times 40}{40 - 30} \times \dfrac{15}{60} = 30$ km

Note : 10 minutes late and 5 minutes earlier make a difference of $10 + 5 = 15$ minutes. As the other units are in km/hr, the difference in time should also be changed into hours.

23. (*d*) Suppose the trains meet at a distance of x km from Delhi. Let the trains from Delhi and Amritsar be A and B respectively. Then,
[Time taken by B to cover $(450 - x)$ km
– [Time taken by A to cover x km] $= \dfrac{40}{60}$,

$$\frac{450 - X}{80} - \frac{X}{60} = \frac{40}{60}$$

So, $3(450 - x) - 4x = 160$
$\Rightarrow 7x = 1190$
$\Rightarrow x = 170$

Thus, the trains meet at a distance of 170 km from Delhi.
Time taken by A to cover 170 km
$= (170/60)$ hrs $= 2$ hrs 50 min.
So, the trains meet at 6:50 p.m.

Note : R.H.S. = 4 : 00 pm – 3:20 p.m. = 40 minutes
$= 40/60$ hr
L.H.S. comes from the fact that the train from Amritsar took 40 minutes more to travel up to the meeting point because it had started its journey at 3:20 p.m. Whereas the train from Delhi had started its journey at 4 p.m. and the meeting time is the same for both the trains.