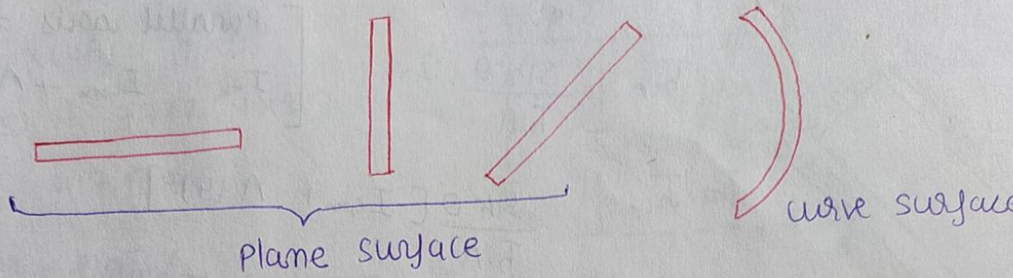
lecture 5

Hydrostatic forces on surfaces :- CHAPTER-2

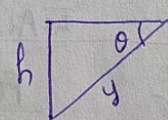


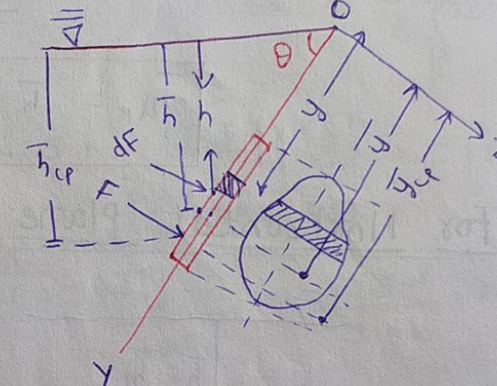
$P = \rho g h$

$dF = P \cdot dA$

$= \rho g \cdot h \cdot dA$

$= \rho g (y \sin\theta) dA$





Int. it $F = \rho g (\sin\theta) \int y \cdot dA$

$\int y \cdot dA = y_1 A_1 + y_2 A_2 + \cdots$

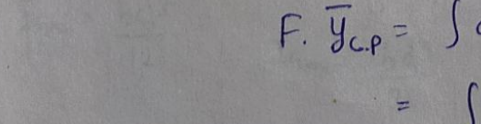
$= \bar{y} \cdot A$

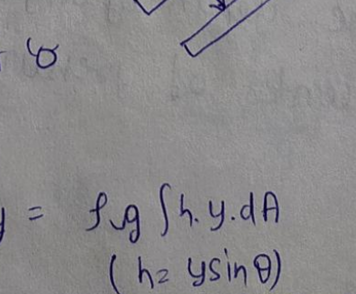
$$F = \rho g \underbrace{(\sin\theta)\bar{y}}_{\bar{h}} \cdot A$$

$$\boxed{F = \rho g \bar{h} \cdot A}$$ *** [this force passes through a point known as center of pressure] ($\bar{h}_{cp}$)

Location of center of pressure ($\bar{h}_{cp}$)

to find x, take moment





taking moment about O

$F \cdot \bar{y}_{c.p} = \int dF \cdot y$

$= \int P \cdot dA \cdot y = \rho g \int h \cdot y \cdot dA$

$(h = y \sin\theta)$

$= \rho \cdot g \cdot \sin\theta \underbrace{\left(\int y^2 \cdot dA\right)}_{I_{xx}}$

$$\cancel{\rho g}\,\bar{h}.A.\frac{\bar{h}_{cp}}{\sin\theta} = \cancel{\rho g}.\sin\theta\, I_{xx}$$

$$\bar{h}_{cp} = \frac{\sin^2\theta}{\bar{h}.A}.I_{xx}$$

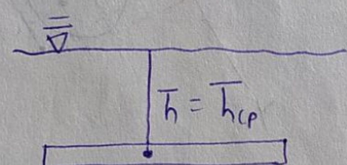
$$\left[\begin{array}{l}\text{Parallel axis theorem} \\ I_{xx} = I_{Gxx} + A\bar{y}^2\end{array}\right]$$

$$\bar{h}_{cp} = \frac{\sin^2\theta\,[I_{Gxx} + A(\bar{y})^2]}{\bar{h}.A}$$

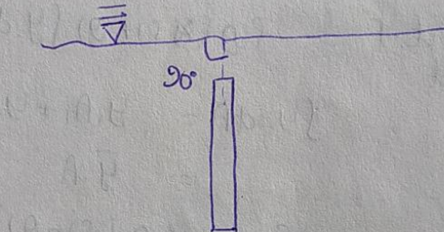
$$= \frac{\sin^2\theta}{\bar{h}.A}\left[I_{Gxx} + \frac{A(\bar{h})^2}{\sin^2\theta}\right]$$

$$\boxed{\bar{h}_{cp} = \bar{h} + \frac{I_{Gxx}.\sin^2\theta}{\bar{h}.A}}$$

For Horizontal Plane :-



For Vertical Plane

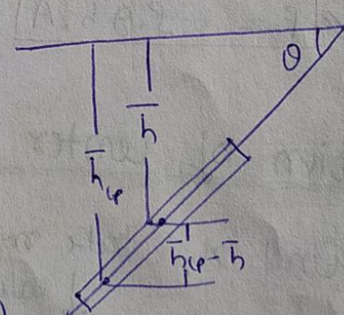


$$\bar{h}_{cp} = \bar{h} + \frac{I_{Gxx}\sin^2 90}{\bar{h}.A} =$$

$$\bar{h}_{cp} = \bar{h} + \frac{I_{Gxx}\sin^2\theta}{\bar{h}.A}$$

$$\bar{h}_{cp} - \bar{h} = \frac{I_{Gxx}\sin^2\theta}{\bar{h}.A}$$

$$\downarrow\ \bar{h}_{cp} - \bar{h} = \frac{\text{constant}}{\bar{h}\uparrow}$$ (increase)



($\bar{h}$ = center of gravity Pe act karta hai)

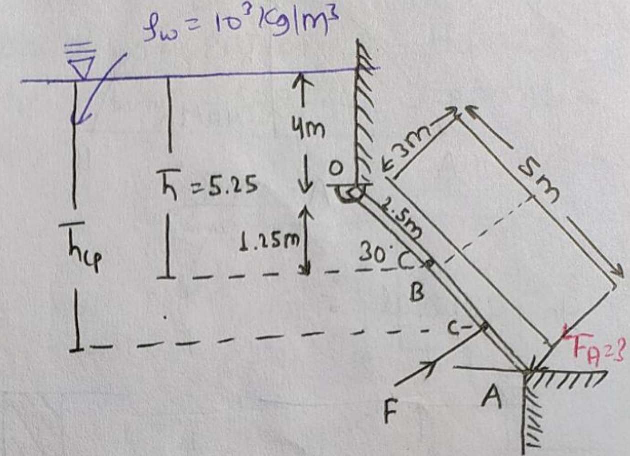
Main

- As the depth of surface$\uparrow$ ($\bar{h}$) increase and $(\bar{h}_{cp} - \bar{h})\downarrow$ so center of pressure shift toward C.G taking moment about O

$$F_A \times (OA) = F \times OC$$

$$F_A \times (5) = 772.54 \times 2.7$$

$$F_A = 417.17\ KN$$



find ① $F = \rho g \bar{h} . A$

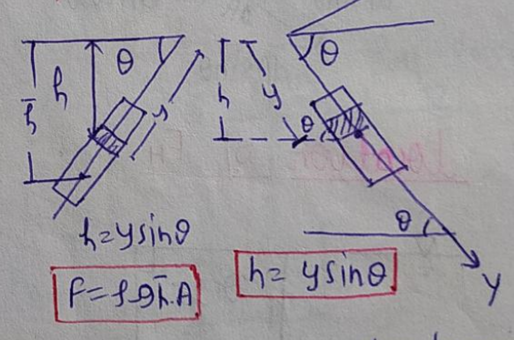
$$= (10)^3 (9.81)(5.25)(15)$$

$$= 772.54\ KN$$

② $$h_{cp} = \bar{h} + \frac{I_{Gxx} \sin^2 30}{\bar{h} . A} = 5.25 + \frac{\frac{(3)(5)^3}{12} \times \left(\frac{1}{2}\right)^2}{5.25 \times 5 \times 3} = 5.35\,m$$

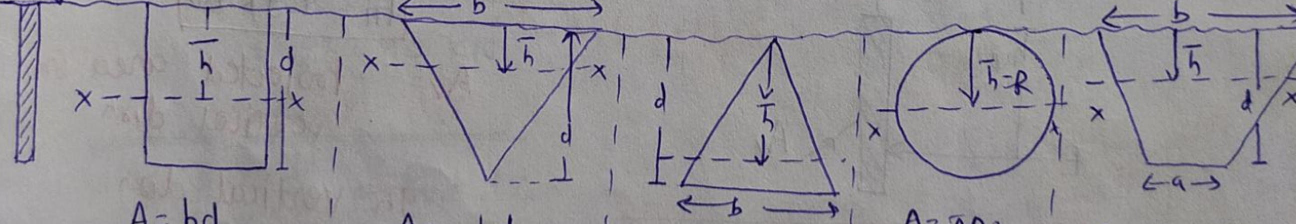
$$OC = \frac{h_{cp} - 4}{\sin 30} = \frac{5.35 - 4}{\sin 30} = 2.7\,m$$

(the force and (h) should be same if we will take plate like this



$h = y \sin\theta$

$\boxed{F = \rho g \bar{h} . A}$ $\boxed{h = y \sin\theta}$



$A = bd$

$\bar{h} = \frac{d}{2}$

$I_{Gxx} = \frac{bd^3}{12}$

$A = \frac{bd}{2}$

$\bar{h} = \frac{d}{3}$

$I_{Gxx} = \frac{bd^3}{36}$

$A = \frac{bd}{2}$

$\bar{h} = \frac{2d}{3}$

$I_{Gxx} = \frac{bd^3}{36}$

$A = \pi R^2$

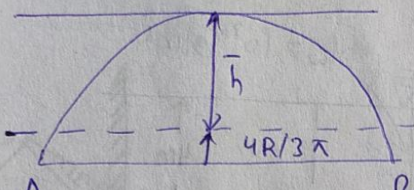
$\bar{h} = R$

$I_{Gxx} = \frac{\pi R^4}{4}$

$A = \frac{(a+b)}{2} . d$

$\bar{h} = \frac{(2a+b)}{(a+b)} . \frac{d}{3}$

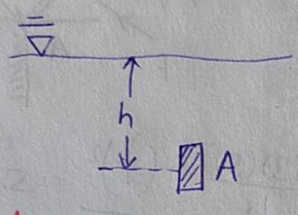
$I_{Gxx} = \left(\frac{a^2 + b^2 + 4ab}{a+b}\right) \frac{d^3}{36}$



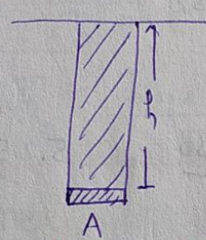
$A = \frac{\pi R^2}{2}$

$I_{Gxx} = 0.11 R^4$

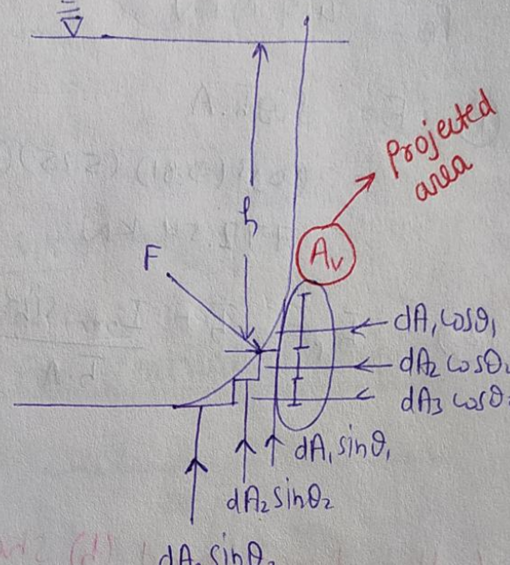
In general :-



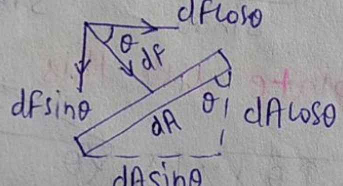
Area moment = A.h



volum = A.h



Force analysis



In horizontal dirn

$F_H = \int dF\cos\theta$

$\left[dA_1\cos\theta_1 + dA_2\cos\theta_2 + dA_3\cos\theta_3 = A_v \right]$

$= \int P.dA\cos\theta$

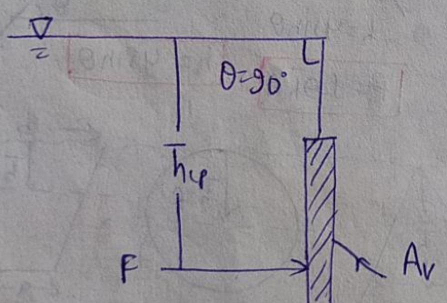
$= \rho g \int h.dA\cos\theta$

$= \rho g \bar{h}.A_v$

$\boxed{F_H = \rho g \bar{h}.A_v}$

A_v = Projected area in vertical dirn

Location of F_H



$$\boxed{h_{cp} = \bar{h} + \frac{I_{Gxx}\sin^2 90}{\bar{h}.A_v}}$$

In vertical dirn

$F_V = \int dF.\sin\theta$

$= \int P.dA.\sin\theta$

$= \rho g \underbrace{\int h.dA.\sin\theta}_{\forall}$

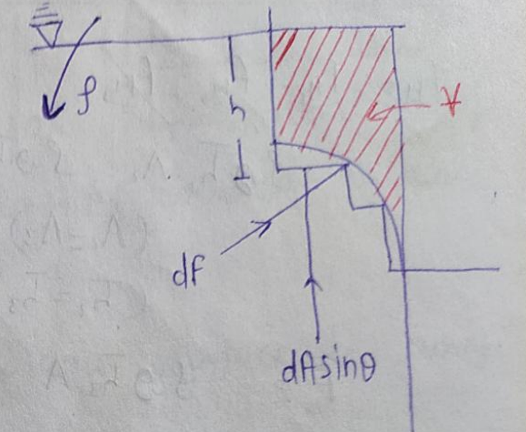
$\forall$ = volume above the curve surface upto free surface level.

$F_V = \rho g \forall$

F_V = wt. of liquid above the curved surface upto free surface

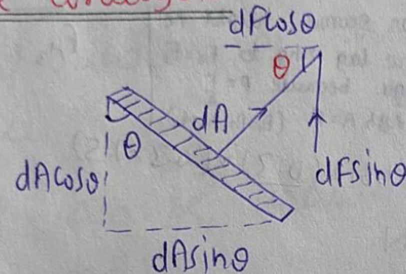
① Location of F_V :-

the line of action of F_V passes to the C.G.S of volume above the curve surface upto free surface level



$$F_V = \rho . g \forall$$

Force analysis

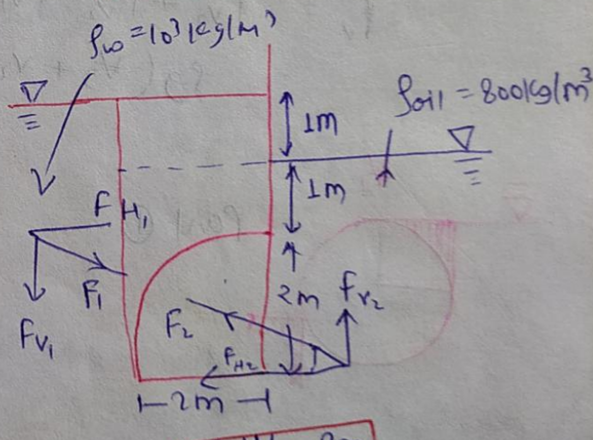


In vertical dirn

$$F_V = \int dF \sin\theta = \int P . dA \sin\theta = \rho g \underbrace{\int h . dA . \sin\theta}_{\forall}$$

$$\boxed{F_V = \rho g \forall}$$

Prob (51)



$\boxed{\text{width} = 3m}$

In Horizontal dirn

$$F_H = F_{H_1} - F_{H_2}$$

$$\left(\frac{2}{2}+1\right)+1)$$

$$= (10)^3 \times (9.81)(2+1).(2\times3)$$

$$- 800(9.81)(1+1)(2\times3) \rightarrow \left(\frac{2}{2}+1\right)$$

$$= 8240 \overrightarrow{KN}$$

In vertical dirn

$$\downarrow F_V = F_{V_1} - F_{V_2}$$

$$= (10)^3(9.81)\forall_1 - (800)(9.81)\forall_2$$

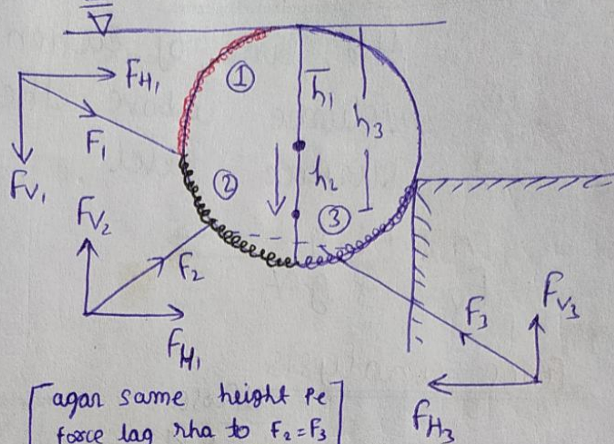
$$= 75.69 KN$$

$$\forall_1 = \left(4\times2 - \frac{\pi(2)^2}{4}\right)\times3 = 14.58 m^3$$

$$\forall_2 = \left(3\times2 - \frac{\pi(2)^2}{4}\right)\times3 = 8.58 m^3$$

(Pb) Radius = 0.3 m

length = 1.5 m



In Horizontal dirn

$$F_H = F_{H_1} + \cancel{F_{H_2}} - \cancel{F_{H_3}}$$

$$\rho g \bar{h}_2 . A_2 \quad \rho g \bar{h}_3 A_3$$

$$(A_2 = A_3)$$

$$(\bar{h}_2 = \bar{h}_3)$$

[agar same height pe force lag rha to $F_2 = F_3$ haga because $P = \frac{F}{A}$, $\rho g h . A = F$ $(A_1 = A_2, h_1 = h_2)$]

$$\vec{F}_h = \rho g \bar{h}_1 . A = (10)^3 \times (9.81)\left(\frac{0.3}{2}\right) \times (0.3 \times 1.5)$$

$$= 622.18\ N$$

In vertical dirn :-

$$\vec{F}_v = F_{V_3} + F_{V_2} - F_{V_1}$$

$$= \rho g (V_3 + V_2 - V_1) ,$$

$$V_2 - V_1 = \frac{\pi (0.3)^2 \times 1.5}{2}$$

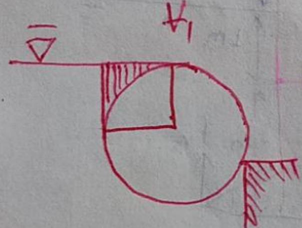
$$= 0.212\ m^3$$

$$V_3 = \left(\frac{\pi (0.3)^2}{4} + (0.3)^2\right) 1.5$$

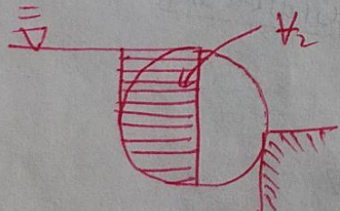
$$= 0.241\ m^3$$

$$\vec{F}_V = (10)^3 (9.81)(0.241 + 0.212)$$

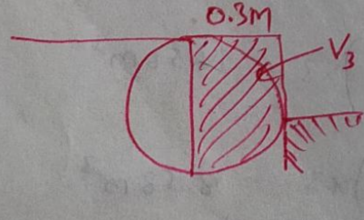
$$= \boxed{4443.93\ N}$$



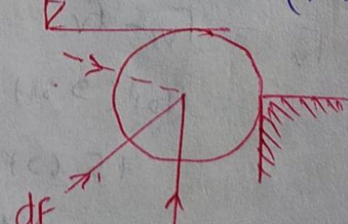
Part ①



Part ②



Part ③



(moment of hydrostatic force about center)

moment = force × ⊥ dist = 0

0

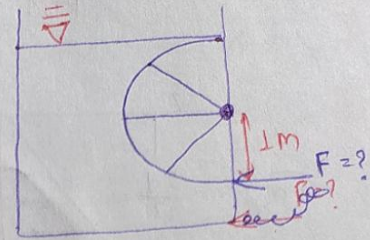
(Pb) (29)

for equilibrium

$\Sigma M = 0$

$F \times 1 = 0$

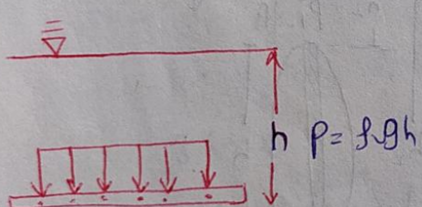
$F = 0$



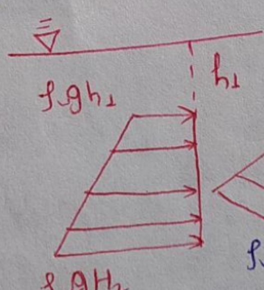
Pressure diagram

is a graphical representation of the variation of pressure intensity over a plane surface

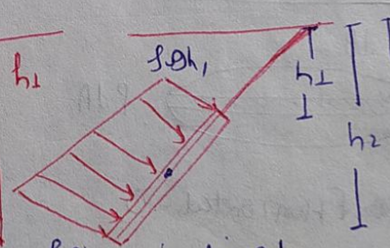
- It can be drawn from plane surface in any orientation.
- total hydrostatic force as well as center of pressure for plane surface may be determined by drawing a pressure diagram



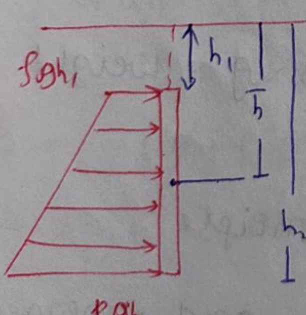
Horizontal plane

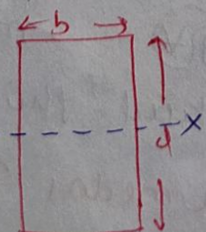


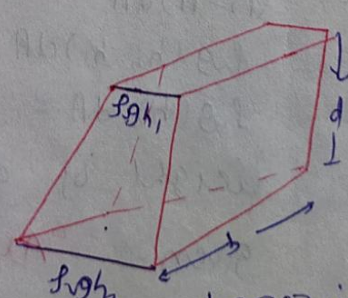
vertical plane



inclined Plane







trapezoidal pism

$\bar{h} = h_1 + \frac{(h_2 - h_1)}{2}$

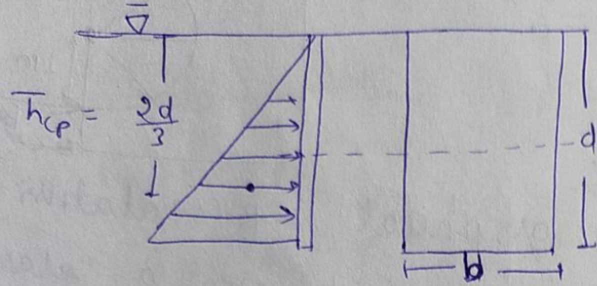
$\bar{h} = \left(\frac{h_1 + h_2}{2}\right)$

$\text{volum}^m = \left(\frac{\rho g h_1 + \rho g h_2}{2}\right) d \cdot b$

$= \rho g \left(\frac{h_1 + h_2}{2}\right)(b \cdot d)$ → Area, → $\bar{h}$

$= \boxed{\rho g \cdot \bar{h} \cdot A}$

For Vertical plane



$$\bar{h}_{cp} = \bar{h} + \frac{I_{xx}\sin^2\theta}{\bar{h}.A}$$

$$= \frac{d}{2} + \frac{\frac{bd^3}{12}\times\sin^2 90}{\frac{d}{2}\times b\times d}$$

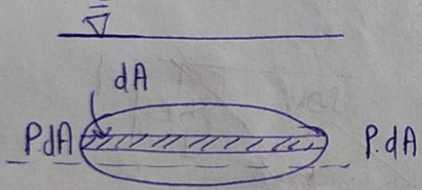
$$= \frac{d}{2} + \frac{\cancel{b}d^{\cancel{3}}\times\cancel{2}}{\cancel{12}_6\, d^{\cancel{2}}\times\cancel{b}}$$

$$= \frac{d}{2} + \frac{d}{6}$$

$$= \left(\frac{2d}{3}\right)$$

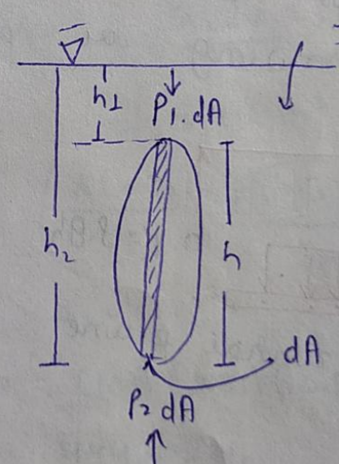
Buoyancy :-

In Horizontal dirn



$F_{Horizontal} = 0$

In vertical dirn



$$\uparrow dF_B = (P_2\,dA - P_1\,dA)$$

$$= (P_2 - P_1)\,dA$$

$$= \rho.g\,(h_2 - h_1)\,dA$$

Inti it, $= \rho.g\,(h).\,dA = \rho g V = \text{mass}\times g = \text{weight}$

Buoyancy force = weight of displaced fluid

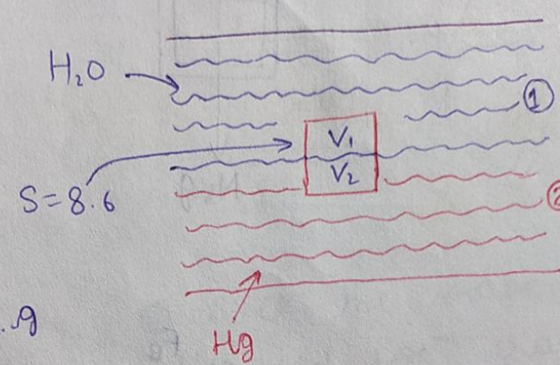
given - "Archimedes Principle"

A.P.- When a body is submerge and emerged fully and partially in a static fluid a net hydro-static force acts on the body in vertical upward dirn

this force is known as upthrust and boyancy force. and the force will equal weight of displaced fluid by a body.

(Pb) Determine,

$$\frac{V_1}{V}\times 100 = ??$$



$$(M_b \cdot g)_{body} = F_{B_1} + F_{B_2}$$

$$\rho_b \cdot \forall \cdot g = \rho_w \forall_1 \cdot g + \rho_{Hg} \cdot \forall_2 \cdot g$$

Divide by ρ_w

$$\frac{\rho_b}{\rho_w} \cdot \forall \cdot g = \forall_1 \cdot g + \frac{\rho_{Hg}}{\rho_w} \forall_2 \cdot g$$

$$\frac{\rho_b}{\rho_w} \forall = V_1 + \frac{\rho_{Hg}}{\rho_w} \forall_2$$

$$8.6 \times \forall = V_1 + 13.6 \times \forall_2$$

$$8.6 \times \forall = V_1 + 13.6 \forall_2$$

$$8.6 (V_1 + V_2) = V_1 + 13.6 \forall_2$$

$$\forall = \forall_1 + \forall_2$$

$$12.6 \forall_1 = 5 \forall$$

$$\frac{\forall_1}{\forall_2} = \frac{5}{12.6}$$

$$\frac{\forall_1}{\forall_2} \times 100 = \frac{5}{12.6} \times 100$$

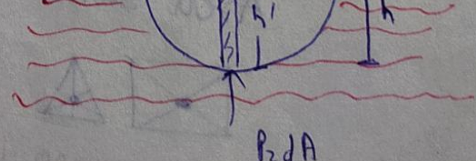
$$= 39.68 \text{ Ans}$$

$$\uparrow d\vec{F}_B = (P_2 - P_1)\, dA$$

$$P_2 = P_1 + \rho_1 g h + \rho_2 g h'$$

$$(\cancel{P_1} + \rho_1 g h + \rho_2 g h' - \cancel{P_1}) \cdot dA$$

$$= \rho_1 g \underbrace{(h \cdot dA)}_{d\forall_1} + \rho_2 g \underbrace{(h' \cdot dA)}_{d\forall_2}$$



Int it

$$\uparrow F_B = \rho_1 g \forall_1 + \rho_2 g \forall_2$$

$$F_{B_1}\uparrow + F_{B_2}\uparrow$$