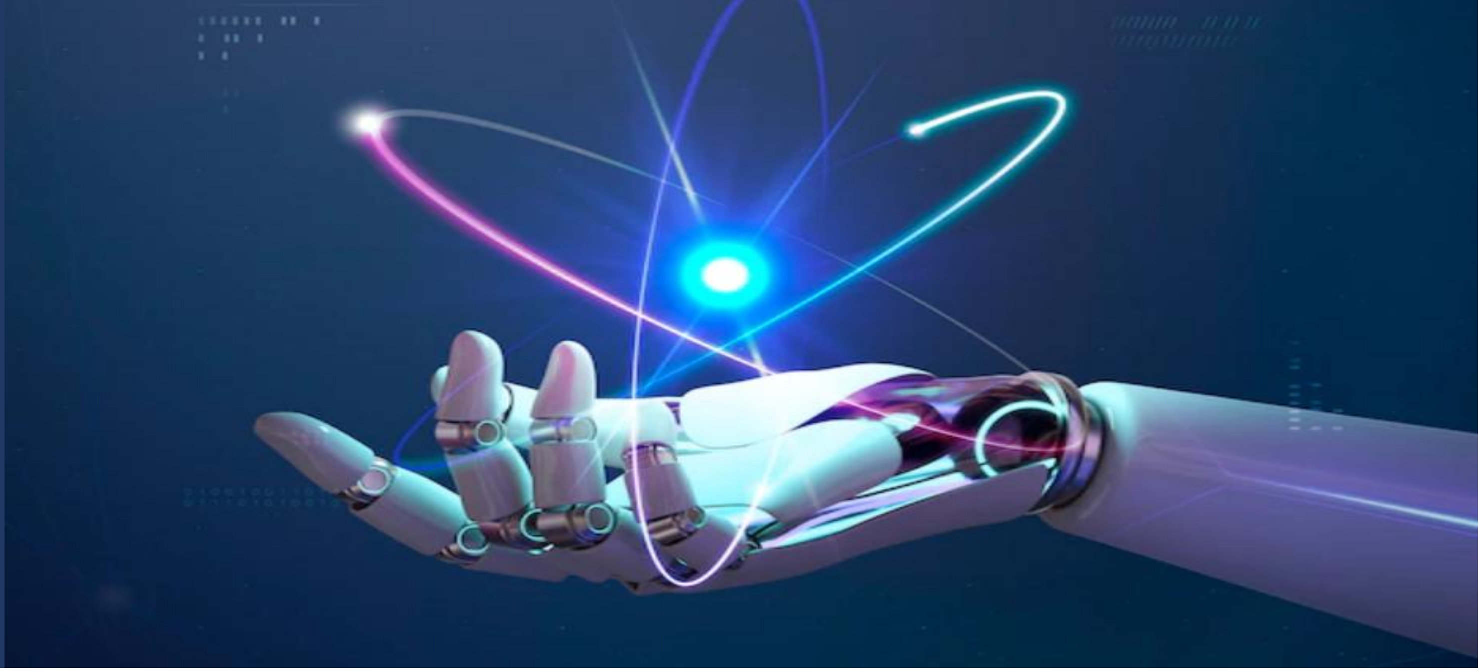


CHAPTER – 13

NUCLEI

In every atom, the positive charge and mass are densely concentrated at the center of the atom forming its nucleus. More than 99.9% mass of the atom is concentrated in the nucleus.

Composition Of Nuclei

the nucleus has two main components: protons and neutrons. The positively-charged entities are protons that are solely present inside the nucleus and neutrons are neutral in charge and do not carry any charge.

Atomic number

The total number of protons present inside a nucleus of an atom is called atomic number. The atomic number is denoted by the letter 'Z'.

Atomic mass

The total combined number of neutrons and protons present inside a nucleus is called atomic mass. While calculating the mass of an atom, the mass of electrons should not be calculated; rather only the mass of neutrons and protons are taken into consideration. This is because the electrons are the lightest particles in a nucleus, and hence their mass is never considered while calculating atomic mass. The atomic mass number is also known as mass number. The atomic mass number is denoted by the letter 'A'.

It is the nearest integer value of mass represented in a.m.u. (atomic mass unit).

1 a.m.u. = $\frac{1}{12}$ [mass of one atom of ${}_6C^{12}$ atom at rest and in ground state]

= 1.6603×10^{-27} kg $\approx$ 931.478 MeV/c^2

mass of proton (m_p) = mass of neutron (m_n) = 1 a.m.u.

No. of Protons, Electrons, nucleons and Neutrons in an Atom:

(a) Number of protons in an atom = Z

(b) Number of electrons in an atom = Z

(c) Number of nucleons in an atom = A

(d) Number of neutrons in an atom = N = A – Z

Isotopes –

The nuclides having the same atomic number (Z) but different mass number (A) are called isotopes.

Isobars-

The nuclides having the same mass number (A), but different atomic number (Z) are called isobars.

Isotones-

The nuclides having the same number of neutrons (A–Z) are called isotones.

Mass – Energy

Einstein showed that mass is another form of energy and one can convert mass-energy into other forms of energy, say kinetic energy and vice-versa. Einstein gave the famous mass-energy equivalence relation $E = mc^2$

Here the energy equivalent of mass m is related by the above equation and c is the velocity of light in vacuum and is approximately equal to 3×10^8 m s

Nuclear Binding Energy

It is the minimum energy required to break the nucleus into its constituent particles.

Or

Amount of energy released during the formation of nucleus by its constituent particles and bringing them from infinite separation.

Binding Energy (B.E.) = Δmc^2
BE = Δm (in amu) × 931 MeV/amu
= Δm × 931 MeV

Note

If binding energy per nucleon is more for a nucleus, then it is more stable.

Example

If $\left(\frac{B.E_1}{A_1}\right) > \left(\frac{B.E_2}{A_2}\right)$

then nucleus 1 would be more stable.

Variation of binding energy per nucleon with mass number:

The binding energy per nucleon first increases on an average and reaches a maximum of about 8.8 MeV for A = 56. For still heavier nuclei, the binding energy per nucleon slowly decreases as **A** increases. Binding energy per nucleon is maximum for $_{26}Fe^{56}$, which is equal to 8.8MeV. Binding energy per nucleon is more for medium nuclei than for heavy nuclei. Hence, medium nuclei are highly stable.

Nuclear Reaction

When a beam of monoenergetic particles (e.g., α-rays, neutrons etc.) collides with a stable nucleus, the original nucleus is converted into a nucleus of new element. This process is called a nuclear reaction. A typical nuclear reaction is

$a + \mathrm{X} \rightarrow \mathrm{Y} + b$

where a is incident energetic particle, X is target nucleus, Y is residual nucleus and b is outgoing particle. This reaction in compact form is expressed as

$X(a, b)Y$

In a nuclear reaction mass number, electric charge, linear momentum, angular momentum and total energy are always conserved. The energy of reaction is

$\mathrm{Q} = (M_a + M_X)c^2 - (M_b + M_Y)c^2$

Nuclear Fission-

The heavier nuclei being unstable have tendency to split into medium nuclei. This process is called **Fission.**

Nuclear Fusion-

The Lighter nuclei being unstable have tendency to fuse into a medium nucleus. This process is called **Fusion.**

Nuclear Force

The nuclear force is a force that acts between the protons and neutrons of atoms.

The nuclear force is the force that binds the protons and neutrons in a nucleus together. This force can exist between protons and protons, neutrons and protons or neutrons and neutrons. This force is what holds the nucleus together. The charge of protons, which is +1e, tends to push them away from each other with a strong electric field repulsive force, following Coulomb's law. But nuclear force is strong enough to keep them together and to overcome that resistance at short range.

Radioactivity

It was discovered by Henry Becquerel. Spontaneous emission of radiations (α, β, γ) from unstable nucleus is called **radioactivity**. Substances which show radioactivity are known as **radioactive substance**. Radioactivity was studied in detail by Rutherford.

Law Of Radioactive Decay

When a radioactive material decays, the number of nuclei decaying per unit time is proportional to the total number of nuclei in the sample material. So,

If N is the total number of nuclei in the sample and ΔN is the number of nuclei that decay in time Δt, then

$\frac{\Delta N}{\Delta t} \propto N$

Or, $\frac{\Delta N}{\Delta t} = \lambda N$

where λ denotes the radioactive decay or disintegration constant. The change in the number of nuclei in the sample is now given by $dN = -\Delta N$ in time Δt. As a result, the rate of change of N (in the limit $\Delta t \rightarrow 0$) is,

$\frac{dN}{dt} = -\lambda N$

Or, $\frac{dN}{N} = -\lambda\, dt$

Now, if we integrate both sides of the above equation, we get,

$\int_{N_0}^{N} \frac{dN}{N} = \lambda \int_{t_0}^{t} dt$

$ln\, N - ln\, N0 = -\lambda(t - t_0)$

Where N_0 denotes the number of radioactive nuclei in the sample at any given time. t_0 is the initial time, and N is the number of radioactive nuclei at any subsequent time t. After that, we set $t_0 = 0$ and rearrange the above equation (3) to get,

$ln\left(\frac{N}{N_0}\right) = -\lambda t$

Or, $N(t) = N_0 e^{-\lambda t}$ ... (4)

This equation is representing the Law of Radioactive Decay.

Radioactive Displacement Laws

(i) When a nuclide emits an α-particle, its mass number is reduced by four and atomic number by two, i.e., ${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He} + \text{Energy}$

(ii) When a nuclide emits a β-particle, its mass number remains unchanged but atomic number increases by one, i.e., ${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\beta + v + \text{Energy}$, where v is the antineutrino.

The β-particles are not present initially in the nucleus but are produced due to the disintegration of neutron into a proton,

i.e., ${}^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{H} + {}^{0}_{-1}\beta + v$ (antineutrion)

When a proton is converted into a neutron, positive β-particle or positron is emitted.

${}^{1}_{1}\mathrm{H} \rightarrow {}^{1}_{0}\mathrm{n} + {}^{0}_{1}\beta + v$ (neutrino)

(iii) When a nuclide emits a gamma photon, neither the atomic number nor the mass number changes.

Half-life and Mean life

The half-life period of a radioactive substance is defined as the time in which one-half of the radioactive substance is disintegrated. If N_0 is the initial number of radioactive atoms present, then in a half life time T, the number of undecayed radioactive atoms will be $N_0/2$ and in next half $N_0/4$ and so on.

That is $t = T$ (half-life), $N = \frac{N_0}{2}$

$\therefore$ From relation $N = N_0 e^{-\lambda T}$

we get, $\frac{N_0}{2} = N_0 e^{-\lambda T}$ or $e^{-\lambda T} = \frac{1}{2}$

From equations (i) and (ii), we get

$$\frac{N}{N_0} = e^{-\lambda t} = \left(\frac{1}{2}\right)^{t/T}$$

Equation (iii) is the basic equation for the solution of half-life problems of radioactive elements.

The half-life T and disintegration constant λ are related as

$$T = \frac{0.6931}{\lambda}$$

The mean life of a radioactive substance is equal to the sum of life time of all atoms divided by the number of all atoms, i.e., Mean life,

$$\tau = \frac{\text{sum of life time of all atoms}}{\text{total number of atoms}} = \frac{1}{\lambda}$$

From equations (iv) and (v), we get

$T = 0.6931\tau$ i.e., $T < \tau$

Activity of Radioactive Substance

The activity of a radioactive substance means the rate of decay (or the number of disintegrations/ sec). This is denoted by

$$A = \left|\frac{dN}{dt}\right| = \left|\frac{d}{dt}\left(N_0 e^{-\lambda}\right)\right| = \lambda N$$

If A_0 is the activity at time $t = 0$, then,

$A_0 = \lambda N_0$

$\therefore \frac{A}{A_0} = \frac{N}{N_0} = e^{-\lambda t}$

i.e., $A = A_0 e^{-\lambda t}$

α-particle

It is a doubly charged helium nucleus. It contains two protons and two neutrons.

Mass of α-particle = Mass of $_2He^4$ atom – $2m_e$ ≈ $4\ m_p$

Charge of α-particle = + 2 e

β-particle

(a) β^- (electron) : Mass = m_e ; Charge = –e

(b) β^+ (positron) : Mass = m_e ; Charge = +e

positron is an antiparticle of electron.

Antiparticle

A particle is called antiparticle of other if on collision both can annihilate (destroy completely) and converts into energy. For example: (i) electron (– e, m_e) and positron (+ e, m_e) are anti particles.

(ii) neutrino (ν) and antineutrino $(\bar{\nu})$ are anti particles.

γ-particle

They are energetic photons of energy of the order of Mev and having rest mass zero.

Q. For the β^+(positron) emission from a nucleus, there is another competing process known as electron capture (electron from inner orbit, say, the K-shell is captured by the nucleus and a neutrino is emitted.

$e^- + {}^A_Z X \longrightarrow {}^A_{Z-1}Y + \nu$

Show that if β^+emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Sol. Consider the two competing processes

Positron emission: $Z{}^X_Z\mathrm{X} \longrightarrow {}^A_{Z-1}\mathrm{Y} + e^+ + \nu + Q_1$ and

Electron capture, $e^- + {}^A_Z\mathrm{X} \longrightarrow {}^A_{Z-1}\mathrm{Y} + \nu + Q_2$

$$Q_1 = \left[m_N\left({}^A_Z\mathrm{X}\right) - m_N\left({}_{Z-1}^{A}\mathrm{Y}\right) - m_e\right]c^2$$

Converting nuclear masses into atomic masses

$$\begin{aligned} Q_1 &= \left[m\left({}^A_Z\mathrm{X}\right) - Zm_e - \{m({}_{Z-1}^{A}\mathrm{Y}) + (\mathrm{Z}-1)m_e\} - m_e\right]c^2 \\ &= \left[m\left({}^A_Z\mathrm{X}\right) - m\left({}^A_{Z-1}\mathrm{Y}\right) - 2m_e\right]c^2 \\ Q_2 &= \left[m_N\left({}^A_Z\mathrm{X}\right) + m_e - m_N({}_{Z-1}^{A}\mathrm{Y})\right]c^2 \\ &= \left[m\left({}^A_Z\mathrm{X}\right) - m_e + m_e - \{m({}_{Z-1}^{A}\mathrm{Y}) + (\mathrm{Z}-1)m_e\}\right]c^2 \\ &= \left[m\left({}^A_Z\mathrm{X}\right) - m({}_{Z-1}^{A}\mathrm{Y})\right]c^2 \end{aligned}$$

This means that $Q_1 > 0$ implies $Q_2 > 0$; but $Q_2 > 0$ does not necessarily imply $Q_1 > 0$. Thus if β^+emission is energetically allowed, electron capture is necessarily allowed, but not vice-versa.

Q. The fission properties of $^{239}{}_{94}\mathrm{Pu}$ are very similar to those of $^{235}{}_{92}\mathrm{U}$. The average energy released per fission is 180MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}\mathrm{Pu}$ undergo fission?

Sol. Average energy released per fission of ${}^{239}_{94}\mathrm{Pu}$, $\mathrm{E_{av}} = 180\mathrm{MeV}$

Amount of pure $^{239}{}_{94}\mathrm{Pu}$, $\mathrm{m} = 1\ \mathrm{kg} = 1000$ grams

Avogadro's number $= \mathrm{N_A} = 6.023 \times 10^{23}$

Mass number of $^{239}{}_{94}\mathrm{Pu} = 239$

Now, 1 mole of $^{239}{}_{94}\mathrm{Pu}$ contains $\mathrm{N_A}$ atoms. Therefore, ' m ' grams of $^{239}{}_{94}\mathrm{Pu}$ will contain $\{(\mathrm{N_A}/\text{Mass number}) \times \mathrm{m}\}$ number of atoms, $= [(6.023 \times 10^{23})/239] \times 1000 = 2.52 \times 10^{24}$ atoms.

Hence, total energy released during the fission of 1 kg of $^{239}{}_{94}\mathrm{Pu}$ is:

$E = E_{av} \times 2.52 \times 10^{24}$

$= 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26}\mathrm{MeV}$

Mass Defect

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$$M_{expected} = Z\,m_p + (A - Z)m_n$$

$$M_{observed} = M_{atom} - Zm_e$$

It is found that

$$M_{observed} < M_{expected}$$

Hence, mass defect is defined as

Mass defect = $M_{expected} - M_{observed}$

$$\Delta m = [Zm_p + (A - Z)m_n] - [M_{atom} - Zm_e]$$

Nuclear Reactor

When $^{235}{}_{92}U$ undergoes a fission after being bombarded by a neutron, it splits into two nuclei and releases a neutron. This extra neutron now initiates the fission of another $^{235}{}_{92}U$ nucleus. For that matter, 2.5 neutrons are released per fission of a uranium nucleus. Also, fission produces more neutrons than what can be consumed.

This increases the chances of a chain reaction with each neutron that is produced, triggering another fission. If this chain reaction is uncontrolled, then it can lead to destruction (like a nuclear bomb). On the other hand, in a controlled manner, it can be harnessed to generate electric power. However, there was a small problem. The neutrons generated in fission were highly energetic. They would escape rather than trigger another fission reaction. Also, it was observed that slow neutrons have a higher possibility of inducing fission in $^{235}{}_{92}U$ than their faster counterparts.

Now, the energy of a neutron produced in fission of $^{235}{}_{92}U$ is around 2 MeV. Unless these neutrons are slowed down, they tend to escape without inducing fission. This simply means that we need a lot of fissionable material to sustain the chain reaction.

Nuclear fusion – energy generation in stars

It has been calculated that the sun radiates energy at the rate of about 1026 J per second. The sun is radiating at this rate for several millions of years. The sources of energy of the sun cannot be the chemical reactions because the energy released in chemical reactions cannot last so long. It has also been found that hydrogen and helium constitute about 90% of the mass of the sun and 10% are other elements. Since heavy elements present in the sun are very small in quantity, so the source of energy of the sun cannot be nuclear fission. Fusion reactions are the source of energy in the sun and the stars, inside which the temperature is of the order of 10^7 -10^8K. The basic energy-producing process in the sun is the fusion of hydrogen nuclei and the same is true for many other stars. The sequence of nuclear reactions is as follows:

$${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + Q_1(\text{MeV})$$

$${}_7N^{13} \rightarrow {}_6C^{13} + {}_1e^0 + v$$

$${}_6C^{13}\ {}_1H^1 \rightarrow {}_7N^{14} + Q_2(\text{MeV})$$

$${}_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15} + Q_3(\text{MeV})$$

$${}_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + v$$

$${}_7N^{15} + {}_1H^1 \rightarrow {}_6C^{12} + {}_2He^4$$

Total energy released in this cycle = $\mathbf{24.68}$MeV

The above combinations give:

$${}_4H^1 \rightarrow_2 He^4 + 2_1e^0 + 2v + 2Y + 24.68\text{MeV}$$

The initial ${}_6C^{12}$ acts as a kind of catalyst for the process since it reappears at its end. The above stated thermo-nuclear reactions take place in the sun and other stars and hence they are the source of energy in the solar system. Thus, energy released by fusion is greater than the energy released by fission.

DID YOU KNOW?

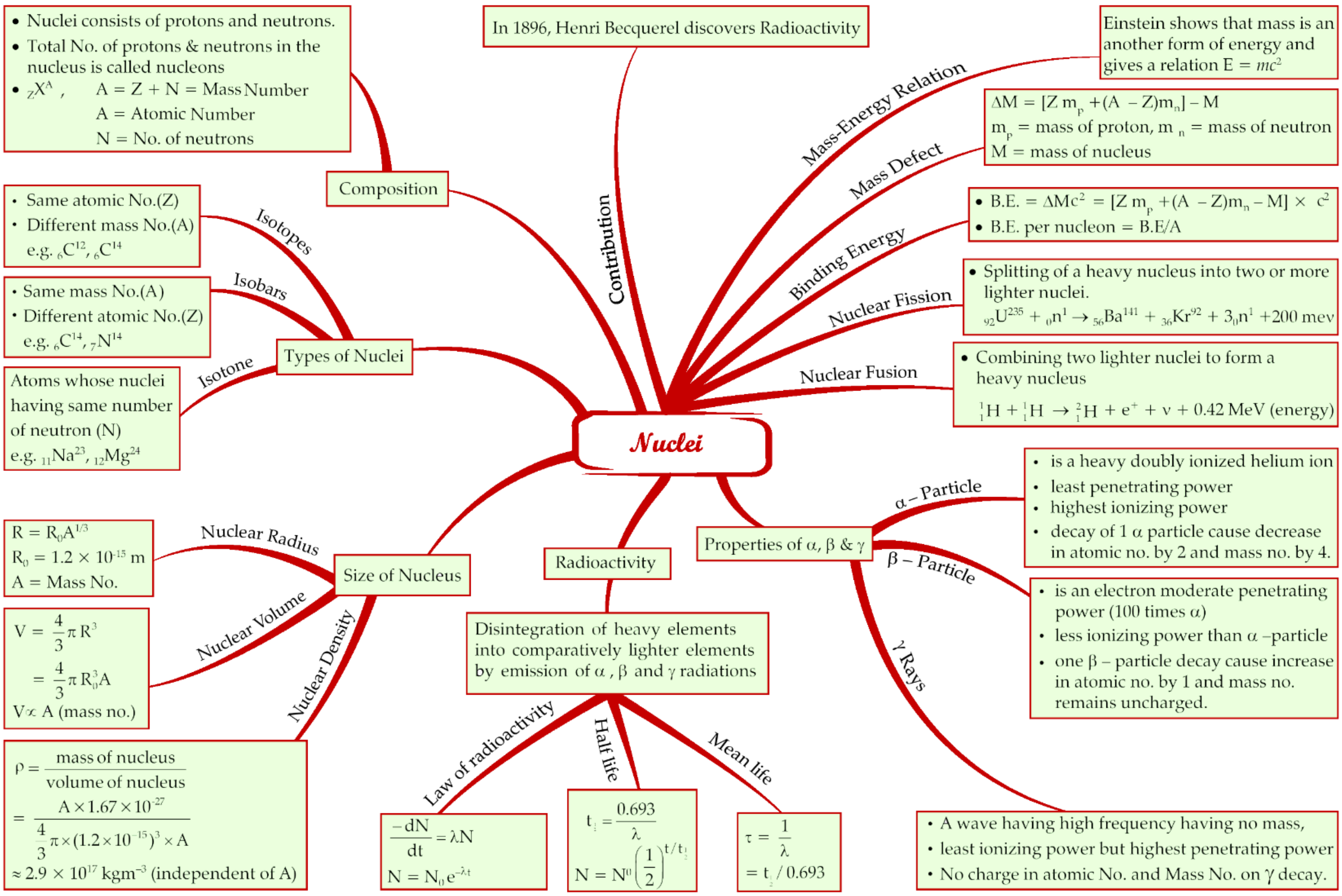
Because of very high temperature ($\sim 10^7$) needed for nuclear fusion cannot be attained by any known method in the laboratory.

SUMMARY

- **Atomic Number:**
The number of protons in the nucleus is called the atomic number. It is denoted by Z.
- **Mass number:**
The total number of protons and neutrons present in a nucleus is called the mass number of the element. It is denoted by A.
- **No. of Protons, Electrons, nucleons and Neutrons in an Atom:**
(a) Number of protons in an atom = Z
(b) Number of electrons in an atom = Z
(c) Number of nucleons in an atom = A
(d) Number of neutrons in an atom = N = A – Z.
- **Nuclear Mass:**
The total mass of the protons and neutrons present in a nucleus is called the nuclear mass.
- **Nuclide:**
A nuclide is a specific nucleus of an atom characterized by its atomic number Z and mass number A. It is represented as, $_ZA^A$
Where X = chemical symbol of the element, Z = atomic number and A = mass number
- **Isotopes:**
(a) The atoms of an element which have the same atomic number but different mass number are called isotopes.
(b) Isotopes have similar chemical properties but different physical properties.
- **Isobars:**
The atoms having the same mass number but different atomic number are called isobars.
- **Isotones:**
The nuclides having the same number of neutrons are called isotones.
- **Isomers:**
These are nuclei with same atomic number and same mass number but in different energy states.
- **Electron Volt:**
It is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt and is denoted by eV.
- **Atomic Mass Unit:**
(a) It is $\frac{1}{12}$ th of the actual mass of a carbon atom of isotope $_6C^{12}$. It is denoted by amu or just by u.
(b) 1 amu = 1.660565 X 10-27 kg
(c) The energy equivalence of 1 amu is 1 amu = 931 MeV
- **Discovery of Neutrons:**
(a) Neutrons were discovered by Chadwick in 1932.
(b) When beryllium nuclei are bombarded by alpha-particles, highly penetrating radiations are emitted, which consists of neutral particles, each having mass nearly that of a proton. These particles were called neutrons.
$$_2^4He + _4^9Be \rightarrow _0^1n + _1^{12}C$$
(c) A free neutron decays spontaneously, with a half- life of about 900 s, into a proton, electron and an antineutrino.
$$_0^1n \rightarrow _1^1H + _{-1}^0e + \bar{v}$$
- **Size of the Nucleus:**
(a) It is found that a nucleus of mass number A has a radius
a. R = R0A1/3
Where, R0 = 1.2 x 10-15 m
(b) This implies that the volume of the nucleus, which is proportional to R^3 is proportional A.
- **Density of the Nucleus:**
Density of nucleus is constant; independent of A, for all nuclei and density of nuclear matter is approximately 2.3 x 10^{17} *kg* m^{-3} which is very large as compared to ordinary matter, say water which is 10^3 kg m^{-3}.
- **Mass-Energy equivalence:**
Einstein proved that it is necessary to treat mass as another form of energy. He gave the mass-energy equivalence relation as,
$E = mc^2$
Where m is the mass and c is the velocity of light in vacuum.
- **Mass Defect:**
The difference between the rest mass of a nucleus and the sum of the rest masses of its constituent nucleons is called its mass defect. It is given by,
$\Delta m = [Zm_p + (A - Z)m_n] - m$
- **Binding Energy:**
(a) It may be defined as the energy required to break a nucleus into its constituent protons and neutrons and to separate them to such a large distance that they may not interact with each other.
(b) It may also be defined as the surplus energy which the nucleus gives up by virtue of their attractions which they become bound together to form a nucleus.
(c) The binding energy of a nucleus $_ZX^A$ is,
$B.E. = [Zm_p + (A - Z)m_n - m]c^2$
- **Binding Energy per Nucleon:**
It is average energy required to extract one nucleon from the nucleus.
It is obtained by dividing the binding energy of a nucleus by its mass number.
$\bar{B} = \frac{B.E}{A} = \frac{[Zm_p + (A-Z)m_n -]c^2}{A}$
- **Nuclear Forces:**
(a) These are the strong in attractive forces which hold protons and neutrons together in a tiny nucleus.
(b) These are short range forces which operate over very short distance of about 2 – 3 fm of separation between any two nucleons.
(c) The nuclear force does not depend on the charge of the nucleon.

- **Nuclear Density:**
The density of a nucleus is independent of the size of the nucleus and is given by,
$\rho_v = \frac{\text{Nuclear mass}}{\text{Nuclear volume}}$
$= \frac{mv}{\frac{4}{3}\pi R^2} = 2.9x10^{17}\text{kg}m^{-3}$
- **Radioactivity:**
(a) It is the phenomenon of spontaneous disintegration of the nucleus of an atom with the emission of one or more radiations like α -particles, β -particles or γ -rays.
(b) The substances which spontaneously emit penetrating radiation are called radioactive substances.
- **Radioactivity Displacement Law:**
It states that,
(a) When a radioactive nucleus emits an α -particle, atomic number decreases by 2 and mass number decreases by 4.
(b) When a radioactive nucleus emits β -particle, its atomic number increases by 1 but mass number remains same.
(c) The emission of a γ -particle does not change the mass number or the atomic number of the radioactive nucleus. The γ -particle emission by a radioactive nucleus lowers its energy state.
- **Alpha Decay:**
It is the process of emission of an α -particle from a radioactive nucleus. It may be represented as,
$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He + Q$
- **Beta Decay:**
It is the process of emission of an electron from a radioactive nucleus. It may be represented as,
$$^{A}_{Z}X \rightarrow _{Z+1}^{A}Y + _{-1}^{0}e + \bar{v}$$
- **Gamma Decay:**
It is the process of emission of a γ -ray photon during the radioactive disintegration of a nucleus. It can be represented as,
$^{A}_{Z}X \rightarrow ^{A}_{Z}X + \lambda$
(Excited State) (Ground State)
- **Radioactive Decay Law:**
It states that the number of nuclei disintegrated of undecayed radioactive nuclei present at that instant. It may be written as,
$N(t) = N(0)e^{-\lambda t}$
Where N(0) is the number of nuclei at t = 0 and λ is disintegration constant.
- **Decay or disintegration Constant:**
It may be defined as the reciprocal or the time interval in which the number of active nuclei in a given radioactive sample reduces to 36.8% of its initial value.
- **Half-life:**
The half-life of a radioactive substance is the time in which one-half of its nuclei will disintegrate. It is inversely proportional to the decay constant of the radioactive substance.
$T_{1/2} = \frac{0.693}{\lambda}$
- **Mean Life:**
The mean-life of a radioactive sample is defined as the ratio of the combined age of all the atoms and the total number of atoms in the given sample. It is given by,
$\tau = \frac{T_{1/2}}{0.693} = 1.44T_{1/2}$
- **Rate of Decay or Activity of a Radioactive Sample:**
It is defined as the number of radioactive disintegrations taking place per second in a given sample. It is expressed as,
$$R(t) = \left[\frac{dN}{dt}\right] = \lambda N(t) = \lambda N(0)e^{-\lambda t}$$
- **Curie:**
(a) It is the SI unit of decay.
(b) One curie is the decay rate of 3.7 X 1010 disintegrations per second.
- **Rutherford:**
One Rutherford is the decay rate of 10^6 disintegrations per second.
- **Natural Radioactivity:**
It is the phenomenon of the spontaneous emission of-α, β and γ radiations from the nuclei of naturally occurring isotopes.
- **Artificial or Induced Radioactivity:**
It is the phenomenon of inducing radioactivity in certain stable nuclei by bombarding them by suitable high energy sub atomic particles.
- **Nuclear Reaction:**
It is a reaction which involves the change of stable nuclei of one element into the nucleus of another element.
- **Nuclear Fission:**
It is the process in which a heavy nucleus when excited gets split into two smaller nuclei of nearly comparable masses. For example,
$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n + Q$$
- **Nuclear Reactor:**
It is a device in which a nuclear chain reaction is initiated, maintained and controlled.
- **Nuclear Fusion:**
It is the process of fusion of two smaller nuclei into a heavier nucleus with the liberation of large amount of energy.

MIND MAP



Nuclei
• Nuclei consists of protons and neutrons.
• Total No. of protons & neutrons in the nucleus is called nucleons
• $_ZX^A$, $A = Z + N$ = Mass Number
A = Atomic Number
N = No. of neutrons
Composition
In 1896, Henri Becquerel discovers Radioactivity
Contribution
Einstein shows that mass is an another form of energy and gives a relation $E = mc^2$
Mass-Energy Relation
$\Delta M = [Z m_p + (A - Z)m_n] - M$
m_p = mass of proton, m_n = mass of neutron
M = mass of nucleus
Mass Defect
• B.E. $= \Delta Mc^2 = [Z m_p + (A - Z)m_n - M] \times c^2$
• B.E. per nucleon = B.E/A
Binding Energy
• Splitting of a heavy nucleus into two or more lighter nuclei.
$_{92}U^{235} + {_0n^1} \rightarrow {_{56}Ba^{141}} + {_{36}Kr^{92}} + 3{_0n^1} + 200$ mev
Nuclear Fission
• Combining two lighter nuclei to form a heavy nucleus
$_1^1H + {_1^1H} \rightarrow {_1^2H} + e^+ + \nu + 0.42$ MeV (energy)
Nuclear Fusion
Types of Nuclei
Isotopes
• Same atomic No.(Z)
• Different mass No.(A)
e.g. $_6C^{12}$, $_6C^{14}$
Isobars
• Same mass No.(A)
• Different atomic No.(Z)
e.g. $_6C^{14}$, $_7N^{14}$
Isotone
Atoms whose nuclei having same number of neutron (N)
e.g. $_{11}Na^{23}$, $_{12}Mg^{24}$
Size of Nucleus
Nuclear Radius
$R = R_0A^{1/3}$
$R_0 = 1.2 \times 10^{-15}$ m
A = Mass No.
Nuclear Volume
$V = \frac{4}{3}\pi R^3$
$= \frac{4}{3}\pi R_0^3 A$
$V \propto A$ (mass no.)
Nuclear Density
$\rho = \frac{\text{mass of nucleus}}{\text{volume of nucleus}}$
$= \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi \times (1.2 \times 10^{-15})^3 \times A}$
$\approx 2.9 \times 10^{17}$ kgm^{-3} (independent of A)
Radioactivity
Disintegration of heavy elements into comparatively lighter elements by emission of α, β and γ radiations
Law of radioactivity
$\frac{-dN}{dt} = \lambda N$
$N = N_0 e^{-\lambda t}$
Half life
$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$
$N = N^0 \left(\frac{1}{2}\right)^{t/t_{\frac{1}{2}}}$
Mean life
$\tau = \frac{1}{\lambda}$
$= t_{\frac{1}{2}} / 0.693$
Properties of α, β & γ
α – Particle
• is a heavy doubly ionized helium ion
• least penetrating power
• highest ionizing power
• decay of 1 α particle cause decrease in atomic no. by 2 and mass no. by 4.
β – Particle
• is an electron moderate penetrating power (100 times α)
• less ionizing power than α –particle
• one β – particle decay cause increase in atomic no. by 1 and mass no. remains uncharged.
γ Rays
• A wave having high frequency having no mass,
• least ionizing power but highest penetrating power
• No charge in atomic No. and Mass No. on γ decay.

PRACTICE EXERCISE

MCQ

Q1. The 'rad' is the correct unit used to report the measurement of
(a) the ability of a beam of gamma ray photons to produce ions in a target
(b) the energy delivered by radiation to a target
(c) the biological effect of radiation
(d) the rate of decay of a radioactive source

Q2. Half-life of radioactive element depends upon
(a) amount of element present
(b) temperature
(c) pressure
(d) nature of element

Q3. Control rods used in nuclear reactors are made of
(a) stainless steel (b) graphite
(c) cadmium (d) plutonium

Q4. Boron rods in a nuclear reactor are used to
(a) absorb excess neutrons
(b) absorb alpha particle
(c) slow down the reaction
(d) speed up the reaction

Q5. A moderator is used in nuclear reactors in order to
(a) slow down the speed of the neutrons
(b) accelerate the neutrons
(c) increase the number of neutrons
(d) decrease the number of neutrons

Q6. Fusion reactions take place at high temperature because (a) atoms are ionised at high temperature (b) molecules break up at high temperature (c) nuclei break up at high temperature (d) kinetic energy is high enough to overcome repulsion between nuclei

Q7. Neutron decay in free space is given as follows
${}_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + []$
Then the parenthesis [ ] represents a
(a) neutrino (b) photon
(c) antineutrino (d) graviton

Q8. Radioactivity is
(a) irreversible process
(b) self-disintegration process
(c) spontaneous
(d) all of the above

Q9. γ-rays are deflected by
(a) an electric field but not by a magnetic field
(b) a magnetic field but not by an electric field
(c) both electric and magnetic field
(d) neither by electric field nor by magnetic field

Q10. The element gold has
(a) 16 isotopes (b) 32 isotopes
(c) 96 isotopes (d) 173 isotopes

Q11. The nuclear radius is of the order of
(a) 10^{-10} m (b) 10^{-6} m
(c) 10^{-15} m (d) 10^{-14} m

Q12. A nucleus represented by the symbol ${}_Z^AX$ has
(a) Z neutrons and $A - Z$ protons
(b) Z protons and $A - Z$ neutrons
(c) Z protons and A neutrons
(d) A protons and $Z - A$ neutrons

Q13. The energy required to break one bond in DNA is 10^{-20} J. This value in eV is nearly
(a) 6 (b) 0.6
(c) 0.06 (d) 0.006

Q14. The half-life of radium is about 1600 years. If 100 g of radium existing now, 25 g will remain unchanged after
(a) 4800 years (b) 6400 years
(c) 2400 years (d) 3200 years

Q15. A sample of radioactive element has a mass of 10 g at an instant $t = 0$. The approximate mass of this element in the sample after two mean lives is
(a) 1.35 g (b) 2.50 g
(c) 3.70 g (d) 6.30 g

ASSERTION AND REASONING

Directions: Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct

Q1. Assertion: Neutrons penetrate matter more readily as compared to protons.
Reason: Neutrons are slightly more massive than protons.

Q2. Assertion: The mass number of a nucleus is always less than its atomic number.
Reason: Mass number of a nucleus may be equal to its atomic number.

Q3. Assertion: The binding energy per nucleon, for nuclei with atomic mass number A > 100, decrease with A.
Reason: The forces are weak for heavier nuclei.

Q4. Assertion: The heavier nuclei tend to have larger N/Z ratio because neutron does not exert electric force.
Reason: Coulomb forces have longer range compared to the nuclear force.

Q5. Assertion: Radioactive nuclei emit β^{-1} particles.
Reason: Electrons exist inside the nucleus.

SHORT ANSWER QUESTIONS

Q1. Why is it found experimentally difficult to detect neutrinos in nuclear β-decay?

Q2. Write two characteristic features of nuclear force which distinguish it from Coulomb's force.

Q3. The radioactive isotope D-decays according to the sequence.

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, what is the
(i) mass number
(ii) atomic number of D ?
Ans. The sequence is represented as ZAD⟶α Z−2A−4D1⟶β− Z−1A−4D2

Q4. What is the nuclear radius of ^{125}Fe, if that of ^{27}Al is 3.6 fermi?

Q5. Two nuclei have mass numbers in the ratio 1: 2. What is the ratio of their nuclei densities?

NUMERICAL TYPE QUESTIONS

Q1. A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6MeV is split into two fragments Y and Z of mass numbers 110 and 130. The binding energy per nucleon in Y and Z is 8.5 MeV per nucleon. Calculate the energy Q released per fission in MeV.

Q2. When four hydrogen nuclei combine to form a helium nucleus, estimate the amount of energy in MeV released in this process of fusion. (Neglect the masses of electrons and neutrinos) Given:
(i) mass of ${}_1^1\mathrm{H} = 1.007825\mathrm{u}$
(ii) mass of helium nucleus $= 4.002603\mathrm{u}, 1\mathrm{u} = 931\mathrm{MeV/c^2}$

Q3. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}_{29}^{63}$Cu atoms (of mass 62.92960u). The masses of proton and neutrons are 1.00783 u and 1.00867 u respectively.

Q4. A 1000 MW fission reactor consumes half of its fuel in 5 years. How much ${}_{92}^{235}$U did it contail initially? Assume that the reactor operates 80% of the time and that all energy generated arise from the fission of ${}_{92}^{235}$U and that this nuclide is consumed only by the fission process. Energ. generated per fission of ${}_{92}^{235}$U is 200MeV.

Q5. How long an electric lamp of 100 W can be kept glowing by fusion of 2.0 kg of deuterium? The fusion reaction can be taken as:

$${}_1^2\mathrm{H} + {}_1^2\mathrm{H} \longrightarrow {}_2^3\mathrm{He} + n + 3.2\mathrm{MeV}$$

Q6. The count rate of a Geiger Muller counter for the radiation of a radioactive material of half-life of 30 minutes decreases to 5 second^{-1} after 2 hours. The initial count rate was

Q7. Half-life of a radioactive element is 12.5 hours and its quantity is 256 g. After how much time its quantity will remain 1 g?

Q8. A certain mass of Hydrogen is changed to Helium by the process of fusion. The mass defect in fusion reaction is 0.02866 u. Then determine the energy liberated per u.
(Given 1 u = 931 MeV)

Q9. What is the respective number of α and β particles emitted in the following radioactive decay?
$^{200}X_{90} \rightarrow {}^{168}Y_{80}$

Q10. The binding energy per nucleon of ${}_3^7$Li and ${}_2^4$He nuclei are 5.60 MeV and 7.06 MeV respectively. In the nuclear reaction
${}_3^7\mathrm{Li} + {}_1^1\mathrm{H} \longrightarrow {}_2^4\mathrm{He} + {}_2^4\mathrm{He} + Q$
Then determine the value of energy Q

HOMEWORK EXERCISE

MCQ

Q1. If radius of the $^{27}_{13}Al$ nucleus is estimated to be 3.6 fermi, then the radius of $^{125}_{52}Te$ nucleus be nearly
(a) 8 fermi (b) 6 fermi
(c) 5 fermi (d) 4 fermi

Q2. The mass of a $^{7}_{3}Li$ nucleus is 0.042u less than the sum of the masses of all its nucleons. The binding energy per nucleon of $^{7}_{3}Li$ nucleus is nearly
(a) 46MeV(b) 5.6McV
(c) 3.9MeV(d) 23MeV

Q3. The mass defect in a particular nuclear reaction is 0.3 grams. The amount of energy liberated in kilowatt hour is (Velocity of light $= 3 \times 10^8$ m/s)
(a) 1.5×10^6(b) 2.5×10^6
(c) 3×10^6(d) 7.5×10^6

Q4. Energy equivalent of 2 g of a substance is
(a) 18×10^{13} mJ(b) 18×10^{13} J
(c) 9×10^{13} mJ(d) 9×10^{13}

Q5. If M_O is the mass of an oxygen isotope g_g^{17}, M_P and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is
(a) $(M_o - 17M_N)c^2$(b) $\left(M_o - 8M_p\right)c^2$
(c) $(M_o - 8M_P - 9M_N)c^2$(d) M_oc^2

Q6. Mass defect of helium (2 He^4) is [use, mass of proton, $m_p = 1.0072676$ u, mass of neutrons, $m_n = 1.008665\ u$ and mass of $_2He^4 = 4.001506\ u$
(a) 0.016767 u (b) 1.00726 u
(c) 2.00686 u (d) 0.0303592 u

Q7. α-particle consists of
(a) 2 protons only
(b) 2 protons and 2 neutrons only
(c) 2 electrons, 2 protons and 2 neutrons
(d) 2 electrons and 4 protons only

Q8. The half-life of a radioactive nucleus is 50 days. The time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and the time t_1 when $\frac{1}{3}$ of it had decayed is
(a) 30 days (b) 50 days
(c) 60 days (d) 15 days

Q9. Two radioactive nuclei P and Q, in a given sample decay into a stable nucleus R. At time t = 0, number of P species are 4 N_0 and that of Q are N_0. Half-life of P (for conversion to R) is 1 minute where as that of Q is 2 minutes. Initially there are no nuclei of R present in the sample. When number of nuclei of P and Q are equal, the number of nuclei of R present in the sample would be
(a) 2 N_0 (b) 3 N_0
(c) $\frac{9N_0}{2}$ (d) $\frac{5N_0}{2}$

Q10. The nucleus $_6C^{12}$ absorbs an energetic neutron and emits a beta particle (β). The resulting nucleus is
(a) $_7N^{14}$ (b) $_7N^{13}$
(c) $_5B^{13}$ (d) $_6C^{13}$

Q11. An element A decays into element C by a two step processes
$A \rightarrow B + {}_2He^4; B \rightarrow C + 2e^-$, Then
(a) A and C are isotopes (b) A and C are isobars
(c) A and B are isotopes (d) A and B are isobars.

Q12. If in a nuclear fusion process the masses of the fusing nuclei be m_1 and m_2 and the mass of the resultant nucleus be m_3, then
(a) $m_3 = m_1 + m_2$ (b) $m_3 = |m_1 - m_2|$
(c) $m_3 < (m_1 + m_2)$ (d) $m_3 > (m_1 + m_2)$

Q13. Which of the following is used as a moderator in nuclear reaction?
(a) Cadmium (b) Plutonium
(c) Uranium (d) Heavy water

Q14. Fission of nuclei is possible because the binding energy per nucleon in them
(a) increases with mass number at low mass numbers
(b) decreases with mass number at low mass numbers
(c) increases with mass number at high mass numbers
(d) decreases with mass number at high mass numbers.

Q15. Atomic weight of Boron is 10.81 and it has two isotopes $_5B^{10}$ and $_5B^{11}$. Then the ratio of $_5B^{10}$: $_5B^{11}$ in nature would be
(a) 15: 16 (b) 10: 11
(c) 19: 81 (d) 81: 19

ASSERTION AND REASONING

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(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct

Q1. Assertion: Cobalt-60 is useful in cancer therapy.
Reason: Cobalt -60 is source of J- radiations capable of killing cancerous cells.

Q2. Assertion: It is not possible to use ^{35}Cl as the fuel for fusion energy.
Reason: The binding energy of ^{35}Cl is too small.

Q3. Assertion: Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion and
Reason: For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

SHORT ANSWER QUESTIONS

Q1. (I) Write the basic nuclear process involved in the emission of β^{+}in a symbolic form, by a radioactive nucleus.
(ii) In the reactions given below:
(a) 611C $\longrightarrow {}_{y}^{z}\mathbf{B} + x + v$
(b) ${}_{6}^{12}\mathrm{C} + {}_{6}^{12}\mathrm{C} \longrightarrow {}_{a}^{20}\mathrm{Ne} + {}_{b}^{c}\mathrm{He}$
Find the values of x, y, z and a, b, c.

Q2. State three properties of nuclear forces.

Q3. Calculate the energy in fusion reaction:
${}_{1}^{2}\mathrm{H} + {}_{1}^{2}\mathrm{H} \longrightarrow {}_{2}^{3}\mathrm{He} + \mathrm{n}$, where BE of ${}_{1}^{2}\mathrm{H} = 2.23\mathrm{MeV}$ and of ${}_{2}^{3}\mathrm{He} = 7.73\mathrm{MeV}$

Q4. Why do stable nuclei never have more protons than neutrons?

Q5. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two — the parent or the daughter nucleus — would have higher binding energy per nucleon?

NUMERICAL TYPE QUESTIONS

Q1. Chlorine has two isotopes having masses 34.98 u and 36.98 u. with relative abundance of 75.4% and 24.6 %, respectively. Then find the average atomic mass of chlorine.

Q2. If the nuclear radius of ^{27}Al is 3.6 fm, then determine the approximate nuclear radius of ^{64}Cu in fm.

Q3. How much mass has to be converted into energy to produce electric power of 200 MW for one hour?

Q4. A nucleus ruptures into two nuclear parts, which have their velocity ratio equal to 2: 1. What will be the ratio of their nuclear size (nuclear radius)?

Q5. The rate of radioactive disintegration at an instant for a radioactive sample of half-life 2.2×10^{9} s is $10^{10}\ \mathrm{s}^{-1}$. Then calculate the number of radioactive atoms in the sample at that instant.

Q6. A radioactive sample with a half-life of 1 month has the label: 'Activity = 2 micro curies on 1 – 8 – 1991'. What would be its activity two months earlier?

Q7. The half-life of a radioactive isotope 'X' is 20 years. It decays to another element 'Y' which is stable. The two elements 'X' and 'Y' were found to be in the ratio 1: 7 in a sample of a given rock. Then determine the age of the rock.

Q8. Half-life of a radioactive element is 12.5 hours and its quantity is 256 g. After how much time its quantity will remain 1 g?

Q9. The half-life of a radioactive isotope X is 50 years. It decays to another element Y which is stable. The two elements X and Y were found to be in the ratio of 1: 15 in a sample of a given rock. Then calculate the age of the rock.

Q10. A radioactive element has half-life period 800 years. After 6400 years what amount will remain?

PRACTICE EXERCISE SOLUTIONS

MCQ

S1. (c) The risk posed to a human being by any radiation exposure depends partly upon the absorbed dose, the amount of energy absorbed per gram of tissue. Absorbed dose is expressed in rad. A rad is equal to 100 ergs of energy absorbed by 1 gram of tissue. The more modern, internationally adopted unit is the gray (named after the English medical physicist L. H. Gray); one gray equal 100 rad.

S2. (d) Half-life of a substance doesn't depend upon amount, temperature and pressure. It depends upon the nature of the substance.

S3. (c) Control rods are made of cadmium.

S4. (a) Boron rods absorb excess neutrons

S5. (c) Moderator slows down neutrons.

S6. (d) Extremely high temps needed for fusion make K.E. large enough to overcome repulsion between nuclei.

S7. (c) An electron is accompanied by an antineutrino.

S8. (d) All the characteristics given are true for radioactivity

S9. (d) γ-rays carry no charge. They are neither deflected by an electric field nor by a magnetic field.

S10. (b) Gold has 32 isotopes ranging from A = 173 to A = 204

S11. (d) Nuclear radius = 10^{-14} m.

S12. (b) Z is number of protons and A is the total number of protons and neutrons.

S13. (c) Given: energy, $E = 10^{-20}$ J

Now, $1\text{ J} = \frac{1}{1.6\times10^{-19}}$ eV

$\therefore \quad E = \frac{10^{-20}}{1.6\times10^{-19}}\text{ eV} = 0.0625\text{ eV} \simeq 0.06\text{ eV}$

S14. (d) Using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{25}{100} = \left(\frac{1}{2}\right)^n \Rightarrow n = 2$

The total time in which radium change to 25 g is = 2 × 1600 = 3200 yr

S15. (a) At $t = 0, M_0 = 10$ g and $t = 2\tau = 2\left(\frac{1}{\lambda}\right)$

$M = M_0e^{-\lambda t} = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.35$ g

ASSERTION AND REASONING

S1. (b) Both statements are separately correct

S2. (d) In case of hydrogen atom mass number and atomic number are equal.

S3. (c) Nuclear force is nearly same for all nucleus

S4. (b) E-particles, being emitted with very high speed compared to D-particles, pass for very little time near the atoms of the medium. So, the probability of the atoms being ionized is comparatively less. But due to this reason, their loss of energy is very slow and they can penetrate the medium through a sufficient depth.

S5. (c) Electrons are not inside nucleus

SHORT ANSWER QUESTIONS

S1. Neutrinos are chargeless (neutral) and almost massless particles that hardly interact with matter.

S2. Characteristic Features of Nuclear Force

(i) Nuclear forces are short range attractive forces (range 2 to 3 fm) while Coulomb's forces have range up to infinity and may be attractive or repulsive.

(ii) Nuclear forces are charge independent forces; while Coulomb's force acts only between charged particles

S3. (i) Given $A-4=176 \Rightarrow$ Mass number of D, $A=180$

(ii) $Z - 1 = 71 \Rightarrow$ Atomic number of $D, Z = \mathbf{72}$

S4. Nuclear radius, $R = R_0A^{1/3} \Rightarrow R \propto A^{1/3}$

For Al, $A = 27, R_{\text{Al}} = 3.6$ fermi, for Fe, $A = 125$

$$\therefore \quad \frac{R_{\text{Fe}}}{R_{\text{Al}}} = \left(\frac{A_{\text{Fe}}}{A_{\text{Al}}}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3}$$

$$\Rightarrow \quad R_{\text{Fe}} = \frac{5}{3}R_{\text{Al}} = \frac{5}{3} \times 3.6\text{ fermi} = \mathbf{6.0}\text{ fermi}$$

S5. Nuclear density is independent of mass number, so ratio 1: 1.

NUMERICAL TYPE QUESTIONS

S1. Energy released $Q = (M_Y + M_Z)c^2 - M_Xc^2$

$= 8.5(110 + 130)\text{MeV} - 7.6 \times 240\text{MeV}$

$= (8.5 - 7.6) \times 240\text{MeV}$

$= 0.9 \times 240\text{MeV} = \mathbf{2}16\text{MeV}$

S2. Energy released $= \Delta m \times 931\text{MeV}$

$\Delta m = 4m(^1_1\text{H}) - m(^4_2\text{He})$

Energy released $(Q) = [4m(^1_1\text{H}) - m(^4_2\text{He})] \times 931\text{MeV}$

$= [4 \times 1.007825 - 4.002603] \times 931\text{MeV}$

$= 26.72\text{MeV}$

S3. Masses of protons and neutrons in 63u of Cu

$= Zm_p + (A - Z)m_n = 29m_p + (63 - 29)m_n$

$= 29 \times 1.00783 + (34 \times 1.00867) =$

$29.22707 + 34.29478 = 63.52185\text{u}$

Mass of $^{63}_{29}$Cu atom $= 62.92960$u

Mass defect $= 63.52185 - 62.92960 = 0.59225$u

Energy released in $^{63}_{29}$Cu atom $= 0.59225 \times 931\text{MeV} = 551.385\text{MeV}$

Number of atoms in 3 g of copper $= \frac{6.02\times10^{23}}{63} \times 3 = 2.87 \times 10^{22}$

$\therefore$ Energy required to separate all nucleons (neutrons and protons) from each other

$= 2.87 \times 10^{22} \times 551.385\text{MeV} = \mathbf{1.6} \times 10^{25}\text{MeV}$

S4. Number of U − 235 atoms in 1 gram $= \frac{1}{235} \times 6 \times 10^{23}$

Energy generated per gram of $^{235}_{92}\text{U} = \frac{1}{235} \times 6 \times 10^{23} \times 200 \times 1.6 \times 10^{-13}\text{Jg}^{-1}$

$P = 1000\text{MW} = 1000 \times 10^6$ W

$t = 5 \times 365 \times 24 \times 60 \times 60 = 5 \times 3.154 \times 10^7$ s

Total energy generated in 5 years with 80% time on

$Q = Pt = \left(1000 \times 10^6 \times \frac{80}{100} \times 5 \times 3.154 \times 10^7\right)$ J

Amount of $^{235}_{92}$U consumed in 5 years.

S5. Number of deuterium atoms in 2 g = 6.02×10^{23}

Number of deuterium atoms in 2.0 kg is = 6.02×10^{26}

Number of reactions $= \frac{6.02\times10^{26}}{2} = 3.01 \times 10^{26}$

Energy released in one reaction = 3.2MeV

Total energy released, $W = 3.01 \times 10^{26} \times 3.2\text{MeV} = 9.632 \times 10^{26}\text{MeV}$

$= 9.632 \times 10^{26} \times 1.6 \times 10^{-13}$ J $= 15.4 \times 10^{13}$ J

If t second is the required time during which the bulb glows, then $W = Pt$ gives

$t = \frac{W}{P} = \frac{15.4\times10^{13}}{100}$

$= 15.4 \times 10^{11}$ s

$= \frac{15.4\times10^{11}}{3.15\times10^{7}}$ years $= 4.9 \times 10^4$ years.

S6. Half-life time = 30 minutes; Rate of decrease (N) = 5 per second and total time = 2 hours = 120 minutes. Relation for initial and final count rate,

$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{\text{time/half-life}} = \left(\frac{1}{2}\right)^{120/30} = \left(\frac{1}{2}\right)^{4} = \frac{1}{16}$

Therefore $R_0 = 16 \times R = 16 \times 5 = 80\ \text{s}^{-1}$

S7. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$; $n \rightarrow$ no. of decays

$\frac{1}{256} = \left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^n \therefore n = 8$

Time for 8 half-life = 100 hours

S8. As $^2_1\text{H} + ^2_1\text{H} \longrightarrow ^4_2\text{He}$

Here, $\Delta M = 0.02866$ u

$\therefore$ The energy liberated per u is = $\frac{\Delta M\times931}{4}$ MeV

$= \frac{0.02866\times931}{4}$ MeV $= \frac{26.7}{4}$ MeV $= 6.675$ MeV

S9. On emission of one α-particle, atomic number decreases by 2 units and mass number decrease by 4 units. While the emission of β-particle does not affect the mass number and atomic number increases by 1 unit. Here, decrease in mass number = 200 – 168 = 32.

$\therefore$ Number of α-particles = 32/4 = 8.

$\therefore$ Number of β-particles = 16 – 10 = 6.

S10. Binding energy of $^7_3$Li nucleus

= 7 × 5.60 MeV = 39.2 MeV

Binding energy of $^4_2$He nucleus

= 4 × 7.06 MeV = 28.24 MeV

The reaction is

$^7_3\text{Li} + ^1_1\text{H} \longrightarrow 2(^4_2\text{He}) + Q$

$\therefore\ Q = 2(\text{BE or } ^4_2\text{He}) - (\text{BE of } ^7_3\text{Li})$

= 2 × 28.24 MeV – 39.2 MeV

= 56.48 MeV – 39.2 MeV = 17.28 MeV

HOMEWORK EXERCISE SOLUTIONS

MCQ

S1. (b) $R = R_0(A)^{1/3}$

$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}$

$R = \frac{5}{3} \times 3.6 = 6$ fermi

S2. (b) B. E $-0.042 \times 931 = 42$MeV

Number of nucleons in ${}_3^7$Li is 7.

$\therefore$ B.E. / nucleon $-\frac{42}{7} - 6$MeV $= 5.6$MeV

S3. (d) $E = \Delta mc^2 \Rightarrow E = \frac{0.3}{1000} \times (3 \times 10^8)^2 = 2.7 \times 10^{13}$ J

$= \frac{2.7\times10^{13}}{3.6\times10^6} = 7.5 \times 10^6$kWh.

S4. (b) According to mass-energy equivalence relation,

Energy, $E = mc^2$

By substituting,

$m = 2\text{g} = 2 \times 10^{-3}$ kg, $c = 3 \times 10^8$ ms^{-1}

We have, $E = 2 \times 10^{-3} \times (3 \times 10^8)^2$

$\Rightarrow \quad E = 2 \times 10^{-3} \times 9 \times 10^{16}\ 18 \times 10^{13}$

S5. (c) Binding energy

$= [7\text{M}_\text{P} + (A - Z)\text{M}_\text{N} - \text{M}]\text{c}^2$

$= [8\text{M}_\text{P} + (17 - 8)\text{M}_\text{N} - \text{M})\text{c}^2$

$= [8\text{M}_\text{P} + 9\text{M}_\text{N} - \text{M}^2\text{c}^2$

$= [8\text{M}_\text{P} + 9\text{M}_\text{N} - \text{M}_\text{o}]\text{c}^2$

S6. (d) Mass defect, Δm

= Mass of nucleons – Mass of nucleus

$= \left[Zm_p + (A - Z)m_n\right] - M_n$

Here, mass number, A = 4

Atomic number, Z = 2

Number of protons = 2

Number of neutrons = A – Z = 4 – 2 = 2

$m_p = 1.0072676$ u

$m_n = 1.008665$ u and $M_n = 4.001506\ u$

Mass of nucleon in ${}_2$He^4 =

Mass of 2 protons + Mass of 2 neutrons

So, mass of nucleons $= 2 \times 1.0072676 + 2 \times 1.008665$

$\therefore \Delta m = 2 \times 1.0072676 + 2 \times 1.008665 - 4.001506$

= 0.0303592 u

S7. (b) Alpha particle is a positively charged particle. It is identical to the nucleus of the helium (${}_2$He^4) atom, so it contains 2 protons and 2 neutrons.

S8. (b) According to radioactive decay law

$N = N_0 e^{-\lambda t}$

where N_0 = Number of radioactive nuclei at time t = 0

N = Number of radioactive nuclei left undecayed at any time t

λ = decay constant

At time t_2, $\frac{2}{3}$ of the sample had decayed

$\therefore N = \frac{1}{3}N_0 \Rightarrow \frac{1}{3}N_0 = N_0 e^{-\lambda t_2}$ ...(i)

At time t_1, $\frac{1}{3}$ of the sample had decayed,

$\therefore N = \frac{2}{3}N_0 \Rightarrow \frac{2}{3}N_0 = N_0 e^{-\lambda t_1}$ ...(ii)

Divide (i) by (ii), we get

$\frac{1}{2} = \frac{e^{-\lambda t_2}}{e^{-\lambda t_1}} \Rightarrow \frac{1}{2} = e^{-\lambda(t_2 - t_1)} \Rightarrow \lambda(t_2 - t_1) = \ln 2$

$t_2 - t_1 = \frac{\ln 2}{\lambda} \Rightarrow T_{\frac{1}{2}} = 50$ days

S9. (c)

	P	Q
	4N$_0$	N$_0$
Half-life	1 min	2 min
No. of nuclei, at t = 0	N$_\text{P}$	N$_\text{Q}$

Let after t min the number of nuclei of P and Q are equal.

$\therefore N_P = 4N_0\left(\frac{1}{2}\right)^{t/1}$ and $N_Q = N_0\left(\frac{1}{2}\right)^{t/2}$

As $N_P = N_Q$

$\therefore 4N_0\left(\frac{1}{2}\right)^{t/1} = N_0\left(\frac{1}{2}\right)^{t/2}$ or $4 = \frac{2^t}{2^{t/2}}$

or $4 = 2^{t/2}$ or $2^2 = 2^{t/2}$ or $\frac{t}{2} = 2$ or $t = 4$ min

After 4 minutes, both P and Q have equal number of nuclei.

$\therefore$ Number of nuclei of R

$= \left(4N_0 - \frac{N_0}{4}\right) + \left(N_0 - \frac{N_0}{4}\right) = \frac{15N_0}{4} + \frac{3N_0}{4} = \frac{9N_0}{2}$

S10. (b) ${}_6\text{C}^{12} + {}_0n^1 \rightarrow {}_6\text{C}^{13} \rightarrow {}_7\text{N}^{13} + {}_{-1}\beta^0$ + Energy

S11. (a) From step (ii), B has 2 units of charge more than C.

From step (i), A loses 2 units of charge by emission of alpha particle. Hence, A and C are isotopes as their charge number is same.

S12. (c) In nuclear fusion the mass of end product or resultant is always less than the sum of initial product, the rest is liberated in the form of energy, like in Sun energy is liberated due to fusion of two hydrogen atoms.

S13. (d) In nuclear fission, the chain reaction is controlled in such way that only one neutron, produced in each fission, causes further fission. Therefore, some moderator is used to slow down the neutrons. Heavy water is used for this purpose.

S14. (d) For nuclei having $A > 56$ binding energy per nucleon gradually decreases

S15. (c) Let ${}_5$B^{10} be present as x% so percentage of ${}_5$B^{11} = (100 – x)

$\therefore$ Average atomic weight

$= \frac{10x + 11(100 - x)}{100} = 10.81 \Rightarrow x = 19$

$\therefore$ % of ${}_5$B^{11} is 100 – 19 = 81. Ratio is 19 : 81.

ASSERTION AND REASONING

S1. (a) Cobalt 60 is the radioactive isotope of cobalt. γ–radiation emitted by it is used in radiation therapy is cancer as it destroys cancerous cells. So, assertion and reason are true and reason explains assertion.

S2. (c) The process of nuclear fusion can be explained with the help of concept of binding energy per nucleon of very light nuclei and the intermediate nuclei. Binding energy per nucleon of very light nuclei is less than that of intermediate nuclei. It means light nuclei are less stable than that of intermediate nuclei. As ^{35}Cl has a large binding energy therefore it cannot be used as fuel for fusion energy.

S3. (d) We know that energy is released when heavy nuclei undergo fission or light nuclei undergo fusion. Therefore statement (1) is correct. The second statement is false because for heavy nuclei the binding energy per nucleon decreases with increasing Z and for light nuclei, B.E/nucleon increases with increasing Z.

SHORT ANSWER QUESTIONS

S1. (i) Basic nuclear reaction for β^+decay is the conversion of proton to neutron.

$p \rightarrow n + e^+ + \text{v}$

(ii) (a) $x = \beta^+ / {}^0_1e, y = 5, z = 11$

(b) $a = 10, b = 2, c = 4$

S2. Properties of nuclear forces

(1) Nuclear forces are the strongest attractive forces.

(2) Nuclear forces are short ranged upto 10^{-15} m.

(3) Nuclear forces are charge independent

S3. Initial binding energy

$BE_1 = (2.23 + 2.23) = 4.46\text{MeV}$

Final binding energy

$BE_2 = 7.73\text{MeV}$

$\therefore$ Energy released $= (7.73 - 4.46)\text{MeV} = 3.27\text{MeV}$

S4. Protons are positively charged and repel one another electrically. This repulsion becomes so great in nuclei with more than 10 protons or so, that an excess of neutrons which produce only attractive forces, is required for stability.

S5. The daughter nucleus would have a higher binding energy per nucleon.

NUMERICAL TYPE QUESTIONS

S1. The average atomic mass of a chlorine atom is obtained by the weighted average of the masses of the two isotopes, which is

Average atomic mass

$= \frac{75.4 \times 34.98 + 24.6 \times 36.98}{100}$

$= 35.47\ u$

S2. Nuclear radius, $R \propto A^{1/3}$, where A is mass number. For ^{27}Al,

$R = R_0 A^{1/3} = R_0(27)^{1/3} = 3R_0$

$R_0 = \frac{3.6}{3} = 1.2$ fm $\quad [\because R = 3.6 \text{ fm, for Al}]$

For ^{64}Cu, $\quad R = R_0\, A^{1/3} = 1.2\,(64)^{1/3} = 4.8$ fm

S3. We have, $P = 200 \text{ MW} = 200 \times 10^6 \text{ W} = 2 \times 10^8 \text{ W}, t = 1h = 3600 \text{ s}$

As, energy, $E = P \times t = 2 \times 10^8 \times 36000$

Also, $\quad E = mc^2$

$\Rightarrow \quad m = \frac{E}{c^2} = \frac{2 \times 10^8 \times 3600}{(3 \times 10^8)^2}$

$= 8 \times 10^{-6}$ kg

S4. Velocity ratio $(v_1 : v_2) = 2 : 1$

Mass $(m) \propto$ Volume $\propto r^3$.

According to law of conservation of momentum,

$m_1 v_1 = m_2 v_2$

Therefore $\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{r_2^3}{r_1^3}$

Or $\frac{r_1}{r_2} = \left(\frac{v_2}{v_1}\right)^{1/3} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{2^{1/3}}$

Or $r_1 : r_2 = 1 : 2^{1/3}$

S5. Given, $t_{1/2} = 2.2 \times 10^9$ s

and rate of radioactive disintegration,

$\frac{dN}{dt} = 10^{10}\ \text{s}^{-1}$

$\therefore \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.2 \times 10^9} = 3.15 \times 10^{-10}\ \text{s}^{-1}$

Now, we know that, $N = N_0 e^{-\lambda t}$

$\Rightarrow \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N$

$\Rightarrow \quad 10^{10} = 3.15 \times 10^{-10} \times N \Rightarrow N = 3.17 \times 10^{19}$

S6. In two half-lives, the activity becomes one fourth.

Activity on 1 - 8 - 91 was 2 micro curies.

$\therefore$ Activity before two months,

4 × 2 micro-Curie = 8 micro curies

S7. There is requirement of three half-lives so age of the rock

$t = nT_{1/2} = 3 \times 20$ years = 60 years

S8. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$; $n \rightarrow$ no. of decays

$\frac{1}{256} = \left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^n \therefore n = 8$

Time for 8 half-life = 100 hours

S9. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

where n is number of half lives

$\therefore \ \frac{1}{16} = \left(\frac{1}{2}\right)^n$ or $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$ or $n = 4$

Let the age of rock be t years.

$\because\ n = \frac{t}{T_{1/2}}$

or $\quad t = nT_{1/2} = 4 \times 50$ years $= 200$ years

S10. Number of half-lives, $n = \frac{t}{T} = \frac{6400}{800} = 8$

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$