INTEREST AND INTEREST FORMULAE

- **Interest**
  The term interest is used to designate a rental for the use of money.
- **Simple Interest**
  When the total interest earned or charged is directly proportional to the principal involved, the interest rate and the no. of interest periods for which the principal is committed, the interest is called simple

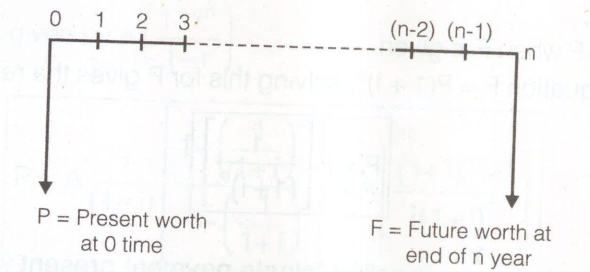
$$\boxed{I = (P)(n)(i)}$$

where I = total interest
P = principal amount rent or borrowed
n = number of years (interest periods)
i = interest rate per year (per interest period)

- **Compound Interest**
  Whenever the interest charge for any interest period (a year, for example) is based on the remaining principal amount plus any accumulated interest charges upto the beginning of that period, the interest is said to be compound.
- **Notations and Cash Flow**
  i = the annual interest rate
  n = the number of annual interest periods
  P = Present sum of money i.e. Present worth at zero time
  F = future sum of money i.e. Future worth equal to the compound amount at the end of n years
  A = a single payment, in a series of n equal payments, made at the end of each annual interest period
  G = uniform period-by-period increase or decrease in amount (the arithmetic gradient)

CASH FLOW DIAGRAM

The graphic presentation of each value plotted at appropriate time is called a cash flow diagram. The normal conventions for cash flow diagrams are as follows;

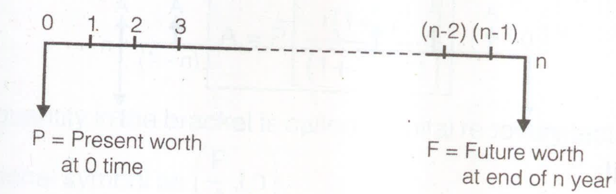


(a) The horizontal line is a time scale with progression of time moving from left to right. The value indicated on time scale (viz., 0, 1,2,...n) indicates the end of the respective period.

(b) The arrows signify cash flow, normally downward arrows represent disbursement or costs and upward arrows represent receipts or benefits.

INTEREST FORMULAE FOR SINGLE PAYMENT SERIES

Figure shows a cash flow diagram involving a present single sum (P), a future single sum (F), separated by n years with interest rate i per year;



- **Case I.**
  Finding F when P is given.
  At the end of n yrs,

$$\boxed{F = P(1+i)^n}$$

$$\boxed{\frac{F}{P} = (1+i)^n}$$

The quantity $(1 + i)^n$ is commonly called "single payment compound amount factor" indicated by the functional notations as $\left(\frac{F}{P}, i, n\right)$

$\therefore \left(\frac{F}{P}, i, n\right)$ = Single payment compound amount factor = $(1 + i)^n$.

- **Case II.**

  Finding P when F is given

  From equation $F = P(1 + i)^n$, solving this for P gives the relation

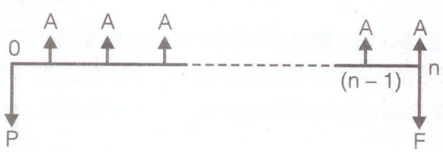
$$P = F\left[\frac{1}{(1+i)^n}\right]$$

The quantity $\frac{1}{(1+i)^n}$ is called **"single payment present worth factor"** indicated by the functional symbol as $\left(\frac{P}{F}, i, n\right)$

$$\therefore \quad \left(\frac{P}{F}, i, n\right) = \text{Single payment present worth factor} = \frac{1}{(1+i)^n}$$

INTEREST FORMULAE FOR EQUAL PAYMENT SERIES

Fig. shows a general cash flow diagram involving a series of uniform (equal) payments, each of amount A, occurring at the end of each year with interest rate i per year.



- **Case III.**

  Finding P when A is given

  If A exists at end of each year for n years with i rate of interest, the present worth P is obtained by summing the present worth of each payment of amount A

$$P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \ldots + \frac{A}{(1+i)^n}$$

$$P = A\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots + \frac{1}{(1+i)^n}\right]$$

The series in the bracket is in the geometric progression whose first term (a) $\frac{1}{(1+i)}$, geometric ratio (r) is $\frac{1}{(1+i)}$.

Sum is given by a $\left(\frac{1-r^n}{1-r}\right)$

$$\therefore \quad P = A\frac{1}{(1+i)}\left[\frac{1-\left(\frac{1}{1+i}\right)^3}{1-\left(\frac{1}{1+i}\right)}\right] = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

The quantity $\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ is called "uniform series present worth factor" indicated by the function notation as $\left[\frac{P}{A}, i, n\right]$

$$\left[\frac{P}{A}, i, n\right] = \text{uniform (equal) series present worth factor} = \frac{(1+i)^n - 1}{i(1+i)^n}$$

- **Case IV.**

  Finding A when P is given

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

The quantity in the bracket is called "capital recovery factor" using the functional symbol as $\left(\frac{P}{A}, i, n\right)$.

- **Case V.**

  Finding F when A is given

$$P = \frac{F}{(1+i)^n} = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$\therefore \quad F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

The factor in the bracket is called "equal payment series compound amount factor" given by fundamental notation as $\left(\frac{F}{A}, i, n\right)$

- **Case VI.**

Finding A when F is given

$$\boxed{A = F\left[\frac{i}{(1+i)^n - 1}\right]}$$

The quantity in the bracket $\left[\frac{i}{(1+i)^n - 1}\right]$ is called "sinking fund factor"

using functional notation as $\left(\frac{A}{F}, i, n\right)$

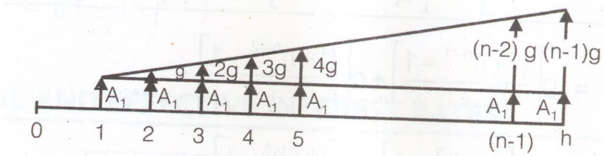
Hence, $\boxed{A = F\left(\frac{F}{A}, i, n\right)}$

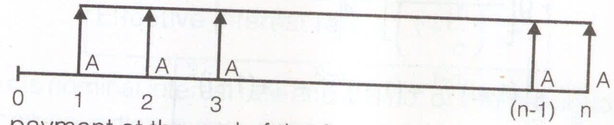
Table: COMPOUND INTEREST FACTORS

	To find	Given	Factor by which given is multiplied	Factor Name	Functional notation
Single Payment	F	P	$(1+i)^n$	Single payment compound amount factor	$\left(\frac{F}{P}, i, n\right)$
	P	F	$\frac{1}{(1+i)^n}$	Single payment present worth factor	$\left(\frac{P}{F}, i, n\right)$
Uniform Series Payment	P	A	$\frac{(1+i)^n - 1}{i(1+i)^n}$	Uniform series present worth factor	$\left(\frac{P}{A}, i, n\right)$
	A	P	$\frac{i(1+i)^n}{(1+i)^n - 1}$	Capital recovery factor	$\left(\frac{A}{P}, i, n\right)$
	F	A	$\frac{(1+i)^n - 1}{i}$	Uniform series compound amount factor	$\left(\frac{F}{A}, i, n\right)$
	A	F	$\frac{i}{(1+i)^n - 1}$	Sinking fund factor	$\left(\frac{A}{F}, i, n\right)$

INTEREST FORMULAE FOR UNIFORM GRADIENT PAYMENT SERIES

Some economic analysis problems involve receipts or disbursements that are projected to increase or decrease by a uniform amount each period, thus contributing an arithmetic series. In general, a uniformly increasing series of payment for n interest periods may be expressed as A_1, $A_1 + g$, $A_1 + 2g$, $A_1 + 3g$.....$A_1 + (n-1)g$ as shown in Fig. Where A_1 denotes the first year end payment in the series and 'g' the annual change in magnitude called gradient amounts.





A_1 = payment at the end of the first year

g = annual change in gradient

n = number of years

A = the equivalent annual payment of the series

A_2 = the equivalent annual payment of the gradient series {0, g, 2g, ... (n – 1)g} at the end of successive years.

$$\boxed{A = A_1 + A_2}$$

where $A_2 = F\left(\frac{A}{F}, i, n\right) = \left[\frac{i}{(1+i)^n - 1}\right]$ and F is the future amount equivalent to the gradient series. The future amount equivalent to the gradient series can be derived from the table as follows,

Table: Gradient Series and an equivalent Set of Series

End of year	Gradient Series	Set of series equivalent to the Gradient series
1	0	0
2	g	g
3	2g	g + g
4	3g	g + g + g
.	.	.
.	.	.
.	.	.
n – 1	(n – 2)g	g + g + g + ...+g
n	(n – 1)g	g + g + g + ...g + g

$$F = g\left[\frac{F}{A}, i, (n-1)\right] + g\left[\frac{F}{A}, i, (n-2)\right] + \ldots + g\left[\frac{F}{A}, i, 2\right] + g\left[\frac{F}{A}, i, 1\right]$$

$$= g\left[\frac{(1+i)^{n-1}-1}{i}\right] + g\left[\frac{(1+i)^{n-2}-1}{i}\right] + \ldots$$

$$+ g\left[\frac{(1+i)^{2}-1}{i}\right] + g\left[\frac{(1+i)^{1}-1}{i}\right]$$

$$= \frac{g}{i}\left[\begin{matrix}(1+i)^{n-1} + (1+i)^{n-2} + \ldots + (1+i)^{2} \\ + (1+i)^{1} - (n-1)1\end{matrix}\right]$$

$$= \frac{g}{i}\left[\begin{matrix}(1+i)^{n-1} + (1+i)^{n-2} + \ldots + (1+i)^{2} \\ + (1+i)^{1} + 1(1+i)^{0}\end{matrix}\right] - \frac{ng}{i}$$

The terms in the brackets consist n terms, 1st term being $(1 + i)^0$ and ratio being $(1 + i)$ of geometric progression.

$$= \frac{g}{i}\left[(1+i)^0 \frac{1-(1+i)^n}{1-(1+i)}\right] - \frac{ng}{i} = \frac{g}{i}\left[\frac{(1+i)^n - 1}{i}\right] - \frac{ng}{i}$$

$$A_2 = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= \frac{g}{i}\left[\frac{(1+i)^n - 1}{i}\right]\left[\frac{i}{(1+i)^n - 1}\right] - \frac{n}{g}\left[\frac{i}{(1+i)^n - 1}\right]$$

$$A_2 = \frac{g}{i} - \frac{ng}{i}\left[\frac{i}{(1+i)^n - 1}\right]$$

$$\boxed{A_2 = g\left[\frac{1}{i} - \frac{n}{i}\left(\frac{A}{f}, i, n\right)\right]}$$

The resulting factor $\left[\frac{1}{i} - \frac{n}{i}\left(\frac{A}{F}, i, n\right)\right]$ is called gradient factor for annual compounding interest and will be designated $\left(\frac{A}{G}, i, n\right)$

$\therefore$ A_2 = Equivalent annual cost of set of gradient series

$$= g\left(\frac{A}{G}, i, n\right)$$

NOMINAL AND EFFECTIVE INTEREST RATES

$$\boxed{\text{Effective interest rate} = \left(1 + \frac{r}{c}\right)^c - 1}$$

where r is nominal interest rate and c is no. of interest periods per year. If the compounding is annual then r = i.

DEPRECIATION

Depreciation is defined as the loss in value of an asset with the passage of time. The main purpose of the depreciation is to provide for the recovery of capital that has been involved in the possession of the physical property.

- **Salvage Value (or Resale Value)**
  It is the value of the property at the end of its utility period without being dismantled. Salvage value implies that the property has further utility.
- **Scrap Value**
  The value of a property realized when it become absolutely useless except for sale as junk is its scrap value.
- **Book Value**
  It is defined as the value of the property shown in the account books in that particular year i.e. the original cost less total depreciation till that year.

METHODS FOR CALCULATING DEPRECIATION

There are several methods of calculating depreciation. The following notation have been use;

C_i = Initial cost of an asset at zero time or original cost (This will include the cost of asset +transporting charge + installation and other charges spent initially)

C_S = Salvage value (or scrape value) to be estimated at the end of utility period or scrap cost

n = the life of the asset

B_m = Book value at the end of period 'm'

(i) Straight Line Method

In this method, the property is assumed to lose value by a constant amount every year. At the end of the life, the salvage value (or scrap value) is let.

$$D_m = \frac{C_i - C_s}{n}$$

$$D_m = D_1 = D_2 = D_3 = D_n$$

$$B_m = C_1 - m\left(\frac{C_i - C_s}{n}\right)$$

This method is recommended for all the equipments/assets which constant demand and do not face any obsolescence during their useful life. It is widely used in the case of all civil engineering appliances and applications.

(ii) Declining Balance Method (or Constant Percentage Method)

In this method, the property is assumed to lose value annually at a constant percentage of its book value.

Fixed Declining Balance $FDB = 1 - \left(\frac{C_s}{C_i}\right)^{1/n}$

(iii) Fixed Double Declining Balance Method

In this method also, the property is assumed to lose value annually by fixed factor of the book value.

FDDB = Fixed factor for double declining balance method.

FDDB is taken as double the straight line rate. i.e. $FDDB = \frac{2}{n}$

$$FDDB = \left(1 - \frac{C_s}{C_i}\right)^{1/n}$$

(iv) Sum of the Years Digit Method

In this method, the digits corresponding to the number of each years of life are listed in reverse order. The general expression for the annual depreciation for any year (m) when the life is n years is expressed as

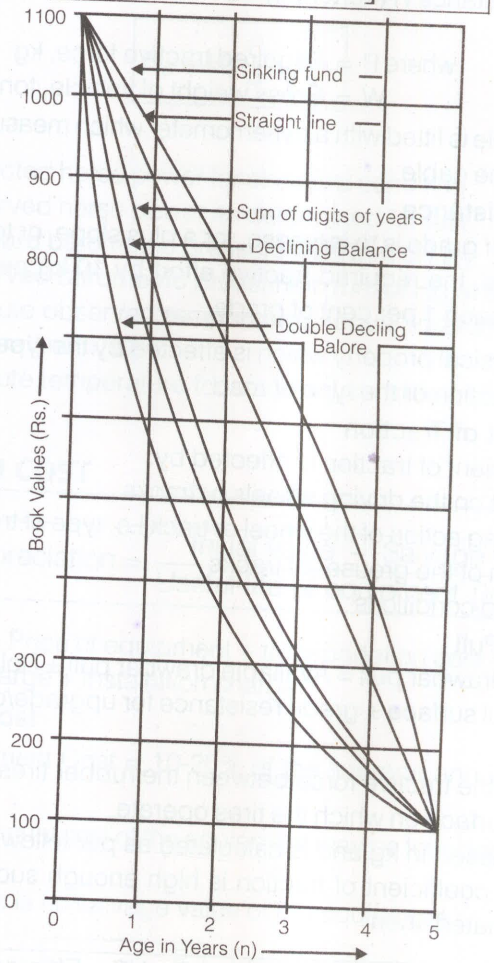
$$D_m = (C_i - C_s)\left[\frac{(n - m + 1)}{\frac{n(n+1)}{2}}\right]$$

m = No. of years of which depreciation is calculated

(iv) Sinking Fund Method

Sinking fund depreciation model assumes that the value of an asset decreases at an increasing rate. Equal amount (D) is assumed to be deposited into a sinking fund at the end of each year of the assets life.

$$D = [C_i - C_s]\left[\frac{i}{(1+i)^n - 1}\right][1+i]^{m-1}$$



DEPLETION

$$\text{Depletion for a year} = \frac{\text{Cost of property}}{\text{No. of units in the property}} \times \text{Units sold during the year}$$

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