Sample Question Paper - 30
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. For what value of k will $k + 11$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P.?

2. In the following distribution:

Marks obtained	More than or equal to 0	More than or equal to 10	More than or equal to 20	More than or equal to 30	More than or equal to 40
No. of students	80	70	65	30	25

Find the frequency of the class 20-30.

3. Find the roots of the quadratic equation $a^2b^2x^2 + b^2x - a^2x - 1 = 0$.

OR

Find the value of k, for which the quadratic equation $x^2 - kx + 4 = 0$ has equal roots.

4. Two parallel lines touch the circle at points A and B. If area of the circle is 16π cm^2, then find the value of AB.

5. In a certain distribution, mean and median are 19.5 and 50 respectively. Find the mode of the distribution, using an empirical relation.

6. A cone and a hemisphere have equal bases and equal volumes. Find the sum of the numerator and denominator of ratio of height of cone and radius of hemisphere.

OR

Three cubes each of edge 3 cm are joined end to end. Find the surface area of the resulting cuboid.

SECTION - B

7. The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers.

8. The 4^{th} term of an A.P. is zero. Prove that the 25^{th} term of the A.P. is three times its 11^{th} term.

OR

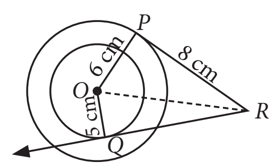
Find the sum of first 8 multiples of 3.

9. AB is a line segment of length 10 cm. Locate a point C on it such that $AC = \frac{1}{5}CB$.

10. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 45°. Find the length of the string, assuming that there is no slack in the string.

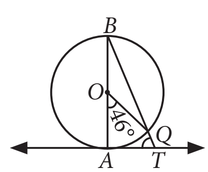
SECTION - C

11. Two concentric circles are of radii 10 cm and 8 cm. RP and RQ are tangents to the two circles from R. If the length of RP is 24 cm, find the length of RQ.



OR

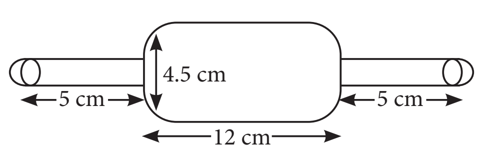
In the given figure, AB is a diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 46°$, find $\angle ATQ$.



12. The angle of elevation of the top of a tower at a point on the ground is 30°. What will be the angle of elevation, if the height of the tower is tripled?

Case Study - 1

13. Arpana is studying in X standard. While helping her mother in kitchen, she saw rolling pin made of steel and empty from inner side, with two small hemispherical ends as shown in the figure.



(i) Find the curved surface area of two identical cylindrical parts, if the diameter is 2.5 cm and length of each part is 5 cm.

(ii) Find the volume of big cylindrical part.

Case Study - 2

14. A group of 71 people visited to a museum on a certain day. The following table shows their ages.

Age (in years)	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60
No. of persons	3	10	22	40	54	71

Based on the above information, answer the following questions.

(i) Find the median class for the given data.

(ii) Find the median age of the persons visited the museum.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. Let $k + 11$, $2k - 1$ and $2k + 7$ are in A.P.

$\therefore$ $(2k - 1) - (k + 11) = (2k + 7) - (2k - 1)$

$\Rightarrow$ $2k - 1 - k - 11 = 2k + 7 - 2k + 1$

$\Rightarrow$ $k - 12 = 8 \Rightarrow k = 20$

2. The frequency distribution table from the given cumulative frequencies can be drawn as :

Marks obtained	Number of students
0–10	80 – 70 = 10
10–20	70 – 65 = 5
20–30	65 – 30 = 35
30–40	30 – 25 = 5
More than or equal to 40	25

Hence, the frequency of the class interval 20–30 is 35.

3. The given quadratic equation is

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$\Rightarrow$ $b^2x(a^2x + 1) - 1(a^2x + 1) = 0$

$\Rightarrow$ $(a^2x + 1)(b^2x - 1) = 0 \Rightarrow a^2x + 1 = 0$ or $b^2x - 1 = 0$

$\Rightarrow$ $x = -\frac{1}{a^2}$ or $x = \frac{1}{b^2}$

Hence, the roots of the given equation are $\frac{-1}{a^2}$ and $\frac{1}{b^2}$.

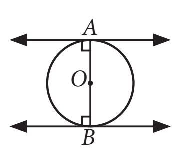
OR

The given equation is, $x^2 - kx + 4 = 0$

For equal roots, $D = b^2 - 4ac = 0$

$\Rightarrow$ $(-k)^2 - 4(1)(4) = 0 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4$

4. Let the radius of the circle be r cm.



Area of circle = 16π [Given]

$\Rightarrow$ $\pi r^2 = 16\pi$

$\Rightarrow$ $r^2 = 16 \Rightarrow r = 4$

$\therefore$ $AB = 2\,OA = 2r = 8$ cm

5. We know that, empirical relation between mean, median and mode is

Mode = 3 Median – 2 Mean ...(i)

From given, we have,

Mean = 19.5, Median = 50

$\therefore$ Mode = 3(50) – 2(19.5) (Using (i))

$\Rightarrow$ Mode = 111

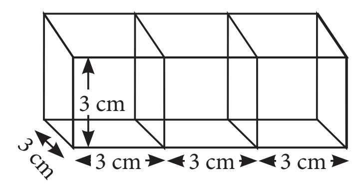
6. Let r be the radius of both the cone and hemisphere and h be the height of the cone.

$\therefore$ Volume of cone = Volume of hemisphere

$\Rightarrow$ $\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3 \Rightarrow h = 2r \Rightarrow \frac{h}{r} = \frac{2}{1}$

$\therefore$ Required sum = 2 + 1 = 3

OR



Length of the resulting cuboid = 3 + 3 + 3 = 9 cm

Breadth of the resulting cuboid = 3 cm

Height of the resulting cuboid = 3 cm

$\therefore$ Surface area of the resulting cuboid = $2(lb + bh + hl)$

$= 2(9 \times 3 + 3 \times 3 + 3 \times 9) = 2 \times 63 = 126\text{ cm}^2$

7. Let the smaller number be x.

$\therefore$ Larger number is $x + 4$.

According to question, $\frac{1}{x} - \frac{1}{x+4} = \frac{4}{21}$

$\Rightarrow$ $\frac{x+4-x}{x(x+4)} = \frac{4}{21} \Rightarrow \frac{1}{x(x+4)} = \frac{1}{21}$

$\Rightarrow$ $x^2 + 4x - 21 = 0 \Rightarrow x^2 + 7x - 3x - 21 = 0$

$\Rightarrow$ $(x + 7)(x - 3) = 0 \Rightarrow x = 3$ or $x = -7$

If $x = 3$, then $x + 4 = 3 + 4 = 7$

If $x = -7$, then $x + 4 = -7 + 4 = -3$

Therefore, the pairs of numbers are 3 and 7 or –7 and –3.

8. Let the first term and common difference of the A.P. be a and d respectively.

Since, $a_n = a + (n - 1)d$

$\therefore$ $a_4 = a + (4 - 1)d = 0$

$\Rightarrow$ $a + 3d = 0 \quad \therefore a = -3d$ ...(i)

Now, $a_{25} = a + 24d$

$\Rightarrow$ $a_{25} = -3d + 24d$ [using (i)] $\therefore a_{25} = 21d$

Now, $a_{11} = a + 10d = -3d + 10d$ [using (i)]

$\therefore$ $a_{11} = 7d$

Multiply both sides by 3, we get

$3a_{11} = 21d \Rightarrow 3a_{11} = a_{25}$

OR

Multiples of 3 are 3, 6, 9, 12, ...

These numbers are in A.P. such that $a = 3$,

$d = 6 - 3 = 3$, $n = 8$

$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$

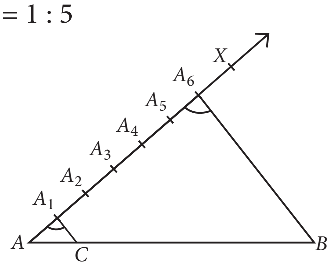
$\Rightarrow\ S_8 = \frac{8}{2}[2 \times 3 + (8-1) \times 3]$

$\Rightarrow\ S_8 = 4[6 + 7 \times 3] = 4[6 + 21] = 4 \times 27 = 108$

$\therefore\ S_8 = 108$

9. We have, $AC = \frac{1}{5}CB \Rightarrow \frac{AC}{CB} = \frac{1}{5}$

$\Rightarrow\ AC : CB = 1 : 5$



So, we have to divide AB in the ratio 1 : 5.

Steps of construction :

Step-I : Draw a line segment AB = 10 cm.

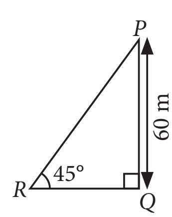
Step-II : Draw a ray AX making an acute angle with AB.

Step-III : Locate (1 + 5 =) 6 points A_1, A_2, A_3, A_4, A_5 and A_6 on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6$.

Step-IV : Join A_6B.

Step-V : Through A_1, draw A_1C parallel to A_6B meeting AB at C, such that $\angle AA_1C = \angle AA_6B$. Thus, C is the required point such that $AC : CB = 1 : 5$.

10. Let P be the position of the kite with height 60 m from the point Q on the ground. Let R is the other point on the ground to which string is temporarily tied.



In ΔPQR, $\sin 45° = \frac{PQ}{PR}$

$\Rightarrow\ \frac{1}{\sqrt{2}} = \frac{60}{PR} \Rightarrow PR = 60\sqrt{2}$

Hence, the length of string is $60\sqrt{2}$ m.

11. Given that, OP = 6 cm, OQ = 5 cm and RP = 8 cm

In ΔOPR, we have

$OP \perp PR$ [$\because$ Tangent is perpendicular to the radius at the point of contact]

$\therefore\ OR = \sqrt{PR^2 + OP^2} = \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36} = \sqrt{100} = 10$ cm

In ΔOQR

$OQ \perp QR$ [$\because$ RQ is tangent at Q]

$\therefore\ OR^2 = RQ^2 + OQ^2 \Rightarrow RQ^2 = OR^2 - OQ^2$

$= (10)^2 - (5)^2 = 100 - 25 = 75$

$\Rightarrow\ RQ = \sqrt{75} = 5\sqrt{3}$ cm

OR

Since AT is a tangent.

$\therefore\ AT \perp OA \Rightarrow \angle BAT = 90°$ ...(i)

Now, angle subtended by an arc at centre is twice the angle subtended by it at remaining part of the circle.

$\therefore\ \angle AOQ = 2\angle ABQ$

$\Rightarrow\ \angle ABQ = \frac{1}{2} \times 46° = 23°$ ...(ii)

In ΔABT, $\angle ABT + \angle BAT + \angle ATB = 180°$ [By angle sum property]

$\Rightarrow\ \angle ABQ + 90° + \angle ATQ = 180°$ [Using (i)]

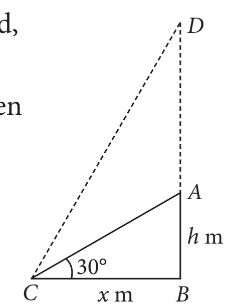
$\Rightarrow\ \angle ATQ = 180° - (90° + 23°)$ [Using (ii)]

$\Rightarrow\ \angle ATQ = 67°$

12. Let $AB = h$ m be the height of the tower and C be the point on the ground such that $BC = x$ m.

In ΔABC, $\tan 30° = \frac{AB}{BC} = \frac{h}{x}$

$\Rightarrow\ \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h$ ...(i)



When the height of tower is tripled, then height of tower, $BD = 3h$ m

Let θ be the angle of elevation when height of tower is tripled.

In ΔDBC, $\tan\theta = \frac{DB}{BC} = \frac{3h}{x}$

$\Rightarrow\ x = \frac{3h}{\tan\theta}$ ...(ii)

From (i) and (ii), we get

$\sqrt{3}h = \frac{3h}{\tan\theta} \Rightarrow \tan\theta = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$

$\Rightarrow\ \tan\theta = \tan 60° \Rightarrow \theta = 60°$

Hence, the angle of elevation is 60° when the height of the tower is tripled.

13. (i) Curved surface area of two identical cylindrical parts $= 2 \times 2\pi rh = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times 5$

$= 78.57\ \text{cm}^2$

(ii) Volume of big cylindrical part $= \pi r^2 h$

$= \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \times 12 = 190.93\ \text{cm}^3$

14. (i) Let us consider the following table :

Age (in years)	**Class interval (x_i)**	**Frequencies (f_i)**	**Cumulative frequency (*c. f.*)**
Less than 10	0-10	3	3
Less than 20	10-20	10 – 3 = 7	10
Less than 30	20-30	22 – 10 = 12	22
Less than 40	30-40	40 – 22 = 18	40
Less than 50	40-50	54 – 40 = 14	54
Less than 60	50-60	71 – 54 = 17	71

Here, $N = 71$, therefore $\frac{N}{2} = 35.5$

Now, the class interval whose cumulative frequency is just greater than 35.5 is 30-40.

∴ Median class = 30-40

(ii) Clearly, the cumulative frequency of the class preceding the median class is 22, *i.e.*, *c.f.* = 22.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h = 30 + \left(\frac{35.5 - 22}{18}\right) \times 10$$

$$= 30 + 13.5 \times \frac{10}{18} = 30 + 7.5 = 37.5$$

Thus, the median age of the persons visited the museum is 37.5 years