HOTS (Higher Order Thinking Skills)

Que 1. The line joining the points (2, 1) and (5, – 8) is trisected by the points P and Q. If the point P lies on the line 2x – y + k = 0, find the value of k.

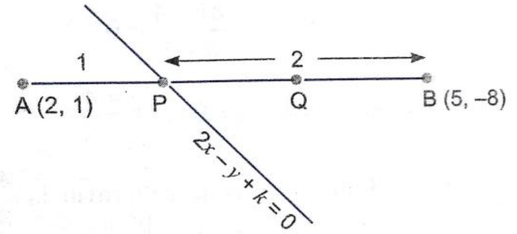

Fig. 6.29

Sol. As line segment AB is trisected by the points P and Q. Therefore,

Case I: When AP: PB = 1: 2.

Then, coordinates of P are $\left\{\frac{1\times5+2\times2}{1+2}, \frac{1\times-8+1\times2}{1+2}\right\}$

$\Rightarrow \quad P\,(3,-2)$

Since the point P (3, – 2) lies on the line

2x – y + k = 0

$\Rightarrow \; 2\times3-(-2)+k=0$

$\Rightarrow$ k = – 8

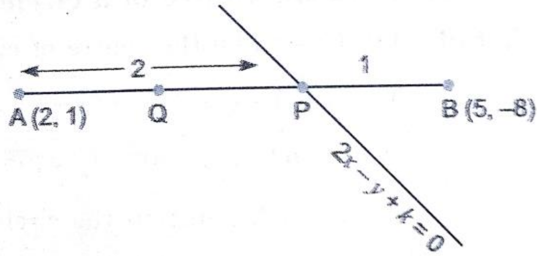

Fig. 6.30

Case II: When AP: PB = 2: 1.

Coordinates of point P are

$$\left\{\frac{2\times5+1\times2}{2+1}, \frac{2\times-8+1\times1}{2+1}\right\} = \{4,-5\}$$

Since the point P (4, -5) lies on the line

2x – y + k = 0

$\therefore$ 2 × 4 – (– 5) + k = 0 $\Rightarrow$ $k = -13.$

Que 2. Prove that the diagonals of a rectangle bisect each other and are equal.

Sol. Let OACB be a rectangle such that OA is along x-axis and OB is along y-axis. Let OA = a and OB = b.

Then, the coordinates of A and B are (a, 0) and (0, b) respectively.

Since, OACB is a rectangle. Therefore,

AC = OB ⇒ $AC = b$

Also, OA = a ⇒ BC = a

So, the coordinates of C are (a, b)

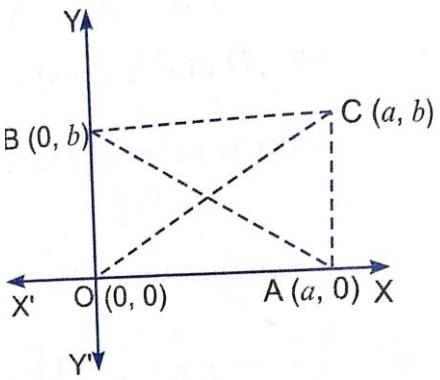

Fig. 6.31

The coordinates of the mid-points of OC are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Also, the coordinates of the mid-points of AB are $\left(\frac{a+0}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$.

Clearly, coordinates of the mid-point of OC and AB are same.
Hence, OC and AB bisect each other.

Also, $OC = \sqrt{a^2+b^2}$ and $BA = \sqrt{(a-0)^2+(0-b)^2} = \sqrt{a^2+b^2}$

∴ OC = AB

Que 3. In what ratio does the y-axis divides the line segment joining the point P (– 4, 5) and Q (3, – 7)? Also, find the coordinates of the point of intersection.

Sol. Suppose y-axis divides PQ in the ration k: 1. Then, the coordinates of the point of

divides are

$R\left(\frac{3k-4}{k+1}, \frac{7k+5}{k+1}\right)$

Since, R lies on y-axis and x-coordinate of every point on y-axis is zero.

∴ $\frac{3k-4}{k+1} = 0$

⇒ $3k - 4 = 0 \Rightarrow k = \frac{4}{3}$

Hence, the required ratio is $\frac{4}{3}$: 1 i.e., 4: 3.

Putting k = 4/3 in the coordinates of R, we find that its coordinates are $\left(0, \frac{-13}{7}\right)$.

Que 4. Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3).

Sol. Let O (x, y) be the centre of circle. Given points are A (6, – 6), B (3, – 7) and C (3, 3).

Then, OA = $\sqrt{(x-6)^2+(y+6)^2}$; $OB = \sqrt{(x-3)^2+(y+7)^2}$

and OC = $\sqrt{(x-3)^2+(y-3)^2}$

Since, each point on the circle is equidistant from centre.

∴ OA = OB = OC = Radius

Since, OA = OB

$\Rightarrow \quad \sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y+7)^2}$

Squaring both sides, we get,

$(x-6)^2+(y+6)^2 = (x-3)^2+(y+7)^2$

$\Rightarrow \quad x^2-12x+36+y^2+12y+36 = x^2-6x+9+y^2+14y+49$

or $-6x-2y = -14$

or 3x + y = 7

Similarly, OB = OC

$\Rightarrow \quad \sqrt{(x-3)^2+(y+7)^2} = \sqrt{(x-3)^2+(y-3)^2}$

Squaring both sides, we get,

$(x-3)^2+(y+7)^2 = (x-3)^2+(y-3)^2 \quad \Rightarrow \quad (y+7)^2 = (y-3)^2$

$\Rightarrow \quad y^2+14y+49 = y^2-6y+9 \quad or \quad 20y = -40$

$\Rightarrow \quad y = -2$

Putting y = – 2 in (i), we get

3x – 2 = 7 $\quad \Rightarrow \quad x = 3$

Hence, the coordinates of the centre of circle are (3, – 2).

Que 5. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, – 3) and (3, 4). Find its centroid.

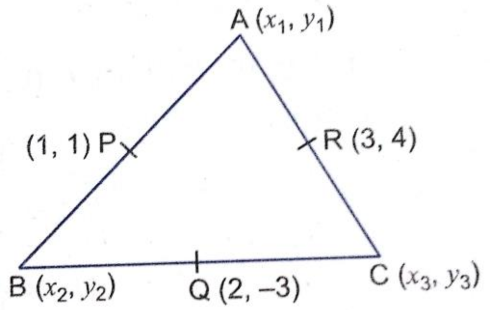

Fig. 6.33

Sol. Let P (1, 1), Q (2, – 3), R (3, 4) be the mid-points of sides AB, BC and CA respectively, of

triangle ABC. Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) be the vertices of triangle ABC.

Then, P is the mid-point of AB.

$\Rightarrow \quad \frac{x_1+x_2}{2} = 1, \quad \frac{y_1+y_2}{2} = 1$

$\Rightarrow \quad x_1+x_2 = 2$ And $y_1+y_2 = 2$ …(i)

Q is the mid-point of BC

$\Rightarrow \quad \frac{x_2+x_3}{2} = 2, \frac{y_2+y_3}{2} = -3 \Rightarrow \quad x_2 + x_3 = 4 \; and \; y_2 + y_3 = -6$ ...(ii)

R is the mid-point of AC

$\Rightarrow \quad \frac{x_1+x_3}{2} = 3 \; and \; \frac{y_1+y_3}{2} = 4 \quad \Rightarrow \quad x_1 + x_3 = 6 \; and \; y_1 + y_3 = 8$ ...(iii)

From (i), (ii) and (iii), we get

$$x_1 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$$

And $\quad y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 - 6 + 8$

$\Rightarrow \quad x_1 + x_2 + x_3 = 6 \; and \; y_1 + y_2 + y_3 = 2$ .....(iv)

The coordinates of the centroid of ΔABC are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{6}{3}, \frac{2}{3}\right) = \left(2, \frac{2}{3}\right)$$ [Using (iv)]