

5.5 Systems of Linear Equations

Variables: $x, y, z, x_1, x_2, \dots$

Real numbers: $a_1, a_2, a_3, b_1, a_{11}, a_{12}, \dots$

Determinants: D, D_x, D_y, D_z

Matrices: A, B, X

$$544. \quad \begin{cases} a_1x + b_1y = d_1 \\ a_2x + b_2y = d_2 \end{cases},$$

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D} \quad (\text{Cramer's rule}),$$

where

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1,$$

$$D_x = \begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix} = d_1b_2 - d_2b_1,$$

$$D_y = \begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix} = a_1d_2 - a_2d_1.$$

545. If $D \neq 0$, then the system has a single solution:

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}.$$

If $D = 0$ and $D_x \neq 0$ (or $D_y \neq 0$), then the system has no solution.

If $D = D_x = D_y = 0$, then the system has infinitely many solutions.

$$546. \quad \begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases},$$

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D} \quad (\text{Cramer's rule}),$$

$$\begin{aligned} \mathbf{D} &= \begin{vmatrix} \mathbf{a}_1 & \mathbf{b}_1 & \mathbf{c}_1 \\ \mathbf{a}_2 & \mathbf{b}_2 & \mathbf{c}_2 \\ \mathbf{a}_3 & \mathbf{b}_3 & \mathbf{c}_3 \end{vmatrix}, \mathbf{D}_x = \begin{vmatrix} \mathbf{d}_1 & \mathbf{b}_1 & \mathbf{c}_1 \\ \mathbf{d}_2 & \mathbf{b}_2 & \mathbf{c}_2 \\ \mathbf{d}_3 & \mathbf{b}_3 & \mathbf{c}_3 \end{vmatrix}, \\ \mathbf{D}_y &= \begin{vmatrix} \mathbf{a}_1 & \mathbf{d}_1 & \mathbf{c}_1 \\ \mathbf{a}_2 & \mathbf{d}_2 & \mathbf{c}_2 \\ \mathbf{a}_3 & \mathbf{d}_3 & \mathbf{c}_3 \end{vmatrix}, \mathbf{D}_z = \begin{vmatrix} \mathbf{a}_1 & \mathbf{b}_1 & \mathbf{d}_1 \\ \mathbf{a}_2 & \mathbf{b}_2 & \mathbf{d}_2 \\ \mathbf{a}_3 & \mathbf{b}_3 & \mathbf{d}_3 \end{vmatrix}. \end{aligned}$$
$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{D_z}{D}.$$

If $D=D_x=D_y=D_z=0$, then the system has infinitely many solutions.

The set of linear equations

$$\left\{ \begin{array}{l} \mathbf{a}_{11}\mathbf{x}_1 + \mathbf{a}_{12}\mathbf{x}_2 + \dots + \mathbf{a}_{1n}\mathbf{x}_n = \mathbf{b}_1 \\ \mathbf{a}_{21}\mathbf{x}_1 + \mathbf{a}_{22}\mathbf{x}_2 + \dots + \mathbf{a}_{2n}\mathbf{x}_n = \mathbf{b}_2 \\ \dots\dots\dots \\ \mathbf{a}_{n1}\mathbf{x}_1 + \mathbf{a}_{n2}\mathbf{x}_2 + \dots + \mathbf{a}_{nn}\mathbf{x}_n = \mathbf{b}_n \end{array} \right.$$

$$\begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \dots & \mathbf{a}_{1n} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \dots & \mathbf{a}_{2n} \\ \vdots & \vdots & & \vdots \\ \mathbf{a}_{n1} & \mathbf{a}_{n2} & \dots & \mathbf{a}_{nn} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{pmatrix} = \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_n \end{pmatrix},$$
$$A \cdot X = B,$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

549. Solution of a Set of Linear Equations $n \times n$

$$X = A^{-1} \cdot B,$$

where A^{-1} is the inverse of A .