Chapter 7

Area Under Curves

CONTENTS

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In earlier method, we have determined the area of a closed region of the plane, when the region is bounded by line segments. However, if the region is bounded, either partially or wholly, by curves, such a computation cannot be performed by earlier methods. Therefore there is a need for a strong mathematical technique for solving such problems. This comes out to be possible by using the concept of the definite integral.

The definite integral is used to solve many interesting types of problems from various disciplines like economics, finance and probability. The area under certain curves used to solve probability problems.

Area Under Curves

7.1 Introduction

We know the methods of evaluating definite integrals. These integrals are used in evaluating certain types of bounded regions. For evaluation of bounded regions defined by given functions, we shall also require to draw rough sketch of the given function. The process of drawing rough sketch of a given function is called **curve sketching.**

7.2 Procedure of Curve Sketching

(1) **Symmetry:**

(i) Symmetry about x-axis: If all powers of y in equation of the given curve are even, then it is symmetric about x-axis *i.e.,* the shape of the curve above x-axis is exactly identical to its shape below x-axis.

For example, $y^2 = 4ax$ is symmetric about x-axis.

(ii) Symmetry about y-axis: If all power of x in the equation of the given curve are even, then it is symmetric about y-axis

For example, $x^2 = 4ay$ is symmetric about y-axis.

(iii) Symmetry in opposite quadrants or symmetry about origin: If by putting $-x$ for x and $-y$ for y, the equation of a curve remains same, then it is symmetric in opposite quadrants.

For example, $x^2 + y^2 = a^2$ and $xy = a^2$ are symmetric in opposite quadrants.

(iv) Symmetry about the line $y = x$: If the equation of a given curve remains unaltered by interchanging x and y then it is symmetric about the line $y = x$ which passes through the origin and makes an angle of 45^0 with the positive direction of x-axis.

(2) **Origin:** If the equation of curve contains no constant terms then it passes through the origin. Find whether the curve passes through the origin or not.

For examples, $x^2 + y^2 + 4ax = 0$ passes through origin.

(3) **Points of intersection with the axes:** If we get real values of x on putting $y = 0$ in the equation of the curve, then real values of x and $y = 0$ give those points where the curve cuts the x-axis. Similarly by putting $x = 0$, we can get the points of intersection of the curve and y-axis.

For example, the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersect the axes at points $(\pm a, 0)$ and $(0, \pm b)$.

(4) **Special points:** Find the points at which $\frac{dy}{dx}=0$, at these points the tangent to the curve is parallel to x-axis. Find the points at which $\frac{dx}{dy}=0$. At these points the tangent to the curve is parallel to y-axis.

(5) **Region:** Write the given equation as $y=f(x)$, and find minimum and maximum values of x which determine the region of the curve.

For example for the curve $xy^2=a^2(a-x) \Rightarrow y=a\sqrt{\frac{a-x}{x}}$

Now y is real, if $0 \le x \le a$, So its region lies between the lines x = 0 and $x = a$

(6) **Regions where the curve does not exist:** Determine the regions in which the curve does not exists. For this, find the value of y in terms of x from the equation of the curve and find the value of x for which y is imaginary. Similarly find the value of x in terms of y and determine the values of y for which x is imaginary. The curve does not exist for these values of x and y.

For example, the values of y obtained from $y^2=4ax$ are imaginary for negative value of x, so the curve does not exist on the left side of y-axis. Similarly the curve $a^2y^2=x^2(a-x)$ does not exist for $x > a$ as the values of y are imaginary for $x > a$.

7.3 Sketching of Some Common Curves

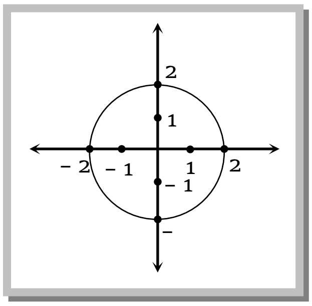
(1) **Straight line:** The general equation of a straight line is $ax+by+c=0$. To draw a straight line, find the points where it meets with the coordinate axes by putting y = 0 and x = 0 respectively in its equation. By joining these two points, we get the sketch of the line.

(2) **Region represented by a linear inequality:** To find the region represented by linear inequalities $ax+by \le c$ and $ax+by \ge c$, we proceed as follows.

(i) Convert the inequality into equality to obtain a linear equation in x, y.

(ii) Draw the straight line represented by it.

(iii) The straight line obtained in (ii) divides the xy-plane in two parts. To determine the region represented by the inequality choose some convenient points, *e.g.* origin or some point on the coordinate axes. If the coordinates of a point satisfy the inequality, then region containing the point is the required region, otherwise the region not containing the point is the required region.

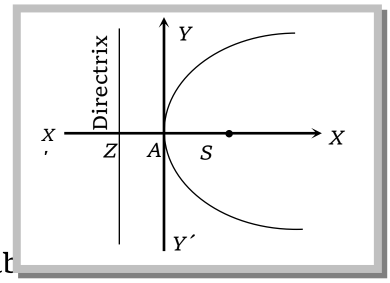


(3) **Circle:** The equation of a circle having centre at (0,0) and radius r is given by $x^2+y^2=r^2$. The equation of a circle having centre at *(h, k)*

and radius r is given by $(x-h)^2+(y-k)^2=r^2$. The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$. This represents the circle whose centre is at (-*g*,-*f*) and radius equal to $\sqrt{g^2+f^2-c}$.

The figure of the circle $x^2+y^2=(2)^2$ is given. Here centre is (0,0) and radius is 2.

(4)**Parabola:** There are four standard forms of parabola with vertex at origin and the axis along either of coordinate axis.

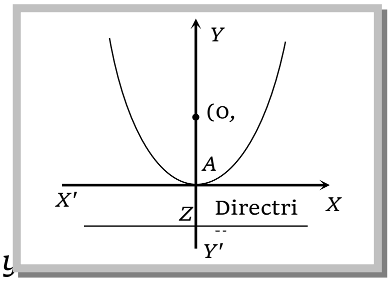


(i) $y^2=\pm 4ax$: For this parabola

(a) Vertex: (0,0) (b) Focus: $(\pm a, 0)$

(c) Directrix: $x \pm a = 0$ (d) Latus rectum: $4a$

(e) Axis $y=0$ (f) Symmetry : It is symmetric ab

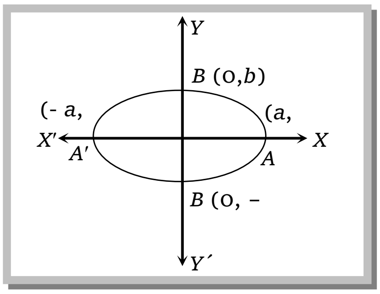


(ii) $x^2=\pm 4ay$: For this parabola

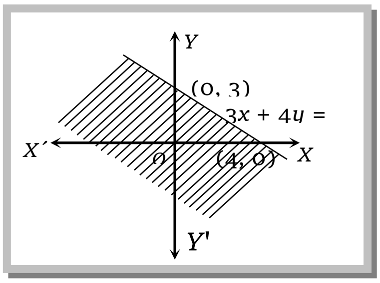
(a) Vertex: (0,0) (b) Focus: $(0, \pm a)$

(c) Directrix: $y \pm a = 0$ (d) Latus rectum: $4a$

(e) Axis *x* = 0 (f) Symmetry: It is symmetric about *y*

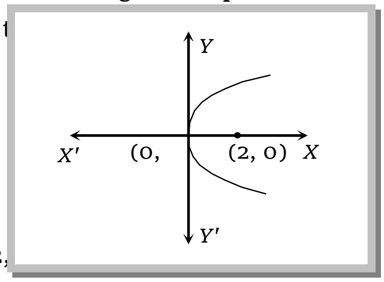


(5) **Ellipse:** The standard equation of the ellipse having its centre at the origin and major and minor axes along the coordinate axes is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Here $a>b$. The figure of the ellipse is given.



Example: 1 Sketch the region bounded by $3x+4y \le 12$

Solution: Converting the inequality into equation we get $3x+4y=12$. This line meets *x*-axis at (4,0) and *y*-axis at (0,3). Joining these two points we obtain the straight line represented by $3x+4y=12$. This straight line divides the plane in two parts. One part contains the origin the other does not contain the origin. Clearly, (0,0) satisfies the inequality $3x+4y \le 12$. So, the region represented by $3x+4y \le 12$ is the region containing t

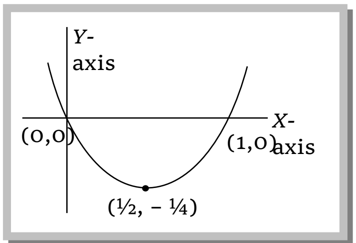


Example: 2 Sketch the parabola $y^2=8x$.

Solution: $y^2=8x$

$y^2=4(2)x$

Here vertex is (0,0) and focus is (2,



Example: 3 Sketch the graph for $y = x^2 - x$.

Solution: We note the following points about the curve.

(i) The curve does not have any kind of symmetry.

(ii) The curve passes through the origin and the tangent at the origin is obtained by equating the lowest degree term to zero.

The lowest degree term is $x + y$. Equation it to zero, we get $x + y = 0$ as the equation of tangent at the origin.

(iii) Putting y=0 in the equation of curve, we get $x^2 - x = 0 \Rightarrow x = 0, 1$. So, the curve crosses x-axis at (0,0) and (1,0).

Putting x=0 in the equation of the curve, we obtain y=0. So, the curve meets y-axis at (0,0) only.

(iv) $y = x^2 - x \Rightarrow \frac{dy}{dx} = 2x - 1$ and $\frac{d^2y}{dx^2} = 2$

Now, $\frac{dy}{dx} = 0 \Rightarrow x = \frac{1}{2}$,

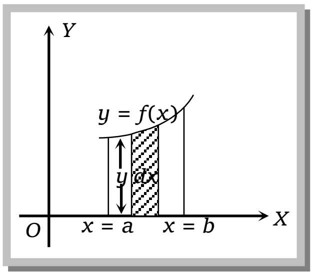
At $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} > 0$, So $x = \frac{1}{2}$ is point of local minima.

(v) $\frac{dy}{dx} > 0 \Rightarrow 2x - 1 > 0 \Rightarrow x > \frac{1}{2}$, So the curve increases for all $x > \frac{1}{2}$ and decreases for all $x < \frac{1}{2}$

7.4 Area of Bounded Regions

(1) The area bounded by a cartesian curve $y = f(x)$, x-axis and ordinates $x = a$ and $x = b$ is given by

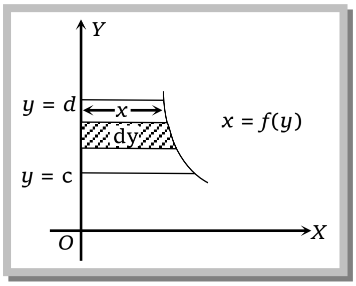
$$\text{Area} = \int_a^b y\, dx = \int_a^b f(x) dx$$



(2) If the curve $y = f(x)$ lies below x-axis, then the area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is negative. So, area is given by $\left|\int_a^b y\, dx\right|$

(3) The area bounded by a cartesian curve $x = f(y)$, y-axis and abscissa $y = c$ and $y = d$ is given by

$$\text{Area} = \int_c^d x\, dy = \int_c^d f(y) dy$$



(4)If the equation of a curve is in parametric form, say $x = f(t)$, $y = g(t)$ then the area $= \int_a^b y\,dx = \int_{t_1}^{t_2} g(t)\,f'(t)\,dt$ where t_1 and t_2 are the values of t respectively corresponding to the values of a and b of x.

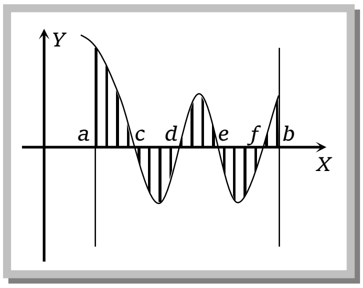
7.5 Sign convention for finding the Areas using Integration

While applying the discussed sign convention, we will discuss the three cases.

Case I: In the expression $\int_a^b f(x)dx$ if $b > a$ and $f(x) > 0$ for all $a \le x \le b$, then this integration will give the area enclosed between the curve $f(x)$, x-axis and the line $x = a$ and $x = b$ which is positive. No need of any modification.

Case II: If in the expression $\int_a^b f(x)dx$ if $b > a$ and $f(x) < 0$ for all $a \le x \le b$, then this integration will calculate to be negative. But the numerical or the absolute value is to be taken to mean the area enclosed between the curve $y = f(x)$, x-axis and the lines $x = a$ and $x = b$.

Case III. If in the expression $\int_a^b f(x)dx$ where $b > a$ but $f(x)$ changes its sign a numbers of times in the interval $a \le x \le b$, then we must divide the region [a, b] in such a way that we clearly get the points lying between [a, b] where $f(x)$ changes its sign. For the region where $f(x)>0$ we just integrate to get the area in that region and then add the absolute value of the integration calculated in the region where $f(x)<0$ to get the desired area between the curve $y = f(x)$, x-axis and the line $x = a$ and $x = b$.



Hence, if $f(x)$ is as in above figure, the area enclosed by $y = f(x)$, x-axis and the lines $x = a$ and $x = b$ is given by

$$A = \int_a^c f(x)dx + \left|\int_c^d f(x)dx\right| + \int_d^e f(x)dx + \left|\int_e^f f(x)dx\right| + \int_f^b f(x)dx$$

Example: 4 The area (in square units) enclosed by the curve $x^2y = 36$, the x-axis and the lines $x = 6$ and $x = 9$ is **[Kerala (Engg.) 2000]**

(a) 2 (b) 1 (c) 4 (d) 3

Solution: (a) Required area $= \int_6^9 y\,dx = \int_6^9 \frac{36}{x^2}dx$ [Given $x^2y = 36 \Rightarrow y = \frac{36}{x^2}$]

$$= \left[-\frac{36}{x}\right]_6^9 = -\left[\frac{36}{9} - \frac{36}{6}\right] = -[4-6] = 2\,.$$

Example: 5 The area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = 1$, $x = b$ is equal to $\sqrt{b^2+1}-\sqrt{2}$ for all $b > 1$, then $f(x)$ is **[MP PET 2000]**

(a) $\sqrt{x-1}$ (b) $\sqrt{x+1}$ (c) $\sqrt{x^2-1}$ (d) $\dfrac{x}{\sqrt{1+x^2}}$

Solution: (d) $\int_1^b f(x)\,dx = \sqrt{b^2+1}-\sqrt{2} = \left[\sqrt{x^2+1}\right]_1^b$

$\Rightarrow f(x) = \dfrac{1}{2}\cdot\dfrac{2x}{\sqrt{x^2+1}}$

Hence $f(x) = \dfrac{x}{\sqrt{1+x^2}}$.

Example: 6 The area of the region bounded by the curve $y = x - x^2$ between $x = 0$ and $x = 1$ is **[Pb. CET 1994, 89]**

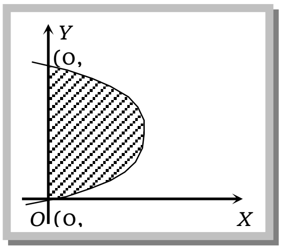
(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{5}{6}$

Solution: (a) Required Area $= \int_0^1 (x-x^2)dx = \left[\dfrac{x^2}{2}-\dfrac{x^3}{3}\right]_0^1 = \dfrac{1}{2}-\dfrac{1}{3} = \dfrac{1}{6}$.

Example: 7 Find the area bounded between the curve $y^2 = 2y - x$ and y-axis .

(a) $\dfrac{4}{3}$ (b) $\dfrac{2}{3}$ (c) $\dfrac{1}{3}$ (d) 5

Solution: (a) The area between the given curve $x = 2y - y^2$ and y- axis will be as shown



$\therefore$ Required Area $= \int_0^2 (2y-y^2)dy$

$= \left[y^2 - \dfrac{y^3}{3}\right]_0^2 = \dfrac{4}{3}$

Example: 8 Find the area bounded by the curves $x = a\cos t$, $y = b\sin t$ in the first quadrant

(a) $\dfrac{\pi ab}{4}$ (b) $\dfrac{\pi a^2 b}{4}$ (c) $\dfrac{\pi ab^2}{4}$ (d) None of these

Solution: (a) Clearly the given equation are the parametric equation of ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$. Curve meet the x-axis in the first quadrant at (a,0)

$\therefore$ Required area $= \int_o^a y\,dx = \int_{\frac{\pi}{2}}^0 (b\sin t)(-a\cos t)dt = ab\int_0^{\pi/2}\sin^2 t\,dt = \left(\dfrac{\pi ab}{4}\right)$

$(\because$ At $x = 0, t = \pi/2$ and $x = a, t = 0)$

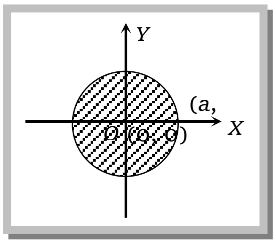
7.6 Symmetrical Area

If the curve is symmetrical about a coordinate axis (or a line or origin), then we find the area of one symmetrical portion and multiply it by the number of symmetrical portions to get the required area.

Example: 9 Find the whole area of circle $x^2+y^2=a^2$

(a) π (b) πa^2 (c) πa^3 (d) a^2

Solution: (b) The required area is symmetric about both the axis as shown in figure



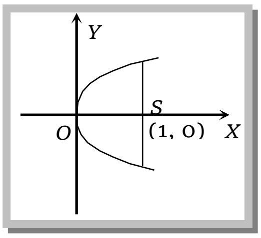
$\therefore$ Required area = $4\int_0^a \sqrt{a^2-x^2}\,dx = 4\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$= 4\left[\frac{\pi}{2}\times\frac{a^2}{2}\right] = \pi a^2$

Example: 10 Find the area bounded by the parabola $y^2=4x$ and its latus rectum **[Rajasthan PET 1997, 94, 92, 84]**

(a) $\frac{8}{3}$ (b) $\frac{4}{3}$ (c) $\frac{16}{3}$ (d) None of these

Solution: (a) Since the curve is symmetrical about x-axis, therefore the required area



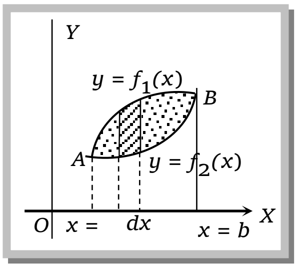
$= 2\int_0^1 y\,dx = 2\int_0^1 \sqrt{4x}\,dx$

$= 4.\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1 = \frac{8}{3}$

7.7 Area between Two curves

(1) **When both curves intersect at two points and their common area lies between these points:**

If the curves $y_1=f_1(x)$ and $y_2=f_2(x)$, where $f_1(x)>f_2(x)$ intersect in two points $A(x=a)$ and $B(x=b)$, then common area between the curves is $=\int_a^b (y_1-y_2)dx$

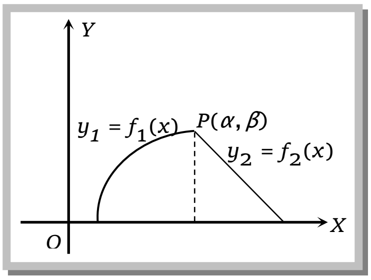


$= \int_a^b [f_1(x)-f_2(x)]dx$

(2) **When two curves intersect at a point and the area between them is bounded by x-axis:**

Area bounded by the curves $y = f_1(x), y_2=f_2(x)$ and $x-$axis is

$$= \int_a^\alpha f_1(x)dx + \int_\alpha^b f_2(x)dx$$



Where $P(\alpha,\beta)$ is the point of intersection of the two curves.

(3) **Positive and negative area :** Area is always taken as positive. If some part of the area lies above the x-axis and some part lies below x-axis, then the area of two parts should be calculated separately and then add their numerical values to get the desired area.

Important Tips

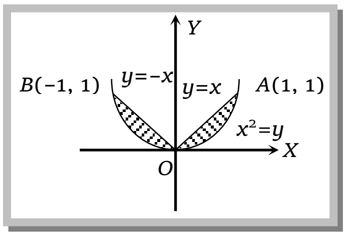
- *The area of the region bounded by* $y^2 = 4ax$ *and* $x^2 = 4by$ *is* $\frac{16ab}{3}$ *square units.*
- *The area of the region bounded by* $y^2 = 4ax$ *and* $y = mx$ *is* $\frac{8a^2}{3m^3}$ *square units*
- *The area of the region bounded by* $y^2 = 4ax$ *and its latus rectum is* $\frac{8a^2}{3}$ *square units*
- *The area of the region bounded by one arch of sin (ax) or cos (ax) and x-axis is* $\frac{2}{a}$ *sq. units*
- *Area of the ellipse* $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ *is* πab *sq. units.*
- *Area of region bounded by the curve y = sin x , x-axis and the line x =0 and x=* 2π *is 4 unit.*

Example: 11 The area of the region bounded by the curves $y = x^2$ and $y = |x|$ is **[Roorkee (Qualifying) 2000]**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{5}{3}$

Solution: (b) Required area $= 2\left[\int_0^1 x dx - \int_0^1 x^2 dx\right] = 2\left[\left\{\frac{x^2}{2}\right\}_0^1 - \left\{\frac{x^3}{3}\right\}_0^1\right]$

$$= 2\left[\left(\frac{1}{2}-0\right)-\left(\frac{1}{3}-0\right)\right] = 2\left[\frac{1}{2}-\frac{1}{3}\right] = 2\left[\frac{1}{6}\right] = \frac{1}{3}.$$



Example: 12 The area (in square units) bounded by the curve $y = x^3$, $y = x^2$ and the ordinates $x = 1$, $x = 2$ is**[EAMCET 2000**

(a) $\frac{17}{12}$ (b) $\frac{12}{17}$ (c) $\frac{2}{7}$ (d) $\frac{7}{2}$

Solution: (a) Required area $= \int_1^2 (x^3 - x^2)dx = \left[\frac{x^4}{4}-\frac{x^3}{3}\right]_1^2 = \left(4-\frac{8}{3}\right)-\left(\frac{1}{4}-\frac{1}{3}\right) = \frac{4}{3}+\frac{1}{12} = \frac{16+1}{12} = \frac{17}{12}$.

Example: 13 The area of the region bounded by the curve $y=2x-x^2$ and line $y = x$ is **[Pb. CET 2000; Roorkee 1992]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

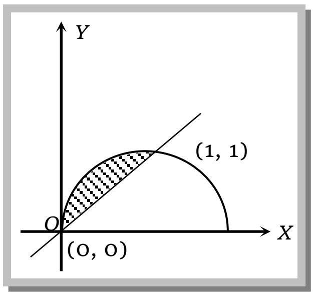
Solution: (d) The given curve is $y = 2x - x^2$

$\Rightarrow y = -(x^2 - 2x + 1) + 1$

$\Rightarrow y - 1 = -(x-1)^2$, it represents a downward parabola with vertex (1,1)

Its points of intersection with the line $y = x$ are (0,0) and (1,1).

Required area = shaded region

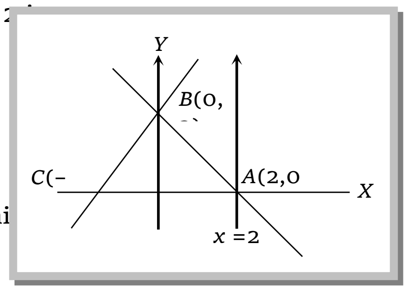


$$= \int_0^1 (2x - x^2)dx - \int_0^1 x\,dx \;= \int_0^1 (x - x^2)dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 \;= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

Example: 14 Area bounded by the lines $y = 2 + x$, $y = 2 - x$ and $x = 2$ is

(a) 3 (b) 4

(c) 8 (d) 16



Solution: (b) Given lines are $y = x + 2$, $y = -x + 2$, $x = 2$

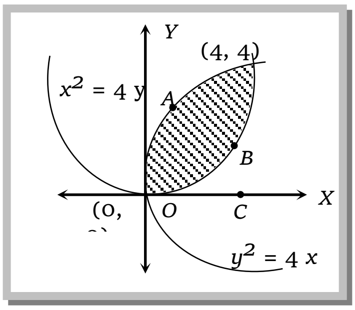
Hence required area = Area of $\Delta CAB = \frac{1}{2}(2)(4) = 4$ sq. unit.

Example: 15 The area bounded by the curve $y^2 = 4x$ and $x^2 = 4y$ is

[Karnataka CET 1999,2003; MP PET 1997; SCRA 1986; Rajasthan PET 1988, 99,97]

(a) $\frac{16}{3}$ sq. units (b) $\frac{3}{16}$ sq. units (c) $\frac{14}{3}$ sq. units (d) $\frac{3}{14}$ sq. units

Solution: (a) Required area $= \int_0^4 (OABC - ODBC)$ Region $= \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right)dx = \frac{16}{3}$ square unit.



Trick : From Important Tips' the area of the region bounded by $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16ab}{3}$ square unit.

Here $y^2 = 4x$ and $x^2 = 4y$, so $a = 1$ and $b = 1$

Required area = $\frac{16}{3}(1)\;(1) = \frac{16}{3}$ square unit.

Example: 16 The area of the bounded region by the curve $y = \sin x$, the x-axis and the line $x = 0$ and $x = \pi$ is

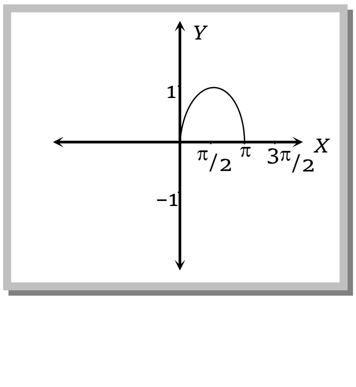
[Rajasthan PET 1989, 92]

(a) 4 (b) 2 (c) 0 (d) None of these

Solution: (b) Required area = $\int_0^\pi \sin x\,dx$

$= 2\int_0^{\pi/2} \sin x\,dx \;=\; 2[-\cos x]_0^{\pi/2} \;= 2\;[\;(-\cos \pi/2) - (-\cos 0)\;] \;= 2(1)$

= 2 square unit.



Trick : For the curve $y = \sin x$ or $\cos x$, the area of

$\int_0^{\pi/2} \sin x\,dx = 1, \int_0^{\pi} \sin x\,dx = 2, \int_0^{3\pi/2} \sin x\,dx = 3, \int_0^{2\pi} \sin x\,dx = 4$ and so on.

Example: 17 The area enclosed by the parabola $y^2 = 8x$ and the line $y = 2x$ is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

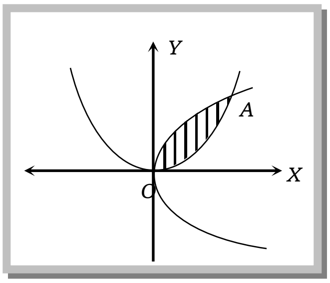
Solution: (a) Solve the equation $y^2 = 8x$ and the line $y = 2x$, we get the point of intersection. Then find the required area bounded by this region. It is $\frac{4}{3}$.

Trick : Required area $=\frac{8(2)^2}{3(2)^3}=\frac{32}{24}=\frac{4}{3}$ [$\because$ Area bounded by $y^2=4ax$ and $y=mx$ is $\frac{8a^2}{3m^3}$. Here $a=2, m=2$]

Example: 18 If the area bounded by $y=ax^2$ and $x=ay^2$, $a>0$, is 1, then a =

(a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{\sqrt{3}}$

Solution: (b) The *x* coordinate of A is $\frac{1}{a}$



According to the given condition

$$1=\int_0^{\frac{1}{a}}\left(\sqrt{\frac{x}{a}}-ax^2\right)dx=\frac{1}{\sqrt{a}}.\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^{\frac{1}{a}}-\frac{a}{3}[x^3]_0^{1/a}$$

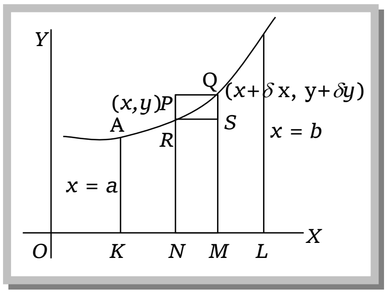
$$\Rightarrow a=\frac{1}{\sqrt{3}}$$

7.8 Volumes and Surfaces of Solids of Revolution

If a plane curve is revolved about some axis in the plane of the curve, then the body so generated is known as solid of revolution. The surface generated by the perimeter of the curve is known as surface of revolution and the volume generated by the area is called volume of revolution.

For example, a right angled triangle when revolved about one of its sides (forming the right angle) generates a right circular cones.

(1) **Volumes of solids of revolution:**



(i) The volume of the solid generated by the revolution, about the *x*-axis, of the area bounded by the curve *y = f(x),* the ordinates at *x = a, x = b* and the *x*-axis is equal to $\pi\int_a^b y^2\,dx$.

(ii) The revolution of the area lying between the curve *x = f(y),* the *y*-axis and the lines $y=a$ and $y=b$ is given by (interchanging *x* and *y* in the above formulae) $\int_a^b \pi x^2\,dy$.

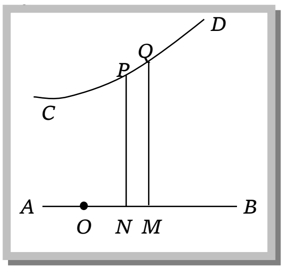
(iii) If the equation of the generating curve be given by $x=f_1(t)$ and $y=f_2(t)$ and it is revolved about *x*-axis, then the formula corresponding to $\int_a^b \pi y^2\,dx$ becomes $\int_{t_1}^{t_2}\pi\{f_2(t)\}^2\,d\{f_1(t)\}$, where f_1 and f_2 are the values of *t* corresponding to *x = a* and *x = b*

(iv) If the curve is given by an equation in polar co-ordinates, say $r=f(\theta)$, and the curve revolves about the initial line, the volume generated

$=\pi\int_a^b y^2\,dx=\pi\int_\alpha^\beta y^2\left(\frac{dx}{d\theta}\right).d\theta$, where α and β are the values of θ corresponding to *x = a* and *x = b*

Now $x = r\cos\theta,\ y = r\sin\theta$. Hence the volume $= \pi\int_{\alpha}^{\beta} r^2 \sin^2\theta \frac{d}{d\theta}(r\cos\theta)d\theta$

(v) If the generating curve revolves about any line AB (which is different from either of the axes), then the volume of revolution is

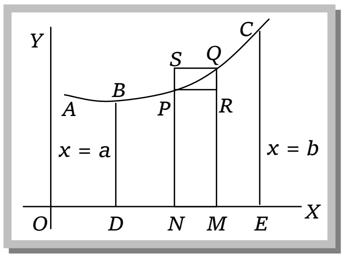


Note : ❑ The volume of the solid generated by revolving the area bounded by the curve $r = f(\theta)$ and the radii vectors $\theta = \alpha$ and $\theta = \beta$ about the initial line is $\frac{2}{3}\pi\int_{\alpha}^{\beta} r^3 \sin\theta\, d\theta$.

❑ The volume in the case when the above area is revolved about the line $\theta = \frac{\pi}{2}$ is $\frac{2}{3}\pi\int_{\alpha}^{\beta} r^3 \cos\theta\, d\theta$.

(2) **Area of surfaces of revolution:**

(i) The curved surface of the solid generated by the revolution, about the *x*- axis, of the area bounded by the curve $y = f(x)$, the ordinates at *x* = *a*, *x* = *b* and the *x*-axis is equal to $2\pi\int_{x=a}^{x=b} y\, ds$.



(ii) If the arc of the curve $y = f(x)$ revolves about *y*-axis, then the area of the surface of revolution (between proper limits) $= 2\pi\int x\, ds$, where $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(iii) If the equation of the curve is given in the parametric form $x = f_1(t)$ and $y = f_2(t)$, and the curve revolves about *x*-axis, then we get the area of the surface of revolution $= 2\pi\int_{t=t_1}^{t=t_2} y ds = 2\pi\int_{t=t_1}^{t=t_2} f_2(t) ds$

$= 2\pi\int_{t_1}^{t_2} f_2(t)\sqrt{\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\}}dt$, where t_1 and t_2 are the values of the parameter t corresponding to *x* = *a* and *x* = *b*.

(iv) If the equation of the curve is given in polar form then the area of the surface of revolution about *x*-axis $= 2\pi\int y\, ds = 2\pi\int (r\sin\theta)\frac{dS}{d\theta}.d\theta = 2\pi\int r\sin\theta.\sqrt{\left\{r^2 + \left(\frac{dr}{d\theta}\right)^2\right\}}d\theta$ between proper limits.

Example: 19 The part of circle $x^2+y^2=9$ in between $y=0$ and $y=2$ is revolved about y-axis. The volume of generating solid will be

[UPSEAT 1999]

(a) $\frac{46}{3}\pi$ (b) 12π (c) 16π (d) 28π

Solution: (a) The part of circle $x^2+y^2=9$ in between $y=0$ and $y=2$ is revolved about y- axis. Then a frustum of sphere will be formed.

The volume of this frustum $=\pi\int_0^2 x^2 dy = \pi\int_0^2 (9-y^2)dy$

$$=\pi\left[9y-\frac{1}{3}y^3\right]_0^2 = \pi\left[9\times 2-\frac{1}{3}(2)^3-(9.0-\frac{1}{3}.0)\right] = \frac{46}{3}\pi \text{ cubic unit.}$$

Example: 20 The part of straight line $y=x+1$ between $x=2$ and $x=3$ is revolved about x-axis, then the curved surface of the solid thus generated is

[UPSEAT 2000]

(a) $\frac{37\pi}{3}$ (b) $\frac{7\pi}{\sqrt{2}}$ (c) 37π (d) $7\pi\sqrt{2}$

Solution: (d) Curved surface $=\int_a^b 2\pi y\sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}dx$. Given that $a=2$, $b=3$ and $y=x+1$ on differentiating with respect to x

$\frac{dy}{dx}=1+0$ or $\frac{dy}{dx}=1$. Therefore, curved surface $=\int_2^3 2\pi(x+1)\sqrt{[1+(1)^2]}\,dx$

$$=\int_2^3 2\pi(x+1)\sqrt{2}\,dx = 2\sqrt{2}\pi\int_2^3 (x+1)dx = 2\sqrt{2}\pi\left[\frac{(x+1)^2}{2}\right]_2^3 = \frac{2\sqrt{2}}{2}\pi[(3+1)^2-(2+1)^2]$$

$$=\sqrt{2}\pi(16-9)=7\sqrt{2}\pi=7\pi\sqrt{2}$$

Assignment

Area of Bounded Region

Basic Level

1. Area under the curve $y = x^2 - 4x$ within the x-axis and the line $x=2$, is **[SCRA 1991]**

(a) $\frac{16}{3}$ sq. units (b) $-\frac{16}{3}$ sq. units (c) $\frac{4}{7}$ sq. units (d) Cannot be calculated

2. The area bounded by the curve $y = 4x - x^2$ and the x-axis is **[MP PET 1999, 2003]**

(a) $\frac{30}{7}$ sq. units (b) $\frac{31}{7}$ sq. units (c) $\frac{32}{3}$ sq. units (d) $\frac{34}{3}$ sq. units

3. The area between the curve $y = 4 + 3x - x^2$ and x-axis is **[Rajasthan PET 2001]**

(a) $\frac{125}{6}$ (b) $\frac{125}{3}$ (c) $\frac{125}{2}$ (d) None of these

4. Area under the curve $y = \sqrt{3x+4}$ between $x = 0$ and $x = 4$, is **[AI CBSE 1979,1980]**

(a) $\frac{56}{9}$ sq. units (b) $\frac{64}{9}$ sq. units (c) 8 sq. units (d) None of these

5. The area bounded by the curve $y = x^3$, x- axis and two ordinates $x = 1$ to $x = 2$ equal to **[MP PET 1999]**

(a) $\frac{15}{2}$ sq. units (b) $\frac{15}{4}$ sq. units (c) $\frac{17}{2}$ sq. units (d) $\frac{17}{4}$ sq. units

6. If the area above the x-axis, bounded by the curves $y = 2^{kx}$ and $x = 0$ and $x = 2$ is $\frac{3}{\ln 2}$, then the value of k is **[Orissa JEE 2003]**

(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) 2

7. Area bounded by curve $y = x^3$, x-axis and ordinates $x = 1$ and $x = 4$, is

(a) 64 sq. units (b) 27 sq. units (c) $\frac{127}{4}$ sq. units (d) $\frac{255}{4}$ sq. units

8. Area bounded by curve $xy = c$, x-axis between $x = 1$ and $x = 4$, is

(a) c log 3 sq. units (b) 2 log c sq. units (c) 2c log 2 sq. units (d) 2c log5 sq. units

9. The measurement of the area bounded by the coordinate axes and the curve $y = \log_e x$ is **[MP PET 1998]**

(a) 1 (b) 2 (c) 3 (d) ∞

10. The area bounded by the curve $y = \log x$, the x- axis and ordinate $x = e$ is **[MP PET 1994]**

(a) e (b) 1 (c) ∞ (d) None of these

11. Area bounded by the curve $y = \log x$, x-axis and the ordinates $x = 1$, $x = 2$ is

(a) log 4 sq. units (b) log 4+1 sq. units (c) log 4-1 sq. units (d) None of these

12. Area bounded by the curve $y = x\,e^{x^2}$, x-axis and the ordinates x=0, x=a is

(a) $\frac{e^{a^2}+1}{2}$ sq. units (b) $\frac{e^{a^2}-1}{2}$ sq. units (c) $e^{a^2}+1$ sq. units (d) $e^{a^2}-1$ sq. units

13. If area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $\frac{a^2}{3}$, then the value of m is

(a) 2 (b) -2 (c) 1/2 (d) None of these

14. The area of the region (in the square units) bounded by the curve $x^2 = 4y$, line $x = 2$ and x-axis is **[MP PET 2002]**

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$

15. Area bounded by the parabola $y = 4x^2$, y- axis and the lines $y = 1$, $y = 4$ is

(a) 3 sq. units (b) $\frac{7}{5}$ sq. units (c) $\frac{7}{3}$ sq. units (d) None of these

16. Area bounded by parabola $y^2 = x$ and straight line $2y = x$ is **[MP PET 1996]**

(a) $\frac{4}{3}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

17. Area enclosed by the parabola $ay = 3(a^2 - x^2)$ and x-axis is

(a) $4a^2$ sq. units (b) $12a^2$ sq. units (c) $4a^3$ sq. units (d) None of these

18. The area enclosed by the curve $y = \sin x$, $y = 0$, $x = 0$ and $x = \frac{\pi}{2}$ is **[MP PET 1995]**

(a) π (b) 2π (c) 1 (d) 2

19. Area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$ is

(a) 2 sq. units (b) 4 sq. units (c) 8 sq. units (d) None of these

20. Area bounded by the curve $y = k \sin x$ between $x = \pi$ and $x = 2\pi$, is

(a) 2κ sq. units (b) 0 (c) $\frac{\kappa^2}{2}$ sq. units (d) κ sq. units

21. The area of the region bounded by the x-axis and the curves defined by $y = \tan x$ $\left(-\frac{\pi}{3} \le x \le \frac{\pi}{3}\right)$ is **[Kurukshetra CEE 1998]**

(a) $\log \sqrt{2}$ (b) $-\log \sqrt{2}$ (c) $2 \log 2$ (d) 0

22. The area between the curve $y = \sin^2 x$, x-axis and the ordinates x=0 and $x = \frac{\pi}{2}$ is **[Rajasthan PET 1996]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) π

23. Area of the region bounded by the curve $y = \tan x$, tangent drawn to the curve at $x = \frac{\pi}{4}$ and the x-axis is **[DCE 2002]**

(a) $\frac{1}{4}$ (b) $\log \sqrt{2} - \frac{1}{4}$ (c) $\log \sqrt{2} + \frac{1}{4}$ (d) None of the above

24. The ratio of the areas bounded by the curves $y = \cos x$ and $y = \cos 2x$ between $x = 0$, $x = \frac{\pi}{3}$ and x- axis, is **[MP PET 1997]**

(a) $\sqrt{2} : 1$ (b) 1:1 (c) 1:2 (d) 2:1

25. The area bounded by the curve $y = \sec x$, the x-axis and the lines x=0 and $x = \frac{\pi}{4}$ is **[Tamilnadu PCEE 2002]**

(a) $\log(\sqrt{2} + 1)$ (b) $\log(\sqrt{2} - 1)$ (c) $\frac{1}{2} \log 2$ (d) $\sqrt{2}$

26. The area bounded by $y = [x]$ and the two ordinates x=1 and x=1.7 is

(a) $\frac{17}{10}$ (b) 1 (c) $\frac{17}{5}$ (d) $\frac{7}{10}$

27. The value of k for which the area of the figure bounded by the curve $y = 8x^2 - x^5$, the straight line x = 1 and x = k and the x-axis is equal to $\frac{16}{3}$

(a) 2 (b) $\sqrt[3]{8 - \sqrt{17}}$ (c) 3 (d) – 1

Advance Level

28. The area of the region bounded by the curves $y = |x-2|$, $x = 1$, $x = 3$ and the x-axis is **[AIEEE 2004]**

(a) 4 (b) 2 (c) 3 (d) 1

29. The area of the region bounded by $y = |x-1|$ and $y = 1$ is **[IIT Screening 1994]**

(a) 2 (b) 1 (c) 1/2 (d) None of these

30. Area bounded by lines $y = 2+x$, $y = 2-x$ and $x = 2$ is **[MP PET1996]**

(a) 3 (b) 4 (c) 8 (d) 16

31. Area enclosed between the curve $y^2(2a-x) = x^3$ and line $x = 2a$ above x-axis is **[MP PET 2001]**

(a) πa^2 (b) $\frac{3\pi a^2}{2}$ (c) $2\pi a^2$ (d) $3\pi a^2$

32. Area bounded by the curve $xy - 3x - 2y - 10 = 0$, x-axis and the lines $x = 3$, $x = 4$ is **[AI CBSE 1991]**

(a) $16 \log 2 - 3$ (b) $16 \log 2 - 13$ (c) $16 \log 2 + 3$ (d) None of these

33. The area of the triangle formed by the tangent to the hyperbola $xy = a^2$ and coordinate axes is **[Rajasthan PET 2000]**

(a) a^2 (b) $2a^2$ (c) $3a^2$ (d) $4a^2$

34. If a curve $y = a\sqrt{x} + bx$ passes through the point (1, 2) and the area bounded by the curve, line $x = 4$ and x-axis is 8 square units, then **[MP PET 2002]**

(a) $a = 3, b = -1$ (b) $a = 3, b = 1$ (c) $a = -3, b = 1$ (d) $a = -3, b = -1$

35. The area bounded by the curve $y = f(x)$, x-axis and ordinates $x = 1$ and $x = $ b is $(b-1)\sin(3b+4)$ then $f(x)$ is **[Rajasthan PET 2000]**

(a) $3(x-1)\cos(3x+4) + \sin(3x+4)$ (b) $(b-1)\sin(3x+4) + 3\cos(3x+4)$

(c) $(b-1)\cos(3x+4) + 3\sin(3x+4)$ (d) None of these

36. The area enclosed by the parabola $y^2 = 4ax$ and the straight line $y = 2ax$, is **[MP PET 1993]**

(a) $\frac{a^2}{3}$ sq. units (b) $\frac{1}{3a^2}$ sq. units (c) $\frac{1}{3a}$ sq. units (d) $\frac{2}{3a}$ sq. units

37. The area bounded by the curve $x = at^2, y = 2at$ and the x-axis in $1 \le t \le 3$ is. **[Pb. CET 1998]**

(a) $26a^2$ (b) $8a^2$ (c) $\frac{26a^2}{3}$ (d) $\frac{104\,a^2}{3}$

38. If A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines x=0, y=0 and $x = \frac{\pi}{4}$, then for n>2 **[IIT 1996, Him. UCET 2002]**

(a) $A_n + A_{n-2} = \frac{1}{n-1}$ (b) $A_n + A_{n-2} < \frac{1}{n-1}$ (c) $A_n - A_{n-2} = \frac{1}{n-1}$ (d) None of these

39. The area between the curve $y = 2x^4 - x^2$, the axis and the ordinates of two minima of the curve is

(a) $\frac{7}{120}$ (b) $\frac{9}{120}$ (c) $\frac{11}{120}$ (d) None of these

40. The slope of the tangent to a curve $y = f(x)$ at $(x, f(x))$ is $2x+1$. If the curve passes through the point (1, 2), then the area of the region bounded be the curve, the x-axis and the line x=1 is **[IIT 1995]**

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$ (c) 6 (d) $\frac{1}{6}$

Symmetrical Area

Basic Level

41. The area bounded by the x-axis and the curve $y = \sin x$ and $x = 0$, $x = \pi$ is **[Kerala (Engg.)2002]**

(a) 1 (b) 2 (c) 3 (d) 4

42. The area of the curve $xy^2 = a^2(a-x)$ bounded by y-axis is **[Rajasthan PET 1996]**

(a) πa^2 (b) $2\pi a^2$ (c) $3\pi a^2$ (d) $4\pi a^2$

43. The area bounded by the parabola $y^2 = 4ax$, its axis and two ordinates $x = 4$, $x = 9$ is

(a) $4a^2$ (b) $4a^2.4$ (c) $4a^2(9-4)$ (d) $\frac{152\sqrt{a}}{3}$

44. Area bounded by the parabola $y^2 = 2x$ and the ordinates $x = 1$, $x = 4$ is

(a) $\frac{4\sqrt{2}}{3}$ sq. units (b) $\frac{28\sqrt{2}}{3}$ sq. units (c) $\frac{56}{3}$ sq. units (d) None of these

45. Area bounded by the parabola $y^2 = 4ax$ and its latus rectum is **[Rajasthan PET 1997, 2000, 2002]**

(a) $\frac{2}{3}a^2$ sq. units (b) $\frac{4}{3}a^2$ sq. units (c) $\frac{8}{3}a^2$ sq. units (d) $\frac{3}{8}a^2$ sq. units

46. The area between the curve $y^2 = 4ax$, x-axis and the ordinates $x = 0$ and $x = a$ is **[Rajasthan PET 1996]**

(a) $\frac{4}{3}a^2$ (b) $\frac{8}{3}a^2$ (c) $\frac{2}{3}a^2$ (d) $\frac{5}{2}a^2$

47. Area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is **[Karnataka CET 1993]**

(a) $\pi\, ab$ sq. units (b) $\frac{1}{2}\pi\, ab$ sq. units (c) $\frac{1}{4}\pi\, ab$ sq. units (d) None of these

48. The area of the smaller segment cut off from the circle $x^2 + y^2 = 9$ by $x = 1$ is **[Rajasthan PET 2002]**

(a) $\frac{1}{2}(9\sec^{-1}3 - \sqrt{8})$ (b) $9\sec^{-1}(3) - \sqrt{8}$ (c) $\sqrt{8} - 9\sec^{-1}3$ (d) None of these

49. The area of the upper half of the circle whose equation is $(x-1)^2 + y^2 = 1$ is given by **[Kurukshetra CEE 1995]**

(a) $\int_0^2 \sqrt{2x - x^2}\,dx$ (b) $\int_0^1 \sqrt{2x - x^2}\,dx$ (c) $\int_1^2 \sqrt{2x - x^2}\,dx$ (d) $\frac{\pi}{4}$

Advance Level

50. The area bounded by the curves $y = \ln x, y = \ln|x|, y = |\ln x|$ and $y = |\ln|x||$ is **[AIEEE 2002]**

(a) 4 sq. units (b) 6 sq. units (c) 10 sq. units (d) None of these

51. Ratio of the area cut off a parabola by any double ordinate is that of the corresponding rectangle contained by that double ordinate and its distance from the vertex is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

52. The area bounded by the curves $x = a\cos^3 t$, $y = a\sin^3 t$ is

(a) $\frac{3\pi a^2}{8}$ (b) $\frac{3\pi a^2}{16}$ (c) $\frac{3\pi a^2}{32}$ (d) $3\pi a^2$

Area between Two curves

Basic Level

53. The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x-axis in the 1st quadrant is **[IIT 2003]**

(a) 9 (b) $\frac{27}{4}$ (c) 36 (d) 18

54. The area of region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$ is **[Kerala (Engg.) 2002]**

(a) $\frac{\pi^2}{5}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi}{4} - \frac{1}{2}$

55. The area bounded by the curve $y = x$, x-axis and ordinates $x = -1$ to $x = 2$ is **[Rajasthan PET 2001]**

(a) 0 (b) 1/2 (c) 3/2 (d) 5/2

56. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is **[IIT Screening 2002]**

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) 4

57. The area bounded by the straight lines $x = 0, x = 2$ and the curves $y = 2^x, y = 2x - x^2$ is **[AMU 2001]**

(a) $\frac{4}{3} - \frac{1}{\log 2}$ (b) $\frac{3}{\log 2} + \frac{4}{3}$ (c) $\frac{4}{\log 2} - 1$ (d) $\frac{3}{\log 2} - \frac{4}{3}$

58. The area of figure bounded by $y = e^x, y = e^{-x}$ and the straight line $x = 1$ is **[Karnataka CET 1999]**

(a) $e + \frac{1}{e}$ (b) $e - \frac{1}{e}$ (c) $e + \frac{1}{e} - 2$ (d) $e + \frac{1}{e} + 2$

59. The area bounded by the curves $y = \log_e x$ and $y = (\log_e x)^2$ is **[Rajasthan PET 2000]**

(a) $3 - e$ (b) $e - 3$ (c) $\frac{1}{2}(3 - e)$ (d) $\frac{1}{2}(e - 3)$

60. The area bounded by the curves $y^2 - x = 0$ and $y - x^2 = 0$ is **[MP PET 1997]**

(a) $\frac{7}{3}$ (b) $\frac{1}{3}$ (c) $\frac{5}{3}$ (d) 1

61. The area enclosed by the parabolas $y = x^2 - 1$ and $y = 1 - x^2$ is **[AMU 1999]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$

62. The area bounded by curve $y^2 = x$, line $y = 4$ and y-axis is **[Roorkee 1995; Rajasthan PET 2003]**

(a) $\frac{16}{3}$ (b) $\frac{64}{3}$ (c) $7\sqrt{2}$ (d) None of these

63. Area included between the two curves $y^2 = 4ax$ and $x^2 = 4ay$, is
[SCRA 1986; Roorkee 1984; Rajasthan PET 1999; Kerala (Engg.)2002]

(a) $\frac{32}{3}a^2$ sq. units (b) $\frac{16}{3}$ sq. units (c) $\frac{32}{3}$ sq. units (d) $\frac{16}{3}a^2$ sq. units

64. Area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$, is **[SCRA 1986; IIT 1981]**

(a) $\frac{8}{9}$ sq. units (b) $\frac{9}{8}$ sq. units (c) $\frac{4}{3}$ sq. units (d) None of these

65. What is the area bounded by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$ **[DCE 1999]**

(a) 0 (b) $\frac{2\sqrt{2}}{3} + \frac{9\pi}{2} - 9\sin^{-1}\left(\frac{1}{3}\right)$ (c) 16π (d) None of these

66. The area bounded by the circle $x^2 + y^2 = 4$, line $x = \sqrt{3}y$ and x- axis lying in the first quadrant, is
[Rajasthan PET 1997, Kurukshetra CEE 1998]

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) π

67. The area in the first quadrant between $x^2 + y^2 = \pi^2$ and $y = \sin x$ is **[MP PET 1997]**

(a) $\frac{(\pi^3 - 8)}{4}$ (b) $\frac{\pi^3}{3}$ (c) $\frac{(\pi^3 - 16)}{4}$ (d) $\frac{(\pi^3 - 8)}{2}$

68. For $0 \le x \le \pi$, the area bounded by $y = x$ and $y = x + \sin x$, is **[Roorkee Quqalifying 1998]**

(a) 2 (b) 4 (c) 2π (d) 4π

69. Area bounded by $y = x\sin x$ and x-axis between $x = 0$ and $x = 2\pi$, is **[Roorkee 1981; Rajasthan PET 1995]**

(a) 0 (b) 2π sq. units (c) π sq. units (d) 4π sq. units

70. The area bounded by curves $y = \cos x$ and $y = \sin x$ and ordinates $x = 0$ and $x = \frac{\pi}{4}$ is **[Karnataka CET 2002]**

(a) $\sqrt{2}$ (b) $\sqrt{2} + 1$ (c) $\sqrt{2} - 1$ (d) $\sqrt{2}(\sqrt{2} - 1)$

71. The area formed by triangular shaped region bounded by the curves $y = \sin x, y = \cos x$ and $x = 0$ is **[MP PET 2000]**

(a) $\sqrt{2} - 1$ (b) 1 (c) $\sqrt{2}$ (d) $1 + \sqrt{2}$

72. Area between the curve $y = \cos x$ and x-axis when $0 \le x \le 2\pi$, is **[MP PET 1997]**

(a) 2 (b) 4 (c) 3 (d) 0

73. AOB is the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $OA = a$, $OB = b$. Then area between the arc AB and chord AB of the ellipse is

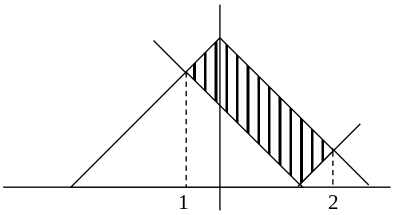
(a) πab (b) $(\pi - 2)ab$ (c) $\frac{ab(\pi - 2)}{4}$ (d) $\frac{ab(\pi + 2)}{4}$

Advance Level

74. For which of the following values of m, the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$ **[IIT 1999]**

(a) – 4 (b) – 2 (c) 2 (d) 4

75. The area of the figure bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$, is **[AIEEE 2003; Orissa JEE 2003]**



(a) 2 sq. units (b) 3 sq. units (c) 4 sq. units (d) 1 sq. units

76. If the ordinate $x = a$ divides the area bounded by the curve $y = \left(1 + \frac{8}{x^2}\right)$, x-axis and the ordinates $x = 2$, $x = 4$ into two equal parts, then a= **[IIT 1983]**

(a) 8 (b) $2\sqrt{2}$ (c) 2 (d) $\sqrt{2}$

77. The area of the region lying inside $x^2 + (y-1)^2 = 1$ and out side $c^2x^2 + y^2 = c^2$, where $c = (\sqrt{2} - 1)$ is **[Roorkee 1999]**

(a) $(4 - \sqrt{2})\frac{\pi}{4} + \frac{1}{\sqrt{2}}$ (b) $(4 + \sqrt{2})\frac{\pi}{4} - \frac{1}{\sqrt{2}}$ (c) $(4 + \sqrt{2})\frac{\pi}{4} + \frac{1}{\sqrt{2}}$ (d) None of these

78. The area enclosed between the curves $y = \log_e(x + e)$, $x = \log_e(x + e)$, $x = \log_e\left(\frac{1}{y}\right)$ and the x-axis, is **[Roorkee 1990; Pb. CET 2002]**

(a) 2 (b) 1 (c) 4 (d) None of these

79. The area of the region formed by $x^2 + y^2 - 6x - 4y + 12 \le 0$, $y \le x$ and $x \le \frac{5}{2}$ is **[Roorkee 1996; PUCET 2002]**

(a) $\frac{\pi}{6} - \frac{\sqrt{3}+1}{8}$ (b) $\frac{\pi}{6} + \frac{\sqrt{3}-1}{8}$ (c) $\frac{\pi}{6} - \frac{\sqrt{3}-1}{8}$ (d) None of these

80. If the area bounded by the curves $y = x - bx^2$ and $y = \frac{1}{b}x^2$, where $b>0$ is maximum, then b= **[IIT 1997]**

(a) 0 (b) 1 (c) 2 (d) None of these

81. Let $f(x)$= Maximum $[x^2, (1 - x^2), 2x(1 - x)]$ where $0 \le x \le 1$. The area of the region bounded by the curves $y = f(x)$, x-axis, $x = 0$ and $x = 1$ is **[IIT 1997; IIIT Hydrabad 2002]**

(a) $\frac{17}{27}$ (b) $\frac{14}{27}$ (c) $\frac{19}{27}$ (d) None of these

82. The area of the closed figure bounded by $x = -1$ and $x = 2$ and $y = \begin{cases} -x^2 + 2, & x \le 1 \\ 2x - 1, & x > 1 \end{cases}$ and the abscissa axis is

(a) $\frac{16}{3}$ sq. units (b) $\frac{10}{3}$ sq. units (c) $\frac{13}{3}$ sq. units (d) $\frac{7}{3}$ sq. units

Volumes and surfaces of Solids of Revolution

Basic Level

83. The volume of the solid formed by rotating the area enclosed between the curve $y = x^2$ and the line $y = 1$ about $y = 1$ is (in cubic units)

(a) $\frac{9\pi}{5}$ (b) $\frac{7\pi}{3}$ (c) $\frac{8\pi}{3}$ (d) None of these

84. The volume of the solid obtained by rotating the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the axis of x is **[MNR 1995]**

(a) $\pi a^2 b$ (b) $\pi - b^2$ (c) $\frac{4}{3}\pi a^2 b$ (d) $\frac{4}{3}\pi ab^2$

85. The part of the parabola between the parabola $y^2 = 4ax$ and the line $x = c$ is revolved about x-axis. The volume of the resulting solid is

(a) $2\pi ac^2$ (b) πac^2 (c) $\frac{\pi c^2}{4}$ (d) $4\pi ac^2$

86. The volume of the solid generated by revolving about the y- axis the figure bounded by the parabola $y = x^2$ and $x = y^2$ is **[UPSEAT 2002]**

(a) $\frac{21}{5}\pi$ (b) $\frac{24}{5}\pi$ (c) $\frac{5}{24}\pi$ (d) None of these

87. The volume of the frustum of a cone of height 6 *cm.*, and radii are 5 *cms* and 8 *cms* is

(a) 258 *cc* (b) 250 *cc* (c) 268 *cc* (d) 275 *cc*

88. The part of the circle $x^2 + y^2 = 4$ between $x = 1$ and $x = 2$ is revolved about x-axis. The curved surface of the resulting solid is

(a) 2π (b) 4π (c) 6π (d) 8π

Advance Level

89. The volume of a solid obtained by revolving about y-axis enclosed between the ellipse $x^2 + 9y^2 = 9$ and the straight line $x + 3y = 3$ in the first quadrant is **[MNR 1994]**

(a) 3π (b) 4π (c) 6π (d) 9π

90. The volume of the frustum of a right circular cone. The radii of whose ends are respectively 10 *cms* and 16 *cms* and thickness is 4 *cms, is*

(a) 1232 π (b) 332 π (c) 1032 π (d) 1132 π

91. The line segment joining the points (1, m) and (2, 2m) is revolved round the y-axis to form a frustum of a cone of the volume of the frustum is 14π then the value of m is equal to

(a) 2 (b) 4 (c) 6 (d) 8

92. A frustum of sphere is made by cutting two parallel planes of any sphere. If radius of sphere is 5 *cm* and distance between the plane is 1*cm*, then what will be the curved surface of frustum when the distance of first plane from the centre of sphere is 2 *cm* **[UPSEAT 1999]**

(a) $5\pi cm^2$ (b) $10\pi cm^2$ (c) $15\pi cm^2$ (d) $40\pi cm^2$

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Answer Sheet

Assignment (Basic and Advance Level)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
a	**c**	**a**	**d**	**b**	**b**	**d**	**c**	**d**	**b**	**c**	**b**	**a**	**b**	**c**	**a**	**a**	**c**	**b**	**a**
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
c	**b**	**d**	**d**	**a**	**d**	**b**	**d**	**b**	**b**	**b**	**c**	**b**	**a**	**a**	**c**	**d**	**a**	**a**	**a**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	**a**	**d**	**b**	**c**	**b**	**a**	**b**	**a**	**a**	**c**	**a**	**a**	**d**	**d**	**b**	**d**	**c**	**a**	**b**
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
d	**b**	**d**	**b**	**b**	**c**	**a**	**a**	**d**	**c**	**a**	**b**	**c**	**b**	**c**	**b**	**a**	**a**	**c**	**b**
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**								
a	**a**	**d**	**d**	**a**	**d**	**a**	**b**	**a**	**c**	**c**	**b**								