CBSE Test Paper 05
Chapter 13 Probability

1. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).
    a. $\frac{11}{3}$
    b. $\frac{14}{3}$
    c. $\frac{16}{3}$
    d. $\frac{10}{3}$
2. Which of the following conditions do Bernoulli trials satisfy?
    a. infinite number of independent trials
    b. finite number of independent trials
    c. finite number of dependent trials
    d. infinite number of dependent trials
3. Let X be a random variable assuming values x_1, x_2,....,x_n with probabilities p_1, p_2, ..., pn, respectively such that $p_i \geq 0, \sum_{i=1}^{n} p_i = 1$. Mean of X denoted by μ is defined as
    a. $\mu = \sum_{i-1}^{n} x_i p_i$
    b. $\mu = \sum_{i-1}^{n} p_i$
    c. $\mu = \sum_{i-1}^{n} x_i p_{i+1}$
    d. $\mu = \sum_{i-1}^{n} x_i$
4. A random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and p, if its probability distribution is given by
    a. $P(X = r) = C_r^n p^r q^{n-r-2}$
    b. $P(X = r) = C_r^n p^r q^{n-r}$
    c. $P(X = r) = C_r^n p^{2r} q^{n-r}$

d. $P(X=r)=C^{n}_{r-2}p^{r}q^{n-r}$

5. Two coins are tossed once ,where E : tail appears on one coin , F : one coin shows head. Find P(E/F)

   a. 0.24
   b. 0.33
   c. 1
   d. 0.23
6. The probability of drawing two clubs from a well shuffled pack of 52 cards is ________.
7. A box has 6 black, 4 red, 2 white and 3 blue shirts. The probability of drawing 2 black shirts if they are picked randomly is ________.
8. The number of times the experiment is repeated is called the number of ________.

9. The probability distribution of a random variable X is given below:

X	0	1	2	3
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

   i. Determine the value of k.
   ii. Determine $P(X\leqslant 2)$ and P(X > 2)
   iii. Find $P(X\leqslant 2)+P(X>2)$.

10. Compute P(A|B), if P (B) = 0.5 and $P(A\cap B)$= 0.32.

11. If P (A) = $\frac{3}{5}$ and P (B) = $\frac{1}{5}$ find P ($A\cap B$) if A and B are independent events.

12. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

13. Events A and B are such that P(A) = $\frac{1}{2}$ P(B) = $\frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$. State whether A and B are independent.

14. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATA NAGAR.

15. Probabilities of solving a specific problem independently by A and Bare $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve problem independently, then find the probability that
    i. problem is solved.
    ii. exactly one of them solves the problem.

16. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3, 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

17. Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

18. Suppose we have four boxes A, B, C and D containing coloured marbles as given below:

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B? box C?

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Solution

1. b. $\frac{14}{3}$

   Explanation: First 6 positive integers are 1,2,3,4,5,6. As 1 is the smallest positive integer. Therefore , X = 2,3,4,5,6.
   P (X = 2) = P{(1,2) or (2,1)} = $\frac{1}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{1}{5} = \frac{2}{30}$
   P(X = 3)= P{(number< 3, 3) or (3, number< 3)}
   $= \frac{2}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{2}{5} = \frac{4}{30}$
   P(X = 4)= P{(number< 4, 4) or (4, number< 4)}
   $= \frac{3}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{3}{5} = \frac{6}{30}$
   P(X = 5)= P{(number< 5, 5) or (5, number< 5)}
   $= \frac{4}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{4}{5} = \frac{8}{30}$
   P(X = 6)= P{(number< 6, 6) or (6, number< 6)}
   $= \frac{5}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{5}{5} = \frac{10}{30}$
   Therefore, the probability distribution is :

X	2	3	4	5	6
P(X)	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$

   Therefore, Expectations of X :
   $$E(X^2) = \sum_{i=1}^{n} X_i P(X_i)$$
   $$= 2 \times \frac{2}{30} + 3 \times \frac{4}{30} + 4 \times \frac{6}{30} + 5 \times \frac{8}{30} + 6 \times \frac{10}{30} = \frac{140}{30} = \frac{14}{3}$$

2. b. finite number of independent trials

   Explanation: Bernoulli trials satisfies the finite number of independent trials.

3. a. $\mu = \sum_{i-1}^{n} x_i p_i$

   Explanation: Let X be a random variable assuming values x1, x2,....,xn with probabilities p1, p2, ...,pn, respectively such that pi ≥ 0, $\sum_{i=1}^{n} p_i = 1$. Mean of X denoted by μ is defined as: $\mu = \sum_{i-1}^{n} x_i p_i$.

4. b. $P(X=r)=C_r^n p^r q^{n-r}$

Explanation: A random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and p, if its probability distribution is given by : $P(X=r)=C_r^n p^r q^{n-r}$.

5. a. 1

Explanation: S = {HH, HT, TH, TT}

E = {HT, TH}

F= {HT, TH}

$\Rightarrow P(E)=\frac{2}{4}=\frac{1}{2}, P(F)=\frac{2}{4}=\frac{1}{2}, P(E\cap F)=\frac{1}{2}$

$\Rightarrow P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{1/2}{1/2}=1$

6. $\frac{1}{17}$
7. $\frac{1}{7}$
8. trials
9. We have,

X	0	1	2	3
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

i. Since, $\sum\limits_{i=1}^{n} P_i=1, i=1,2,\ldots\ldots,n$ and $p_i\geqslant 0$

$\therefore k+\frac{k}{2}+\frac{k}{4}+\frac{k}{8}=1$

$\Rightarrow 8k+4k+2k+k=8$

$\therefore k=\frac{8}{15}$

ii. $P(X\leqslant 2)=P(0)+P(1)+P(2)=k+\frac{k}{2}+\frac{k}{4}$

$=\frac{(4k+2k+k)}{4}=\frac{7k}{4}=\frac{7}{4}\cdot\frac{8}{15}=\frac{14}{15}$

and $P(X>2)=P(3)=\frac{K}{8}=\frac{1}{8}\cdot\frac{8}{15}=\frac{1}{15}$

iii. $P(X\leqslant 2)+P(X>2)=\frac{14}{15}+\frac{1}{15}=1$

10. Given: P (B) = 0.5, $P(A\cap B)$ = 0.32

$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0.32}{0.50}=\frac{32}{50}=0.64$

11. As, A and B are independent events.

Therefore, $P(A\cap B)$ = P (A). P (B)= $\frac{3}{5}\times\frac{1}{5}=\frac{3}{25}$

12. $p=\frac{1}{3}$ and q = 1 - p $=1-\frac{1}{3}=\frac{2}{3}$

n = 5, r = 4, 5 and P (X = r) = ${}^{n}C_{r}p^{r}q^{n-r}$

P (Four or more success) = P (X = 4) + P (X = 5)

$$={}^{5}C_{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{1}+{}^{5}C_{5}\left(\frac{1}{3}\right)^{5}=5\times 2\times(\frac{1}{3})^{5}+\left(\frac{1}{3}\right)^{5}=\frac{11}{243}$$

13. $P(\overline{A}\cup\overline{B})=1-P(A\cap B)\Rightarrow\frac{1}{4}=1-P(A\cap B)$

$$P\left(\bar{A}\cap\bar{B}\right)=1-\frac{1}{4}=\frac{3}{4}$$

P (A) = $\frac{1}{2}$ and P (B) = $\frac{1}{12}$

P (A). P (B) = $\frac{1}{2}\times\frac{7}{12}=\frac{7}{24}$

Therefore, $P\left(A\cap B\right)\neq P(A)\cdot P(B)$, i.e., A and B are not independent.

14. Let E_1be the event that letter is from TATA NAGAR and E_2be the event that letter is from CALCUTTA.

Also, let E_3be the event on the letter, two consecutive letters TA are visible.

$\therefore P(E_1)=\frac{1}{2}$ and $P(E_2)=\frac{1}{2}$

And $P\left(\frac{E_3}{E_1}\right)=\frac{2}{8}$ and $P\left(\frac{E_3}{E_2}\right)=\frac{1}{7}$

[Since, if letter is from TATA NAGAR, we see that the events of two consecutive letters visible are {TA, AT, TA, AN, NA, AG, GA, AR}. So, $P\left(\frac{E_3}{E_1}\right)=\frac{2}{8}$ and if letter is from CALCUTTA, we see that the events of two consecutive letters to visible are {CA, AL, LC, CU, UT, TT, TA}. So, $P\left(\frac{E_3}{E_2}\right)=\frac{1}{7}$

$$\therefore P\left(\frac{E_1}{E_3}\right)=\frac{P(E_1)\cdot P\left(\frac{E_3}{E_1}\right)}{P(E_1)\cdot P\left(\frac{E_3}{E_1}\right)+P(E_2)\cdot P\left(\frac{E_3}{E_2}\right)}$$

$$=\frac{\frac{1}{2}\cdot\frac{2}{8}}{\frac{1}{2}\cdot\frac{2}{8}+\frac{1}{2}\cdot\frac{1}{7}}=\frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{14}}=\frac{\frac{1}{8}}{\frac{22}{8\times 14}}=\frac{\frac{1}{8}}{\frac{11}{56}}=\frac{7}{11}$$

15. Let P(A) = Probability that A solves the problem

P(B) = Probability that B solves the problem

$P(\overline{A})$= Probability that A does not solve the problem

and $P(\overline{B})$= Probability that B does not solve the problem

According to the question, we have

$P(A)=\frac{1}{2}$

then $P(\overline{A})=1-P(A)=1-\frac{1}{2}=\frac{1}{2}$

$[\because \quad P(A)+P(\overline{A})=1]$
and $P(B)=\frac{1}{3}$
then $P(\overline{B})=1-P(B)=1-\frac{1}{3}=\frac{2}{3}$

i. P (problem is solved)
$=P(A\cap\overline{B})+P(\overline{A}\cap B)+P(A\cap B)$
$=P(A)P(\overline{B})+P(\overline{A})\cdot P(B)+P(A)\cdot P(B)$
[∵ A and Bare independent events]
$=\left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)$
$=\frac{2}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$
Hence, probability that the problem is solved, is $\frac{2}{3}$

ii. P (exactly one of them solve the problem)
= P (A solve but B do not solve) + P (A do not solve but B solve)
$=P(A\cap\overline{B})+P(\ \overline{A}\cap B)$
$=P(A)\quad P(\overline{B})+P(\overline{A})P(B)$
$=\left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)=\frac{2}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$

16. E_1 : 1, 2, 3, 4 is shown on dice

E_2 : 5 or 6 is shown on dice

$P(E_1)=\frac{4}{6}=\frac{2}{3}, P(E_2)=\frac{2}{6}=\frac{1}{3}$
Let, A exactly one head shown up
$P(A/E_1)=\frac{1}{2}, P(A/E_2)=\frac{3}{8}$
$P(E_1/A)=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}=\frac{8}{11}$

17. Let A_1: Event that the bulb is produced by machine E_1

A_2: Event that the bulb is produced by machine E_2

A_3: Event that the bulb is produced by machine E_3

A: Event that the picked up bulb is defective (1)

Here, $P\left(A_{1)}=50\%=\frac{50}{100}=\frac{1}{2}\right.$
$P\left(A_{2)}=25\%=\frac{25}{100}=\frac{1}{4}\right.$
$P\left(A_3\right)=25\%=\frac{25}{100}=\frac{1}{4}$

Also, $P\left(\frac{A}{A_1}\right) = 4\% = \frac{4}{100} = \frac{1}{25}$

$P\left(\frac{A}{A_2}\right) = 4\% = \frac{4}{100} = \frac{1}{25}$

and $P\left(\frac{A}{A_3}\right) = 5\% = \frac{5}{100} = \frac{1}{20}$

$\therefore$ The probability that the picked bulb is defective,

$$P(A) = P(A_1) \times P\left(\frac{A}{A_1}\right) + P(A_2) \times P\left(\frac{A}{A_2}\right) + P(A_3) \times P\left(\frac{A}{A_3}\right)$$

$$= \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$

$$= \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = \frac{8+4+5}{400} = \frac{17}{400} = 0.0425$$

18. Let R represents the drawing of red ball and the four boxes are represented by A, B, C and D.

So $P(R|A) = \frac{1}{10}, P(R|B) = \frac{6}{10},$

$P(R|C) = \frac{8}{10}, P(R|D) = \frac{0}{10} = 0$

Since there are 4 bags.

Therefore, P (A) = $\frac{1}{4}$, P (B) = $\frac{1}{4}$, P (C) = $\frac{1}{4}$, P (D) = $\frac{1}{4}$

$$P(A|R) = \frac{P(A)\cdot P(R|A)}{P(A)\cdot P(R|A)+P(B)\cdot P(R|B)+P(C)\cdot P(R|C)+P(D)\cdot(R|D)}$$

$$= \frac{\frac{1}{4}\times\frac{1}{10}}{\frac{1}{4}\times\frac{1}{10}+\frac{1}{4}\times\frac{6}{10}+\frac{1}{4}\times\frac{8}{10}+\frac{1}{4}\times 0}$$

$$= \frac{\frac{1}{10}}{\frac{1}{10}+\frac{6}{10}+\frac{8}{15}} = \frac{1}{15}$$

$$P(B/R) = \frac{P(B)\cdot P(R|B)}{P(A)P(R|A)+P(B)P(R|B)+P(C)P(R|C)+P(D)P(R|D)}$$

$$= \frac{\frac{1}{4}\times\frac{6}{10}}{\frac{1}{4}\times\frac{1}{10}+\frac{1}{4}\times\frac{6}{10}+\frac{1}{4}\times\frac{8}{10}+\frac{1}{4}\times 0} = \frac{6}{15} = \frac{2}{5}$$

$$P(C|R) = \frac{P(C)\cdot P(R|C)}{P(A)P(R|A)+P(B)P(R|B)+P(C)P(R|C)+P(D)P(R|D)}$$

$$= \frac{\frac{1}{4}\times\frac{8}{10}}{\frac{1}{4}\times\frac{1}{10}+\frac{1}{4}\times\frac{6}{10}+\frac{1}{4}\times\frac{8}{10}+\frac{1}{4}\times 0} = \frac{8}{15}$$