

# Mathematics

(Chapter - 13)(Statistics)

(Class – X)

## Exercise 13.1

### Question 1:

A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

### Answer 1:

To find the class mark ( $x_i$ ) for each interval, the following relation is used.

$$\text{Class mark } x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

$x_i$  and  $f_i x_i$  can be calculated as follows.

Number of plants	Number of houses ( $f_i$ )	$x_i$	$fix_i$
0 – 2	1	1	$1 \times 1 = 1$
2 – 4	2	3	$2 \times 3 = 6$
4 – 6	1	5	$1 \times 5 = 5$
6 – 8	5	7	$5 \times 7 = 35$
8 – 10	6	9	$6 \times 9 = 54$
10 – 12	2	11	$2 \times 11 = 22$
12 – 14	3	13	$3 \times 13 = 39$
Total	20		162

From the table, it can be observed that

$$\sum f_i = 20$$

$$\sum f_i x_i = 162$$

Mean,  $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$$= \frac{162}{20} = 8.1$$

Therefore, mean number of plants per house is 8.1.

Here, direct method has been used as the values of class marks ( $x_i$ ) and  $f_i$  are small.

### Question 2:

Consider the following distribution of daily wages of 50 worker of a factory.

Daily wages (in Rs)	<b>500 – 520</b>	<b>520 – 540</b>	<b>540 – 560</b>	<b>560 – 580</b>	<b>580 – 600</b>
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

### Answer 2:

To find the class mark for each interval, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Class size ( $h$ ) of this data = 20

Taking 550 as assured mean ( $a$ ),  $d_i$ ,  $u_i$ , and  $f_i u_i$  can be calculated as follows.

<b>Daily wages (in Rs)</b>	<b>Number of workers (<math>f_i</math>)</b>	<b><math>x_i</math></b>	<b><math>d_i = x_i - 150</math></b>	<b><math>u_i = \frac{d_i}{20}</math></b>	<b><math>f_i u_i</math></b>
<b>500 – 520</b>	12	510	– 40	– 2	– 24
<b>520 – 540</b>	14	530	– 20	– 1	– 14
<b>540 – 560</b>	8	550	0	0	0
<b>560 – 580</b>	6	570	20	1	6
<b>580 – 600</b>	10	590	40	2	20
<b>Total</b>	<b>50</b>				<b>– 12</b>

From the table, it can be observed that

$$\sum f_i = 50$$

$$\sum f_i u_i = -12$$

$$\begin{aligned}
 \text{Mean } \bar{x} &= a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) h \\
 &= 550 + \left( \frac{-12}{50} \right) 20 \\
 &= 550 - \frac{24}{5} \\
 &= 550 - 4.8 \\
 &= 545.2
 \end{aligned}$$

Therefore, the mean daily wage of the workers of the factory is Rs 545.20.

**Question 3:**

The following distribution shows the daily pocket allowance of children of a locality.

The mean pocket allowance is Rs.18. Find the missing frequency  $f$ .

Daily pocket allowance (in Rs)	<b>11 – 13</b>	<b>13 – 15</b>	<b>15 – 17</b>	<b>17 – 19</b>	<b>19 – 21</b>	<b>21 – 23</b>	<b>23 – 25</b>
Number of workers	7	6	9	13	$f$	5	4

**Answer 3:**

To find the class mark ( $x_i$ ) for each interval, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Given that, mean pocket allowance,

Taking 18 as assured mean ( $a$ ),  $d_i$  and  $f_i d_i$  are calculated as follows.

<b>Daily pocket allowance (in Rs)</b>	<b>Number of children <math>f_i</math></b>	<b>Class mark <math>x_i</math></b>	<b><math>d_i = x_i - 18</math></b>	<b><math>f_i d_i</math></b>
<b>11 – 13</b>	7	12	– 6	– 42
<b>13 – 15</b>	6	14	– 4	– 24
<b>15 – 17</b>	9	16	– 2	– 18
<b>17 – 19</b>	13	18	0	0
<b>19 – 21</b>	$f$	20	2	$2f$
<b>21 – 23</b>	5	22	4	20
<b>23 – 25</b>	4	24	6	24
<b>Total</b>	<b><math>\sum f_i = 44 + f</math></b>			<b><math>2f - 40</math></b>

From the table, we obtain

$$\sum f_i = 44 + f$$

$$\sum f_i u_i = 2f - 40$$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$18 = 18 + \left( \frac{2f - 40}{44 + f} \right)$$

$$0 = \left( \frac{2f - 40}{44 + f} \right)$$

$$2f - 40 = 0$$

$$2f = 40$$

$$f = 20$$

Hence, the missing frequency,  $f$ , is 20.

#### Question 4:

Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute	<b>65 – 68</b>	<b>68 – 71</b>	<b>71 – 74</b>	<b>74 – 77</b>	<b>77 – 80</b>	<b>80 – 83</b>	<b>83 – 86</b>
Number of women	2	4	3	8	7	4	2

#### Answer 4:

To find the class mark of each interval ( $x_i$ ), the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Class size,  $h$ , of this data = 3

Taking 75.5 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ ,  $f_i u_i$  are calculated as follows.

Number of heart beats per minute	Number of women $f_i$	$x_i$	$d_i = x_i - 75.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
65 – 68	2	66.5	– 9	– 3	– 6
68 – 71	4	69.5	– 6	– 2	– 8
71 – 74	3	72.5	– 3	– 1	– 3
74 – 77	8	75.5	0	0	0
77 – 80	7	78.5	3	1	7
80 – 83	4	81.5	6	2	8
83 – 86	2	84.5	9	3	6
Total	30				4

From the table, we obtain

$$\sum f_i = 30$$

$$\sum f_i u_i = 4$$

$$\begin{aligned}\text{Mean } \bar{x} &= a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 75.5 + \left( \frac{4}{30} \right) \times 3 \\ &= 75.5 + 0.4 = 75.9\end{aligned}$$

Therefore, mean hear beats per minute for these women are 75.9 beats per minute.

#### Question 5:

In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

#### Answer 5:

Number of mangoes	Number of boxes $f_i$
50 – 52	15
53 – 55	110
56 – 58	135
59 – 61	115
62 – 64	25

It can be observed that class intervals are not continuous. There is a gap of 1 between two class intervals. Therefore,  $1/2$  has to be added to the upper class limit

and  $1/2$  has to be subtracted from the lower class limit of each interval.

Class mark ( $x_i$ ) can be obtained by using the following relation.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Class size ( $h$ ) of this data = 3

Taking 57 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ ,  $f_i u_i$  are calculated as follows.

Class interval	$f_i$	$x_i$	$d_i = x_i - 57$	$u_i = \frac{d_i}{3}$	$f_i u_i$
49.5 – 52.5	15	51	– 6	– 2	– 30
52.5 – 55.5	110	54	– 3	– 1	– 110
55.5 – 58.5	135	57	0	0	0
58.5 – 61.5	115	60	3	1	115
61.5 – 64.5	25	63	6	2	50
<b>Total</b>	400				25

It can be observed that

$$\sum f_i = 400$$

$$\sum f_i u_i = 25$$

$$\text{Mean, } \bar{x} = a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 57 + \left( \frac{25}{400} \right) \times 3$$

$$= 57 + \frac{3}{16} = 57 + 0.1875$$

$$= 57.1875$$

$$= 57.19$$



Mean number of mangoes kept in a packing box is 57.19.

Step deviation method is used here as the values of  $f_i$ ,  $d_i$  are big and also, there is a common multiple between all  $d_i$ .

**Question 6:**

The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in Rs)	<b>100 – 150</b>	<b>150 – 200</b>	<b>200 – 250</b>	<b>250 – 300</b>	<b>300 – 350</b>
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

**Answer 6:**

To find the class mark ( $x_i$ ) for each interval, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Class size = 50

Taking 225 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ ,  $f_i u_i$  are calculated as follows.

Daily expenditure (in Rs)	$f_i$	$x_i$	$d_i = x_i - 225$	$u_i = \frac{d_i}{50}$	$f_i u_i$
<b>100 – 150</b>	4	125	– 100	– 2	– 8
<b>150 – 200</b>	5	175	– 50	– 1	– 5
<b>200 – 250</b>	12	225	0	0	0
<b>250 – 300</b>	2	275	50	1	2
<b>300 – 350</b>	2	325	100	2	4
<b>Total</b>	25				– 7

From the table, we obtain

$$\sum f_i = 25$$

$$\sum f_i u_i = -7$$

$$\begin{aligned}
 \text{Mean, } \bar{x} &= a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) \times h \\
 &= 225 + \left( \frac{-7}{25} \right) \times (50) \\
 &= 225 - 14 \\
 &= 211
 \end{aligned}$$

Therefore, mean daily expenditure on food is Rs 211.

### Question 7:

To find out the concentration of SO<sub>2</sub> in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

concentration of SO <sub>2</sub> (in ppm)	Frequency
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4
0.20 – 0.24	2

Find the mean concentration of SO<sub>2</sub> in the air.

**Answer 7:**

To find the class marks for each interval, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Class size of this data = 0.04

Taking 0.14 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ ,  $f_i u_i$  are calculated as follows.

Concentration of SO <sub>2</sub> (in ppm)	Frequency $f_i$	Class mark $x_i$	$d_i = x_i - 0.14$	$u_i = \frac{d_i}{0.04}$	$f_i u_i$
<b>0.00 – 0.04</b>	4	0.02	– 0.12	– 3	– 12
<b>0.04 – 0.08</b>	9	0.06	– 0.08	– 2	– 18
<b>0.08 – 0.12</b>	9	0.10	– 0.04	– 1	– 9
<b>0.12 – 0.16</b>	2	0.14	0	0	0
<b>0.16 – 0.20</b>	4	0.18	0.04	1	4
<b>0.20 – 0.24</b>	2	0.22	0.08	2	4
<b>Total</b>	<b>30</b>				<b>–31</b>

From the table, we obtain

$$\sum f_i = 30$$

$$\sum f_i u_i = -31$$

$$\begin{aligned}
 \text{Mean, } \bar{x} &= a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) \times h \\
 &= 0.14 + \left( \frac{-31}{30} \right) (0.04) \\
 &= 0.14 - 0.04133 \\
 &= 0.09867 \\
 &\approx 0.099 \text{ ppm}
 \end{aligned}$$

Therefore, mean concentration of SO<sub>2</sub> in the air is 0.099 ppm.

**Question 8:**

A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

<b>Number of days</b>	<b>0 – 6</b>	<b>6 – 10</b>	<b>10 – 14</b>	<b>14 – 20</b>	<b>20 – 28</b>	<b>28 – 38</b>	<b>38 – 40</b>
<b>Number of students</b>	11	10	7	4	4	3	1

**Answer 8:**

To find the class mark of each interval, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Taking 17 as assumed mean ( $a$ ),  $d_i$  and  $f_i d_i$  are calculated as follows.

Number of days	Number of students $f_i$	$x_i$	$d_i = x_i - 17$	$f_i d_i$
<b>0 – 6</b>	11	3	– 14	– 154
<b>6 – 10</b>	10	8	– 9	– 90
<b>10 – 14</b>	7	12	– 5	– 35
<b>14 – 20</b>	4	17	0	0
<b>20 – 28</b>	4	24	7	28
<b>28 – 38</b>	3	33	16	48
<b>38 – 40</b>	1	39	22	22
<b>Total</b>	<b>40</b>			<b>– 181</b>

From the table, we obtain

$$\sum f_i = 40$$

$$\sum f_i d_i = -181$$

$$\begin{aligned}\text{Mean, } \bar{x} &= a + \left( \frac{\sum f_i d_i}{\sum f_i} \right) \\ &= 17 + \left( \frac{-181}{40} \right) \\ &= 17 - 4.525 \\ &= 12.475 \\ &\approx 12.48\end{aligned}$$

Therefore, the mean number of days is 12.48 days for which a student was absent.

#### Question 9:

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

#### Answer 9:

To find the class marks, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Class size ( $h$ ) for this data = 10

Taking 70 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ , and  $f_i u_i$  are calculated as follows.

Literacy rate (in %)	Number of cities $f_i$	$x_i$	$d_i = x_i - 70$	$u_i = \frac{d_i}{10}$	$f_i u_i$
<b>45 – 55</b>	3	50	– 20	– 2	– 6
<b>55 – 65</b>	10	60	– 10	– 1	– 10
<b>65 – 75</b>	11	70	0	0	0
<b>75 – 85</b>	8	80	10	1	8
<b>85 – 95</b>	3	90	20	2	6
<b>Total</b>	<b>35</b>				<b>– 2</b>

From the table, we obtain

$$\sum f_i = 35$$

$$\sum f_i u_i = -2$$

$$\text{Mean, } \bar{x} = a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 70 + \left( \frac{-2}{35} \right) \times (10)$$

$$= 70 - \frac{20}{35}$$

$$= 70 - \frac{4}{7}$$

$$= 70 - 0.57$$

$$= 69.43$$

Therefore, mean literacy rate is 69.43%.

# Mathematics

(Chapter - 13) (Statistics)

(Class – X)

## Exercise 13.2

### Question 1:

The following table shows the ages of the patients admitted in a hospital during a year:

age (in years)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

### Answer 1:

To find the class marks ( $x_i$ ), the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Taking 30 as assumed mean (a),  $d_i$  and  $f_i d_i$  are calculated as follows.

Age (in years)	Number of patients $f_i$	Class mark $x_i$	$d_i = x_i - 30$	$f_i d_i$
5 – 15	6	10	– 20	– 120
15 – 25	11	20	– 10	– 110
25 – 35	21	30	0	0
35 – 45	23	40	10	230
45 – 55	14	50	20	280
55 – 65	5	60	30	150
Total	80			430



From the table, we obtain

$$\sum f_i = 80$$

$$\sum f_i d_i = 430$$

$$\begin{aligned}\text{Mean, } \bar{x} &= a + \frac{\sum f_i d_i}{\sum f_i} \\ &= 30 + \left( \frac{430}{80} \right) \\ &= 30 + 5.375 \\ &= 35.375 \\ &\approx 35.38\end{aligned}$$

Mean of this data is 35.38. It represents that on an average, the age of a patient admitted to hospital was 35.38 years.

It can be observed that the maximum class frequency is 23 belonging to class interval 35 – 45.

Modal class = 35 – 45

Lower limit ( $l$ ) of modal class = 35

Frequency ( $f_1$ ) of modal class = 23

Class size ( $h$ ) = 10

Frequency ( $f_0$ ) of class preceding the modal class = 21

Frequency ( $f_2$ ) of class succeeding the modal class = 14

$$\begin{aligned}\text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 35 + \left( \frac{23 - 21}{2(23) - 21 - 14} \right) \times 10 \\ &= 35 + \left[ \frac{2}{46 - 35} \right] \times 10 \\ &= 35 + \frac{20}{11} \\ &= 35 + 1.81 \\ &= 36.8\end{aligned}$$

Mode is 36.8. It represents that the age of maximum number of patients admitted in hospital was 36.8 years.

### Question 2:

The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

<b>Lifetimes (in hours)</b>	<b>0 – 20</b>	<b>20 – 40</b>	<b>40 – 60</b>	<b>60 – 80</b>	<b>80 – 100</b>	<b>100 – 120</b>
<b>Frequency</b>	10	35	52	61	38	29

Determine the modal lifetimes of the components.

### Answer 2:

From the data given above, it can be observed that the maximum class frequency is 61, belonging to class interval 60 – 80.

Therefore, modal class = 60 – 80

Lower class limit ( $l$ ) of modal class = 60

Frequency ( $f_1$ ) of modal class = 61

Frequency ( $f_0$ ) of class preceding the modal class = 52

Frequency ( $f_2$ ) of class succeeding the modal class = 38

Class size ( $h$ ) = 20

$$\begin{aligned}
 \text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 60 + \left( \frac{61 - 52}{2(61) - 52 - 38} \right) (20)
 \end{aligned}$$

$$\begin{aligned}
 &= 60 + \left( \frac{9}{122 - 90} \right) (20) \\
 &= 60 + \left( \frac{9 \times 20}{32} \right) \\
 &= 60 + \frac{90}{16} = 60 + 5.625 \\
 &= 65.625
 \end{aligned}$$

Therefore, modal lifetime of electrical components is 65.625 hours.

### Question 3:

The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

<b>Expenditure (in Rs)</b>	<b>Number of families</b>
<b>1000 – 1500</b>	24
<b>1500 – 2000</b>	40
<b>2000 – 2500</b>	33
<b>2500 – 3000</b>	28
<b>3000 – 3500</b>	30
<b>3500 – 4000</b>	22
<b>4000 – 4500</b>	16
<b>4500 – 5000</b>	7

**Answer 3:**

It can be observed from the given data that the maximum class frequency is 40,

belonging to 1500 – 2000 intervals.

Therefore, modal class = 1500 – 2000

Lower limit ( $l$ ) of modal class = 1500

Frequency ( $f_1$ ) of modal class = 40

Frequency ( $f_0$ ) of class preceding modal class = 24

Frequency ( $f_2$ ) of class succeeding modal class = 33

Class size ( $h$ ) = 500

$$\begin{aligned}\text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 1500 + \left( \frac{40 - 24}{2(40) - 24 - 33} \right) \times 500 \\ &= 1500 + \left( \frac{16}{80 - 57} \right) \times 500 \\ &= 1500 + \frac{8000}{23} \\ &= 1500 + 347.826 \\ &= 1847.826 = 1847.83\end{aligned}$$

Therefore, modal monthly expenditure was Rs 1847.83.

To find the class mark, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Class size ( $h$ ) of the given data = 500

Taking 2750 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ , and  $f_i u_i$  are calculated as follows.

<b>Expenditure (in Rs)</b>	<b>Number of families <math>f_i</math></b>	<b><math>x_i</math></b>	<b><math>d_i = x_i - 2750</math></b>	<b><math>u_i = \frac{d_i}{500}</math></b>	<b><math>f_i u_i</math></b>
<b>1000 – 1500</b>	24	1250	– 1500	– 3	– 72
<b>1500 – 2000</b>	40	1750	– 1000	– 2	–80
<b>2000 – 2500</b>	33	2250	– 500	– 1	–33
<b>2500 – 3000</b>	28	2750	0	0	0
<b>3000 – 3500</b>	30	3250	500	1	30
<b>3500 – 4000</b>	22	3750	1000	2	44
<b>4000 – 4500</b>	16	4250	1500	3	48
<b>4500 – 5000</b>	7	4750	2000	4	28
<b>Total</b>	<b>200</b>				<b>–35</b>

From the table, we obtain

$$\sum f_i = 200$$

$$\sum f_i u_i = -35$$

$$\bar{x}(\text{mean}) = a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$\bar{x} = 2750 + \left( \frac{-35}{200} \right) \times 500$$

$$= 2750 - 87.5$$

$$= 2662.5$$

Therefore, mean monthly expenditure was Rs 2662.50.

#### Question 4:

The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	Number of states/U.T
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0
45 – 50	0
50 – 55	2

**Answer 4:**

It can be observed from the given data that the maximum class frequency is 10 belonging to class interval 30 – 35.

Therefore, modal class = 30 – 35

Class size ( $h$ ) = 5

Lower limit ( $l$ ) of modal class = 30

Frequency ( $f_1$ ) of modal class = 10

Frequency ( $f_0$ ) of class preceding modal class = 9

Frequency ( $f_2$ ) of class succeeding modal class = 3

$$\begin{aligned}\text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 30 + \left( \frac{10 - 9}{2(10) - 9 - 3} \right) \times (5) \\ &= 30 + \left( \frac{1}{20 - 12} \right) 5 \\ &= 30 + \frac{5}{8} = 30.625\end{aligned}$$

$$\text{Mode} = 30.6$$

It represents that most of the states/U.T have a teacher-student ratio as 30.6.

To find the class marks, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Taking 32.5 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ , and  $f_i u_i$  are calculated as follows.

Number of students per teacher	Number of states/U.T ( $f_i$ )	$x_i$	$d_i = x_i - 32.5$	$u_i = \frac{d_i}{5}$	$f_i u_i$
15 – 20	3	17.5	– 15	– 3	– 9
20 – 25	8	22.5	– 10	– 2	–16
25 – 30	9	27.5	– 5	– 1	– 9
30 – 35	10	32.5	0	0	0
35 – 40	3	37.5	5	1	3
40 – 45	0	42.5	10	2	0
45 – 50	0	47.5	15	3	0
50 – 55	2	52.5	20	4	8
Total	<b>35</b>				<b>–23</b>

$$\begin{aligned}
 \text{Mean, } \bar{x} &= a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) h \\
 &= 32.5 + \left( \frac{-23}{35} \right) \times 5 \\
 &= 32.5 - \frac{23}{7} = 32.5 - 3.28 \\
 &= 29.22
 \end{aligned}$$

Therefore, mean of the data is 29.2.

It represents that on an average, teacher–student ratio was 29.2.



**Question 5:**

The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

<b>Runs scored</b>	<b>Number of batsmen</b>
<b>3000 – 4000</b>	4
<b>4000 – 5000</b>	18
<b>5000 – 6000</b>	9
<b>6000 – 7000</b>	7
<b>7000 – 8000</b>	6
<b>8000 – 9000</b>	3
<b>9000 – 10000</b>	1
<b>10000 – 11000</b>	1

Find the mode of the data.

**Answer 5:**

From the given data, it can be observed that the maximum class frequency is 18, belonging to class interval 4000 – 5000. Therefore, modal class = 4000 – 5000

Lower limit ( $l$ ) of modal class = 4000

Frequency ( $f_1$ ) of modal class = 18

Frequency ( $f_0$ ) of class preceding modal class = 4

Frequency ( $f_2$ ) of class succeeding modal class = 9

Class size ( $h$ ) = 1000

$$\begin{aligned}
 \text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 4000 + \left( \frac{18 - 4}{2(18) - 4 - 9} \right) \times 1000 \\
 &= 4000 + \left( \frac{14000}{23} \right) \\
 &= 4000 + 608.695 \\
 &= 4608.695
 \end{aligned}$$

Therefore, mode of the given data is 4608.7 runs

#### Question 6:

A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	14	13	12	20	11	15	8

#### Answer 6:

From the given data, it can be observed that the maximum class frequency is 20, belonging to 40 – 50 class intervals. Therefore, modal class = 40 – 50

Lower limit ( $l$ ) of modal class = 40

Frequency ( $f_1$ ) of modal class = 20

Frequency ( $f_0$ ) of class preceding modal class = 12

Frequency ( $f_2$ ) of class succeeding modal class = 11

Class size = 10

$$\begin{aligned}\text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 40 + \left[ \frac{20 - 12}{2(20) - 12 - 11} \right] \times 10 \\ &= 40 + \left( \frac{80}{40 - 23} \right) \\ &= 40 + \frac{80}{17} \\ &= 40 + 4.7 \\ &= 44.7\end{aligned}$$

Therefore, mode of this data is 44.7 cars.

# Mathematics

(Chapter - 13)(Statistics)

(Class – X)

## Exercise 13.3

### Question 1:

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 – 85	4
85 – 105	5
105 – 125	13
125 – 145	20
145 – 165	14
165 – 185	8
185 – 205	4

### Answer 1:

To find the class marks, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Taking 135 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ ,  $f_i u_i$  are calculated according to step deviation method as follows.

Monthly consumption (in units)	Number of consumers ( $f_i$ )	$x_i$ class mark	$d_i = x_i - 135$	$u_i = \frac{d_i}{20}$	$f_i u_i$
<b>65 – 85</b>	4	75	– 60	– 3	–12
<b>85 – 105</b>	5	95	– 40	– 2	– 10
<b>105 – 125</b>	13	115	– 20	– 1	– 13
<b>125 – 145</b>	20	135	0	0	0
<b>145 – 165</b>	14	155	20	1	14
<b>165 – 185</b>	8	175	40	2	16
<b>185 – 205</b>	4	195	60	3	12
Total	<b>68</b>				<b>7</b>

From the table, we obtain

$$\sum f_i u_i = 7$$

$$\sum f_i = 68$$

$$\text{Class size } (h) = 20$$

$$\begin{aligned}\text{Mean, } \bar{x} &= a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 135 + \frac{7}{68} \times 20 \\ &= 135 + \frac{140}{68} \\ &= 137.058\end{aligned}$$

From the table, it can be observed that the maximum class frequency is 20, belonging to class interval 125 – 145.

Modal class = 125 – 145

Lower limit ( $l$ ) of modal class = 125

Class size ( $h$ ) = 20

Frequency ( $f_1$ ) of modal class = 20

Frequency ( $f_0$ ) of class preceding modal class = 13

Frequency ( $f_2$ ) of class succeeding the modal class = 14

$$\begin{aligned}\text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 125 + \left[ \frac{20 - 13}{2(20) - 13 - 14} \right] \times 20 \\ &= 125 + \frac{7}{13} \times 20 \\ &= 125 + \frac{140}{13} = 135.76\end{aligned}$$

To find the median of the given data, cumulative frequency is calculated as follows.

Monthly consumption (in units)	Number of consumers	Cumulative frequency
<b>65 – 85</b>	4	4
<b>85 – 105</b>	5	4 + 5 = 9
<b>105 – 125</b>	13	9 + 13 = 22
<b>125 – 145</b>	20	22 + 20 = 42
<b>145 – 165</b>	14	42 + 14 = 56
<b>165 – 185</b>	8	56 + 8 = 64
<b>185 – 205</b>	4	64 + 4 = 68

From the table, we obtain

$$n = 68$$

Cumulative frequency ( $cf$ ) just greater than  $\frac{n}{2}$  (i.e.,  $\frac{68}{2} = 34$ ) is 42, belonging to interval 125 – 145.

Therefore, median class = 125 – 145

Lower limit ( $l$ ) of median class = 125

Class size ( $h$ ) = 20

Frequency ( $f$ ) of median class = 20

Cumulative frequency ( $cf$ ) of class preceding median class = 22

$$\begin{aligned}
 \text{Median} &= l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h \\
 &= 125 + \left( \frac{34 - 22}{20} \right) \times 20 \\
 &= 125 + 12 \\
 &= 137
 \end{aligned}$$

Therefore, median, mode, mean of the given data is 137, 135.76, and 137.05 respectively.

The three measures are approximately the same in this case.

### Question 2:

If the median of the distribution is given below is 28.5, find the values of  $x$  and  $y$ .

Class interval	Frequency
<b>0 – 10</b>	5
<b>10 – 20</b>	$x$
<b>20 – 30</b>	20
<b>30 – 40</b>	15
<b>40 – 50</b>	$y$
<b>50 – 60</b>	5
<b>Total</b>	60



**Answer 2:**

The cumulative frequency for the given data is calculated as follows.

Class interval	Frequency	Cumulative frequency
0 – 10	5	5
10 – 20	$x$	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	$y$	$40 + x + y$
50 – 60	5	$45 + x + y$
Total ( $n$ )	60	

From the table, it can be observed that  $n = 60$

$$45 + x + y = 60 \quad \text{or} \quad x + y = 15 \dots\dots\dots(1)$$

Median of the data is given as 28.5 which lies in interval 20 – 30.

Therefore, median class = 20 – 30

Lower limit ( $l$ ) of median class = 20

Cumulative frequency ( $cf$ ) of class preceding the median class =  $5 + x$

Frequency ( $f$ ) of median class = 20

Class size ( $h$ ) = 10

$$\text{Median} = l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$28.5 = 20 + \left[ \frac{\frac{60}{2} - (5 + x)}{20} \right] \times 10$$

$$8.5 = \left( \frac{25 - x}{2} \right)$$

$$17 = 25 - x$$

$$x = 8$$

From equation (1),

$$8 + y = 15 \quad y = 7$$

Hence, the values of x and y are 8 and 7 respectively

### Question 3:

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

**Answer 3:**

Here, class width is not the same. There is no requirement of adjusting the frequencies according to class intervals. The given frequency table is of less than type represented with upper class limits. The policies were given only to persons with age 18 years onwards but less than 60 years. Therefore, class intervals with their respective cumulative frequency can be defined as below.

Age (in years)	Number of policy holders ( $f_i$ )	Cumulative frequency ( $cf$ )
18 – 20	2	2
20 – 25	$6 - 2 = 4$	6
25 – 30	$24 - 6 = 18$	24
30 – 35	$45 - 24 = 21$	45
35 – 40	$78 - 45 = 33$	78
40 – 45	$89 - 78 = 11$	89
45 – 50	$92 - 89 = 3$	92
50 – 55	$98 - 92 = 6$	98
55 – 60	$100 - 98 = 2$	100
Total ( $n$ )		

From the table, it can be observed that  $n = 100$ .

Cumulative frequency ( $cf$ ) just greater than  $\frac{n}{2}$  (i.e.,  $\frac{100}{2} = 50$ ) is 78, belonging to interval 35 – 40.

Therefore, median class = 35 – 40

Lower limit ( $l$ ) of median class = 35

Class size ( $h$ ) = 5

Frequency ( $f$ ) of median class = 33

Cumulative frequency ( $cf$ ) of class preceding median class = 45

$$\begin{aligned}\text{Median} &= l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 35 + \left( \frac{50 - 45}{33} \right) \times 5 \\ &= 35 + \frac{25}{33} \\ &= 35.76\end{aligned}$$

Therefore, median age is 35.76 years.

#### Question 4:

The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	Number of leaves $f_i$
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the median length of the leaves.

(**Hint:** The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 – 126.5, 126.5 – 135.5... 171.5 – 180.5)

**Answer 4:**

The given data does not have continuous class intervals. It can be observed that the difference between two class intervals is 1. Therefore,  $\frac{1}{2} = 0.5$  has to be added and subtracted to upper class limits and lower class limits respectively.

Continuous class intervals with respective cumulative frequencies can be represented as follows.

Length (in mm)	Number or leaves $f_i$	Cumulative frequency
<b>117.5 – 126.5</b>	3	3
<b>126.5 – 135.5</b>	5	$3 + 5 = 8$
<b>135.5 – 144.5</b>	9	$8 + 9 = 17$
<b>144.5 – 153.5</b>	12	$17 + 12 = 29$
<b>153.5 – 162.5</b>	5	$29 + 5 = 34$
<b>162.5 – 171.5</b>	4	$34 + 4 = 38$
<b>171.5 – 180.5</b>	2	$38 + 2 = 40$

From the table, it can be observed that the cumulative frequency just greater than

$$\frac{n}{2} \left( \text{i.e., } \frac{40}{2} = 20 \right)$$

is 29, belonging to class interval 144.5 – 153.5.

Median class = 144.5 – 153.5

Lower limit ( $l$ ) of median class = 144.5

Class size ( $h$ ) = 9

Frequency ( $f$ ) of median class = 12

Cumulative frequency ( $cf$ ) of class preceding median class = 17

$$\begin{aligned} &= l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h \\ \text{Median} &= 144.5 + \left( \frac{20 - 17}{12} \right) \times 9 \\ &= 144.5 + \frac{9}{4} = 146.75 \end{aligned}$$

Therefore, median length of leaves is 146.75 mm.

**Question 5:**

Find the following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
<b>1500 – 2000</b>	14
<b>2000 – 2500</b>	56
<b>2500 – 3000</b>	60
<b>3000 – 3500</b>	86
<b>3500 – 4000</b>	74
<b>4000 – 4500</b>	62
<b>4500 – 5000</b>	48

Find the median life time of a lamp.

**Answer 5:**

The cumulative frequencies with their respective class intervals are as follows.

Life time	Number of lamps ( $f_i$ )	Cumulative frequency
<b>1500 – 2000</b>	14	14
<b>2000 – 2500</b>	56	$14 + 56 = 70$
<b>2500 – 3000</b>	60	$70 + 60 = 130$
<b>3000 – 3500</b>	86	$130 + 86 = 216$
<b>3500 – 4000</b>	74	$216 + 74 = 290$
<b>4000 – 4500</b>	62	$290 + 62 = 352$
<b>4500 – 5000</b>	48	$352 + 48 = 400$
<b>Total (<math>n</math>)</b>	400	

It can be observed that the cumulative frequency just greater than  $\frac{n}{2}$  (i.e.,  $\frac{400}{2} = 200$ ) is 216, belonging to class interval 3000 – 3500.

Median class = 3000 – 3500

Lower limit ( $l$ ) of median class = 3000

Frequency ( $f$ ) of median class = 86

Cumulative frequency ( $cf$ ) of class preceding median class = 130

Class size ( $h$ ) = 500

$$\begin{aligned}\text{Median} &= l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 3000 + \left( \frac{200 - 130}{86} \right) \times 500 \\ &= 3000 + \frac{70 \times 500}{86}\end{aligned}$$

$$= 3406.976$$

Therefore, median life time of lamps is 3406.98 hours.

#### Question 6:

100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 – 4	4 – 7	7 – 10	10 – 13	13 – 16	16 – 19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

#### Answer 6:

The cumulative frequencies with their respective class intervals are as follows.



Number of letters	Frequency ( $f_i$ )	Cumulative frequency
<b>1 – 4</b>	6	6
<b>4 – 7</b>	30	$30 + 6 = 36$
<b>7 – 10</b>	40	$36 + 40 = 76$
<b>10 – 13</b>	16	$76 + 16 = 92$
<b>13 – 16</b>	4	$92 + 4 = 96$
<b>16 – 19</b>	4	$96 + 4 = 100$
<b>Total (<math>n</math>)</b>	100	

It can be observed that the cumulative frequency just greater than  $\frac{n}{2}$  (i.e.,  $\frac{100}{2} = 50$ ) is 76, belonging to class interval 7 – 10.

Median class = 7 – 10

Lower limit ( $l$ ) of median class = 7

Cumulative frequency ( $cf$ ) of class preceding median class = 36

Frequency ( $f$ ) of median class = 40

Class size ( $h$ ) = 3

$$\begin{aligned}
 \text{Median} &= l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h \\
 &= 7 + \left( \frac{50 - 36}{40} \right) \times 3 \\
 &= 7 + \frac{14 \times 3}{40}
 \end{aligned}$$

$$= 8.05$$

To find the class marks of the given class intervals, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Taking 11.5 as assumed mean ( $a$ ),  $d_i$ ,  $u_i$ , and  $f_i u_i$  are calculated according to step deviation method as follows.

Number of letters	Number of surnames $f_i$	$x_i$	$d_i = x_i - 11.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
1 – 4	6	2.5	– 9	– 3	– 18
4 – 7	30	5.5	– 6	– 2	– 60
7 – 10	40	8.5	– 3	– 1	– 40
10 – 13	16	11.5	0	0	0
13 – 16	4	14.5	3	1	4
16 – 19	4	17.5	6	2	8
<b>Total</b>	100				–106

From the table, we obtain

$$\sum f_i u_i = -106$$

$$\sum f_i = 100$$

$$\begin{aligned} \text{Mean, } \bar{x} &= a + \left( \frac{\sum f_i u_i}{\sum f_i} \right) h \\ &= 11.5 + \left( \frac{-106}{100} \right) \times 3 \end{aligned}$$

$$= 11.5 - 3.18 = 8.32$$

The data in the given table can be written as

Number of letters	Frequency ( $f_i$ )
1 – 4	6
4 – 7	30
7 – 10	40
10 – 13	16
13 – 16	4
16 – 19	4
<b>Total (<math>n</math>)</b>	100

From the table, it can be observed that the maximum class frequency is 40 belonging to class interval 7 – 10. Modal class = 7 – 10

Lower limit ( $l$ ) of modal class = 7

Class size ( $h$ ) = 3

Frequency ( $f_1$ ) of modal class = 40

Frequency ( $f_0$ ) of class preceding the modal class = 30

Frequency ( $f_2$ ) of class succeeding the modal class = 16

$$\begin{aligned}
 \text{Mode} &= l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 7 + \left[ \frac{40 - 30}{2(40) - 30 - 16} \right] \times 3 \\
 &= 7 + \frac{10}{34} \times 3 \\
 &= 7 + \frac{30}{34} = 7.88
 \end{aligned}$$

Therefore, median number and mean number of letters in surnames is 8.05 and 8.32 respectively while modal size of surnames is 7.88.

### Question 7:

The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
Number of students	2	3	8	6	6	3	2

### Answer 7:

The cumulative frequencies with their respective class intervals are as follows.

Weight (in kg)	Frequency ( $f_i$ )	Cumulative frequency
40 – 45	2	2
45 – 50	3	$2 + 3 = 5$
50 – 55	8	$5 + 8 = 13$
55 – 60	6	$13 + 6 = 19$
60 – 65	6	$19 + 6 = 25$
65 – 70	3	$25 + 3 = 28$
70 – 75	2	$28 + 2 = 30$
Total ( $n$ )	30	

Cumulative frequency just greater than  $\frac{n}{2}$  (i.e.,  $\frac{30}{2} = 15$ ) is 19, belonging to class interval 55 – 60.

Median class = 55 – 60

Lower limit ( $l$ ) of median class = 55

Frequency ( $f$ ) of median class = 6

Cumulative frequency ( $cf$ ) of median class = 13

Class size ( $h$ ) = 5

$$\begin{aligned}\text{Median} &= l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 55 + \left( \frac{15 - 13}{6} \right) \times 5 \\ &= 55 + \frac{10}{6}\end{aligned}$$

= 56.67

Therefore, median weight is 56.67 kg.