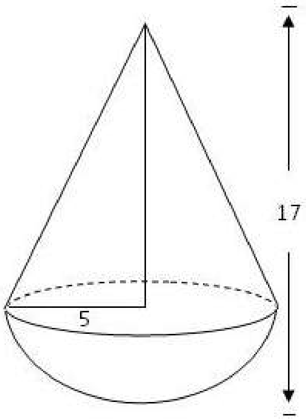
Surface Areas and Volumes

Exercise-14.1

Question 1:

A toy is made by mounting a cone onto a hemisphere. The radius of the cone and a hemisphere is 5 cm. The total height of the toy is 17 cm. Find the total surface area of the toy.

Solution :



Radius of the hemisphere

= Radius of the cone = r = 5cm.

Height of the cone

h = Total height of the toy − Radius of the hemisphere

= 17 − 5 = 12cm

Then, Slant height of cone

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{12^2 + 5^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169} = 13 \text{ cm}$$

Total Surface Area of the toy

= Curved Surface Area of the hemisphere

+ Curved Surface Area of the cone

$$= 2\pi r^2 + \pi r l$$

$$= \pi r(2r + l)$$

$$= \frac{22}{7} \times 5(2 \times 5 + 13)$$

$$= \frac{22}{7} \times 5 \times 23$$

$$= \frac{2530}{7}$$

$$= 361.42 \text{ cm}^2$$

∴ Total surface area of the toy is 361.42 cm^2

Question 2:

A show-piece shown in figure 14.10 is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 7 cm and the hemisphere fixed on the top has diameter 5.2 cm. Find the total surface area of the piece.

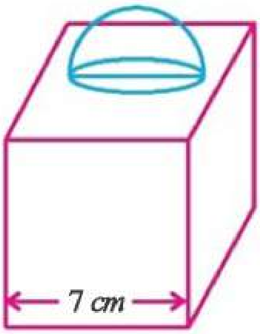
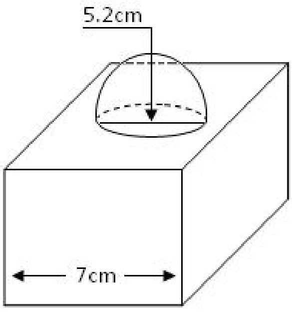


Figure 14.10

Solution :



Here, length of the cube l = 7 cm

Radius of the hemisphere

$r = \frac{\text{diameter}}{2} = \frac{5.2}{2} = 2.6$ cm

TSA of the show-piece

= TSA of cube + CSA of hemisphere – Area of base of the hemisphere

$= 6l^2 + 2\pi r^2 - \pi r^2$

$= 6l^2 + \pi r^2$

$= 6 \times (7)^2 + \frac{22}{7} \times (2.6)^2$

$= 294 + 21.25$

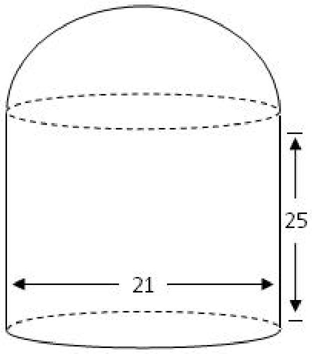
$= 315.25$ cm²

$\therefore$ The total surface area of the show-piece is 315.25 cm^2

Question 3:

A vessel is in the form of a hemisphere mounted on a hollow cylinder. The diameter of the hemisphere is 21 cm and the height of vessel is 25 cm. If the vessel is to be painted at the rate of ₹ 3.5 per cm^2, then find the total cost to paint the vessel from outside.

Solution :



Here it is given that hemisphere is mounted on a cylinder.

$\therefore$ Radius of the cylinder = radius of the hemisphere

$\therefore r = \frac{\text{diameter}}{2} = \frac{21}{2}$ cm

Also, height of the cylinder h = 25 cm

TSA of the vessel

= CSA of the cylinder + CSA of the hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2\pi r (h + r)$

$= 2 \times \frac{22}{7} \times \frac{21}{2}\left(25 + \frac{21}{2}\right)$

$= 66 \times 35.5$

$= 2343 \text{ cm}^2$

Cost of painting 1 cm^2 = Rs 3.5

$\therefore$ Cost of painting 2343 cm^2 = Rs. (2343×3.5)

= Rs. 8200.50

$\therefore$ The total cost of painting the vessel from outside is Rs. 8200.50.

Question 4:

Chirag made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end, (see the figure 14.11). The height of the cylinder is 1.5 m and its radius is 50 cm. Find the total area of the bird-bath. (π = 3.14)

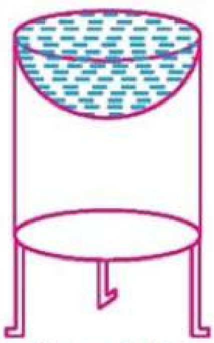


Figure 14.11

Solution :

Height of the cylinder h = 1.5 m = 1.5 x 100 cm = 150 cm

Radius of the cylinder = Radius of the hemisphere = r = 50 cm.

TSA of the bird-bath

= CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2 \times 3.14 \times 50(150+50)$

$= 3.14 \times 100 \times 200$

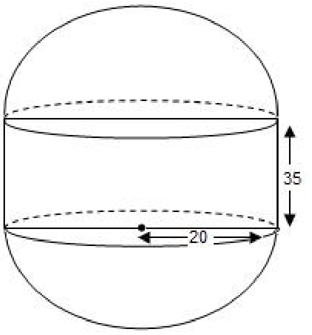
$= 62{,}800\ cm^2$

$\therefore$ Total surface area of the bird-bath is $62{,}800\ cm^2$.

Question 5:

A solid is composed of a cylinder with hemisperical ends on both the sides. The radius and the height of the cylinder are 20 cm and 35 cm respectively. Find the total surface area of the solid.

Solution :



Radius of the cylinder = radius of each hemisphere = r = 35 cm

Height of cylinder h = 20 cm.

Then,

TSA of the solid

= CSA of the cylinder + 2 x CSA of the hemisphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 35(20 + 2 \times 35)$

$= 2 \times 22 \times 5 \times 90$

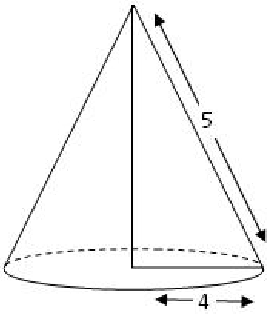
$= 19{,}800\ cm^2$

$\therefore$ Total surface area of the given solid is $19{,}800\ cm^2$.

Question 6:

The radius of a conical tent is 4 m and slant height is 5 m. How many meters of canvas of width 125 cm will be used to prepare 12 tents ? If the cost of canvas is ₹ 20 per meter, then what is total cost of 12 tents ? (π = 3.14)

Solution :



Radius of each conical tent, $r = 4$ m and slant height $l = 5$ m.
Area of the canvas required to prepare 1 tent
= Curved Surface Area of the conical tent
$= \pi rl$
$= 3.14 \times 4 \times 5$
$= 62.8\ m^2$
$\therefore$ Area of the canvas required to prepare 12 tents $= 12 \times 62.8$
$= 753.60\ m^2$

Width of the canvas = 125 cm $= \dfrac{125}{100} = 1.25$ m.
Area of the canvas = Length x Width
$\therefore 753.60 = \text{Length} \times 1.25$
$\therefore \text{Length} = \dfrac{753.60}{1.25}$
$\therefore$ Length = 602.88 m
Thus, 602.88 m canvas is required to prepare 12 tents.
Cost of 1 m canvas = Rs. 20
$\therefore$ Cost of 602.88 m canvas = Rs.(602.88×20)
= Rs. 12,057.60
$\therefore$ The total cost of 12 tents is Rs. 12,057.60.

Question 7:

If the radius of a cone is 60 cm and its curved surface area is 23.55 m^2, then find its slant height. ($\pi = 3.14$)

Solution :

Radius of the cone r = 60 cm $= \dfrac{60}{100}$ m = 0.60 m
Curved surface area of the cone $= \pi rl$
$\therefore 23.55 = 3.14 \times 0.60 \times l$

$$\therefore l = \frac{23.55}{3.14 \times 0.60}$$

$$= \frac{2355 \times 100 \times 100}{314 \times 60 \times 100}$$

$$\therefore l = \frac{\cancel{2355}_{\cancel{157}} \times 100}{\cancel{314}_2 \times \cancel{60}_4}$$

$$\therefore l = \frac{100}{2 \times 4}$$

$= 12.50$ m
$\therefore$ The slant height of the cone is 12.50 m.

Question 8:

The cost of painting the surface of sphere is ₹ 1526 at the rate of ₹ 6 per m^2. Find the radius of sphere.

Solution :

Area of the painted part at the cost of Rs. 6 = $1 m^2$

$\therefore$ Area of the painted part at the cost of Rs. 1526

$= \frac{1526}{6} = \frac{763}{3} m^2$

$\therefore$ Surface area of the sphere = $\frac{763}{3} m^2$

Surface area of the sphere = $4\pi r^2$

$\therefore \frac{763}{3} = 4 \times 3.14 \times r^2$

$\therefore r^2 = \frac{763}{3} \times \frac{1}{4 \times 3.14}$

$\therefore r^2 = 20.25$

$\therefore r = \sqrt{20.25}$

$\therefore r = 4.5$ m

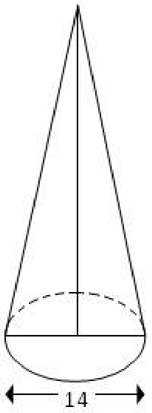
$\therefore$ Radius of the sphere is 4.5 m.

Exercise-14.2

Question 1:

The curved surface area of a cone is 550 cm^2. If its diameter is 14 cm, find its volume.

Solution :



Here, C.S.A. of the cone = 550 cm^2

Next, Radius r = $\frac{\text{diameter}}{2} = \frac{14}{2} = 7$ cm

C.S.A. of a cone = πrl

$\therefore 550 = \frac{22}{7} \times 7 \times l$

$\therefore \frac{550}{22} = l$

$\therefore l = 25$ cm

We know that, $l^2 = r^2 + h^2$

$\therefore 25^2 = 7^2 + h^2$

$\therefore 625 = 49 + h^2$

$\therefore h^2 = 625 - 49$

$\therefore h^2 = 576$

$\therefore h = 24$ cm

Now,

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$

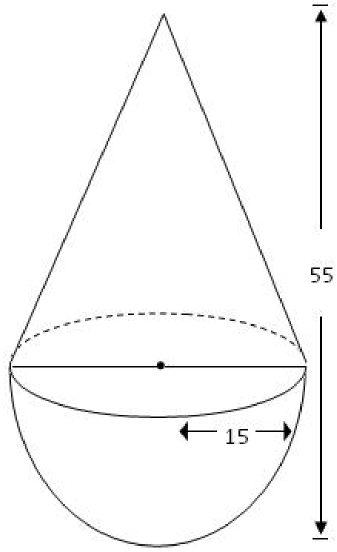
$= 1232 \; cm^3$

$\therefore$ Volume of the cone is 1232 cm^3

Question 2:

A solid is in the form of cone with hemispherical base. The radius of the cone is 15 cm and the total height of the solid is 55 cm. Find the volume of the solid. (π = 3.14)

Solution :



Radius of the cone
= radius of the hemisphere
= r = 15 cm
Height of the cone
h = Total height of the solid − Radius of the hemisphere
= 55 − 15
= 40 cm
Volume of the solid = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}(h + 2r)$$

$$= \frac{22}{7} \times \frac{15 \times 15}{3}(40 + 2 \times 15)$$

$$= \frac{22}{7} \times 15 \times 5 \times 70$$

$$= 16,500 \text{ cm}^2$$

Volume of the given solid is 16,500 cm^2.

Question 3:

How many litres of milk can be stored in a cylindrical tank with radius 1.4 m and height 3 m ?

Solution :

For the cylindrical tank,
Radius r = 1.4 m
Height h = 3 m
Then,
Volume of the cylindrical tank = $\pi r^2 h$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 3$$

$$= 18.48 \text{ m}^3$$

But, 1 m^3 = 1000 litres

$\therefore$ 18.48 m^3 = (18.48×1000) litres
= 18, 480 litres

$\therefore$ 18,480 litres of milk can be stored in the tank.

Question 4:

The spherical balloon with radius 21 cm is filled with air. Find the volume of air contained in it.

Solution :

Radius of the spherical balloon is given as 21 cm

Now,

Volume of air contained in the spherical balloon

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

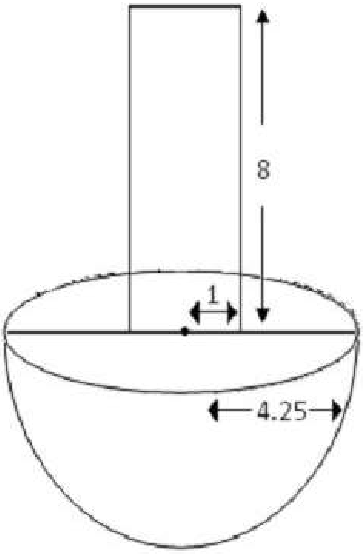
$= 38,808 \text{ cm}^3$

$\therefore$ The volume of air contained in the balloon is $38,808 \text{ cm}^3$.

Question 5:

A solid has hemi-spherical base with diameter 8.5 cm and it is surmounted by a cylinder with height 8 cm and diameter of cylinder is 2 cm. Find the volume of this solid. (π = 3.14)

Solution :



Here, For cylinder,

Radius $r = \frac{d}{2} = \frac{2}{2} = 1$ cm

Height h = 8 cm

Then,

Radius of the hemisphere $= R = \frac{d}{2} = \frac{8.5}{2} = 4.25$ cm

Then,

Volume of the solid = Volume of the cylinder + Volume of the hemishpere

$= \pi r^2 h + \frac{2}{3}\pi R^3$

$= 3.14 \times 1 \times 1 \times 8 + \frac{2}{3} \times 3.14 \times 4.25 \times 4.25 \times 4.25$

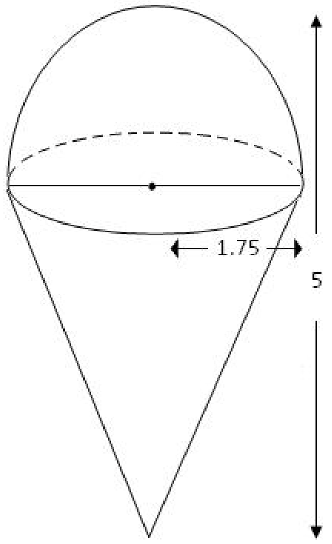
$= 25.12 + 160.70$

$= 185.82 \text{ cm}^3$

$\therefore$ The volume of the solid is 185.82 cm^3.

Question 6:

A playing top is made up of steel. The top is shaped like a cone surmounted by a hemisphere. The total height of top is 5 cm and the diameter of the top is 3.5 cm. Find the volume of the top.



For the cone,

Radius of cone = Radius of the hemisphere

$$= r = \frac{d}{2} = \frac{3.5}{2} = 1.75 \text{ cm}$$

Height of cone = Total height of the top − Radius of hemisphere

$= 5 - 1.75$

$= 3.25$ cm

Volume of top = Volume of cone + Volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{\pi r^2}{3}(h + 2r)$

$= \frac{22}{7} \times \frac{1.75 \times 1.75}{3}(3.25 + 2 \times 1.75)$

$= 22 \times 0.25 \times 1.75 \times 2.25$

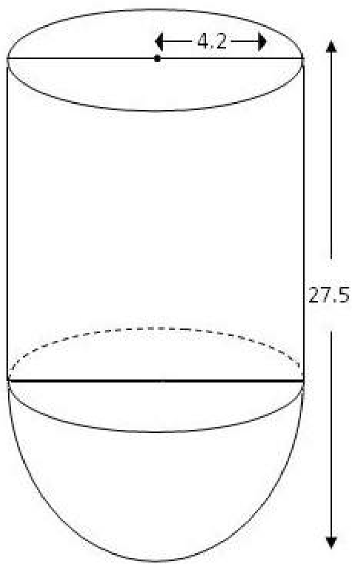
$= 21.66 \text{ cm}^3$

$\therefore$ Volume of the top is 21.66 cm^3.

Question 7:

How many litres of petrol will be contained in a closed cylindrical tank with hemisphere at one end having radius 4.2 cm and total height 27.5 cm ?

Solution :



For the given cylindrical tank,
Radius of cylinder = Radius of hemisphere = r = 4.2 cm
Height of the cylinder = Total height – Radius of the hemisphere
= 27.5 – 4.2
= 23.3 cm
Now,
Volume of the tank = Volume of cylinder + Volume of hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi r^2\left(h + \frac{2}{3}r\right)$$

$$= \frac{22}{7} \times 4.2 \times 4.2\left(23.3 + \frac{2}{3} \times 4.2\right)$$

$$= 22 \times 0.6 \times 4.2(26.1)$$

$$= 1446.984 \text{ cm}^3$$

$$= \frac{1446.984}{1000} \text{ litres} \quad \left(\because 1\text{litre} = 1000 \text{ cm}^3\right)$$

$$= 1.446984 \text{ litres}$$

$$= 1.45 \text{ litres}$$

Thus, 1.45 litres of petrol will be contained in the given closed cylindrical tank.

Question 8:

The capacity of a cylindrical tank at a petrol pump is 57750 litres. If its diameter is 3.5 m, find the height of cylinder.

Solution :

Capacity of the cylinderical tank is given to be 57,750 litres.

$$\text{But, } 57{,}750 \text{ litres} = \frac{57750}{1000} \text{ m}^3$$

$$= 57.75 \text{ m}^3.$$

$\therefore$ The volume of the cylinderical tank = 57.75 m^3

Radius of the cylinderical tank $r = \frac{d}{2} = \frac{3.5}{2} = 1.75 \text{ m}$

Let height of the cylinder be h m.
Now,
Volume of the cylindrical tank = $\pi r^2 h$

$$\therefore 57.75 = \frac{22}{7} \times 1.75 \times 1.75 \times h$$

$$\therefore \frac{5775}{100} \times \frac{7}{22} \times \frac{100}{175} \times \frac{100}{175} = h$$

$\therefore h = 6 \text{ m}$

$\therefore$ The height of the cylindrical tank is 6 m.

Question 9:

A hemispherical pond is filled with 523.908 m^3 of water. Find the maximum depth of pond.

Solution :

Volume of the hemispherical pond = 523.908 m^3
Maximum depth (radius) = r m

Volume of a hemisphere = $\frac{2}{3}\pi r^3$

$\therefore 523.908 = \frac{2}{3} \times \frac{22}{7} \times r^3$

$\therefore r^3 = \frac{523908}{1000} \times \frac{3}{2} \times \frac{7}{22}$

$\therefore r^3 = \frac{11907 \times 3 \times 7}{1000}$

$\therefore r^3 = \frac{63 \times 63 \times 63}{10 \times 10 \times 10}$

$\therefore r^3 = \left(\frac{63}{10}\right)^3$

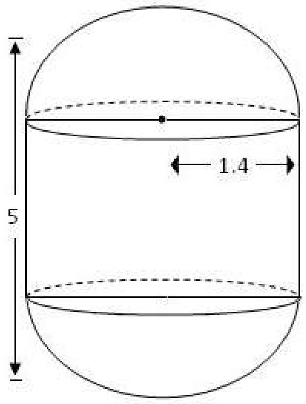
$\therefore r = 6.3$ m

$\therefore$ The maximum depth of the hemispherial pond is 6.3 m.

Question 10:

A gulab-jamun contain 40 % sugar syrup in it. Find how much syrup would be there in 50 gulab-jamuns, each shaped like a cylinder with two hemispherical ends with total length 5 cm and diameter 2.8 cm.

Solution :



Radius of the cylindrical part of the gulab-jamun
= radius of each hemispherical end

$\therefore r = \frac{d}{2} = \frac{2.8}{2} = 1.4$ cm

Height of the cylindrical part of the gulab-jamun
= Total height of the gulab-jamun – (2 x radius of hemisphere)
= 5 – (2 x 1.4)
= 5 – 2.8
= 2.2 cm

Volume of one gulab-jamun

$= \text{Volume of the cylinderical part} + \begin{pmatrix} 2 \times \text{Volume of the} \\ \text{hemispherical part} \end{pmatrix}$

$= \pi r^2 h + \left(2 \times \frac{2}{3}\pi r^3\right)$

$= \pi r^2 \left(h + \frac{4}{3}r\right)$

$= \frac{22}{7} \times 1.4 \times 1.4 \left(2.2 + \frac{4}{3} \times 1.4\right)$

$= \frac{22}{7} \times 1.4 \times 1.4 (2.2 + 1.87)$

$= 6.16 \times 4.07$

$= 25.07 \text{ cm}^3$

$\therefore$ Volume of 50 gulab-jamuns $= (50 \times 25.07) \text{ cm}^3$

But, there is 40% syrup in the volume of each gulab-jamun.

$\therefore$ Volume of the syrup in 50 gulab-jamuns

= 40% of the volume of 50 gulab-jamuns

= 40% of (50×25.07)

$= \frac{40}{100} \times 50 \times 25.07$

$= 501.4 \text{ cm}^3$

$= \frac{501.4}{1000}$ litre $\quad (\because 1 \text{ litre} = 1000 \text{ cm}^3)$

= 0.5014 litre

= 0.5 litre

$\therefore$ The volume of syrup in 50 gulab-jamuns is 0.5 litre

Question 11:

The height and the slant height of a cone are 12 cm and 20 cm respectively. Find its volume. ($\pi = 3.14$)

Solution :

For the given cone,

Height h = 12 cm

Slant height l = 20 cm

Radius = r cm.

We know that,

$l^2 = h^2 + r^2$

$\therefore 20^2 = 12^2 + r^2$

$\therefore r^2 = 400 - 144$

$= 256$

$\therefore r = 16$ cm

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 16 \times 16 \times 12$

$= 3215.36 \text{ cm}^3$

$\therefore$ The volume of the cone is 3215.36 cm^3.

Question 12:

Find the total volume of a cone having a hemispherical base. If the radius of the base is 21 cm and height 60 cm.

Solution :

Here,
Radius of cone = Radius of hemisphere = r = 21 cm
Height of the cone, h = 60 cm.
Volume of the solid = Volume of the cone + Volume of the hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times (60 + 2 \times 21)$

$= 462 \times 102$

$= 47,124 \text{ cm}^3$

Thus, volume of the solid is $47,124 \text{ cm}^3$.

Question 13:

If the slant height of a cone is 18.7 cm and the curved surface area is 602.8 cm^2, find the volume of cone. ($\pi = 3.14$)

Solution :

For the cone,
Slant height = l = 18.7 cm
C.S.A. = 602.8 cm^2
But , we know that,
C.S.A. of a cone = πrl

$\therefore 602.8 = 3.14 \times r \times 18.7$

$\therefore r = \frac{602.8}{3.14 \times 18.7}$

$= \frac{6028}{10} \times \frac{100}{314} \times \frac{10}{187}$

$\therefore r = 10.27$ cm (by logarithm)

In a cone, $l^2 = h^2 + r^2$

$\therefore h^2 = l^2 - r^2$

$= (18.7)^2 - (10.27)^2$

$= 349.69 - 105.4729$

$\therefore h^2 = 244.2171$

$\therefore h = 15.63$ cm (by logarithm)

Now,

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 10.27 \times 10.27 \times 15.63$

$= 1725.47 \text{ cm}^3$

Thus, volume of the cone is 1725.47 cm^3.

Question 14:

If the surface area of a spherical ball is 1256 cm^2, then find the volume of sphere. (Take π = 3.14)

Solution :

Let radius of the spherical ball be r cm
Surface area of the spherical ball = 1256 cm^2
Surface area of the sphere = $4\pi r^2$
$\therefore 1256 = 4\pi r^2$
$\therefore 1256 = 4 \times 3.14 \times r^2$
$\therefore r^2 = \dfrac{1256}{4 \times 3.14}$
$\therefore r^2 = 100$
$\therefore r = 10$ cm
Volume of sphere = $\dfrac{4}{3}\pi r^3$
$= \dfrac{4}{3} \times 3.14 \times 10 \times 10 \times 10$
$= 4186.67$ cm^3
$\therefore$ Volume of the spherical ball is 4186.67 cm^3.

Exercise-14.3

Question 1:

A hemispherical bowl of internal radius 12 cm contains some liquid. This liquid is to be filled into cylindrical bottles of diameter 4 cm and height 6 cm. How many bottles can be filled with this liquid ?

Solution :

The radius of the hemispherical bowl r = 12 cm,
Radius of each cylindrical bottle R = $\dfrac{d}{2} = \dfrac{4}{2} = 2$ cm
Height of each cylindrical bottle H = 6 cm
Assume, n bottles can be filled with the liquid in the hemispherical bowl.
$\therefore$ Volume of n cylindrical bottles = Volume of the hemispherical bowl
$\therefore n \times \pi R^2 H = \dfrac{2}{3}\pi r^3$
$\therefore n \times 2 \times 2 \times 6 = \dfrac{2}{3} \times 12 \times 12 \times 12$
$\therefore n = \dfrac{2}{3} \times \dfrac{12 \times 12 \times 12}{2 \times 2 \times 6}$
$\therefore n = 48$
$\therefore$ 48 bottles can be filled with the liquid in the hemispherical bowl.

Question 2:

A cylindrical container having diameter 16 cm and height 40 cm is full of ice-cream. The ice-cream is to be filled into cones of height 12 cm and diameter 4 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with the ice-cream.

Solution :

For the cylindrical container,

Radius $r = \frac{d}{2} = \frac{16}{2} = 8$ cm

Height h = 40 cm

Volume of ice-cream in the cylindrical container $= \pi r^2 h$

$= (\pi \times 8 \times 8 \times 40)$ cm^2

Now,

Radius of the conical part for each cone = Radius of the hemispherical top

$\therefore R = \frac{d}{2} = \frac{4}{2} = 2$ cm

Height of the conical part H = 12 cm.

Volume of ice-cream filled in one cone = Volume of the cone + Volume of the hemisphere

$= \frac{1}{3}\pi R^2 H + \frac{2}{3}\pi R^3$

$= \frac{1}{3}\pi R^2 (H + 2R)$

$= \frac{1}{3} \times \pi \times 2 \times 2(12 + 2 \times 2)$

$= \frac{4}{3} \times \pi \times 16$

$= \left(\frac{64}{3} \times \pi\right)$cm^3

Number of cones that can be filled = $\frac{\text{Volume of ice-cream in the container}}{\text{Volume of ice-cream filled in one cone}}$

$$= \frac{\pi \times 8 \times 8 \times 40}{\frac{64}{3} \times \pi}$$

$$= \frac{64 \times 40 \times 3}{64}$$

$= 120$

$\therefore$ 120 cones can be filled with the ice-cream.

Question 3:

A cylindrical tank of diameter 3 m and height 7 m is completely filled with groundnut oil. It is to be emptied in 15 tins each of capacity 15 litres. Find the number of such tins required.

Solution :

Radius of the cylindrical tank,

$r = \frac{d}{2} = \frac{3}{2}$ m and height h = 7 m

Volume of the groundnut oil in the cylindrical tank $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 7$$

$$= \frac{99}{2} \text{ m}^3$$

$$= \left(\frac{99}{2} \times 1000\right) \text{ litres}$$

$= 49,500$ litres

Number of tins filled with 15 litres of oil = 1

$\therefore$ Number of tins filled with 49,500 litres of oil $= \frac{49500}{15} = 3300$

$\therefore$ 3300 tins are required to hold the oil.

Question 4:

A cylinder of radius 2 cm and height 10 cm is melted into small spherical balls of diameter 1 cm. Find the number of such balls.

Solution :

Radius of the cylinder, $r = 2$ cm and height $h = 10$ cm.

For each spherical ball,

radius $R = \frac{\text{diameter}}{2} = \frac{1}{2}$ cm

Assume, n balls can be made by melting the cylinder,

$\therefore$ Volume of n balls = Volume of the cylinder

$\therefore n \times \frac{4}{3}\pi R^3 = \pi r^2 h$

$\therefore n \times \frac{4}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 2 \times 2 \times 10$

$\therefore n \times \frac{1}{6} = 40$

$\therefore n = 40 \times 6$

$\therefore n = 240$

Hence, 240 balls can be made.

Question 5:

A metallic sphere of radius 15 cm is melted and a wire of diameter 1 cm is drawn from it. Find the length of the wire.

Solution :

Radius of the metallic sphere, $r = 15$ cm.

For the cylindrical wire,

Radius $R = \frac{d}{2} = \frac{1}{2}$ cm and Height = H cm.

Further, the metallic sphere is melted to form a wire.

$\therefore$ Volume of the sphere = Volume of the wire

$\therefore \frac{4}{3}\pi r^3 = \pi R^2 H$

$\therefore \frac{4}{3} \times 15 \times 15 \times 15 = \frac{1}{2} \times \frac{1}{2} \times H$

$\therefore H = \frac{4}{3} \times 15 \times 15 \times 15 \times 2 \times 2$

$\therefore H = 18000$ cm

$\therefore H = 180$ m

$\therefore$ The length of the wire is 180 m.

Question 6:

There are 45 conical heaps of wheat, each of them having diameter 80 cm and height 30 cm. To store the wheat in a cylindrical container of the same radius, what will be the height of cylinder ?

Solution :

Radius of each conical heap

= Radius of cylindrical container

$\therefore r = \frac{d}{2} = \frac{80}{2} = 40$ cm

Height of each conical heap $h = 30$ cm

Height of cylindrical container = H cm.

Volume of 45 conical heaps of wheat = Volume of the cylindrical container

$\therefore 45 \times \frac{1}{3}\pi r^2 h = \pi r^2 h$

$\therefore 15 \times 40 \times 40 \times 30 = 40 \times 40 \times H$

$\therefore H = 450$ cm

$\therefore$ Thus, height of the cylindrical container is 450 cm.

Question 7:

A cylindrical bucket, 44 cm high and having radius of base 21 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 33 cm, find the radius and the slant height of the heap.

Solution :

For the cylindrical bucket,
Radius r = 21 cm and height h = 44 cm.
For the conical heap,
Height H = 33 cm and radius R cm.
Volume of sand in the conical heap = Volume of sand in the cylindrical bucket

$\therefore \frac{1}{3}\pi R^2 H = \pi r^2 h$

$\therefore \frac{1}{3} \times R^2 \times 33 = 21 \times 21 \times 44$

$\therefore R^2 = \frac{21 \times 21 \times 44 \times 3}{33}$

$\therefore R^2 = 1764$

$\therefore R = 42$ cm

For a conical heap, if l is the slant height, then

$l^2 = R^2 + H^2$

$\therefore l = \sqrt{R^2 + H^2}$

$= \sqrt{(42)^2 + (33)^2}$

$= \sqrt{1764 + 1089}$

$= \sqrt{2853}$

$= 53.41$ cm

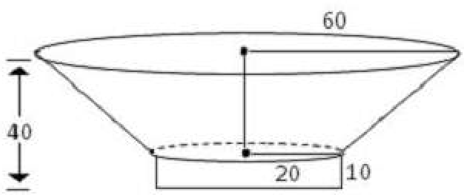
$\therefore$ The radius of the conical heap is 42 cm and its slant height is 53.41 cm.

Exercise-14.4

Question 1:

A metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metalic sheet. The total vertical height of the bucket is 40 cm and that of cylindrical base is 10 cm, radii of two circular ends are 60 cm and 20 cm. Find the area of the metalic sheet used. Also find the volume of water the bucket can hold. ($\pi = 3.14$)

Solution :



For the cylindrical portion of the bucket,
Radius R = 20 cm and height H = 10 cm.
For the frustum of a cone,
Bigger radius $r_1 = 60$ cm,
Smaller radius $r_2 = 20$ cm
Height h = Total height of the bucket − Height of the cylindrical portion
$= 40 - 10$
$= 30$ cm

Let l be the slant height of the cone.

We know that, $l = \sqrt{h^2+(r_1 - r_2)^2}$

$= \sqrt{30^2 + (60 - 20)^2}$

$= \sqrt{900 + 1600} = \sqrt{2500} = 50$

$\therefore$ Slant height of the frustum of a cone l = 50 cm

Area of the metal sheet used to make the bucket

= C.S.A. of the cylinder + C.S.A. of the frustum of a cone

+ Area of base of the frustum

$= 2\pi RH + \pi l(r_1+r_2) + \pi r_2^2$

$= \pi\left(2RH + l(r_1+r_2) + r_2^2\right)$

$= 3.14\left(2 \times 20 \times 10 + 50(60+20) + (20)^2\right)$

$= 3.14(400 + 4000 + 400)$

$= 3.14 \times 4800$

$= 15,072 \text{ cm}^2$

Volume of the water the bucket can hold

= Volume of the frustum of a cone

$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times 3.14 \times 30\left(60^2 + 20^2 + 60 \times 20\right)$

$= 31.4 \times 5200$

$= 163280 \text{ cm}^3$

$= \frac{163280}{1000}$ litres

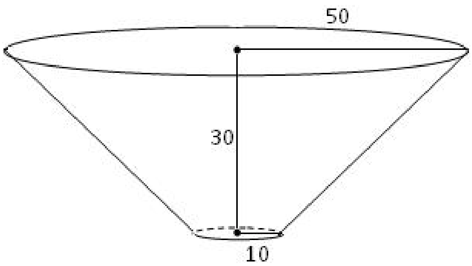
= 163.280 litres

$\therefore$ The area of the metal sheet used to make the bucket is 15,072 cm^2 and the volume of water the bucket can hold is 163.280 litres.

Question 2:

A container, open from the top and made up of a metal sheet is the form of frustum of a cone of height 30 cm with radii 30 cm and 10 cm. Find the cost of the milk which can completely fill container at the rate of ₹ 30 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 50 per 100 cm^2. ($\pi = 3.14$)

Solution :



For the container in the shape of the frustum of a cone,

Bigger radius r_1 = 50 cm

Smaller radius r_2 = 10 cm

Height h = 30 cm

Now, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(30)^2 + (50 - 10)^2}$

$= \sqrt{900 + 1600}$

$= \sqrt{2500}$

= 50 cm

Volume of the container in the shape of frustum of a cone

$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times 3.14 \times 30\left(50^2 + 10^2 + 50 \times 10\right)$

$= \frac{1}{3} \times 3.14 \times 30(2500 + 100 + 500)$

$= 31.4 \times 3100$

$= 97340\ cm^2$

$= \frac{97340}{1000}$ litres

$= 97.34$ litres

$\therefore$ 97.34 litres of milk will fill the container completely.

Cost of 1 litre of milk = Rs. 30

$\therefore$ Cost of 97.34 litres of milk = Rs. (97.34×30) = Rs. 2920.20

Hence, the total cost of milk to fill the container completely is Rs. 2920.20.

Area of the metal sheet used to make the container

= C.S.A. of the frustum of a cone + Area of the base

$= \pi l\left(r_1 + r_2\right) + \pi r_2^2$

$= 3.14 \times 50(50 + 10) + 3.14 \times 10^2$

Next,

Cost of 100 cm^2 metal sheet = Rs. 50

$\therefore$ Cost of 9734 cm^2 metal sheet = Rs. $\left(\frac{9734 \times 50}{100}\right)$

= Rs. 4867

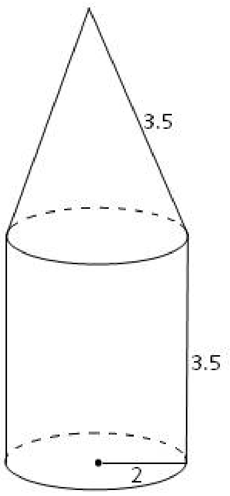
$\therefore$ The cost of metal sheet used to make the container is Rs 4867.

Exercise-14

Question 1:

A tent is in the shape of cylinder surmounted by a conical top. If the height and the radius of the cylindrical part are 3.5 m and 2 m respectively and the slant height of the top is 3.5 m, find the area of the canvas used for making the tent. Also find the cost of canvas of the tent at the rate of ₹ 1000 per m^2.

Solution :



Radius of the cylindrical part

= Radius of the conical part

$\therefore$ r = 2 m

Height of the cylindrical part h = 3.5 m and

Slant height of the conical part l = 3.5 m

Area of the canvas used for making the tent

= C.S.A. of cylindrical part + C.S.A. of conical part

$= 2\pi rh + \pi rl$

$= \pi r(2h + l)$

$= \frac{22}{7} \times 2(2 \times 3.5 \times 3.5)$

$= \frac{44}{4} \times \frac{105}{10}$

$= 66\ m^2$

Next,

Cost of 1 m^2 canvas = Rs. 1000

$\therefore$ Cost of 66 m^2 canvas = Rs. (66×1000)

= Rs. 66,000

$\therefore$ The area covered by the canvas tent is 66 m^2 and its cost is Rs. 66,000.

Question 2:

A metallic sphere of radius 5.6 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution :

Radius of the metallic sphere r = 5.6 cm

Radius of the cylinder R = 6 cm

Height of the cylinder = H cm

But, the sphere is melted and recast into the shape of a cylinder.

$\therefore \frac{4}{3}\pi r^3 = \pi R^2 H$

$\therefore \frac{4}{3}r^3 = R^2 H$

$\therefore \frac{4}{3} \times 5.6 \times 5.6 \times 5.6 = 6 \times 6 \times H$

$\therefore H = \frac{4 \times 5.6 \times 5.6 \times 5.6}{3 \times 6 \times 6}$

$\therefore$ H = 6.5 cm

$\therefore$ The height of the cylinder is 6.5 cm.

Question 3:

How many spherical balls of radius 2 cm can be made out of a solid cube of lead whose side measures 44 cm ?

Solution :

Length of the cube l = 44 cm

Radius of each spherical ball = 2 cm

Assume, n spherical balls can be made from a solid cube of lead.

$\therefore$ Volume of n balls = Volume of cube

$$\therefore n \times \frac{4}{3}\pi r^3 = l^3$$

$$\therefore n \times \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 = 44 \times 44 \times 44$$

$$\therefore n = \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 2 \times 2 \times 2}$$

$\therefore n = 2541$

$\therefore$ 2541 spherical balls can be made out of the solid cube.

Question 4:

A hemispherical bowl of internal radius 18 cm contains an edible oil to be filled in cylindrical bottles of radius 3 cm and height 9 cm. How many bottles are required to empty the bowl ?

Solution :

Radius of hemispherical bowl R = 18 cm

Radius of cylindrical bottle r = 3 cm

Height of the bottle h = 9 cm

Assume, n bottles are required to empty the bowl,

$\therefore$ Volume of n bottles = Volume of the bowl

$$\therefore n \times \pi r^2 h = \frac{2}{3}\pi R^3$$

$$\therefore n \times r^2 h = \frac{2}{3} R^3$$

$$\therefore n \times 3 \times 3 \times 9 = \frac{2}{3} \times 18 \times 18 \times 18$$

$$\therefore n = \frac{2 \times 18 \times 18 \times 18}{3 \times 3 \times 3 \times 9}$$

$\therefore n = 48$

$\therefore$ 48 bottles are required to empty the bowl.

Question 5:

A hemispherical tank of radius 2.4 m is full of water. It is connected with a pipe which empties it at the rate of 7 litres per second. How much time will it take to empty the tank compeletely ?

Solution :

Radius of the hemisherical tank r = 2.4 m

Volume of the water in the hemispherical tank

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{24}{10} \times \frac{24}{10} \times \frac{24}{10}$

$= \frac{44 \times 8 \times 576}{7 \times 1000} m^3$

$= \left(\frac{44 \times 8 \times 576}{7 \times 1000} \times 1000\right)$ litres

$= \frac{44 \times 8 \times 576}{7}$ litres

$= 28964.57$ litres

Time required to empty 7 litres of water = 1 second

$\therefore$ Time required to empty 28964.57 litres of water

$= \frac{28964.57}{7}$ seconds

$= \frac{28964.57}{7 \times 60}$ minutes

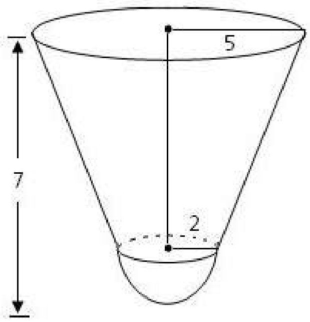
$= 68.96$ minutes

$\therefore$ It will take 68.96 minutes to empty the tank.

Question 6:

A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameter of the frustum are 5 cm and 2 cm. The height of the entire shuttle cock is 7 cm. Find its external surface area.

Solution :



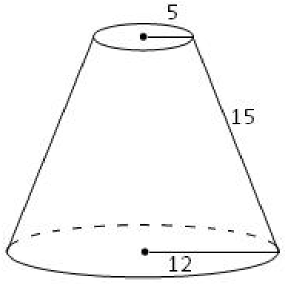
Let r_1 and r_2 be the larger and smaller radius of the frustum of a cone respectively.
Radius of the hemisphere = smaller radius of the frustum of a cone
$\therefore\ r = r_2 = 2$ cm
Bigger radius for the frustum of a cone, $r_1 = 5$ m and
Also,
Height h = total height of the shuttle cock − radius of the hemisphere
$= 7 - 2 = 5$ cm.

But, $l = \sqrt{h^2 + (r_1 - r_2)^2}$
$= \sqrt{5^2 + (5-2)^2}$
$= \sqrt{25 + 9}$
$= \sqrt{34}$
$\therefore\ l = 5.83$ cm
$\therefore$ Slant height of the frustum of a cone $l = 5.83$ cm.
Now,
External surface area of the shuttle cock
= C.S.A. of the frustum of a cone + C.S.A. of the hemisphere
$= \pi l(r_1 + r_2) + 2\pi r^2$
$= \pi\left[l(r_1 + r_2) + 2r^2\right]$
$= \frac{22}{7}\left[5.83(5 + 2) + 2 \times 4\right]$
$= \frac{22}{7}(5.83 \times 7 + 8)$
$= \frac{22}{7} \times 48.81$
$= 153.40\ cm^2$
$\therefore$ The external surface area of the shuttle cock is $153.40\ cm^2$.

Question 7:

A fez, the headgear cap used by the trucks is shaped like the frustum of a cone. If its radius on the open side is 12 cm and radius at the upper base is 5 cm and its slant height is 15 cm, find the area of material used for making it. (π = 3.14)

Solution :

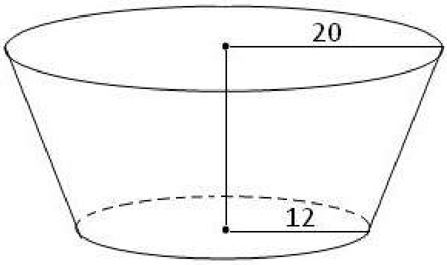


For the fez in the shape of the frustum of a cone,
Bigger radius $r_1 = 12$ cm
Smaller radius $r_2 = 5$ cm
Slant height $l = 15$ cm.
Assume that the top of the fez is made of some different material.
$\therefore$ Area of the cloth used for making the fez
= C.S.A. of the frutum of a cone
$= \pi l(r_1 + r_2)$
$= 3.14 \times 15(12 + 5)$
$= 3.14 \times 15 \times 17$
$= 800.7\ cm^2$
$\therefore$ The area of the cloth used to make the fez is $800.7\ cm^2$.

Question 8:

A bucket is in the form of a frustum of a cone with capacity of 12308.8 cm^3 of water. The radius of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of bucket and the cost of making it at the rate of ₹ 10 per cm^2.

Solution :



For the bucket in the form of the frustum of a cone,

Bigger radius r_1 = 20 cm

Smaller radius r_2 = 12 cm

Capacity of the bucket = 12,308.8 cm^3.

Capacity of the bucket = Volume of the frustum of a cone

$\therefore 12308.8 = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1r_2\right)$

$\therefore 12308.8 = \frac{1}{3} \times \frac{22}{7} \times h\left(20^2 + 12^2 + 20 \times 12\right)$

$\therefore 12308.8 = \frac{22}{21} \times h \times 784$

$\therefore h = \frac{12308.8 \times 21}{22 \times 784} = 15(\text{approx})$

$\therefore h = 15$ cm

We know that, $l = \sqrt{h^2 + (r^1 - r^2)^2}$

$= \sqrt{15^2 + (20-12)^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289}$

$\therefore l = 17$ cm

$\therefore$ The slant height of the frustum of a cone is 17 cm.

We don't consider the area of the base while calculating the cost of making the bucket.

C.S.A. of the bucket $= \pi l(r_1 + r_2)$

$= \frac{22}{7} \times 17\,(20 + 12)$

$= \frac{22}{7} \times 17 \times 32$

$= 1709.7\ cm^2$

Cost of making 1 cm^2 = Rs. 10

$\therefore$ Cost of making for 1709.7 cm^2 = Rs. (1709.7×10)

= Rs. 17097

$\therefore$ Height of the bucket is 15 cm and the cost of making it is Rs. 17,097.

Question 9:

Select a proper option (a), (b), (c) or (d) from given option :

Question 9(1):

The volume of sphere with diameter 1 cm is ……. cm^3.

Solution :

b. $\frac{1}{6}\pi$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \qquad \left(\because r = \frac{d}{2}\right)$

$= \frac{1}{6}\pi \text{ cm}^3$

Question 9(2):

The volume of hemisphere with radius 1.2 cm is …… cm^3.

Solution :

a. 1.152π

Volume of a hemisphere

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 1.2 \times 1.2 \times 1.2$

$= 1.152\pi \text{ cm}^3$

Question 9(3):

The volume of sphere is $\frac{4}{3}$ It cm^3. Then its diameter is …… cm .

Solution :

c. 2

Solution:

Volume of a sphere $= \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi = \frac{4}{3}\pi r^3$

$\therefore r^3 = 1$

$\therefore r = 1 \text{ cm}$

Diameter $= 2r = 2 \times 1 = 2$ cm

Question 9(4):

The volume of cone with radius 2 cm and height 6 cm is ……. cm^3.

Solution :

a. 8π

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 2 \times 2 \times 6$

$= 8\pi \text{ cm}^3$

Question 9(5):

The diameter of the base of cone is 10 cm and its slant height is 17 cm. Then the curved surface area of the cone is ……. cm^2.

Solution :

a. 85π

$$\text{Radius of the cone } r = \frac{d}{2} = \frac{10}{2} = 5 \text{ cm}$$

$$\text{C.S.A. of a cone } = \pi rl$$
$$= \pi \times 5 \times 17$$
$$= 85\pi \text{ cm}^2$$

Question 9(6):

The diameter and the height of the cylinder are 14 cm and 10 cm respectively. Then total surface area is ………. cm^2.

Solution :

c. 748

$$\text{Radius of the cylinder } r = \frac{d}{2} = \frac{14}{2} = 7 \text{ cm}$$

$$\text{T.S.A. of a cylinder } = 2\pi r(h + r)$$
$$= 2 \times \frac{22}{7} \times 7(10 + 7)$$
$$= 44 \times 17$$
$$= 748 \text{ cm}^2$$

Question 9(7):

The ratio of the radii of two cones having equal height is 2 : 3. Then, the ratio of their volumes is……..

Solution :

d. 4 : 9

Height of both the cones $= h$

Let r_1 and r_2 be radii of the two cones respectively.

Here $r_1 = 2x$ and $r_2 = 3x$

$$\text{Ratio of volumes of cones} = \frac{\text{Volume of the first cone}}{\text{Volume of the second cone}}$$
$$= \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h}$$
$$= \frac{(2x)^2}{(3x)^2}$$
$$= \frac{4x^2}{9x^2}$$
$$= \frac{4}{9}$$
$$= 4 : 9$$

Question 9(8):

If the radii of a frustum of a cone are 7 cm and 3 cm and the height is 3 cm, then the curved surface area is …… cm_2.

Solution :

a. 50π

The radii of a frustum of a cone are r_1 and r_2 respectively.

Here $r_1 = 7$ cm and $r_2 = 3$ cm and $h = 3$ cm.

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{3^2 + (7-3)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$\therefore l = 5$ cm

C.S.A. of the frustum of a cone

$$= \pi l (r_1 + r_2)$$

$$= \pi \times 5(7+3)$$

$$= 50\pi \text{ cm}^2$$

Question 9(9):

The radii of a frustum of a cone are 5 cm and 9 cm and height is 6 cm, then the volume is ……. cm^3.

Solution :

c. 320π

The radii of a frustum of a cone are r_1 and r_2 respectively.

Here $r_1 = 9$ cm, $r_2 = 5$ cm and $h = 6$ cm

$$\text{Volume of the frustum of a cone} = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3} \times \pi \times 6(81 + 25 + 45)$$

$$= 2\pi(151)$$

$$= 302\pi \text{ cm}^3$$