CBSE Test Paper 03

CH-07 Permutations & Combinations

1. The number of all possible positive integral solutions of the equation xyz = 30 is
   a. 25
   b. none of these
   c. 27
   d. 26
2. 5 persons board a lift on the ground floor of an 8 storey building. In how many ways can they leave the lift?
   a. ${}^7P_5$
   b. none of these
   c. 5^7
   d. 7^5
3. The sum of all the numbers which can be formed by using the digits 1 , 3 , 5 , 7 ,9 all at a time and which have no digit repeated is
   a. 33333000
   b. 266664
   c. 600
   d. 666660
4. The number of five digit telephone numbers having atleast one of their digits repeated is
   a. 30240
   b. 69760
   c. 90000
   d. 66500
5. The number of diagonals that can be drawn by joining the vertices of an octagon is :
   a. 12
   b. 20
   c. 28
   d. 48
6. Fill in the blanks:

The number of words which can be formed out of letters of the word ARTICLE, so that the vowels occupy the even place is ________.

7. Fill in the blanks:

   If 2P(5, 3) = P(n, 4), then the value of n is ________.

8. Find the number of chords that can be drawn through 16 points on a circle.
9. How many chords can be drawn through 21 points on a circle?
10. In how many ways, can 3 people be seated in a row containing 7 seats?
11. In how many ways 9 pictures can be hung from 6 picture nails on a wall?
12. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?
13. Determine n if
    (i) ${}^{2n}C_3 : {}^{n}C_2 = 12 : 1$
    (ii) ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$
14. In how many different ways, the letters of the word 'ALGEBRA' can be arranged in a row if

    i. the two A's are together?
    ii. the two A's are not together?

15. A group consist of 4 girls and 7 boys. In how many ways, a team of 5 members be selected, if the team has

    i. no girl?
    ii. at least one boy and one girl?
    iii. at least 3 girls?

Solution

1. (c) 27

   Explanation: Given xyz=30

   We have the possible values of x ,y,z are the following triads

   1,1,30

   1,2,15

   1,3,10

   1,5,6

   2,3,5

   First one can have 3!/2! = 3 ways and the remaining four triads can have 3! combinations

   Hence total combinations = 3 + 4*3! = 27
2. (d) 7^5

   Explanation:

   Since they are boarding from ground floor and we are considering the number of ways they leave the lift ,we can consider there are 7 floor sas we exclude the ground floor)

   As each of the 5 persons can leave the lift in 7 ways, required number of ways= 7^5

3. (d) 666660

   Explanation:

   First we will fix any one digit in a fixed position .Then we have the remaining 4 digits can be arranged in 4! different ways.

   Which means each of the five digits can appear in each of the five places in 4! times.

   Hence the sum of the digits in each position is $4!(1+3+5+7+9)=25\times4!$Now to find the sum of these numbers formed we have to consider the place values for these digits

   So the sum of all the numbers which can be formed by using the digits 1 , 3 , 5 , 7 ,9=

$25 \times 4! \times (1 + 10 + 100 + 1000 + 10000) = 25 \times 4! \times 11111 = 666660$

4. (b) 69760 Explanation:

   As we have the ten digits from 0 to 9 which can be used

   Number of five digit telephone numbers in which the digits can be repeated= $10^5 = 100000$

   Number of five digit telephone numbers in which the digits cannot be repeated= ${}^{10}P_5 = \frac{10!}{(10-5)!} = 6 \times 7 \times 8 \times 9 \times 10 = 30240$

   Therefore the number of five digit telephone numbers in which atleast one digit is repeated=$100000 - 30240 = 69760$

5. (b) 20 Explanation:

   We have octagon is an eight sided polygon which has 8 vertices.

   A diagonal is obtained by joining two points .

   Thus the number of diagonals obtained by joining any two points out of 8 is given by

   $8C_2 \quad -8 = \frac{8!}{2!(8-2)!} - 8 = \frac{1\times2\times3\times4\times5\times6\times7\times8}{1\times2\times1\times2\times3\times4\times5\times6} - 8 = \frac{7\times8}{1\times2} - 8 = 28 - 8 = 20$

6. 144

7. 5

8. Since, the points lies on the circumference of the circle. So, no three of them are collinear.

   Thus, number of chords formed by 16 points by taking 2 at time = ${}^{16}C_2$

   $= \frac{16!}{2!14!} = \frac{16\times15}{2\times1}$ = 120

9. There are 21 points on the circumference on a circle. Since one and only one chord can be drawn by joining 2 distinct points, the required number of chords is given by $21_{C_2} = \frac{21!}{2!19!} = \frac{21\times20\times19!}{2\times19!} = 210$

10. The first person can be seated in 7 ways. When the first person has taken his seat,

then the number of seats for the second person =7 - 1 = 6

The second person can be seated in 6 ways and the third person can be seated in 3 (or 6 - 1) ways.

$\therefore$ By fundamental principle of multiplication, the total number of ways in which three persons can be seated in a row containing seven seats $= 7 \times 6 \times 5$ = 210 ways.

11. Required number of ways $={}^9P_6$
$= \frac{9!}{(9-6)!} = \frac{9!}{3!}$
$= 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 60480$

12. There are 5 boys and 4 girls. We have to select 3 out of 5 boys and 3 out of 4 girls.
$\therefore$ Number of ways of selection $={}^5C_3 \times {}^4C_3$
$= \frac{5!}{3!2!} \times \frac{4!}{3!2!} = 10 \times 4 = 40$

13. Here ${}^{2n}C_3 : {}^nC_2 = 12 : 1$
$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{2!(n-2)!}{n!} = \frac{12}{1}$
$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3 \times 2!(2n-3)!} \times \frac{2!(n-2)!}{n(n-1)(n-2)!} = \frac{12}{1}$
$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{12}{1}.$
$\Rightarrow \frac{4(2n-1)}{3} = \frac{12}{1} \Rightarrow 8n - 4 = 36 \Rightarrow n = 5$
(ii) Here ${}^{2n}C_3 : {}^nC_3 = 11 : 1$
$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{11}{1}$
$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$
$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$
$\Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1} \Rightarrow 8n - 4 = 11n - 22$
$\Rightarrow$ 3n = 18 $\Rightarrow$ n = 6

14. Clearly, ALGEBRA has seven letters, where two A’s, one L, one G, one E, one B, one R.

    i. Since, two A’s are always together. We take both the A’s as one letter.
    If p is the number of arrangements, then
    p $= 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$ = 720
    ii. Total number of arrangements of given word
    q $= \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2!}$

$= 7 \times 6 \times 5 \times 4 \times 3$ = 2520

In these arrangements, some arrangements have two A's are together while in the rest they are not together.

Hence, the number of arrangements in which two A's are not together is q - p = 2520 - 720 = 1800

15. Given, number of boys = 7, number of girls = 4 and members in team = 5

    i. If S_1 is the number of combinations in this case, then

       $S_1 = {}^7C_5 = \frac{7!}{5!2!} \Rightarrow S_1 = \frac{7 \times 6 \times 5!}{5! \times 2 \times 1}$ = 21 ways

    ii. Here, the following are the ways of selecting the team

       a. 4 boys and 1 girl

          This can be done in ${}^7C_4 \times {}^4C_1$ ways.

       b. 3 boys and 2 girls

          This can be done in ${}^7C_3 \times {}^4C_2$ ways.

       c. 2 boys and 3 girls

          This can be done in ${}^7C_2 \times {}^4C_3$ ways.

       d. 1 boy and 4 girls

          This can be done in ${}^7C_1 \times {}^4C_4$

          The total number of ways of selecting atleast one boy and one girl

          $= {}^7C_4 \times {}^4C_1 + {}^7C_3 \times {}^4C_2 + {}^7C_2 \times {}^4C_3 + {}^7C_1 \times {}^4C_4$

          $= \frac{7!}{4!3!} \times \frac{4!}{1!3!} + \frac{7!}{3!4!} \times \frac{4!}{2!2!} + \frac{7!}{2!5!} \times \frac{4!}{3!!!} + \frac{7!}{1!6!} \times \frac{4!}{4!0!}$

          $= 35 \times 4 + 35 \times 6 + 21 \times 4 + 7$

          = 140 + 210 + 84 + 7 = 441 ways

    iii. Here, the team can be formed in the following ways

       a. 3 girls and 2 boys

       b. 4 girls and 1 boy

          So, required number of combination

          $= {}^7C_2 \times {}^4C_3 + {}^7C_1 \times {}^4C_4$

          $= \frac{7!}{2!5!} \times \frac{4!}{3!1!} + \frac{7!}{1!6!} \times \frac{4!}{4!0!}$

          $= \frac{7 \times 6 \times 5!}{2 \times 1 \times 5!} \times \frac{4 \times 3!}{3!} + \frac{7 \times 6!}{6!} \times 1$

          $= 21 \times 4$ + 7 = 84 + 7 = 91 ways