# 6. PERMUTATIONS AND COMBINATIONS

***EXERCISE 6.1***

**1) A teacher wants to select the class monitor in a class of 30 boys and 20 girls. In how many ways can he select a student if the monitor can be a boy or a girl?**

**Sol:** There are 30 boys and 20 girls in the class.

∴ Total number of students in the class = 50
Now, one class monitor is to be selected.
Since, the monitor can be a boy or a girl, any one of the 50 students can be the monitor.

∴ Number of ways to select the monitor = 50

**(2) In question (1), in how many ways can the monitor be selected if the monitor must be a boy? What is the answer if the monitor must be a girl?**

**Sol:** Since teacher wants to select class monitor which must be a boy and there are
30 boys in a class.

∴ Total number of ways of selecting boy monitor

= 30 ways.
Since teacher wants to select class monitor which must be a girl and there are 20 girls in a class.

∴ Total number .of ways of selecting girl monitor

= 20 ways.

**3) A Signal is generated from 2 flags by putting one flag above the other. If 4 flags of different colours are available, how many different signals can be generated?**

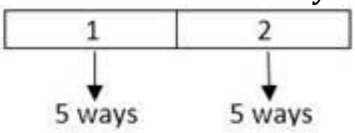
**Sol:** Number of flags available = 4
A signal is generated from 2 flags one above the other.
Number of ways of arranging lower flag = 4

∴ Number of ways of arranging upper flag = 3
Therefore, number of ways of generating different signals = 4 X 3 = 12 ways.

**(4) How many two letter words can be formed using letters from the word**

**SPACE, when repetition of letters (i) is allowed (ii) is not allowed?**

**Sol:** The given word is SPACE, which has five letters.
Using these five letters, two letter words are to be formed.
**(i)** If repetition of letters is allowed, we have two positions of two letters as follows:
Each position can be arranged in 5 ways. So, the number of two letter words formed = 5x5=25ways.




Each position can be arranged in 5 ways.
So, the number of two letter words formed
$= 5 \times 5 = 25$ ways.

**(ii)** Since repetition of letters is not allowed, the first position can be arranged using any one of the
5 letters in 5 ways.
The second position can be arranged in 4 ways using remaining 4 letters.

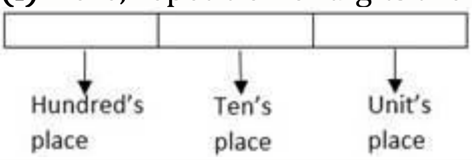

By the principle of multiplication, this can be done in 5 X 4 = 20 ways.

**(5) How many three-digit numbers can be formed from the digits 0, l, 3, 5, 6 if repetitions of digits**

**(i) Are allowed (ii) are not allowed?**

**Sol.** We have to form three digit numbers using the 3 digits 0, 1, 3, 5, 6. 1

**(I)** Here, repetition of digits are allowed.



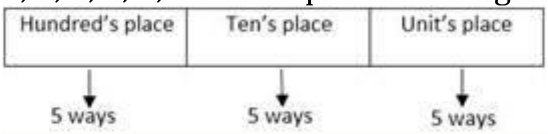

Zero cannot be in hundred's place.
The hundred's place can be filled 1n 4 ways by 4 non-zero digits.
Since repetition is allowed, the ten's place and unit's place can be filled in by all 5 digits in 5 ways.
Hence, all three places can be arranged in $4 \times 5 \times 5 = 100$ Ways.
Thus, 100 three digit numbers can be formed.

**(II)** Here, repetition of digits 1s not allowed.
As before, the hundred's place can be filled in 4 ways by 4 nonzero digits.

After filling in the hundred's place, we now have 4 digits left to fill in the ten's place.
This can be done in 4 ways.
For unit's place 3 digits are available.
Hence, three places can be arranged in $4 \times 4 \times 3 = 48$ ways.
Thus, 48 three digit numbers can be formed.

**6) How many three digit numbers can be formed using the digits 2, 3, 4, 5, 6 if digits can be repeated?**

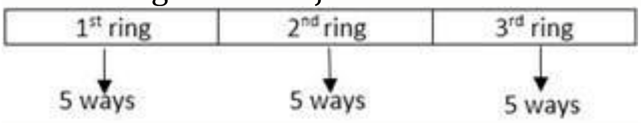
**Sol:** We have to form 3 digit numbers using the digits
2, 3, 4, 5, 6, where repetition of digits is allowed.




Here, all the places can be filled in 5 ways each.
Hence, the three places can be arranged in $5 \times 5 \times 5 = 125$ ways.
Thus, in all, 125 three digit numbers can be formed.

**7) A letter lock has 3 rings and each ring has 5 letters. Determine the maximum number of trials that may be required to open the lock.**

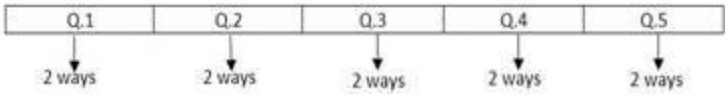
**Sol:** There are three rings in the lock and each ring has 5 letters.
∴ Each ring can be adjusted in' 5 different ways.




∴ By the principle of multiplication, the 3 rings can be arranged in $5 \times 5 \times 5 = 125$ ways.
Thus, in all 125 maximum number of trials are required to open the lock.

**8) In a test that has 5 true/false questions, no student has got all correct answers and no sequence of answers is repeated. What is the maximum number of students for this to be possible?**

Sol: There are 5 questions whose answers are either true or false.
Since each question has 2 answers, in all $2 \times 2 \times 2$ X $2 \times 2 = 25 = 32$ answers are there.

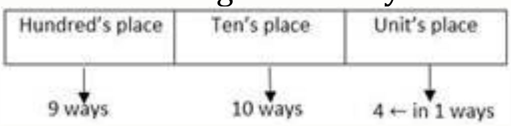



Since, no student has got all answers correct. Also, the answer of every student is different.
This is possible only if the number of students appeared for the test are 31.

**9) How many numbers between 100 and 1000 have 4 in the unit's place?**

Sol. The numbers between 100 and 1000 have 3-digits each. Here, we need to find 3 digit numbers which have 4 in the unit's place.

It can be arranged in 1 way.




Zero cannot be in hundred's place. Also, hundred's place can be arranged by any non-zero digit from 1 to 9 in nine ways. The ten's place can be arranged using any one digit from 0 to 9 in ten ways.

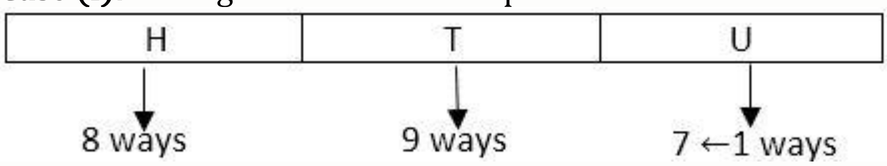
∴ The required numbers can be formed in $9 \times 10 \times 1 = 90$ ways.

∴ There are 90 numbers between 100 and 1000 that have '4' in the units place.

**10) How many numbers between 100 and 1000 have the digit 7 exactly once?**

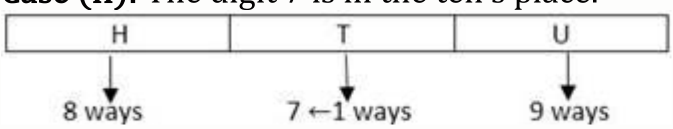
**Sol:** A number between 100 and 1000 has 3 digits.
So, we have to find three digit numbers having the digit 7 exactly once.
Let us consider the three cases separately.

**Case (I):** the digit 7 is in the unit's place.



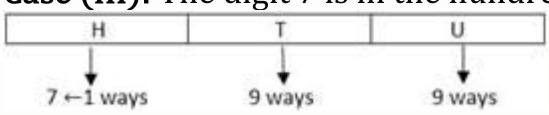

The ten's place is filled by one digit from 0 to excluding 7 in 9 ways.
Here, there are $8 \times 9 \times 1 = 72$ three digit numbers with required condition.

**Case (II):** The digit 7 is in the ten's place.




Unit's place can be filled by digit from 0 to 9 excluding 7 in 9 ways.
Zero is not allowed at hundred's place.
Hundred's place can be filled by digit from 1 to 9 excluding 7 in 8 ways.
The hundred's place can be filled in by any digit from 1 to 9 excluding 7 in 8 ways.
Here, there will be 8 X 1 X 9 9: 72 three digit numbers with required condition

**Case (III):** The digit 7 is in the hundred's place.




Then, there are $1 \times 9 \times 9 = 81$ three digit numbers with required condition.

Hence, the numbers between 100 and 1000 having the digit 7 exactly once are 72 + 72 + 81 = 225.

**11) How many four digit numbers Will not exceed 7432 if they are formed using the digits 2, 3, 4, 7 without repetition?**

**Sol:** Four digit number is to be formed using the digits 2, 3, 4 and 7 without repetition.
However, the 4-digit number should not exceed the number 7432.
The highest 4-digit number than can be formed using the digits 2, 3, 4, and 7 is 7432.
(i.e.) All 4-digit numbers formed using the digits 2, 3, 4 and 7 Will be less than or equal to 7432.
Starting with the thousands place, 4 digits are available for this place.
Since, repetition is not allowed, for the hundreds place, 3 digits are available.
Similarly, 2 digits will be available for the tens place and 1 digit for the units place.

| | | | |
|---|---|---|---|
| 4 | 3 | 2 | 1 |

∴ Number of 4-digit numbers that can be formed:

$= 4 \times 3 \times 2 \times 1$

$= 24$

**12) If numbers are formed using digits 2, 3, 4, 5, 6 without repetition, how many of them will exceed 400?**

**Sol:** We are required to form numbers exceeding 400 using the digits 2, 3, 4, 5, 6 without repetition
Such numbers can have either 3 digits, 4 digits or 5 digits.

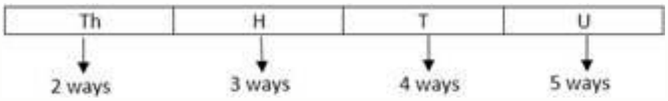
**Case (I):** The number has 3 digits.

| H | T | U |
|---|---|---|
| ↓ | ↓ | ↓ |
| 3 ways | 4 ways | 3 ways |

(Digit 4, 5 or 6)
Since the number has to be greater than 400 the hundred's place can 'be filled in 3 ways with digits 4, 5 or 6 and the ten's place in 4 ways by the remaining 4 digits

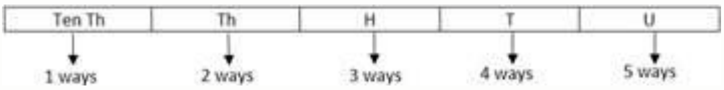
and units place in 3 ways.
Hence, the numbers formed = $3 \times 4 \times 3 = 36$

**Case (II):** The number has 4 digits.




The unit's place can be filled in 5 ways, ten's place in 4 ways, hundred's place in 3 ways and f thousand's place in 2 ways.
Hence, the numbers formed = $2 \times 3 \times 4 \times 5 = 120$

**Case (III):** The number has 5 digits.



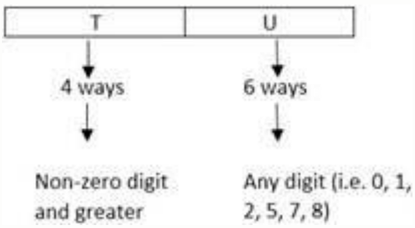

The units place can be filled is 5 ways, ten's place in 4 ways, hundred's place in 3 ways, thousand's place in 2 ways and ten thousand's place in 1 way.
Hence, the numbers formed = $2 \times 3 \times 4 \times 5 = 120$
From Case (I), Case (II) and Case (III),
the total numbers exceeding 400 are
$36 + 120 + 120 = 276$.

**13) How many numbers formed with digits 0, 1, 2, 5, 7, 8 will fall between 13 and 1000 if digits can be repeated?**

**Sol:** We need to find numbers between 13 and 1000
using the digits 0, 1, 2, 5, 7, 8 where repetition is allowed:
A number falling between 13 and 1000 will be either a 2 digit number or a 3 digit number.

**Case (I):** The number has 2 digits.

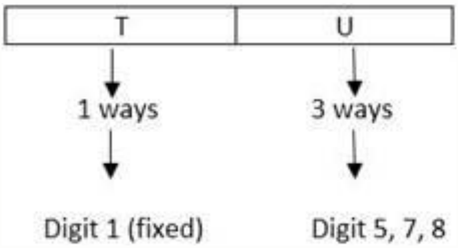
**(a)**




The ten's place can be filled by any non-zero digit, greater than 1 in 4 ways. The unit's place can be filled in 6 ways.

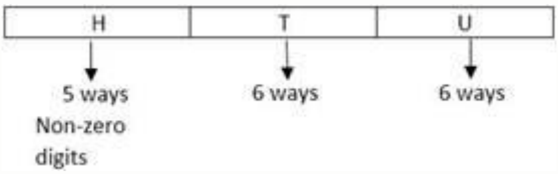
∴ Numbers formed $= 4 \times 6 = 24$

(b)




The ten's place is filled in 1 way by the digit 1, the unit's place is filled in 3 ways by the digits 5 or 7 or 8.
Hence, the numbers formed $= 1 \times 3 = 3$

**Case (II):** The number has three digits.



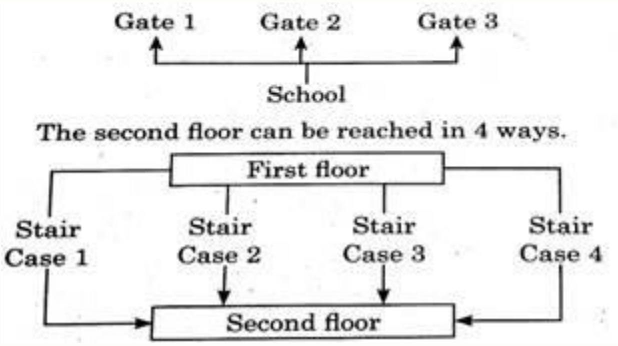

As before, the hundred's place, ten's place and unit's place can be filled in 5 ways, 6
ways and 6 ways respectively.

∴ Number of 3 digit numbers formed

$5 \times 6 \times 6 = 180$
Total numbers which fall between 13 and 100 are $24 + 3 + 180 = 207$

**14) A school has three gates and four staircases from the first floor to the second floor. How many ways does a student have to go from outside the school to his classroom on the second floor?**

**Sol:** There are three ways of entering the school.

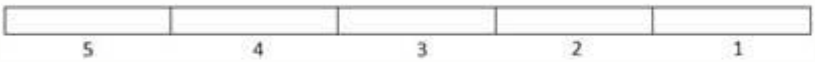



*So, a student can go from outside the school to a classroom on the second floor in $3 \times 4 = 12$ ways.

**(15) How many five-digit numbers formed using the digit 0, 1, 2, 3, 4, 5 are divisible by 3 if digits are not repeated?**

**Sol:** Five-digits numbers divisible by 3 are to be formed using the digits 0, 1, 2, 3, 4, and 5 without repetition.
For a number to be divisible by 3, the sum of its 'digits should be divisible by 3,
Consider the digits: 1, 2, 3, 4 and 5 Sum of the digits = 1+2+3+4+5 L =
15 which is divisible by 3.

∴ Any 5-digit number formed using the digits 1, 2, 1
Starting with the most significant digit, 5 digits are available for this place.
Since, repetition is not allowed, for the next significant place, 4 digits are available.
Similarly, all the places can be filled as:



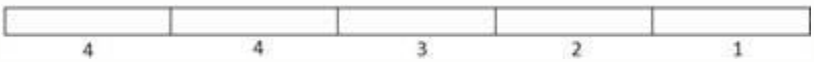

Number of 5-digit numbers

$=^* 5 \times 4 \times 3 \times 2 \times 1 = 120$

**Now, consider the digits: 0, 1, 2, 4 and 5**
Sum of the digits $= 0 + 1 + 2 + 4 + 5 = 12$ which is divisible by 3.

∴ Any 5-digit number formed using the digits

0, 1, 2, 4 and 5 will be divisible by 3.
Starting with the most significant digit, 4 digits are available for this place (since 0 cannot be used).
Since, repetition is not allowed, for the next significant place, 4 digits are available (since 0 can now be used).
Similarly, all the places can be filled as:




Number of 5-digit numbers

$= 4 \times 4 \times 3 \times 2 \times 1 = 96$

**Next, consider the digits: 0, 1, 2, 3, 4**
Sum of the digits $=0 +1+2+3+4 = 10$ which is not divisible by 3.

∴ *None of the 5-digit numbers formed using the digits 0, 1, 2, 3 and 4 will not be divisible by 3.

**Further, no other selection of 5 digits (out of the given 6) will give a 5-digit number, which is divisible by 3.**

$\therefore$ Total number of 5adigit numbers divisible by3 $= 120 + 96 = 216$

***EXERCISE 6.2***

**1) Evaluate:**

**(I) $8^!$**

**Sol:** $8^! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$\therefore 8^! \ = 40{,}320$

**(II) $6^!$**

**Sol:** $6^! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

**(III) $8^! - 6^!$**

**Sol:** *$8^! - 6^! = 8 \times 7 \times 6^! - 6^!$

$= 6^! \,(8 \times 7^{-\,1})$

$= 6^! \,(56^{-\,1})$

$= 6^! \times 55$

$= 720 \times 55$

$8^! - 6^! = 39600$

**(IV) $(8^! - 6)!$**

**Sol:** $(8^! - 6)! = 2!$

$= 2 \times 1$

$(8! - 6)! = 2$

**2) Compute:**

**(I)** $\frac{12!}{6!}$

**Sol:** $\frac{12!}{6!} = \frac{12\times11\times10\times9\times8\times7\times6!}{6!}$

$\therefore \frac{12!}{6!} = 665280$

**(II)** $\left(\frac{12!}{6!}\right)!$

**Sol:** $\left(\frac{12!}{6!}\right)! = 2! = 2 \times 1 = 2$

$\therefore \left(\frac{12!}{6!}\right)! = 2$

**(III)** $(3 \times 2)!$

**Sol:** $(3 \times 2)! = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$\therefore (3 \times 2)! = 720$

**(IV)** $3! \times 2!$

**Sol:** $3! \times 2! = (3 \times 2 \times 1) \times (2 \times 1) = 6 \times 2 = 12$

$\therefore 3! \times 2! = 12$

**(3) Compute**

**(I)** $\frac{9!}{3!\,6!}$

**Sol:** $\frac{9!}{3!\,6!} = \frac{9\times8\times7\times6!}{3!\,6!} = \frac{9\times8\times7}{3\times2\times1}$

$= 3 \times 4 \times 7 = 84$

$\therefore \frac{9!}{3!\,6!} = 84$

(II) $\frac{6!-4!}{4!}$

Sol: $\frac{6!-4!}{4!} = \frac{6\times5\times4!-4!}{4!} = \frac{4!(6\times5-1)}{4!}$

$= 30 - 1 = 29$

$\therefore \frac{6!-4!}{4!} = 29$

(III) $\frac{8!}{6!-4!}$

Sol: $\frac{8!}{6!-4!} = \frac{8\times7\times6\times5\times4!}{6\times5\times4!-4!}$

$= \frac{4!(8\times7\times6\times5)}{4!(30-1)} = \frac{1680}{29}$

$\therefore \frac{8!}{6!-4!} = \frac{1680}{29}$

(IV) $\frac{8!}{(6-4)!}$

Sol: $\frac{8!}{(6-4)!} = \frac{8!}{2!}$

$= \frac{8\times7\times6\times5\times4\times3\times2!}{2!}$

$= 8\times7\times6\times5\times4\times3$

$= 20,160$

**4) Write in terms of factorials.**

(I) $5\times6\times7\times8\times9\times10$

Sol: $5\times6\times7\times8\times9\times10$

$= \frac{10\times9\times8\times7\times6\times5\times(4\times3\times2\times1)}{(4\times3\times2\times1)}$

$= \frac{10!}{4!}$

$\therefore 5 \times 6 \times 7 \times 8 \times 9 \times 10 = \frac{10!}{4!}$

**(II)** $\mathbf{3 \times 6 \times 9 \times 12 \times 15}$

**Sol:** $3 \times 6 \times 9 \times 12 \times 15$

$= (3 \times 1) \times (3 \times 2) \times (3 \times 3) \times (3 \times 4)$

$= (3 \times 3 \times 3 \times 3 \times 3) \times (1 \times 2 \times 3 \times 4 \times 5)$

$= 35 \times 5!$

$\therefore \mathbf{3 \times 6 \times 9 \times 12 \times 15 = 3^5 \times 5!}$

**(III)** $\mathbf{6 \times 7 \times 8 \times 9}$

**Sol:** $6 \times 7 \times 8 \times 9 = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9}{1 \times 2 \times 3 \times 4 \times 5}$

$= \frac{9!}{5!}$

$\therefore 6 \times 7 \times 8 \times 9 = \frac{9!}{5!}$

**(IV)** $\mathbf{5 \times 10 \times 15 \times 20 \times 25}$

**Sol:** $5 \times 10 \times 15 \times 20 \times 25$

$= 5 \times 10 \times 15 \times 20 \times 25$

$= (5 \times 1) \times (5 \times 2) \times (5 \times 3) \times (5 \times 4) \times (5 \times 5)$

$= (5 \times 5 \times 5 \times 5 \times 5) \times (1 \times 2 \times 3 \times 4 \times 5)$

$= 5^5 \times 5!$

**5) Evaluate** $\frac{n!}{r!(n-r)!}$ **For:**

**(I)** $\mathbf{n = 8, r = 6}$

**Sol:** For $n = 8, r = 6$

$\frac{n!}{r!(n-r)!} = \frac{8!}{6!(8-6)!} = \frac{8!}{6!\ 2!}$

$= \frac{8\times7\times6!}{2!\ 6!} = \frac{8\times7}{2\times1}$

$= 4\times7$

$= 28$

**(II) $n = 12, r = 12$**

**Sol:** For $n = 12, r = 12$

$\frac{n!}{r!(n-r)!} = \frac{12!}{12!(12-12)!} = \frac{12!}{12!\ 0!}$

$= 1$ ... $(\because 0! = 1)$

**6) Find $n$, if:**

**(I) $\frac{n}{8!} = \frac{3}{6!} + \frac{1!}{4!}$**

**Sol:** $\frac{n}{8!} = \frac{3}{6!} + \frac{1!}{4!}$

$\therefore \frac{n}{8\times7\times6\times5\times4!} = \frac{3}{6\times5\times4!} + \frac{1!}{4!}$

$\therefore \frac{1!}{4!}\left[\frac{n}{1680}\right] = \frac{1!}{4!}\left[\frac{3}{30} + 1\right]$

$\therefore \frac{n}{1680} = \frac{33}{30}$

$\therefore n = \frac{3\times1680}{30} = \frac{55440}{30}$

$\therefore n = 1848$

**(II) $\frac{n}{6!} = \frac{4}{8!} + \frac{3}{6!}$**

**Sol:** $\frac{n}{6!} = \frac{4}{8!} + \frac{3}{6!}$

$\frac{n}{6!} = \frac{4}{8\times7\times6!} + \frac{3}{6!}$

$\therefore \frac{1}{6!}[n] = \frac{1}{6!}\left[\frac{4}{56} + 3\right]$

$\therefore n = \frac{4+168}{56} = \frac{172}{56} = \frac{43}{14}$

$\therefore n = \frac{43}{14}$

**(III)** $\frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$

**Sol:** $\frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$

$\therefore \frac{1}{n!} = \frac{1}{4!} - \frac{4}{5\times 4!}$

$\therefore \frac{1}{n!} = \frac{1}{4!}\left[1 - \frac{4}{5}\right]$

$\therefore \frac{1}{n!} = \frac{1}{4!}\left[\frac{5-4}{5}\right]$

$\therefore \frac{1}{n!} = \frac{1}{4!}\left(\frac{1}{5}\right)$

$\therefore \frac{1}{n!} = \frac{1}{5\times 4!}$

$\therefore \frac{1}{n!} = \frac{1}{5!}$

$\therefore n = 5$

**7) Find $n$, if:**

**(I)** $(n+1)! = 42\ (n-1)!$

**Sol:** $(n+1)! = 42\ (n-1)!$

$\therefore (n+1)(n)(n-1)! = 42\ (n-1)!$

$\therefore (n+1)\ (n) = 42$

$\therefore n^2 + n - 42 = 0$

$\therefore (n+7)\ (n-6) = 42$

$\therefore n + 7 = 0$ or $n - 6 = 0$

$\therefore n = -7$ or $n = 6$

Since $n \in N$, $n \neq -7$

$\therefore n = 6$

**(II) $(n+3)! = 110(n+1)!$**

**Sol:** $(n+3)! = 110(n+1)!$

$\therefore (n+3)(n+2)(n+1)! = 110(n+1)!$

$\therefore (n+3)(n+2) = 110$

$\therefore n^2 + 5n + 6 = 110$

$\therefore n^2 + 5n - 104 = 0$

$\therefore (n + 13)(n - 8) = 0$

$\therefore n + 13 = 0$ or $n - 8 = 0$

$\therefore n = -13$ or $n = 8$

Since $n \in N$, $n \neq -13$

$\therefore n = 8$

**8) Find n if:**

**(I) $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5:3$**

**Sol:** $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5:3$

$\therefore \dfrac{\frac{n!}{3!(n-3)!}}{\frac{n!}{5!(n-5)!}} = \frac{5}{3}$

$\therefore \frac{5!(n-5)!}{3!(n-3)!} = \frac{5}{3}$

$\therefore \frac{5\times4\times3!}{3!} \times \frac{(n-5)!}{(n-3)\times(n-4)\times(n-5)!} = \frac{5}{3}$

$\therefore \frac{20}{(n-3)(n-4)} = \frac{5}{3}$

$\therefore (n-3)(n-4) = \frac{20\times3}{5}$

$\therefore (n-3)(n-4) = 12$

$\therefore (n-3)(n-4) = 4\times3$

(Both LHS and RHS represent product of two consecutive natural numbers.

$\therefore$ comparing the large numbers)

$\therefore n-3 = 4$

$\therefore n = 7$

**(II)$\frac{n!}{3!(n-5)!} : \frac{n!}{5!(n-7)!} = 10:3$**

**Sol:**$\frac{n!}{3!(n-5)!} : \frac{n!}{5!(n-7)!} = 10:3$

$\therefore \frac{\frac{n!}{3!(n-5)!}}{\frac{n!}{5!(n-7)!}} = \frac{10}{3}$

$\therefore \frac{5!(n-7)!}{3!(n-5)!} = \frac{10}{3}$

$\therefore \frac{5\times4\times3!}{3!} \times \frac{(n-7)!}{(n-5)\times(n-6)\times(n-7)!} = \frac{10}{3}$

$\therefore \frac{20}{(n-5)(n-6)} = \frac{10}{3}$

$\therefore (n-5)(n-6) = \frac{20\times3}{10}$

$\therefore (n-5)(n-6) = 6$

$\therefore (n-5)(n-6) = 3\times2$

(Both LHS and RHS represent product of two consecutive natural numbers.

$\therefore$ comparing the large numbers)

$\therefore n - 5 = 3$
$\therefore n = 8$

**9) Find $n$, if:**

**(I) $\frac{(17-n)!}{(14-n)!} = 5!$**

**Sol:** $\frac{(17-n)!}{(14-n)!} = 5!$

$\therefore \frac{(17-n)(16-n)(15-n)(14-n)!}{(14-n)!} = 5!$

$\therefore (17-n)(16-n)(15-n) = 5 \times 4 \times 3 \times 2 \times 1$

$\therefore (17-n)(16-n)(15-n) = 6 \times 5 \times 4$

$\therefore 17 - n = 6$

$\therefore n = 11$

**(II) $\frac{(15-n)!}{(13-n)!} = 12$**

**Sol:** $\frac{(15-n)!}{(13-n)!} = 12$

$\therefore \frac{(15-n)(14-n)(13-n)!}{(13-n)!} = 12$

$\therefore (15-n)(14-n) = 4^{4 \times 3}$

$\therefore 15 - n = 4$ or $14 - n = 3$

$\therefore n = 11$

**10) Find $n$, if:**

**(I) $\frac{(2n)!}{7!(2n-7)} : \frac{n!}{4!(n-4)!} = 24:1$**

**Sol:** $\frac{(2n)!}{7!(2n-7)}:\frac{n!}{4!(n-4)!}=24:1$

$\therefore \frac{2n(2n-1)(2n-2)(2n-3)(2n-4)(2n-5(2n-6)(2n-7)!}{7!(2n-7)!}:\frac{n(n-1)(n-2)(n-3)(n-4)!}{4!(n-4)!}=24:1$

$\therefore \frac{2n(2n-1)(2n-2)(2n-3)(2n-4)(2n-5(2n-6)}{7\times6\times5\times4\times3\times2\times1}:\frac{n(n-1)(n-2)(n-3)}{4\times3\times2\times1}=24:1$

$\therefore \frac{2n(2n-1)(2n-2)(2n-3)2(n-2)(2n-5)2(n-3)}{7\times6\times5\times4\times3\times2\times1}\frac{4\times3\times2\times1}{n(n-1)(n-2)(n-3)}=24$

$\therefore \frac{8(2n-1)(2n-3)(2n-5)}{105}\quad 24$

$\therefore (2n-1)(2n-3)(2n-5)=\frac{24\times105}{8}$

$\therefore (2n-1)(2n-3)(2n-5)=315$

$\therefore (2n-1)(2n-3)(2n-5)=9\times7\times5$

$\therefore (2n-1)=9$ [or $(2n-3)=7$ or $(2n-5)=5$]

$\therefore n=5$

## 11) Show that:

$\frac{n!}{r!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)!}=\frac{(n+1)!}{r!(n-r+1)!}$

**Sol:** L.H.S.

$=\quad \frac{n!}{r!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)!}$

$=\quad \frac{n!}{r(r-1)!(n-r)!}+\frac{n!}{(r-1)!\times(n-r+1)(n-r)!}$

$=\frac{n!}{(r-1)!(n-r)!}+\left[\frac{1}{r}+\frac{1}{n-r+1}\right]$

$=\quad \frac{n!}{(r-1)!(n-r)!}+\left[\frac{n-r+1+r}{r(n-r+1)}\right]$

$= \frac{n! \times (n+1)}{[(r-1)! \times r][(n-r+1) \times (n-r)!]}$

$= \frac{(n+1)!}{r!(n-r+1)!}$

$=$ R.H.S.

## 12) Show that:

$\frac{9!}{3!6!} + \frac{9!}{4!5!} = \frac{10!}{4!6!}$

**Sol:** L.H.S $= \frac{9!}{3!6!} + \frac{9!}{4!5!}$

$= \frac{9!}{3!6 \times 5!} + \frac{9!}{4 \times 3! \times 5!}$

$= \frac{9!}{5!3!} \left[\frac{1}{6} + \frac{1}{4}\right]$

$= \frac{9!}{5! \times 3!} \left[\frac{4+6}{24}\right]$

$= \frac{9!}{5! \times 3!} \left[\frac{10}{6 \times 4}\right]$

$= \frac{10 \times 9!}{(6 \times 5!)(4 \times 3!)}$

$= \frac{10!}{6!4!}$

$=$ R.H.S.

## 13) Find the value of:

(I) $\frac{8!+5(4!)}{4!-12}$

**Sol:** $\frac{8!+5(4!)}{4!-12}$

$= \frac{8 \times 7 \times 6 \times 5! + 5!}{24-12} \left[\begin{array}{l} \because 5 \times 4! = 5! \\ 4! = 24 \end{array}\right]$

$= \frac{336 \times 5! + 5!}{12}$

$= \frac{5!(336+1)}{12}$

$= \frac{120(337)}{12}$

$= 3370$

**(II)** $\frac{5(26!)+(27!)}{4(27!)-8(26!)}$

**Sol:** $\frac{5(26!)+(27!)}{4(27!)-8(26!)}$

$= \frac{5(26!)+27(26!)}{4\times 27(26!)-8(26!)}$

$= \frac{26!(5+27)}{26!(108-8)}$

$= \frac{32}{100}$

$= \frac{8}{25}$

**14) Show that:**

$\frac{(2n)!}{n!} = 2^n(2n-1)(2n-3)\ldots 5\cdot 3\cdot 1.$ $=$

**Sol:** L.H.S. $= \frac{(2n)!}{n!}$

$= \frac{(2n)(2n-1)(2n-2)!(2n-3)(2n-4)\ldots 4\cdot 3\cdot 2\cdot 1}{n!}$

$= \frac{(2n)(2n-1)[2(n-1)](2n-3)2[(n-2)]\ldots 4\cdot 3\cdot 2\cdot 1}{n!}$

$= \frac{[(2n\times 2(n-1)\times 2(n-2)\ldots 6\times 4\times 2][(2n-1)(2n-3)(2n-5)\ldots 5\times 3\times 1]}{n(n-1)(n-2)(n-3)\ldots 5\times 4\times 3\times 2\times 1}$

$= \frac{(2\times 2\times 2\times \ldots n\ times)\ [n(n-1)(n-2)\ldots 3\times 2\times 1][(2n-1)(2n-3)(2n-5)\ldots 5\times 3\times 1}{n(n-1)(n-2)(n-3)\ldots 5\times 4\times 3\times 2\times 1}$

$= 2^n(2n-1)(2n-30\ldots 5\cdot 3\cdot 1.$

R.H.S.

## EXERCISE 6.3

**1) Find $n$, if $^nP_6 : {^nP_3} = 120 : 1$.**

Sol: $\frac{^nP_6}{^nP_3} = \frac{120}{1}$

$\therefore \frac{\frac{n!}{(n-6)!}}{\frac{n!}{(n-3)!}} = \frac{120}{1}$

$\therefore \frac{n!}{(n-6)!} \times \frac{n!}{(n-3)!} = 120$

$\therefore \frac{(n-3)(n-4)(n-5)(n-6)!}{(n-6)!} = 120$

$\therefore (n-3)(n-4)(n-5) = 120$

$\therefore (n-3)(n-4)(n-5) = 6 \times 5 \times 4$

$\therefore n-3 = 6$ or $n-4 = 5$ or $n-5 = 4$

$\therefore n = 9$

**2) Find $m$ and $n$, if $^{(m+n)}P_2 = 56$ and $^{(m-n)}P_2 = 12$.**

Sol: $^{(m+n)}P_2 = 56$

$\therefore \frac{(m+n)!}{(m+n-2)!} = 56$

$\therefore \frac{(m+n)(m+n-1)(m+n-2)!}{(m+n-2)!} = 56$

$\therefore (m+n)(m+n-1) = 56$

$\therefore (m+n)(m+n-1) = 8 \times 7$

$\therefore (m+n) = 8$ ... (I)

$\therefore$ and $^{(m-n)}P_2 = 12$

$\therefore \frac{(m-n)!}{(m-n-2)!} = 12$

$\therefore \frac{(m-n)(m-n-1)(m-n-2)!}{(m+n-2)!} = 12$

$\therefore (m-n)(m-n-1) = 12$

$\therefore (m-n)(m-n-1) = 4\times 3$

$\therefore (m-n) = 4$ ... (II)

From equations (I) and (II), we get after solving $m = 6$ and $n = 2$.

**3) Find $r$, if $^{12}P_{r-2} : {}^{11}P_{r-1} = 3:14$**

**Sol:** $\frac{^{12}P_{r-2}}{^{11}P_{r-1}} = \frac{3}{14}$

$\therefore \frac{\frac{12!}{(12-r+2)!}}{\frac{11!}{(11-r+1)!}} = \frac{3}{14}$

$\therefore \frac{12!}{(14-r)!} \times \frac{(12-r)!}{11!} = \frac{3}{14}$

$\therefore \frac{12\times 11!}{(14-r)(13-r)(12-r)!} \times \frac{(12-r)!}{11!} = \frac{3}{14}$

$\therefore \frac{12}{(14-r)(13-r)} = \frac{3}{14}$

$\therefore \frac{12\times 14}{3} = (14-r)(13-r)$

$\therefore (14-r)(13-r) = 56$

$\therefore (14-r)(13-r) = 8\times 7$

$\therefore 14-r = 8$ or $13-r = 7$

$\therefore r = 6$

**4) Show that $(n+1)({}^{n}P_r) = (n-r+1)[{}^{(n+1)}P_r]$**

**Sol:** L.H.S. $= (n+1)({}^{n}P_r)$

$= (n+1) \times \frac{n!}{(n-r)!}$

$= \frac{(n+1)!}{(n-r)!}$ ... (I)

$=$ R.H.S. $= (n-r+1) \times {}^{(n+1)}P_r$

$= (n-r+1) \times \frac{(n+1)!}{(n+1-r)}$

$= \frac{(n-r+1)(n+1)!}{(n-r+1)(n-r)!}$

$= \frac{(n+1)!}{(n-r)!}$ ... (II)

From (I) and (II), L.H.S. = R.H.S.
$\therefore (n+1)( {}^{n}P_r ) = (n-r+1)[ {}^{(n+1)}P_r ]$

**5) How many 4 letter words can be formed using letters in the word MADHURI if (i) letters can be repeated (ii) letters cannot be repeated?**

**Sol.** We have to 'form 4 letter words using letters M, A, D, H, U, R, I.
Number of letters = 7.

**(I)** Letters can be repeated.
So, each of the 4 places can be arranged in 7 ways.

| 1st place | 2nd place | 3rd place | 4th place |
|---|---|---|---|
| 7 ways | 7 ways | 7 ways | 7 ways |

Hence, this can be done in
74 = 7×7×7×7 = 2401 ways.

**(II)** Letters cannot be repeated.
So, we have to arrange 7 letters in 4 places.
This can be done in 7P4 ways.

$\therefore$ 7P4 $= \frac{7!}{4!} = \times 6 * \times 5 \times 4 = 840$ ways

**6) Determine the number of arrangements of letters of the word ALGORITHM if**

**(I) vowels are always together.**

**(II) No two vowels are together.**

**(III) Consonants are at even positions.**

**(IV) O is the first and T is the last letter. (*Answers differ from textbook)**

**Sol:** The word 'ALGORITHM' has 9 different letters of which 3 are vowels (A, I, O) and 6 are consonants.

**(I)** Here, the vowels are always together. So, we consider them as one unit (vowel).

∴ Now, the total number of letters = 1 + 6 = 7.
Hence, the number of ways of arranging them = 7!
Further, the three vowels can be arranged amongst themselves in 3! ways.

∴ The number of arrangements in which the three vowels are together = 7! × 3! = 30,240.

**(II)** Here, no two vowels are together. So, there has to be a consonant between two vowels. The 3 vowels can be arranged in any 3 out of 7 available places as shown below.
$\underline{1}$, C1, $\underline{2}$, C2, $\underline{3}$, C3, $\underline{*4}$, C4, $\underline{5}$, C5, $\underline{6}$, C6, $\underline{7}$,
Where C1, C1, C1,... denote consonants. This can be done in 7P3 Ways.
Having arranged the vowels in any three such places, the 6 consonants can be arranged in the remaining 6 places in 6P6 ways. Both of these arrangements are independent of each other.
Hence, the required number of arrangements
7P3 × 6P6 = (7×6×5) × (6×5×4×3×2×1)

= 210 × 720

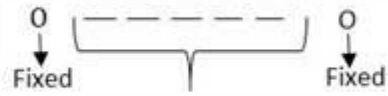
= 1, 51,200

**(III)** Here the consonants are at even position.
The even positions are the second, fourth, sixth and eighth, since there are nine letters in the word. These four places can be filled in by the six 'Consonants in 6P4 ways.
Having arranged the consonants in four places, the other five places can be filled in by the remaining five letters in 6P4 = 5! ways.
Both these arrangements are independent of each other.
Hence the required number of arrangements

= 6P4×5! = (6×5×4×3)×(5×4×3×2×1) = 43,200.

**(IV)** Since each word has 'O' as the first letter and 'T' as the last letter.

∴ The first and last place can be filled in 1 way each.




Middle 7 places can be filled in by the 7 letters in 7P7 $\times$ = 7! .ways.
Hence, the total number of required arrangements = 1 × 7! × 1 =
7×6×5×4×3×2×1 = 5040.

**7) In a group photograph, 6 teachers are in the first row and 18 students are in the second row. There are 12 'boys* and 6 girls among the students. If the middle position is reserved for the principal and if no two girls are 5 together, find the number of arrangements.**

**Sol:** In the group photograph, all 6 teachers are in the first row and the middle seat is reserved for the principal.
They can be arranged in 6P6 × 1 = 6! ways.
12 boys and 6 girls are in the second row such that no two girls are together.
So, the 6 girls can be arranged in any'6 out of 13 available places as show below.
$\underline{1}$, B1,$\underline{2}$, B2, $\underline{3}$, B3,$\underline{4}$, B4, $\underline{5}$, B5,$\underline{6}$, B6, $\underline{7}$, B7,$\underline{8}$, B8, $\underline{9}$, B9,$\underline{10}$, B10, $\underline{11}$, B11,$\underline{12}$, B12, $\underline{13}$,
Where B1, B2, B3,... denote boys
This can be done in 13P6 ways.
Having arranged the girls in any 6 places, the boys can be arranged among themselves in 12P12 ways.
Therefore, the second row can be arranged in 13P6 × 12P12 ways.
Hence, the number of ways of arranging both the rows = 6! × 13P6 × 12P12

$= 6! \times \frac{13!}{(13-6)!} \times 12!$

$= 6! \times \frac{13!}{7!} \times 12!$

$= 6! \times \frac{13!}{7\times 6!} \times 12!$

$= \frac{12!\times 13!}{7}$

**8) Find the number of ways so that letters of the word HISTORY can be arranged if,**

**(I) Y and T are together**

**(II) Y is next to T.**

**Sol:** The word 'HISTORY' has 7 different letters of which 2 are vowels (I, O).

**(I)** Here Y and T are together.
Since Y and T are together, we consider them as one letter.

$\therefore$ The total number of letters $= 1 + 5 = 6$
Hence, the number of ways of arranging them $= 6!$
Further, the letters Y and T can be arranged amongst themselves in $2!$ ways.

$\therefore$ The number of arrangements in which letters
Y and T are together $= 6! \times 2! = 720 \times 2$
$= 1440$

**(II)** Here! Y is next to T.


We consider them as one letter.

$\therefore$ The total number of letters $= 1 + 5 = 6$
Hence, the number of ways of arranging them $= 6!$
Further Y is next to T, this arrangement is possible in only one way.

$\therefore$ The total number of required arrangements

$= 6! \times 1 = 720$

**9) Find the number of arrangements of the letters in the word BERMUDA so that consonants and vowels are in the same relative positions.**
**Sol.** The word BERMUDA has 4 consonants and 3 vowels.

The current arrangement has consonants and vowels arranged as follows.

| | | | | | | |
|---|---|---|---|---|---|---|
| C | V | C | C | V | C | V |

To maintain their relative positions, the 4 consonants need to be arranged amongst themselves and the 3 vowels need to be arranged amongst themselves.
No. of ways of arranging 4 consonants $= 4!$
No. of ways of arranging 3 vowels $= 3!$

$\therefore$ Total 'No. of ways of arranging the word BERMUDA

$= 4! \times 3! = 24 \times 6 = 144$

**10) Find the number of 4-digit numbers that can be formed using the digits 1, 2, 4, 5, 6, 8 if**

**(I) digits Can be repeated**

**(II) digits cannot be repeated.**

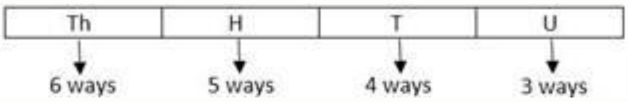
**Sol.** we have to form 4 digit numbers using the digits

1, 2, 4, 5, 6, 8.

**(I)** digits can be repeated


All the places, thousand's place, hundred's, ten's and unit's place can each be filled in 6 ways.
Hence, 6 X 6 X 6 X 6 = 1296, four digit numbers can be formed.

**(II)** digits cannot be repeated.



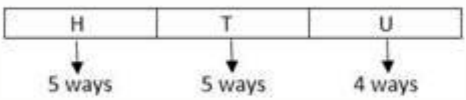

This can be done by arranging 6 digits in 4 places
As 6P4 ways.

$6P4 = \frac{6!}{2!} = 6\times5\times4\times3 = 360$

∴ 360 four digit numbers can be formed.

**11) How many numbers can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition so that resulting numbers are between 100 and 1000?**

Sol: Any number between 100 and 1000 will have 3 digits.




Since we re*quire a 3 digit number, zero cannot be in hundred's place. Hence, hundred's place can be filled in by any one of the five non-zero digits. Thus, hundred's place can be filled in 5 ways.
After filling in the hundred's place, we have now 5 digits left to fill in the ten's and unit's place.
Thus, these places can be filled in $5P2 = 5\times4 = 20$ ways.

Hence, by the fundamental principle of multiplication, the three places can be filled in $5\times5\times4 = 100$ ways.
Thus, 100 three digit numbers can be formed if repetition of digits is not allowed.

**12) Find the number of 6-digit numbers using the digits 3, 4, 5, 6, 7, 8 without repetition. How many of these numbers are:**

**(i) divisible by 5, (ii) not divisible by 5.**

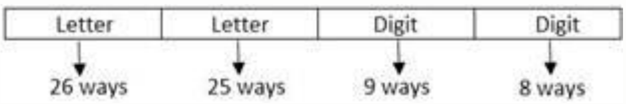
**Sol:** We have to form 6 digit numbers using the digits 3, 4, 5, 6, 7, 8 without repetition. Total number of ways of arranging 6 digits in six places = 6P6 = 6!
$= 6\times5\times4\times3\times2\times1 = 720$

**(I)** Here, the number is divisible by 5. So it will have the digit 5 in the unit's place.
Hence, the unit's place can be filled in 1 way.
The other five places can be filled in by the remaining 5 digits
(Since repetition is not allowed) in 5P5 = 5! Ways.
Thus, there are $5! \times 1 = 120$ numbers with 6 digits which are divisible by 5.

**(II)** Here, the number is not divisible by 5. We have, numbers not divisible by 5 = total numbers divisible by 5
$= 720 - 120 = 600$
Thus there are 600 numbers which are not divisible by 5.

**13) A code word is formed by two distinct English letters followed by two non-zero distinct digits. Find the number of such code words. Also, find the number of such code words that end with an even digit.**

**Sol:** The code word consists of two different English letters followed by two non-zero distinct digits (i.e. from 1 to 9).

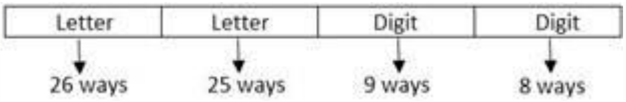



∴ Total number of code words available

$= 26\times25\times9\times8$

$= 46{,}800$

To find the code words that end with an even digit.
The even digits from 1 to 9 are 2, 4, 6, 8.



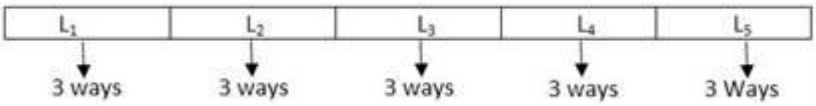

Hence, the total number of required code words

$= 26 \times 25 \times 8 \times 4$

$= 20{,}800$

**14) Find the number of ways in which 5 letters can be posted in 3 post boxes if any number of letters can be posted in a post box.**

**Sol:** Here, any number of letters can be posted in all the three post boxes.
Hence each letter can be posted in 3 different ways.
Let L1, L2, L3, L4, L5 denote the five letters. We assign the 3 post boxes to each letter as follows:




By the fundamental principle of multiplication, the 5 letters can be posted in $3 \times 3 \times 3 \times 3 \times 3 = 35 = 243$ways Thus, the five letters can be posted in 243 ways.

**15) Find the number of arranging 11 distinct objects taken 4 at a time so that a specified object. '**

**(I) always occurs (II) never occurs.**

**Sol:** There are 11 distinct objects and 4 objects are arranged at a time.

**(I)** Here, one particular object out of 4 always occurs.
It can be arranged in 4 ways.
The other 3 out of 4 objects can be arranged from the remaining 10 objects in 10P3 ways. Number of ways of arranging all 4 objects

$= 10P3 \times 4 = 10 \times 9 \times 8 \times 4$

$= 2880$ ways

**(ii)** Here, One particular object never occurs. 80, we exclude it, from 11 distinct object. Now, we have 10 distinct objects from which we have to arrange any 4 objects.
This can be done in $14P4 = 10 \times 9 \times 8 \times 7$

= 5040 ways

***EXERCISE 6.4***

**1) Find the number of permutations of letters in each of the following words:**

**(I) DIVYA**

**Sol:** The word DIVYA has 5 letters, all are different.
Hence, the number of distinct permutations of the letters

$= n! = 5! = 5 \times 4 \times 3 \times 2 \times 1$

= 120

**(II) SHANTARAM**

**Sol:** The word SHANTARAM has 9 letters of which 'A' is repeated 3 times and rest all are different.
Hence, the number of permutations

$= \frac{n!}{p!} = \frac{9!}{3!} \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!}$

= 60,480

**(III) REPRESENT**

*(*Answer differ from textbook)*

**Sol:** The word REPRESENT has 9 letters of which 'E' is repeated 3 times, 'R' is repeated 2 times and
rest all are different. .

∴ The number of distinct permutations

$= \frac{n!}{p!q!} = \frac{9!}{3!2!} = \frac{60480}{2}$

= 30,240

**(IV) COMBINE**
Sol: The word COMBINE has 7 letters, all are different
The number of permutations

$= n! = 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 5040$

**2) You have 2 identical books on English, 3 identical books in Hindi, and 4 identical books on Mathematics. Find the number of distinct ways of arranging them on a shelf.**

Sol: Here, total books $= 2 + 3 + 4 = 9$

$\therefore n = 9$

There are, 2 identical books on English. $\therefore p = 2$

3 identical books on Hindi. $\therefore q = 3$

4 identical books on Mathematics. $\therefore r = 4$

$\therefore$ Total number of distinct arrangements

$= \frac{n!}{p!q!r!} = \frac{9!}{2!3!4!}$

$= \frac{9\times8\times7\times6\times5\times4!}{(2\times1)(3\times2\times1)4!}$

$= 9 \times 4 \times 7 \times 5$

$= 1260$

**3) A coin is tossed 8 times. In how many ways can we obtain (i) 4 heads and 4 tails?**

**(II) at least 6 heads?**

**Sol:** A coin is tossed 8 times.

$\therefore n = 8$

**(I)** We require 4 heads and 4 tails.

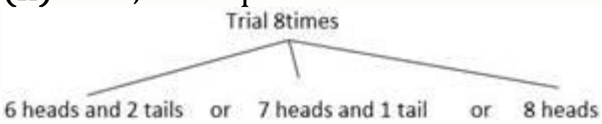
$\therefore p = 4, q = 4$

$\therefore$ Number of ways in which we obtain 4 heads and 4 tails

$= \frac{n!}{p!q!} = \frac{8!}{4!4!} = \frac{8\times7\times6\times5\times4!}{(4\times3\times2\times1)4!}$

$= 2 \times 7 \times 5 = 70$

**(II)** Here, we require at least 6 heads when coin is tossed 8 times.




The number of distinct permutations

$\frac{8!}{6!2!}$ or $\frac{8!}{7!1!}$ or $\frac{8!}{8!}$

i.e. $\frac{8\times7}{2}$ or $8$ or $1$

∴ Total number of required permutations

$= 28 + 8 + 1 = 37$

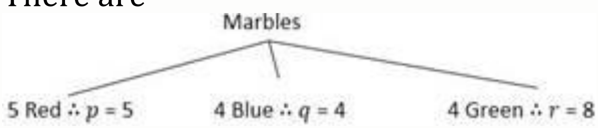
**4) A bag has 5 red, 4 blue and 4 green marbles. If all are drawn one by one and their colours are recorded, how many different arrangements can be found?**

**Sol:** Here, the total number of marbles

$=5 + 4 + 4 = 13$

∴ $n = 13$

There are




∴ Total number of distinct arrangements

$= \frac{n!}{p!q!r!} = \frac{13!}{5!4!4!}$

**5) Find the number of ways of arranging letters of the word MATHEMATICAL. How many of these arrangements have all vowels together?**

**Sol:** The word 'MATHEMATICAL' has 12 letters of which A is repeated 3 times, M is repeated 2 times, T is repeated 2 times.

∴ Number of arrangements of word

MATHEMATICAL $= \frac{12!}{3!2!2!}$

The given word has 5 vowels – A, E, A, I, A.

Taking vowels together, we consider them as a single letter, say P.

Now, we have 8 letters – P, M, T-, H, M, T, C, L of Which M is repeated 2 times, T is repeated 2 times.

The number of ways of arranging these 8 Letters

$= \frac{8!}{2!2!}$

After this is done, 5 vowels (in which A is repeated

3 times) can be arranged in $\frac{5!}{3!}$ ways.

∴ The number of required arrangements

$= \frac{8!}{2!2!} \times \frac{5!}{3!}$

**6) Find the number of different arrangements of letters in the word MAHARASHTRA. How many of these arrangements have (I) letters M and T never together? (II) All vowels together?**

**Sol:** Total number of letters in the word MAHARASHTRA = 11
The letter 'A' is repeated '4' times.
The letter 'H' is repeated twice.
The letter 'R' is repeated twice.

∴ Number of arrangements = $\frac{11!}{4!2!2!}$

**(a)** For letters 'M' and 'T' to he never together,
remaining 9 letters need to be arranged first, (represented by $-$)
$\times - \times - \times - \times - \times - \times - \times$
The letters 'M' and 'T' can occupy any position marked by ×
No. of ways of arranging 9 letters a 9!
Since 10 positions are available for the letters 'M' and 'T'.
No. of ways of arranging these letters = 10P2
Considering the repetitions of the letters 'A', 'H' and 'R',

**(b)** Here, all vowels are together. The given word has 4 vowels A, A, A, A.
We consider 4 vowels together as a single letter, say G.
We have 8 letters G, M, H, R, S, H, T, R of which R and H are repeated 2 times each.
The number of ways of arranging 8 letters
$= \frac{8!}{2!2!}$
After this is done, 4 vowels (in which A is repeated 4

4 times) can be arranged in $\frac{4!}{4!}$ = 1 way.
∴ The number of arrangements in which the vowels

are together = $\frac{8!}{2!2!}$

**7) How many different words are formed if the letter R is used thrice and letters S and T are used twice each?**

**Sol:** Here, the letter R is used thrice, letter S and T are used twice each.
Total letters = 3 + 2 + 2 = 7
∴ The number of different words formed

$= \frac{7!}{3!2!2!} = \frac{7\times6\times5\times4\times3!}{3!(2\times1)(2\times1)}$

= 210

**8) Find the number of arrangements of letters in the word MUMBAI so that the letter B is always next to A.**

**Sol:** The word MUMBAI has 6 letters of which M is repeated twice.
∵ B is always next to A, consider them as a single letter.
Now, we have 5 letters M, U, M, AB ,I in which M
is repeated twice.

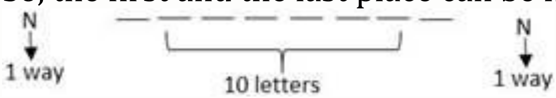
∴ The number of ways of arranging them $= \frac{5!}{2!}$
After this is done, we arrange A and B such that B is next to A.
This can be done in 1 way.

∴ The number of required arrangements

$= \frac{5!}{2!} \times 1 = \frac{5\times4\times3\times2!}{2!} \times 1$

= 60

**9) Find the number of arrangements of letters in the word CONSTITUTION that begin and end with N.**
**Sol:** We have to find the number of arrangements of letters in the word CONSTITUTION that begin and end with N.
So, the first and the last place can be filled in 1 way.




After this is done, we have 10 letters of which
O and I are repeated 2 times and T is repeated 3 times.

So, The 10 places can be arranged in $\frac{10!}{2!3!2!}$ ways.

∴ The total number of required arrangements

$= 1 \times \frac{10!}{2!3!2!} \times 1$

$= \frac{10!!}{2!3!2!}$

**10) Find the number of different ways of arranging letters in the word ARRANGE.**

**How many of these arrangements the two R's and two A's are not together?** ***(*Answer differ from textbook)***

**Sol:** The word ARRANGE has 7 letters of which A and R are repeated 2 times.

$\therefore$ The number of ways of arranging letters of the word

$= \frac{7!}{2!2!} = \frac{7\times6\times5\times4\times3\times2!}{2!2!}$

$= 1260$

Here, we have to find the number of arrangement in which two R's nor A's are together.
We will first find the arrangements in which two R's and two A's are together. _ '
Two R's together, say P and two A's together, say $Q$.
Now we have 5 letters. They can be arranged in 5! 2! 2!

$\frac{5!}{2!2!}$ Ways.

i.e. $\frac{5!}{2!2!} = \frac{5\times4\times3\times2!}{2!2!} = 30$

As two A's and two R's are repeated.

$\therefore$ Arrangements do not have two R's nor A's together

$=$ Total Arrangements $-$ Number of arrangements in which two R's and ' two A's are together

$= 1260 - 30$

$= 1230$

**11) How many distinct 5 digit numbers can be formed using the digits 3, 2, 3, 2, 4, 5?**

**Sol:** The given digits are 3, 2, 3, 2, 4, 5 of which 3 and 2 are repeated twice each.

$\therefore n = 6, \quad p = 2, \quad q = 2$

We can form $\frac{6!}{2!2!}$ distinct 5 digit numbers

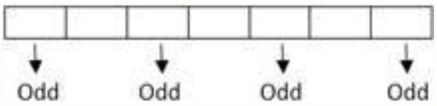
$\therefore \frac{6!}{2!2!} = \frac{6\times5\times4\times3\times2!}{2!2!}$

$= 6 \times 5 \times 2 \times 3$

$= 180$

**12) Find the number of distinct numbers formed using the digits 3, 4, 5, 6, 7, 8, 9 so that odd positions are occupied by odd digits?**

**Sol:** The given digits are 3, 4, 5, 6, 7, 8, 9, all are different.
We need to form 7 digit numbers such that odd digits 3, 5, 7, 9 occupy the odd places namely 1st, 3rd, 5th and 7th.




This can be done in 4P4 = 4! Ways.
Having done this, the other 3 places can be filled in by the remaining digits 4, 6, 8 in 3P3 = 3! ways.
Hence, total number of required arrangements

$= 4! \times 3! = (4 \times 3 \times 2 \times 1) \times (3 \times 2 \times 1)$

$= 144$

**13) How many different 6-digit numbers can be formed using digits in the number 659942? How many of them are divisible by 2?**
**Sol:** Total number of digits in the dumber = 6 the digit '9' is repeated twice.

$\therefore$ Number of arrangements $= \frac{6!}{2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$

$= 360$

For a number to be divisible by 2, the digit in the units place must be an even number.
There are 3 even digits available: 2, 4, 6

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 3 |

2, 4, 6

$\therefore$ 3 options are available for the unit's place.
The remaining 5 places can be occupied by the remaining 5 digits.
Considering the repetition of the digit 9, the total

number of arrangements $= \frac{3\times120}{2}$

$= \frac{360}{2}$

$= 180$

**14) Find the number of distinct words formed from letters in the word INDIAN. How many of them have the two N's together?**

**Sol:** The word INDIAN has 6 letters of which I and N are repeated 2 times.

∴ Number of distinct words that can be formed

$= \frac{6!}{2!2!} = \frac{6\times5\times4\times3\times2!}{2!2!}$

$= 6 \times 5 \times 2 \times 3 = 180$

Next, the two N's together.

So, we consider them as one letter.
Now, the total number of letters $= 1 + 4 = 5$
In these 5 letters, I is repeated twice.

∴ The number of ways of arranging them

$= \frac{5!}{2!} \quad = \frac{5\times4\times3\times2!}{2!}$

$= 5 \times 2 \times 4 \quad = 60$

**15) Find the number of different ways 0f arranging letters in the word PLATOON if**

**(I) the two 0's are never together**

**(II) consonants and vowels occupy alternate positions.**

**Sol:** The word 'PLATOON' has 7 letters, in which 0 is repeated 2 times.

**(I)** The two 0's are never together.
Let us consider the two O's are together as one letter.
Now, we have 6 letters. They can be arranged in 6! Ways.

Also, the number of ways of arranging the letters of the word PLATOON is $\frac{7!}{2!}$

∴ The number of required arrangements

= Total Arrangements − Number of arrangements in which two R's and ' two A's are together

$= \frac{7!}{2!} - 6!$ $\quad = \frac{7 \times 6!}{2!} - 6!$

$= 6! \frac{(7-2)}{2}$ $\quad = \frac{720}{2} \times 5$

$= 1800$

**(II)** Consonants and vowels occupy alternate positions.
There are 3 vowels (A, O, O) and 4 consonants.

| | | | | | | |
|---|---|---|---|---|---|---|
| C1 | C2 | C2 | C2 | C2 | C2 | C2 |

Places → 1 2 3 4 5 6 7

The three vowels can be arranged in 2nd, 4th, 6th positions in 3!
Ways and the 4 consonants can be arranged in 1st, 3rd, 5th and 7th positions in 4! Ways.
Hence, the consonants and vowels occupy alternate places in $3! \times 4! = (6) \times (24) = 144$ ways.
But the vowel 'O' is repeated 2 times.

∴ Total number of distinct arrangements

$= \frac{144}{2!}$ $\quad = \frac{144}{2}$ $\quad = 72$

***EXERCISE 6.5***

**1) In how many different ways can 8 friends sit around a table?**

**Sol:** The number of different ways in which 8 friends sit around a table

= (8-1)!

= 7!

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

**2) A party has 20 participants and a host. Find the number of distinct ways for the host to sit with them around a circular table. How many of these ways have two specified persons on either side of the host?**

**Sol:** Number of persons = 20

Number of hosts = 1

$\therefore$ Total number of persons = 21
They can be seated around a circular table in
$(21 - 1)! = 20!$ Ways.
Two particular persons sit on either side of the host.
These two persons can be arranged in 2! Ways.
Now, there are 19 persons including the host.
They can be seated in $(19 - 1)! = 18!$ Ways.

$\therefore$ Total number of required arrangements

**3) Delegates from 24 countries participate in a round table discussion. Find the number of seating arrangements where two specified delegates are (I) always together (II) never together.**
*(*Answers differ from textbook)*

**Sol:** There are 24 delegates.

**(I)** Two specified delegates are always together. Consider them as one member. We have 23 members. They can be seated around the table in $(23 - 1)! = 22!$ Ways and two specified persons can be arranged among themselves in 2! Ways.

$\therefore$ Total number of required arrangement $= 2! \times 22!$

$= 2 \times 22!$

(II) Two Specified delegates are never together

= Total number of arrangements.

$-$Number of arrangements in which
two specified delegates are together.

$= 23! - 2 \times 22!$

$= 23 \times 22! - 2 \times 22!$

$= 22! \, (23 - 2)$

$= 21 \times 22!$

**4) Find the number of ways for 15 people to sit around the table so that no two arrangements have the same neighbours.**

**Sol:** 15 people can sit around a table in $(15 - 1)! = 14!$ ways.
Total number of arrangements $= 14!$
Now, the number of arrangements in which any person can have the same neighbours on either side by clockwise or anticlockwise arrangements $= \frac{14!}{2!}$

∴ The number of arrangements in which no two arrangements have the same neighbours

$= 14! - \frac{14!}{2!} = 14!\left(1 - \frac{1}{2}\right) = 14! \times \frac{1}{2}$

$= \frac{14!}{2!}$

**5) A committee of 20 members sits around a table. Find the number of arrangements that have the president and the vice president together.**

**Sol:** Number of members in the committee $= 20$
Since the president and vice president are to be seated together, consider them as 1 unit.

∴ Number of people to be arranged $= 19$

∴ Number of ways of arranging 19 people around a table $= (19 - 1)!$

$= 18!$
Further, the president and vice president can be arranged among each other in 2! ways.

∴ Total Number of arrangements $= 18! \; 2!$

**6) Five men, two women, and a child sit around a table. Find the number of arrangements where the child is seated (I) between the two women (II) between two men.**

**Sol:** Five men, two women and a child sit around a table.

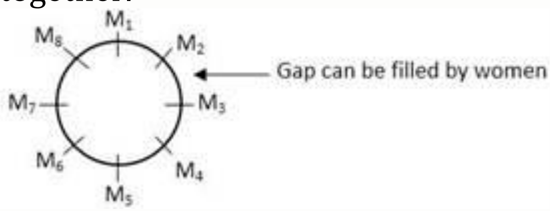
**(I)** Child is seated between the two women.

So, we consider them as a single person. Now, we have $5 + 1 = 6$ persons.
They can be arranged around a table in
$(6^{-1})! = 5!$ Ways.
Also, the two women can be arranged among themselves in 2! ways.
So, the number of required arrangements $= 5!\ 2! = 240$

**(II)** Here, the child is seated between 2 men.
Since there are 5 men, such a group can be formed in
$5P2 = \frac{5!}{3!} = 5 \times 4 = 20$ ways
Thus, there are 20 ways in which the child can be seated between 2 men.
We consider the 2 men and the child between as one unit.
Also, we have 3 more men and 2 women.
Thus, we have $1 + 3 + 2 = 6$ persons.
These 6 persons can be arranged around a table in $(6 - 1)! = 5! -$ ways.
∴ Total number of required arrangements $= 20 \times 5! = 20 \times 120$
$= 2400$

**7) Eight men and six women sit around a table. How many of sitting arrangements will have no two women together?**

**Sol:** Eight men and six women sit around a table, in which no two women are together.




There are 8 gaps between the eight men.
Eight men can sit around the table in

$= (8^{-}1)! = 7!$ Ways
To fulfill our condition, the six women can be
seated in eight gaps between eight men in 8P6 ways.

∴ Total number of required arrangements

$= 7! \times 8P6 \quad = 7! \times \frac{8!}{2!}$

$= \frac{7! \times 8!}{2!}$

**8) Find the number of sitting arrangements for 3 men and 3 women to sit around a table, so that exactly two women are together.**

**Sol:** There are 3 men and 3 women sitting around a table, with the condition exactly two women are together.
Group of exactly two women can be formed in

$3P2 = \frac{3!}{1!} = 3 \times 2 \times 1 = 6$ ways

Thus, there are 6 ways in which exactly two women can be seated together.
After choosing two women, consider them as one unit.
Also, we have 1 remaining women and 3 men. '
Thus, we have $1 + 1 + 3 = 5$ persons.
These 5 persons can be arranged around a table in $(5-1)! = 4!$ ways.

$\therefore$ Total number of required arrangements

$= 6 \times 4! = 6 \times 24 = 144$

**9) Four objects in a set of ten objects are alike. Find the number of ways of arranging them in a circular order.**

Sol: There are 10 objects.
These 10 objects can be arranged in a circular order in $(10-1)! = 9!$ ways.

$\therefore n = 9!$
Out of 10 objects, 4 are alike.
$r = 4$

$\therefore$ Required number of arrangements

$= \frac{n!}{r!} = \frac{9!}{4!}$

**10) Fifteen persons sit .around a table. Find the number of arrangements that have two specified persons not sitting side by side.**

**Sol:** There are 15 persons. They can be arranged around a table in $(15-1)! = 14!$ ways.
We have to find the number of arrangements in which two specified persons not sitting side by side.
Consider two specified persons sitting side by side as a single person, we have 14 persons.
They can be arranged round the table in $(14-1)! = 13!$ ways.
Further, the two particular persons can be arranged among themselves in 2! ways.

$\therefore$ Arrangements in which two specified persons are side by side $= 2! \times 13!$
Now, arrangements in which two specified persons not sitting side by side

= Total Arrangements − Number of arrangements in which two R's and ' two A's are together

$= 14! - 2! \times 13!$

$= 14 \times 13! - 2 \times 3!$

$= 13!\,[14 - 2]$

$= 13!\,(12)$

$= 12 \times 13!$

***EXERCISE 6.6***

**1) Find the value of:**

**(I)** $^{15}C_4$

**Sol:** $^{15}C_4 = \frac{15!}{4!(15-4)!} \quad \ldots\left(\because \quad {}^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= \frac{15!}{4!11!}$

$= \frac{15\times14\times13\times12\times11!}{(4\times3\times2\times1)11!}$

$= 15 \times 7 \times 13$

$= 1365$

**(II)** $^{80}C_2$

**Sol:** Here, $n = 80; \quad r = 2$

$^{80}C_2 = \frac{80!}{2!(80-2)!} \quad \ldots\left(\because \quad {}^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= \frac{80!}{2!78!}$

$= \frac{80\times79\times78\times12\times11!}{(2\times1)78!}$

$= 40 \times 79$

$= 3160$

**(III)** $\boldsymbol{{}^{15}C_4 + {}^{15}C_5}$

**Sol:** Use ${}^{n}C_r + {}^{n}C_r = {}^{n+1}C_r$

Here $n = 15;\quad r = 5$

$\therefore\ {}^{15}C_4 + {}^{15}C_5]$

$= {}^{15+1}C_5$

$= {}^{16}C_5$

**(IV)** $\boldsymbol{{}^{20}C_{16} - {}^{19}C_{16}}$

**Sol:** ${}^{n+1}C_r - {}^{n}C = {}^{n}C_{r-1}$

Here $n = 19;\ \ r = 16$

$\therefore\ \boldsymbol{{}^{20}C_{16} - {}^{19}C_{16}}$

$= {}^{19}C_{16-1}$

$= \boldsymbol{{}^{19}C_{15}}$

**2) Find $n$ if:**

**(I)** ${}^{6}P_2 = n\ {}^{6}C_2$

**Sol:** We have ${}^{6}P_2 = n\ {}^{6}C_2$

$\therefore \frac{6!}{(6-2)!} = n.\frac{6!}{2!(6-2)!}$

$\therefore \frac{6!}{4!} = n.\frac{6!}{(2\times1)4!}$

$\therefore \frac{n}{2} = 1$

$\therefore n = 2$

**(II)** $\boldsymbol{{}^{2n}C_3 : {}^{n}C_2 = 52:3}$

**Sol:** we have $\quad {}^{2n}C_3 : \quad {}^{n}C_2 = 52:3$

$\therefore \frac{{}^{2n}C_3}{{}^{n}C_2} = \frac{52}{3}$

$\therefore \frac{\frac{2n!}{(2n-3)!3!}}{\frac{n!}{(n-2)!2!}} = \frac{52}{3}$

$\therefore \frac{2n!}{(2n-3)!3!} \times \frac{(n-2)!2!}{n!} = \frac{52}{3}$

$\therefore \frac{2n(2n-1)(2n-2)(2n-3)!}{(2n-3)!(3\times2\times1)} \times \frac{(n-2)!(2\times1)}{n(n-1)(n-2)!} = \frac{52}{3}$

$\therefore \frac{n(2n-1)\times2(n-1)}{3} \times \frac{2}{n(n-1)} = \frac{52}{3}$

$\therefore \frac{4}{3} \times (2n-1) = \frac{52}{3}$

$\therefore 2n - 1 = \frac{52}{3} \times \frac{3}{4}$

$\therefore 2n - 1 = 13$

$\therefore 2n = 14$

$\therefore n = 7$

**(III)** $\quad \boldsymbol{{}^{n}C_{n-3} = 84}$

**Sol:** We have, $\quad {}^{n}C_{n-3} = 84$

$\therefore \frac{n!}{(n-n+3)!(n-3)!} = 84$

$\therefore \frac{n(n-1)(n-2(n-3)!}{(3\times2\times1)(n-3)!} = 84$

$\therefore n(n-1)(n-2) = 84 \times 6$

$= 12 \times 7 \times 6$

$= 9 \times 8 \times 7$

$\therefore n = 9 \quad (or\, n-1 = 8\, or\, n-2 = 7)$

Thus, $n = 9$

**3) Find $r$ if $\;{}^{14}C_{2r} : \;{}^{10}C_{2r-4} = 143{:}10$**

**Sol:** We have $\;{}^{14}C_{2r} : \;{}^{10}C_{2r-4} = 143{:}10$

$$\therefore \frac{{}^{14}C_{2r}}{{}^{10}C_{2r-4}} = \frac{143}{10}$$

$$\therefore \frac{\frac{14!}{(14-2r)!(2r)!}}{\frac{10!}{(10-2r+4)!(2r-4)}} = \frac{143}{10}$$

$$\therefore \frac{14!}{(14-2r)!(2r)!} \times \frac{(14-2r)!(2r-4)!}{10!} = \frac{143}{10}$$

$$\therefore \frac{14\times13\times12\times11\times10}{10!} \times \frac{(2r\times4)!}{2r(2r-1)(2r-2)(2r-3)(2r-4)!} = \frac{143}{10}$$

$$\therefore \frac{14\times13\times12\times11}{2r(2r-1)(2r-2)(2r-3)} \times \frac{2}{n(n-1)} = \frac{143}{10}$$

$\therefore 2r(2r-1)(2r-2)(2r-3)$

$= 14 \times 13 \times 12 \times 11 \times \frac{10}{143}$

$= 14 \times 12 \times 109$

$= 7 \times 6 \times 5 \times 8$

$= 8 \times 7 \times 6 \times 5$

$\therefore 2r = 8 \quad (or\; 2r-1 = 7 \quad or \quad 2r-2 = 6$
$or\; 2r-3 = 5$
$r = 4$

**4) Find $n$ and $r$ if:**

**(I) $\;{}^{n}P_{r} = 720$ and $\;{}^{n}C_{n-r} = 120$**

**(II) $\;{}^{n}C_{r-1} : \;{}^{n}C_{r} : \;{}^{n}C_{n-r} = 20{:}35{:}42$**

**Sol: (I)** We have $^{n}P_r = 720$ and $^{n}C_{n-r} = 120$

i.e. $\frac{n!}{(n-r)!} = 720$ and $\frac{n!}{(n-r)!r!} = 120$

$\therefore \frac{^{n}P_r}{^{n}C_r} = \frac{720}{120}$

$\therefore \frac{n!}{(n-r)!} \times \frac{n!}{(n-r)!r!} = 6$

$\therefore r! = 6$

$= 3 \times 2 \times 1$

$\therefore r! = 3!$

$\therefore r = 3$

Substituting $r = 3$ in $^{n}P_r = 720,$

We get $^{n}P_3 = 720$

$\therefore \frac{n!}{(n-3)!} = 720$

$\therefore \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 720$

$\therefore n(n-1)(n-2) = 10 \times 9 \times 8$

$\therefore n = 10$

Thus, $n = 10$ and $r = 3$

**(II)** We have $^{n}C_{r-1}: \ ^{n}C_r: \ ^{n}C_{n-r} = 20:35:42$

$\therefore = \frac{^{n}C_{r-1}}{^{n}C_r} = \frac{20}{35}$

$\therefore \frac{\frac{n!}{(n-r+1)!(r-1)!}}{\frac{n!}{(n-r)!r!}} = \frac{4}{7}$

$\therefore \frac{n!}{(n-r+1)(n-r)!(r-1)!} \times \frac{(n-r)! \times r(r-1)!}{n!} = \frac{4}{7}$

$\therefore \frac{r}{n-r+1} = \frac{4}{7}$

$\therefore 4n - 4r + 4 = 7r$

$\therefore 11r - 4n = 4 \quad ...(I)$

Also, ${}^{n}C_{r}: \ {}^{n}C_{n-r} = 35:42$

$\therefore \frac{{}^{n}C_{r}}{{}^{n}C_{n-r}} = \frac{35}{42}$

$\therefore \frac{\frac{n!}{(n-r)!r!}}{\frac{n!}{(n-r-1)!(r+1)!}} = \frac{5}{6}$

$\therefore \frac{n!}{(n-r)!r!} \times \frac{(n-r-1)!(r+1)!}{n!} = \frac{5}{6}$

$\therefore \frac{(n-r-1)!(r+1)!}{(n-r)(n-r-1)!r!} = \frac{5}{6}$

$\therefore \frac{r+1}{n-r}$

$\therefore 5n - 5r = 6r + 6$

$\therefore 5n - 11r = 6 \quad ...(II)$

Solving equations (I) and (II), we get $n$

$= 10$ and $r = 4$

**5) If ${}^{n}P_{r} = 1814400$ and ${}^{n}C_{r} = 45$, find $r$.**

**Sol: ${}^{n}P_{r} = 1814400$**

$\boldsymbol{{}^{n}C_{r} = 45}$

$\boldsymbol{{}^{n}P_{r} = \frac{n!}{(n-r)!}; \quad {}^{n}C_{r} = \frac{n!}{(n-r)!}}$

$\therefore \frac{n!}{(n-r)!} = \mathbf{1814400}$ **... (I)**

$\therefore \frac{n!}{r!(n-r)!} = 45$ ... (II)

Dividing (I) by (II),

$$\frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{1814400}{45}$$

$\therefore r! = 40320$

$\therefore r! = 8!$

$\therefore r = 8$

**6) If** $\boldsymbol{{}^nC_{r-1} = 6435, \quad {}^nC_r = 5005, \quad {}^nC_{r+1} = 3003}$, **find** $\boldsymbol{{}^rC_5}$

**Sol:** Given: ${}^nC_{r-1} = 6435, \quad {}^nC_r = 5005, \quad {}^nC_{r+1} = 3003$

$\therefore \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{6435}{5005}$

$\therefore \frac{\frac{n!}{[n-(r-1)]!(r-1)!}}{\frac{n!}{(n-r)!r!}} = \frac{13\times11\times9\times5}{13\times11\times7\times5}$

$\therefore \frac{(n-r)!r!}{(n-r+1)!(r-1!} = \frac{9}{7}$

$\therefore \frac{(n-r)!r(r-1)!}{(n-r+1)(n-r)!(r-1)!} = \frac{9}{7}$

$\therefore \frac{r}{n-r+1} = \frac{9}{7}$

$\therefore 7r \quad = 9n - 9r + 9$

$\therefore 16r - 9n = 9 \qquad \ldots(I)$

Also,

$\therefore \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5005}{3003}$

$\therefore \frac{\frac{n!}{(n-r)!r!}}{\frac{n!}{[n-(r+1)]!(r+1)!}} = \frac{5\times1001}{3\times1001}$

$\therefore \frac{(n-r-1)!(r+1)!}{(n-r)!r!} = \frac{5}{3}$

$\therefore \frac{(n-r-1)!(r+1)r!}{(n-r)(n-r-1)!r!} = \frac{5}{3}$

$\therefore \frac{r+1}{n-r} = \frac{5}{3}$

$\therefore 3(r+1) = 5(n-r)$

$\therefore 3r + 3 = 5n - 5r$

$\therefore 8r - 5n = -3 \quad \dots (II)$

Multiplying equations (II), by 2 and
Subtracting equation (I) from (III)

$16r - 10n = -6 \quad \dots$ (III)
$16r - 9n = 9 \quad \dots$ (I)

- ___________ $+ \qquad -$

$- \quad n = -15$

$\therefore n = 15$

Substituting $n = 15$ in equation (I)
$16r - 9(15) = 9$

$\therefore 16r - 135 = 9$

$\therefore 16r = 144$

$\therefore r = \frac{144}{16}$

$\therefore r = 9$

$\therefore {}^{r}C_5 = {}^{9}C_5 = \frac{9!}{4!5!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!}$

$\therefore {}^{r}C_5 = 126$

**7) Find the number of ways of drawing 9 balls from a bag that has 6 red balls, 5 green balls, and 7 blue balls so that 3 balls of every colour are drawn.**

**Sol:** Total number of balls = 6 + 5 + 7 = 18
Number of red balls = 6
Number of green balls = 5
Number of blue balls = 7

9 balls are to be drawn, 3 of each Colour,
No. of ways of drawing 3 red balls out of 6 red balls = 6C3
Similarly, No. of ways of drawing 3 green balls out of
5 green balls = 5C3
No. of ways of drawing 3 blue balls out of
7 blue balls = 7C3

∴ Total number of ways = 6C3 × 5C3 ×7C3

$= \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{7!}{3!4!}$

$= \frac{120}{6} \times \frac{20}{2} \times \frac{210}{6}$
= 7000

**8) Find the number 6f selecting a team of 3 boys and 2 girls from 6 boys and 4 girls.**

**Sol:** Number of boys = 6

∴ Number of ways of selecting 3 boys = 6C3
Number of girls = 4

∴ Number of ways of selecting 2 girls = 4C2
Hence, the number of ways of selecting a team of 3 boys and 2 girls

= 6C3 × 4C2

$= \frac{6!}{(6-3)!3!} \times \frac{4!}{(4-2)!2!}$

$= \frac{6!}{3!3!} \times \frac{4!}{2!2!}$

$= \frac{6\times5\times4\times3!}{(3\times2\times1)3!} \times \frac{4\times3\times2!}{(2\times1)2!}$

$= (5 \times 4) \times (2 \times 3)$

$= (20) \times (6)$

= 120

**9) After a meeting, every participant shakes hands with every .other participants.**

**If, the number of handshakes is 66, find the number of participants in the meeting.**

**Sol:** For shaking hands, minimum two persons are needed.
So, the total number of handshakes will be the same as the number of ways of selecting

2 persons from those who are present.
Let $n$ be the number of members present at the meeting.
Then, the number of ways of selecting any 2 persons from them = $^{n}C_2$
Now, in all 66 handshakes were exchanged.

$\therefore \quad ^{n}C_2 = 66$

$\therefore \frac{n!}{(n-2)!2!} = 66$

$\therefore \frac{n\times(n-1)\times(n-2)!}{(n-2)!} = 66 \times 2$

$\therefore n(n-1) = 132$

$\therefore n(n-1) = 12 \times 11$

$\therefore n = 12\ (or\ n-1 = 11)$

$\therefore$ The number of participants in the meeting = 12.

**10) If 20 points are marked on a circle, how many chords can be drawn?**

**Sol:** To draw a chord of a circle, we need two points lying on the circle.
There are 20 points on the circle.

$\therefore$ We can draw 20C2 chords of the circle.

Now, 20C2 $= \frac{20!}{(20-2)!2!} = \frac{20!}{18!2!}$
$= \frac{20\times19}{2} = 190$

190 chords can be drawn.
**11) Find the number of diagonals of an n-shaded polygon. In particular, find the number of diagonals when:**

**(I) $n = 10$ (II) $n = 15$ (III) $n = 12$**

**Sol:** Two points are needed to draw a segment.
A polygon of n sides has $n$ vertices.
So, in a polygon of n sides, there will be
$^nC_2$ segments, which include its sides and diagonals both.
Since the polygon has n sides, the number of its diagonals is $^nC_2 - n$

**(I)** Here, $n = 10$

∴ The number of diagonals = 10C2 − 10

$= \frac{10!}{(10-2)!2!} - 10$ $\quad = \frac{10\times9}{2} - 10$

$= 45 - 10$ $\quad = 35$

**(II)** Here, $n = 15$

∴ The number of diagonals = 15C2 − 15
$= \frac{15!}{(15-2)!2!} - 15$ $\quad = \frac{15\times14}{2} - 15$

$= 105 - 15$ $\quad = 90$

**(III)** Here, $n = 12$

∴ The number of diagonals = 12C2 − 12

$= \frac{12!}{(12-2)!2!} - 12$ $\quad = \frac{12\times11}{2} - 12$

$= 66 - 12$ $\quad = 54$

**12) There are 20 straight lines in a plane so that no two lines are parallel and no three lines are concurrent. Determine the number of points of intersection.**

**Sol:** Two coplanar lines which are not parallel intersect each other in a point.
There are 20 straight lines, no two of them 'are parallel and no three of them are concurrent. .
So, the number of points of intersection

$= 20C2 = \frac{20!}{(20-2)!2!} = \frac{20!}{18!2!}$

$= \frac{20\times19}{2} = 190$
190 points of intersection

**13) Ten points are plotted on a plane. Find the number of straight lines obtained by joining**

**these points if :**

**(i) No three points are collinear**

**(ii) Four points are collinear.**

**Sol:** To draw a line, two points are needed.

**(I)** There are 10 points in a plane such that no three of them are collinear. .
Hence, the number of lines formed
10C2 $= \frac{10!}{(10-2)!2!} = \frac{10\times 9}{2}$
$= 5 \times 9 \qquad = 45$
45 straight lines are obtained if no three points are collinear.

**(ii)** There are 10 points in a plane such that four points are collinear.
If no three of the given 10 points are collinear,
we will get 10C2 lines. But 4 points are collinear.
So, we will not get 4C2 lines from these points. Instead,
we get only one line containing the 4 points.

∴ Number of straight lines formed

$= \quad {}^{10}C_2 - \quad {}^{4}C_2 = \frac{10\times 9}{2} + \frac{4\times 3}{2} + 1$

$= 45 - 6 + 1 \quad = 40$

40 straight lines are obtained if four points are collinear

**14) Find the number of triangles formed by joining 12 points if:**

**(I) no three points are collinear (II) four points are collinear.**

Sol: **(I)** There are 12 points in a plane. No three of them are collinear.
We need three non-collinear points to 'form a triangle.

∴ The number of triangles formed

12C3 $= \frac{12!}{(12-3)!3!} = \frac{12\times 11\times 10}{3\times 2}$

$= 220$

**(II)** There are 12 points 1n a plane of which four points are collinear.
If no three points are collinear, we will get 1003 triangles. Since four points are collinear, the number of triangles will reduce by 403.

∴ The number of triangles formed

$= \quad {}^{12}C_3 - \quad {}^{4}C_3 = 220 - \frac{4\times3\times2}{3\times2} = 220 - 4$

$= 216$

**15) A word has 8 consonants and 3 vowels, How many distinct words can be formed if 4 consonants and 12 vowels are chosen?**

**Sol:** There are 3 consonants and 3 vowels.

So, 4 consonants and 2 vowels can be selected in 8C4 X 3C2 ways.
Now, 8C4 X 3C2 $= \frac{(8\times7\times6\times5)}{(4\times3\times2)} \times \frac{(3\times2\times1)}{(2\times10}$

$= 35 \times 2 \times 3$

$= 210$
Thus, there are 210 groups consisting of 4 consonants and 2 vowels.
We need to form different words from these 210 groups.
Now, each group has 6 letters.
These 6 letters can be arranged amongst themselves m 6! Ways.

∴ The number of required words

$= (210) \times 6! \quad = (210) \times 720$

$= 151{,}200$

***<u>EXERCISE 6.7</u>***

**1) Find $n$, if $\quad {}^{n}C_8 = \quad {}^{n}C_{12}$**

**Sol:** We have $\quad {}^{n}C_8 = \quad {}^{n}C_{12}$
But, $\quad {}^{n}C_{12} = \quad {}^{n}C_{n-12} \quad \ldots (\because \quad {}^{n}C_r = \quad {}^{n}C_{r-1})$

$\therefore \qquad {}^{n}C_8 = \quad {}^{n}C_{n-12}$

$\therefore \quad 8 = n - 12$

$\therefore \quad 8 + 12 = n$

$\therefore \quad 20 = n$

$\therefore \; n = 20$

**2) Find $n$, if $\;{}^{23}C_{3n} = {}^{23}C_{2n+3}$**

**Sol: We have $\;{}^{23}C_{3n} = {}^{23}C_{2n+3}$**

$\therefore \mathbf{3n = 2n + 3}$ ... ($\because$ ${}^{n}C_{x}$ **then** $x = y$)

$\therefore 3n - 2n = 3$

$\therefore n = 3$

**3) Find $n$, if $\;{}^{21}C_{6n} = {}^{21}C_{(n^2+5)}$**

*(*Answer differ from textbook)*

**Sol:** We have $\;{}^{21}C_{6n} = {}^{21}C_{(n^2+5)}$

$\therefore 6n = n^2 + 5$

$\therefore n^2 - 6n + 5 = 0$

$\therefore (n - 5)(n - 1) = 0$

If $n = 5$ , $\;{}^{21}C_{6n} = {}^{21}C_{30}$

Which is possible,

$\therefore n = 5$ is discarded.

Also $\;{}^{21}C_{6n} = {}^{21}C_{(n^2+5)}$

$\therefore \;{}^{21}C_{21-6n} = {}^{21}C_{(n^2+5)}$

$\therefore n^2 + 6n - 16 = 0$

$\therefore (n+8)(n-2) = 0$

$\therefore n+8 = 0 \quad or \quad n-2 = 0$

$\therefore n = -8 \quad or \quad n = 2$

But $n \in N$

$\therefore n = 2$

**4) Find $n$, if $^{2n}C_{r-1} = {}^{2n}C_{r+1}$**

**Sol:** $^{2n}C_{r-1} = {}^{2n}C_{r+1}$

But $^{2n}C_{r-1} = {}^{2n}C_{2n-r+1}$

$\therefore {}^{2n}C_{2n-r+1} = {}^{2n}C_{r+1}$

$\therefore 2n - r + 1 = r + 1$

$\therefore 2n = r + 1 + r - 1$

$\therefore 2n = 2r$

$\therefore n = r$

**5) Find $n$, if $^{n}C_{n-2} = 15$**

**Sol:** $^{n}C_{n-2} = 15$

$\therefore \frac{n!}{[n-(n-2)]!(n-2)!} = 15$

$\therefore \frac{n!}{2!(n-2)!} = 15$

$\therefore \frac{n(n-1)(n-2)!}{2 \times (n-2)!} = 15$

$\therefore n(n-1) = 15 \times 2$

$\therefore n(n-1) = 30$

$\therefore n(n-1) = 6 \times 5$

$\therefore n = 6\ (or\ n-1 = 5)$

**6) Find $x$ if ${}^{n}P_r = x\,{}^{n}C_r.$**

*(*Answer differ from textbook)*

**Sol:** ${}^{n}P_r = x\,{}^{n}C_r$

$\therefore \frac{n!}{(n-r)!} = x\frac{n!}{(n-r)!\,r!}$

$\therefore x = \frac{n!}{(n-r)!} \times \frac{(n-r)!\,r!}{n!}$

$\therefore x = r!$

**7) Find $r$ if ${}^{11}C_4 + {}^{11}C_5 + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r.$**

**Sol:** Use ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\therefore {}^{11}C_4 + {}^{11}C_5 + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r$

$\therefore ({}^{11}C_4 + {}^{11}C_5) + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r$

$\therefore ({}^{11}C_5 + {}^{12}C_6) + {}^{13}C_7 = {}^{14}C_r$

$\therefore {}^{13}C_6 + {}^{13}C_7 = {}^{14}C_r$

$\therefore {}^{14}C_7 = {}^{14}C_r$

$\therefore = r = 7$

**8) find the value of $\sum_{r=1}^{4} {}^{21-r}C_4 + {}^{17}C_5$**

**Sol:** $\sum_{r=1}^{4} {}^{21-r}C_4 + {}^{17}C_5$

$= {}^{20}C_4 + {}^{19}C_4 + {}^{18}C_4 + {}^{17}C_4 + {}^{17}C_5$

$= {}^{20}C_4 + {}^{19}C_4 + {}^{18}C_4 + {}^{18}C_5$

$= {}^{20}C_4 + {}^{19}C_4 + {}^{19}C_5$

$= {}^{20}C_4 + {}^{20}C_5$

$= {}^{21}C_5$

$\because {}^{n}C_r + {}^{n}C_{r+1}$

$= {}^{(n+1)}C_{(r+1)}$

**9) Find the differences between the largest values in the following:**

**(I)** $\boldsymbol{{}^{14}C_r}$ **and** $\boldsymbol{{}^{12}C_r}$

**(II)** $\boldsymbol{{}^{13}C_r}$ **and** $\boldsymbol{{}^{8}C_r}$

**(III)** $\boldsymbol{{}^{15}C_r}$ **and** $\boldsymbol{{}^{11}C_r}$

**Sol:** Use ${}^{n}C_r$ has maximum value if,

(a) $r = \frac{n}{2}$ when $n$ is even
(b) $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$ when $n$ is odd

**(I)** ${}^{14}C_r$ and ${}^{12}C_r$

Maximum value of ${}^{14}C_r$ occurs at $r = \frac{14}{2} = 7$
∴ Maximum value of

${}^{14}C_r = {}^{14}C_7$

$= \frac{14\times13\times12\times11\times10\times9\times8}{7\times6\times5\times4\times3\times2\times1}$

$= 3432$

Maximum value of $^{12}C_r$ occurs at $r = \frac{12}{2} = 6$

∴ Maximum value of

$^{12}C_r = {}^{12}C_6$

$= \frac{12\times11\times10\times9\times8\times7}{8\times7\times6\times5\times4\times3\times2}$

$= 924$

∴ The difference between the maximum values of

$^{14}C_r$ and $^{12}C_r = 3432 - 924$

$= 2508$

**(II)** $^{13}C_r$ and $^{8}C_r$

Maximum value of $^{13}C_r$ occurs at

$r = \frac{12}{2} = 6$ or $r = \frac{14}{2} = 7$

∴ Maximum value of

$^{13}C_r = {}^{13}C_6$

$= \frac{13\times12\times11\times10\times9\times8}{6\times5\times4\times3\times2\times1}$

$= 1716$

Maximum value of $^{8}C_r$ occurs at $r = \frac{8}{2} = 6$

∴ Maximum value of

$^{8}C_r = {}^{8}C_4$

$= \frac{8\times7\times6\times5}{4\times3\times2\times1}$

$= 70$

∴ The difference between the maximum values of

$^{13}C_r$ and $^{8}C_r = 1716 - 70$

$= 1646$

**(III)** $^{15}C_r$ and $^{11}C_r$

Maximum value of $^{15}C_r$ occurs at

$r = \frac{14}{2} = 7$ or $r = \frac{16}{2} = 8$

∴ Maximum value of

$^{15}C_r = {}^{15}C_7 = {}^{15}C_8$

$= \frac{15\times14\times13\times12\times11\times10\times9}{7\times6\times5\times4\times3\times2\times1}$

$= 6435$

Maximum value of $^{11}C_r$ occurs at

$r = \frac{10}{2} = 5$ or $r = \frac{12}{2} = 6$

∴ Maximum value of $^{11}C_r = {}^{11}C_5 = {}^{11}C_6$

$^{11}C_r = {}^{11}C_5 = \frac{11\times10\times9\times8\times7}{5\times4\times3\times2\times1}$

$= 462$

∴ The difference between the maximum values of

$^{15}C_r$ and $^{11}C_r = 6435 - 462$

$= 5973$

**10) In how many ways can a boy invite his 5 friends to a party so that at least three join the party?**

**Sol:** The number of ways of inviting at least three friends from 5 friends

$= {}^5C_3 + {}^5C_4 + {}^5C_5$

$= \frac{5!}{(5-3)!\,3!} + \frac{5!}{(5-4)!\,4!} + 1$

$= \frac{5\times4}{2\times1} + 5 + 1$

$= 10 + 6$

$= 16$

**11) A group consists of 9 men and 6 women. A team of 6 is to be selected. How many of possible selections will have at least 3 women?**

**Sol:** Number of men = 9
Number of women = 6
Number of persons in the team = 6
A team of 6 persons consisting of at least 3 women can be formed as follows:

(I) 3 women and 3 men or

(II) 4 women and 2 men or

(III) 5 women and 1 man or

(IV) 6 women

(I) 3 women and 3 men: The number of ways of forming the team

$= {}^6C_3 \times {}^9C_3 = \frac{6\times5\times4}{3\times2\times1} \times \frac{9\times8\times7}{3\times2\times1}$

$= 20 \times 84$

$= 1680$

(II) 4 women and 2 men: The number of ways of forming the team

$= {}^6C_4 \times {}^9C_2 = \frac{6\times5}{2\times1} \times \frac{9\times8}{2\times1}$

$= 15 \times 36$

$= 540$

(III) 5 women and 1 man: The number of ways of forming the team

$= {}^6C_5 \times {}^9C_1$

$= 6 \times 9$

$= 54$

(IV) 6 women: The number of ways of forming the team

$= {}^{6}C_{6} = 1$

∴ The total number of ways of forming the team

$= 1680+540+54+1$

$= 2275$

**12) A committee of 10 persons is to be formed from a group of 10 women and 8 men. How many possible committees will have at least 5 women? How many possible committees will have men in majority?**

**Sol:** Number of women = 10

Number of men = 8

Number of persons in the team = 10

A committee of 10 persons consisting of at least 5 women can be formed as follows:

(I) 5 women and 5 men or

(II) 6 women and 4 men or

(III) 7 women and 3 man or

(IV) 8 women and 2 man or

(V) 9 women and 1 man or

(IV) 10 women

The number of ways of forming the committee:
(I) 5 women and 5 men

$= {}^{10}C_{5} \times {}^{8}C_{5}$

$= \frac{10\times9\times8\times7\times6\times5\times4}{5\times4\times3\times2\times1} \times \frac{8\times7\times6}{3\times2\times1}$

$= (2 \times 9 \times 2 \times 7) \times (8 \times 7)$

$= 14112$

(II) 6 women and 4 men

$= {}^{10}C_6 \times {}^{8}C_4$

$= \frac{10\times9\times8\times7}{4\times3\times2\times1} \times \frac{8\times7\times6\times5}{4\times3\times2\times1}$

$= (5 \times 2 \times 3 \times 7) \times (2 \times 7 \times 5)$

$= 14700$

(III) 7 women and 3 men

$= {}^{10}C_7 \times {}^{8}C_3$

$= \frac{10\times9\times8}{3\times2\times1} \times \frac{8\times7\times6}{3\times2\times1}$

$= (5 \times 3 \times 8) \times (8 \times 7)$

$= 6720$

(IV) 8 women and 2 men

$= {}^{10}C_8 \times {}^{8}C_2$

$= \frac{10\times9}{2\times1} \times \frac{8\times7}{2\times1}$

$= (5 \times 9) \times (4 \times 7)$

$= 1260$

(V) 9 women and 1 men

$= {}^{10}C_9 \times {}^{8}C_1$

$= \frac{10}{1} \times \frac{8}{1}$

$= 80$

(VI) 10 women

= ${}^{10}C_{10}$

= 1
Hence, the number of ways of forming the required committee

= $14112 + 14700 + 6720 + 1260 + 80 + 1$

= 36873
For men to be in majority, the committee should have 6 or more men.
Following are the possibilities:

(I) 6 men and 4 women or

(II) 7 men and 3 women or

(III) 8 men and 2 women
The number of ways of forming the committee:

(I) 6 men and 4 women

= ${}^{8}C_{6} \times {}^{10}C_{4}$

$= \frac{8\times7}{1\times2} \times \frac{10\times9\times8\times7}{1\times2\times3\times4}$

= 5880

(II) 7 men and 3 Women

= ${}^{8}C_{7} \times {}^{10}C_{3}$

$= 8 \times \frac{10\times9\times8}{1\times2\times3}$

= 960

(III) 8 men and 2 women

= ${}^{8}C_{8} \times {}^{10}C_{2}$

$= 1 \times \frac{10\times9}{1\times2}$

=45

Hence, number of ways of forming the required committee

$= 5880 + 960 + 45$

$= 6885$

**13) A question paper has two sections, section I has 5 questions and section II has 6 questions. A student must answer at least two questions from each section among 6 questions he answers. How many different choices does the student have in choosing questions?**
**Sol:** A question paper has two sections.
Number of questions in section I = 5
Number of questions in section II = 6

At least 2 questions have to be selected from each

section and in all 6 questions are to be selected.
This can be done as follows:
(I) 2 questions (out of 5) from section I and
4 questions (out of 6) from section II are selected.
or (II) 3 questions from section I and
3 questions from section II are selected.
or (III) 4 questions from section I and
2 questions from section II are selected.
Now, number of selections in (I)

$= {}^5C_2 \times {}^6C_4$

$= \frac{5\times4}{2\times1} \times \frac{6\times5}{2\times1}$

$= 10 \times 15$

$= 150$

Number of selections in (II)

$= {}^5C_3 \times {}^6C_3$

$= \frac{5\times4}{2\times1} \times \frac{6\times5\times}{3\times2\times1}$

$= 10 \times 20$

$= 200$

Number of selections in (III)

$= {}^5C_4 \times {}^6C_2$

$= 5 \times \frac{6\times5}{2\times1}$

$= 75$

By addition principle, total number of required selections $= 150 + 200 + 75$
$= 425$

**14) There are 3 wicketkeepers and 5 bowlers among 22 cricket players. A team of 11 players is to be selected so that there is exactly one wicketkeeper and at least 4 bowlers in the team. How many different teams can be formed?**
**Sol:** Total number of cricket players = 22

Number of wicketkeepers = 3

Number of bowlers = 5
Remaining players = 14
A team of 11 players consisting of exactly one wicketkeeper and at least 4 bowlers can be selected as follows
(I) 1 wicketkeeper, 4 bowlers, 6 players
or (II) 1 wicketkeeper, 5 bowlers, 5 players
Now, number of selections in (I)

$= {}^3C_1 \times {}^5C_4 \times {}^{14}C_6$

$= 3 \times 5 \times \frac{14\times13\times12\times11\times10\times9}{6\times5\times4\times3\times2\times1}$

$= \frac{15\times14\times13\times2\times11\times3}{4}$

$= 45045$

Number of selections in (II)

$= {}^3C_1 \times {}^5C_5 \times {}^{14}C_5$

$= 3 \times 1 \times \frac{14\times13\times12\times11\times10}{5\times4\times3\times2\times1}$

$= 6006$

By a6dition principle, total number of ways choosing a team of 11 players

= 45045 + 6006

= 51051

15) Five students are selected from 11. How many ways can these students be selected if:

(I) two specified students are selected

(II) two specified students are not selected.

Sol: Number of students = 11
Number of students to be selected = 5
Here, 2 specified students are included.
So, we need to select 3 more students from the remaining 9 students.
This can be done in:
$= \ {}^9C_3$

$= \frac{9\times8\times7}{3\times2\times1}$

= 84ways

∴ Number of required selections = 84 × 1 × 1 = 84
Thus, 84 selections. can be made such .that 2 specified students are included.

(II) Here, 2 specified students are not included.
So, we need to select 5 students from the remaining 9 students.
This can be done in:

$= \ {}^9C_5$

$= \frac{9\times8\times7\times6}{4\times3\times2\times1}$

= 126 ways
Thus, 126 selections can be made such that 2 specified students are not included.