03

Motion in a straight line is also called rectilinear motion. It involves no change in direction so its study is relatively simple. Under this head, we will study motion of object either along *X*-axis or along *Y*-axis.

MOTION IN A STRAIGHT LINE

|TOPIC 1|
General Introduction of Motion

Mechanics is a branch of physics in which we study the motion of objects.

There are three branches of mechanics that examine the motion of an object such as

Statics It is a branch of mechanics in which we study the objects at rest. In statics, time factor does not play any role.

Kinematics It describes the motion of objects without looking at the cause of the motion. Here, time factor plays an important role.

Dynamics It relates the motion of objects to the forces which cause them. The time factor also plays an important role.

REST

If the position of an object does not change w.r.t. its surrounding with the passage of time, it is said to be at **rest**. e.g. Book lying on the table, a person sitting on a chair, etc.

CHAPTER CHECKLIST

- Rest
- Motion
- Point Object
- Scalar and Vector Quantities
- Position, Path Length and Displacement
- Velocity
- Acceleration
- Position-Time Graphs for Positive, Negative and Zero Acceleration
- Velocity-Time Graphs for an Accelerated Motion
- Kinematic Equations for Uniformly Accelerated Motion

MOTION

If the position of an object is continuously changing w.r.t. its surrounding, then it is said to be in the **state of motion**. Thus, motion can be defined as a change in position of an object with time. It is common to everything in the universe. In our daily life, we see a train moving on rails, the walking man, the crawling insects, water flowing down a dam, etc., showing that the object is in motion.

Types of Motion

On the basis of the nature of path followed, motion is classified as

Rectilinear Motion The motion in which a particle moves along a straight line is called **rectilinear motion.**

e.g. Motion of a sliding body on an inclined plane.

Circular Motion The motion in which a particle moves in a circular path is called **circular motion.**

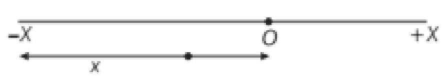
e.g. A string whirled in a circular loop.

Oscillatory Motion The motion in which a particle moves to and fro about a given point is known as **oscillatory motion.**

e.g. Simple pendulum

On the basis of the number of coordinates required to define the motion, motion is classified as

One-dimensional Motion The motion of an object is considered as 1-D if only one coordinate is needed to specify the position of the object.

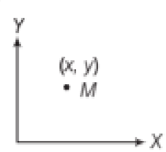


Motion in 1-D

In 1-D motion, the object moves along a straight line. In this type of motion, there are only two directions (backward and forward, upward and downward) in which an object moves and these directions are specified by + and – signs.

e.g. A boy running on a straight road, etc.

Two-dimensional Motion The motion of an object is considered as 2-D if two coordinates are needed to specify the position of the object. In 2-D motion, the object moves in a plane.

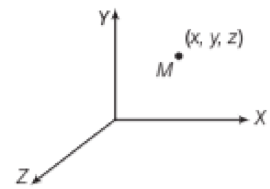


Motion in 2-D

e.g. A satellite revolving around the earth, etc.

Three-dimensional Motion The motion of an object is considered as 3-D if all the three coordinates are needed to specify the position of the object. This type of motion takes place in three-dimensional space.

e.g. Butterfly flying in garden, the motion of water molecules, etc.



Motion in 3-D

POINT OBJECT

An object is considered as point object if the size of the object is much smaller than the distance it moves in a reasonable duration of time.

e.g.

(i) A train under a journey of several hundred kilometres can be regarded as a point object.

(ii) Earth can be regarded as a point object for studying its motion around the sun.

The following example helps us to decide about a point object.

EXAMPLE |1| Body as a Point Object

In which of the following examples of motion, can the body be considered approximately a point object? **[NCERT]**

(i) A railway carriage moving without jerks between two stations.

(ii) A monkey sitting on the top of a man cycling smoothly on a circular track.

(iii) A spinning cricket ball that turns sharply on hitting the ground.

Sol. Any object can be considered as a point object if the distance travelled by it is very large in comparison to its dimensions.

(i) A railway carriage is moving without jerks between two stations, it means stations are at large distance, therefore railway carriage can be taken as a point object.

(ii) Man along with monkey is cycling smoothly which indicates that the distance travelled by the man is very large, therefore monkey can be taken as a point object.

(iii) The distance travelled by the ball is not so large therefore, spinning cricket ball cannot be taken as a point object.

SCALAR AND VECTOR QUANTITIES

Physical quantities are studied under two heads, i.e. scalars and vectors. Both types of quantities can be defined as follows

(i) **Scalars** If only the magnitude is required to specify a physical quantity, that physical quantity is known as **scalar quantity** or **scalars.**

e.g. Mass, length, time, speed, etc.

(ii) **Vectors** If magnitude as well as direction both are required to specify a physical quantity, that physical quantity is known as **vector quantity** or **vectors.**

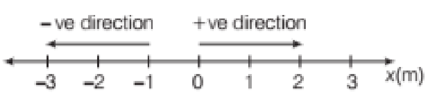
e.g. Displacement, velocity, acceleration, etc.

POSITION, PATH LENGTH AND DISPLACEMENT

In order to specify position, we need to use a reference point, a set of axes and a frame of reference.

Position

It is defined as the point where an object is situated. Position can be determined by the coordinate axis that is, marked in units of length and that has positive and negative directions.



Position in +ve and –ve direction

e.g. If an object is situated at – 1 m, then minus sign indicates that the position has negative direction but if the object is at 0 m position then it will be said to be at rest.

The position of the object can be specified with reference to a conveniently chosen origin. For motion in a straight line, position to the right of the origin is taken as positive and to the left as negative.

Frame of Reference

We choose a rectangular coordinate system of three mutually perpendicular axes as *x*, *y* and *z*. The point of intersection of these three axes is called **origin** (*O*) and considered as the **reference point**.

The *x*, *y*, *z*-coordinates describe the position of the object w.r.t. the coordinate system. To measure time, we need a clock. This coordinate system alongwith a clock constitutes a **frame of reference.**

So, the frame of reference is a coordinate system with a clock w.r.t. which, an observer can describe the position, displacement, acceleration of an object.

Frame of reference are of two types

(i) **Inertial frame of reference** These are the frame of reference in which, Newton's first law of motion is applicable.

(ii) **Non-inertial frame of reference** These are the frame of reference in which, Newton's first law of motion is not applicable.

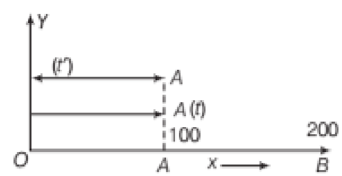
> ***Accelerated and Unaccelerated Frames***
>
> - Generally, accelerated frames (with respect to earth) are non-inertial and unaccelerated frames (with respect to earth) are inertial.
> - The frames at rest or in uniform motion are inertial and frames in non-uniform motion are non-inertial.
> - To apply Newton's second law in the non-inertial frames, we use the concept of pseudo force.

Path Length/Distance

The length of the path covered by an object in a given time-interval, is known as its **path length** or **distance travelled**. Its SI unit is metre.

e.g. Suppose an object moves along *x*-axis to a distance of 100 m from the origin *O* in time (t). Then, the path length is 100 m.

Now, if the object returns to the origin in time (t'), then the path length is $100 + 100 = 200$ m.

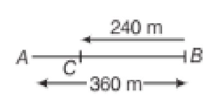


Path length of object

EXAMPLE |2| Total Path Length of a Scooter

A scooter is moving along a straight line *AB* covers a distance of 360 m in 24 s and returns back from *B to C* and coveres 240 m in 18 s. Find the total path length travelled by the scooter.

Sol. From the above question, we draw the following figure.



Total path length = $AB + BC = 360 + 240 = 600$ m

Displacement

The change in position of an object in a particular direction is termed as **displacement**, i.e. the difference between the final and initial positions of the object. It is denoted by Δx. Mathematically, it is represented by

$$\text{Displacement, } \Delta x = x_2 - x_1$$

where, x_1 and x_2 are the initial and final position of the object, respectively.

Cases

(i) If $x_2 > x_1$, then Δx is positive.
(ii) If $x_1 > x_2$, then Δx is negative.
(iii) If $x_1 = x_2$, then Δx is zero.

i.e. **the displacement of an object in motion can be positive, negative or zero.**

Displacement is a **vector quantity** as it possesses both, the magnitude and direction.

Note

- The magnitude of displacement may or may not be equal to the path length traversed by an object.
- The magnitude of the displacement for a course of motion may be zero but the corresponding path length will never be zero.

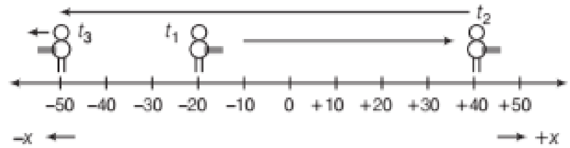
Differences between Distance and Displacement

S.No.	Distance	Displacement
1.	Length of the path is distance.	Magnitude of displacement is the length of the shortest possible path between initial and final positions.
2.	It is a scalar quantity.	It is a vector quantity.
3.	The distance covered by an object for some time interval cannot be zero.	The displacement of an object can be zero.
4.	The distance travelled by an object is always positive.	The displacement of an object can be positive, negative and zero

EXAMPLE |3| Motion of a Boy

A boy starts moving from −20 m towards $+x$-axis as shown in figure. He turns at time instant t_2 and starts moving towards $-x$-axis. At time t_3, he reached at −50 m as shown in the figure.

Find the displacement and distance for the time interval (i) t_1 to t_2 and (ii) t_1 to t_3.



For distance, we observe the actual path length and for displacement we observe the change in position.

Sol. (i) For t_1 to t_2,

Distance covered $= 20 + 40 = 60$ m

Displacement $= 40 - (-20)$
$= +60$ m (towards $+\ x$-axis)

(ii) For t_1 to t_3,

Distance covered $= 60 + 90 = 150$ m

Displacement $= -50 - (-20)$
$= -30$ m (towards $-\ x$-axis)

UNIFORM MOTION IN A STRAIGHT LINE

A body is said to be in a uniform motion if it travels equal distance in equal intervals of time along a straight line.

e.g. A vehicle running with the constant speed of 10 m/s will cover equal distances of 10 m in every second, so its motion will be uniform.

Note

For a uniform motion along a straight line in a given direction, the magnitude of the displacement is equal to the actual distance covered by the object.

Non-uniform Motion

A body is in non-uniform motion if it travels equal displacement in unequal intervals of time.

During the non-uniform motion, the speed of the body or its direction of motion or both change with time.

e.g. The velocity of the vehicle is different at different instants, so it has non-uniform motion.

SPEED AND VELOCITY

Speed

The distance covered by an object divided by the time taken by the object to cover that distance is called the **speed** of that object.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Speed is a **scalar quantity**. The speed of the object for a given interval of time is always positive.

Unit of speed The unit of speed in MKS (SI) is m/s and in CGS as cm/s.

Dimensional formula $[M^0LT^{-1}]$.

Uniform Speed

If an object is moving with a uniform speed, it means that it covers equal distance in equal interval of time.

Variable or Non-Uniform Speed

If an object is moving with a non-uniform speed, it means that it covers equal distances in unequal intervals of time.

Velocity

The rate of change of position of an object with time is called the **velocity** of that object.

i.e. $$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Velocity is a **vector** quantity.

Unit of velocity The unit of velocity is cms^{-1} in CGS and ms^{-1} in MKS or SI.

Dimensional formula The dimensional formula of velocity is $[M^0LT^{-1}]$.

Uniform Velocity

An object could have uniform velocity if it covers equal displacement in equal interval of time. If an object have equal displacement in equal interval of time, it means that it is moving with uniform velocity.

Non-uniform Velocity

If an object is moving with a non-uniform velocity, then it will have unequal displacements in equal interval of time.

Note The velocity of an object can be positive, zero and negative according to its displacement is positive, zero and negative.

Average Speed and Average Velocity

Average Speed

Average speed of an object is defined as the total distance travelled by the object divided by the total time taken.

$$\text{Average speed, } v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Average Velocity

Average velocity of a body is defined as the change in position or displacement (Δx) divided by the time interval (Δt) in which that displacement occur.

$\therefore$ Average velocity of the body is given by

$$\mathbf{v}_{av} = \frac{\Delta x}{\Delta t}$$

Average Velocity *vs* Average Speed

- For average speed, find net distance covered and divide it by time taken.
- Average velocity is net displacement divided by time taken.
- So, just find out the net displacement and divide it by time taken for that displacement.
- Average velocity could be zero or positive or negative but average speed is always positive for a moving body.

Instantaneous Speed and Instantaneous Velocity

Instantaneous Speed

Speed at an instant is defined as the limit of the average speed as the time interval (Δt) becomes infinitesimally small or approaches to zero.

Mathematically, instantaneous speed at any instant of time (t) is expressed as

$$\text{Instantaneous speed, } s_i = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \text{ or } s_i = \frac{ds}{dt}$$

[ds is the distance covered in time dt]

where, $\frac{ds}{dt}$ is the differential coefficient of s w.r.t. t.

EXAMPLE |4| Instantaneous Speed of the Particle

If the average speed of the particle is $[2t^2\,\hat{i} + 3t\,\hat{j}]$, then find out the instantaneous speed of the particle.

Sol. Given, position of the particle, $s = [2t^2\hat{i} + 3t\hat{j}]$

$$s_i = \frac{ds}{dt} = \frac{d}{dt}[2t^2\hat{i} + 3t\hat{j}]$$

Instantaneous speed of the particle is $s_i = 4t\,\hat{i} + 3\hat{j}$

Instantaneous Velocity

Velocity at an instant is defined as the limit of average velocity as the time interval (Δt) becomes infinitesimally small or approaches to zero.

Mathematically, instantaneous velocity at instant of time (t) is given by

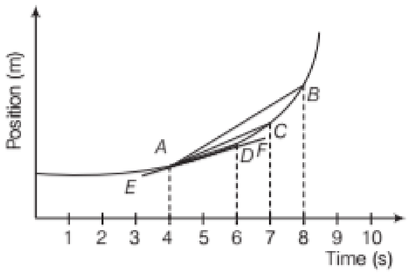
$$v_i = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \text{ or } v_i = \frac{dx}{dt}$$

where, dx is displacement for time dt.

Note $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ is called **differential coefficient** of displacement x w.r.t. time t. i.e. $\frac{dx}{dt}$.

The value of instantaneous velocity can be calculated graphically also as given below.

Suppose an object is moving along a straight line with variable velocity. Let the position-time graph of this motion is represented by a curve as shown in the figure.



Graphical interpretation of instantaneous velocity

From the graph, the average velocity of the object over the time interval $\Delta t = 4$ s, (i.e. from 4s to 8s) is the slope of straight line AB. If we decrease the interval of time Δt from 4s to 3s, then the line becomes AC. Similarly, at time interval Δt is 2s and the line becomes AD. As Δt approaches to zero, the point B approaches point A.

Thus, the line AB becomes the tangent of EF to the curve at D. Hence, the slope of the tangent with time axis gives the value of instantaneous velocity. Generally, we use numerical method to find the value of instantaneous velocity as the limiting process becomes clear.

Note The magnitude of instantaneous velocity is always equal to the instantaneous speed for a particular instant.

EXAMPLE | 5| Instantaneous velocity of a particle

The displacement (in m) of a particle moving along x-axis is given by $x = 18t + 15t^2$. Find the instantaneous velocity at $t = 0$ and $t = 2$s.

Sol. Given, Displacement,

$$x = 18t + 15t^2$$

Instantaneous velocity,

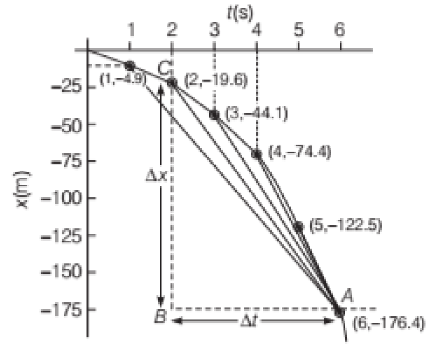
$$v_i = \frac{dx}{dt} = 18 + 30t$$

Instantaneous velocity at

$$t = 0,\ v = 18 + 30 \times 0 = 18 \text{ m/s}$$
$$t = 2\text{s},\ v = 18 + 30 \times 2 = 78 \text{ m/s}$$

EXAMPLE |6| Instantaneous velocity by graphical method

Find the instantaneous velocity at $t = 3.5$ s from the graph given below.



Sol. At $t = 3.5$,

We know that instantaneous velocity of a particle

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

In the words to calculate instantaneous velocity, we have to reduce Δt to approximately zero (i.e. infinitesimal small) and find the corresponding value of $\Delta x / \Delta t$. Following table will illustrate the process.

$\Delta t'$	t_1'	t_2'	$x'(t_1)$	$x'(t_2)$	Δx	$\Delta x/\Delta t$
2	4.5	2.5	−103	−34	69	34.5
1	4	3	86	56	30	30.0
.5	3.75	3.25	119	60	59	29.55
.1	3.55	3.45	78	64	14	29.52
.01	3.505	3.495	81	69	12	29.51

So, the value of $\frac{\Delta x}{\Delta t}$ tends to come close to the value of 29.5 when we reduce Δt to very small around 3.5. Therefore, instantaneous value of velocity at 3.5 will be 29.5 m/s. The value of $x(t_1)$ and $x(t_2)$ have been obtained from the given graph.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Which of the following statements is true for a car moving on the road?

(a) With respect to the frame of reference attached to the ground, the car is at rest
(b) With respect to the frame of reference attached to the person sitting in the car, the car is at rest
(c) With respect to the frame of reference attached to the person outside the car, the car is at rest
(d) None of the above

Sol. (b) For a car in motion, if we describe this event w.r.t. a frame of reference attached to the person sitting inside the car, the car will appear to be at rest as the person inside the car (observer) is also moving with same velocity and in the same direction as car.

2. The coordinates of object with respect to a frame of reference at $t = 0$ are $(-1, 0, 3)$. If $t = 5$ s, its coordinates are $(-1, 0, 4)$, then the object is in

(a) motion along z-axis (b) motion along x-axis
(c) motion along y-axis
(d) rest position between $t = 0$ s and $t = 5$ s

Sol. (a) Given, at $t = 0$ s, position of an object is $(-1,0,3)$ and at $t = 5$ s, its coordinate is $(-1,0,4)$. So, there is no change in x and y-coordinates, while z-coordinate changes from 3 to 4. So,the object is in motion along z-axis.

VERY SHORT ANSWER Type Questions

3. What is the condition for an object to be considered as a point object?

Sol. An object can be considered as a point object, if the distance travelled by it is very large than its size.

4. Can a tumbling beaker that has slipped off the edge of a table considered as a point object? **[NCERT]**

Sol. No, because the size of the beaker is not negligible as compared to the height of the table.

5. Does the displacement of an object depend on the choice of the position of origin of the coordinate system?

Sol. No, the displacement of the object does not depend on the choice of the position of the origin.

6. For which condition, the distance and the magnitude of displacement of an object have the same values?

Sol. The distance and the magnitude of displacement of an object have the same values, when the body is moving along a straight line path in a fixed direction.

7. Which speed is measured by speedometer of your scooter?

Sol. Instantaneous speed of the scooter is measured by the speedometer.

8. The position coordinate of a moving particle is given by $x = 6 + 18t + 9t^2$, where x is in metres and t in seconds. What is the velocity at $t = 2$s?

We know that, velocity is rate of change of displacement i.e. $v = \frac{dx}{dt}$

Sol. Given, $x = 6 + 18t + 9t^2$

$$v_t = \frac{dx}{dt} = 18 + 18t$$

At $t = 2$,

$$v_2 = 18 + 18 \times 2$$
$$= 54 \text{ m/s}$$

9. For which condition, the average velocity will be equal to the instantaneous velocity?

Sol. When a body moves with a uniform velocity, then

$$v_{av} = v_{inst}$$

SHORT ANSWER Type Questions

10. From the given example, find if the motion is one or two or three-dimensional.

(i) A kite flying in the sky
(ii) A cricket ball hit by a player
(iii) Moon revolving around the earth and
(iv) The motion of a stone in a circle

Sol. (i) A flying kite in the sky comes under **three**-dimensional motion.

(ii) A cricket ball hit by a player comes under **two**-dimensional motion.

(iii) Moon revolving around the sun-earth comes under **two**-dimensional motion.

(iv) The motion of the stone in circular motion comes under **two**-dimensional motion.

11. For what condition, an object could be considered as a point object? Describe in brief.

Sol. An object could be considered as a point object if it covers a distance much larger than its own size.

e.g. If a bus of 5 m in size move 100 km, then the bus can be considered as a point object.

12. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Determine how long the drunkard takes to fall in a pit 13 m away from the start. **[NCERT]**

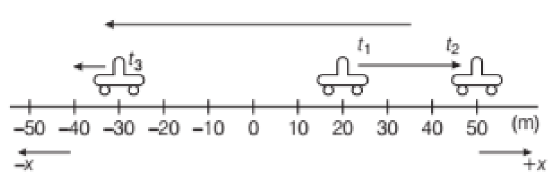
Sol. The effective distance travelled by drunkard in 8 steps $= 5 - 3 = 2$ m

Therefore, he takes 32 steps to move 8 m.

Now, he will have to cover 5 m more to reach the pit, for which he has to take only 5 forward steps.

Therefore, he will have to take $= 32 + 5 = 37$ steps to move 13 m. Thus, he will fall into the pit after taking 37 steps i.e. after 37 s from the start.

13. For the motion shown in the figure, find the displacement of car between the time intervals t_1 and t_3.



Sol. A car moving towards $+x$-axis. At time t_1 it is at +20 m. It turns at +50 m and starts moving towards $-x$-axis. At time t_3, it reached at −30 m.

The displacement of car in the interval $(t_3 - t_1) = \Delta t$ is $-30-(+20) = -50$ m (negative sign shows the direction of displacement is towards $-x$-axis).

14. The position x of a body is given by $x = A \sin(\omega t)$. Find the time at which the displacement is maximum.

Sol. The value of position x will be maximum, when the value of $\sin(\omega t)$ is maximum, for this

$$\sin(\omega t) = 1 = \sin \pi/2$$

or $$\omega t = \frac{\pi}{2} \Rightarrow t = \left(\frac{\pi}{2\omega}\right)$$

15. If the displacement of a body is zero, is distance necessarily zero? Answer with one example.

Sol. No, because the distance covered by an object is the path length of the path covered by the object. The displacement of an object is given by the change in position between the initial position and final position.

e.g. A boy starts from his home and moves towards market along a straight path. Then, he returns to home from the same path. Here, displacement is zero but distance is non-zero.

16. The position of an object is given by $x = 2t^2 + 3t$. Find out that its motion is uniform and non-uniform.

Sol. As given, $$x = 2t^2 + 3t$$

By differentiating x w.r.t. t, we get

Velocity, $$v = \frac{dx}{dt} = \frac{d}{dt}(2t^2 + 3t)$$

$$v = (4t + 3)$$

As velocity is time dependent, it means that motion is non-uniform.

17. The data regarding the motion of two different objects P and Q are given in the following table. Examine them carefully and state whether the motion of the objects is uniform or non-uniform.

Time	Distance travelled by object P (in m)	Distance travelled by object Q (in m)
9:30 am	10	12
9:45 am	20	19
10:00 am	30	23
10:15 am	40	35
10:30 am	50	37
10.45 am	60	41
11:00 am	70	44

Sol. We can see that the object P covers a distance of 10 m in every 15 min. In other words, it covers equal distance in equal intervals of time. So, the motion of object P is uniform. On the other hand, the object Q covers 7 m from 9:30 am to 9:45 am, 4 m from 9:45 am to 10:00 am and so on. In other words, it covers unequal distances in equal intervals of time. So, the motion of object Q is non-uniform.

18. Is earth inertial or non-inertial frame of reference?

Sol. Since, earth revolves around the sun and also spins about its own axis, so it is an accelerated frame of reference. Hence, earth is a non-inertial frame of reference.

However, if we do not take large scale motion such as wind and ocean currents into consideration, we can say that approximation the earth is an inertial frame.

19. A body is moving in a straight line along x-axis. Its distance from the origin is given by the equation $x = at^2 - bt^3$, where x is in metre and t is in second. Find its instantaneous speed at $t = 2$ s.

Sol. The given equation $x = at^2 - bt^3$

Instantaneous speed $$v = \frac{dx}{dt} = \frac{d}{dt}(at^2 - bt^3) = 2at - 3bt^2$$

At $t = 2$ s, $v = 4a - 12b$ m/s

20. The position of an object moving along x-axis is given by $x = a + bt^2$, where $a = 8.5$ m, $b = 2.5$ m and t is measured in seconds. What is its velocity at $t = 0$ s and $t = 2.0$ s? **[NCERT]**

Sol. We know that, $v = \frac{dx}{dt}$

On differentiating w.r.t. t, we get

$$v = \frac{d}{dt}(a + bt^2) = 2bt = 5t \text{ m/s} \quad [\because b = 2.5 \text{ m}]$$

At $t = 0$, $v = 0$, $t = 2$ s and $v = 10$ m/s

LONG ANSWER Type I Question

21. No distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Sol. Instantaneous speed (v_{ins}) of the particle at an instant is the first derivative of the distance with respect to time at that instant of time i.e. $v_{ins} = \frac{dx}{dt}$.

Since, in instantaneous speed, we take only a small interval of time (dt) during which direction of motion of a body is not supposed to change, hence there is no difference between total path length and magnitude of displacement for small interval of time dt.

Hence, instantaneous speed is always equal to magnitude of instantaneous velocity.

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Question

1. The displacement of a car is given as – 240 m. Here, negative sign indicates
(a) direction of displacement
(b) negative path length
(c) position of car is at point whose coordinate is – 120
(d) no significance of negative sign

Answer

1. (a)

SHORT ANSWER Type Question

2. A particle moves from one position to another position to the left of the origin in a straight line. Can the displacement of a particle be positive? Explain.

LONG ANSWER Type I Questions

3. What do you understand by non-uniform motion? Explain instantaneous velocity of an object in one-dimensional motion.

4. The displacement x of a particle moving in one dimension is related to time t by the relation $x = \sqrt{2t^2 - 3t}$, where x is in metre and t in second. Find the displacement of the particle when its velocity is zero. $\left[\textbf{Ans.} \sqrt{-\frac{9}{8}}\ \text{m}\right]$

|TOPIC 2| Uniformly Accelerated Motion

ACCELERATION

The existence of acceleration was given by Galileo in his different thoughts. **Acceleration of a body can be expressed as the rate of change of velocity with time.** By acceleration, we can understand that how fast or slow the velocity of an object is changing. Acceleration is a **vector quantity**.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Its SI unit is metre per second square (m/s^2) and CGS is cm/s^2. The dimensional formula is $[M^0L^1T^{-2}]$.

Note

- Acceleration therefore, may result from a change in speed (magnitude), a change in direction or changes in both.
- If the signs of velocity and acceleration is same (both positive or both negative), the body will accelerate and when the signs of velocity and acceleration are opposite, it means that the body is retarding.

EXAMPLE |1| Retarding Bus

Starting from a stationary position, a bus attains a velocity of 6 m/s in 30 s. Then, the driver of the bus applies a brake such that the velocity of the bus comes down to 4 m/s in the next 5 s. Calculate the acceleration of the bus in both the cases.

Sol. **Case I** Initial velocity of the bus, $u = 0$

Final velocity, $v = 6$ m/s; Time taken, $t = 30$ s

$$\therefore \text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$= \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken}} = \frac{v-u}{t} = \frac{6-0}{30}$$

$$= 0.2\ \text{m/s}^2$$

Case II Initial velocity, $u = 6$ m/s

Final velocity, $v = 4$ m/s

$$\therefore \quad \text{Acceleration} = \frac{v-u}{t} = \frac{4-6}{5} = -0.4\ \text{m/s}^2$$

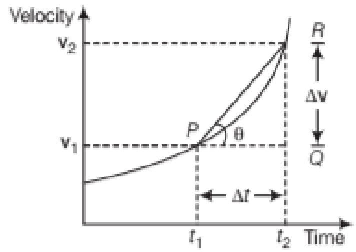
Thus, the acceleration in both the cases are $0.2\ \text{m/s}^2$ and $-0.4\ \text{m/s}^2$

Average Acceleration

The average acceleration over a time interval is defined as **the change in velocity divided by the time interval.**

Suppose that at any time (t_1) a body has velocity $(\mathbf{v}_1)$. At a later time (t_2), it has velocity $(\mathbf{v}_2)$. Thus, Change in velocity, $\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$

Time during which velocity has changed, $\Delta t = t_2 - t_1$



Velocity-time graph to calculate average acceleration

$$\mathbf{a}_{av} = \text{slope of straight line } PR = \frac{RQ}{PQ}$$

$$\text{Average acceleration, } \mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1}$$

The slope of velocity-time graph gives acceleration.

Instantaneous Acceleration

It is defined as the acceleration of a body at a certain instant or the limiting value of average acceleration when time interval becomes very small or tends to zero.

So, $$\text{Instantaneous acceleration, } \mathbf{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

where $\frac{d\mathbf{v}}{dt}$ is the differential coefficient of **v** w.r.t. t.

Elementary Concept of Integration for Describing Motion

We know that, $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$ or $v\frac{dv}{ds}$

If the displacement is given and we have to find the velocity and acceleration, then we use differentiation. If the acceleration is given and we have to find the velocity and displacement, then we use integration. To find the average value of square of velocity

$$\overline{v^2} = \frac{\int_{t_1}^{t_2} v^2 dt}{\int_{t_1}^{t_2} dt} \Rightarrow \overline{v^4} = \frac{\int_{t_1}^{t_2} v^4 dt}{\int_{t_1}^{t_2} dt}$$

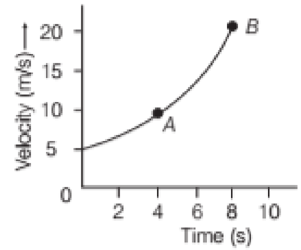
If velocity is a function of displacement, $v = f(x)$, for average of v from $x = x_1$ to $x = x_2$

$$\Rightarrow \quad \bar{v} = \frac{\int_{x_1}^{x_2} v\, dt}{\int_{x_1}^{x_2} dt}$$

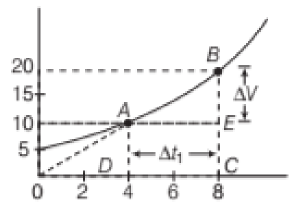
The above procedure can be applied to find the average value of any quantity like velocity, acceleration, force, etc.

EXAMPLE |2| Calculation of Average Acceleration

From the figure given, find the average acceleration between points A and B.



Sol. We draw the graph as



$$\text{Average acceleration, } \mathbf{a}_{av} = \text{slope of line } AB = \frac{BE}{AE}$$

$$BE = \Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = 20 - 10 = 10 \text{ m/s}$$

$$AE = \Delta t = t_2 - t_1 = 8 - 4 = 4 \text{ s}$$

$$\therefore \quad \mathbf{a}_{av} = \frac{10}{4} = 2.5 \text{ m/s}^2$$

EXAMPLE |3| Calculation of instantaneous acceleration

The velocity of a particle is given by $\mathbf{v} = 2t^2 - 3t + 10$ m/s. Find the instantaneous acceleration at $t = 5$ s.

Sol. Given, $\mathbf{v} = 2t^2 - 3t + 10$ m/s

$$\mathbf{a}_{in} = \frac{d\mathbf{v}}{dt} = 4t - 3 \text{ m/s}^2$$

If $t = 5$, $\mathbf{a}_{in} = 5 \times 4 - 3 = 17 \text{ m/s}^2$

EXAMPLE |4| Average and Instantaneous Acceleration

A particle is moving in a straight line. Its displacement at any instant t is given by $x = 10t + 15t^3$, where x is in metres and t is in seconds. Find

(i) the average acceleration in the interval $t = 0$ to $t = 2$ s and

(ii) instantaneous acceleration at $t = 2$ s.

Sol. Given equation, $x = 10t + 15t^3$

and the variables are (i) $t = 0$ to $t = 2$s (ii) $t = 2$ s

Velocity of particle, $\mathbf{v} = \frac{dx}{dt}$

$$\mathbf{v} = \frac{d}{dt}(10t + 15t^3) = 10 + 45t^2$$

At $t = 0$, $\mathbf{v}_0 = 10 + 45(0) = 10$ m/s,

At $t = 2$ s, $\mathbf{v}_2 = 10 + 45 \times (2)^2$

$= 10 + 180 = 190$ m/s

$$\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_0 = 190 - 10 = 180 \text{ m/s}$$

$$\Delta t = 2 - 0 = 2 \text{ s}$$

$$\therefore \quad \mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{180}{2} = 90 \text{ m/s}^2$$

$$\mathbf{a} = \frac{d}{dt}(10 + 45t^2) = 90t$$

At $t = 2$ s, $\mathbf{a} = 90 \times 2 = 180 \text{ m/s}^2$

The instantaneous acceleration of a particle at 2s is 180 m/s^2.

Uniform Acceleration

If an object is moving with uniform acceleration, it means that the change in velocity is equal for equal intervals of time.

Non-uniform Acceleration

If an object has variable or non-uniform acceleration, it means that the change in velocity is unequal in equal intervals of time.

EXAMPLE |5| Uniform Acceleration

The displacement x of a particle varies with time t as $x = 4t^2 - 15t + 25$.

(i) Find the position, velocity and acceleration of the particle at $t = 0$.

(ii) Can we call the motion of the particle as one with uniform acceleration?

Sol. (i) Given position, $x = 4t^2 - 15t + 25$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(4t^2 - 15t + 25) = 8t - 15$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(8t - 15) = 8$

At time $t = 0$, we have

$x = 4t^2 - 15t + 25$

$= 4(0) - 15(0) + 25 = 25$ m

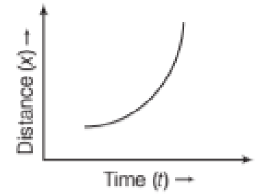
$v = 8t - 15 = 8(0) - 15$

$= -15$ m/s and $a = 8 \text{ m/s}^2$

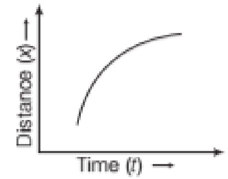
(ii) Yes, the particle has a uniform acceleration because it does not depend on time t.

POSITION-TIME GRAPHS FOR POSITIVE, NEGATIVE AND ZERO ACCELERATION

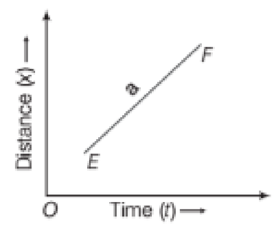
(i) **Positive Acceleration** When distance covered by a moving object goes on increasing with time, the object is said to have positive acceleration.



(ii) **Negative Acceleration** When distance covered by a moving object goes on decreasing with time, the object is said to have negative acceleration.

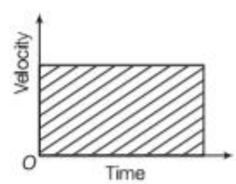


(iii) **Zero Acceleration** When the moving object covers equal distance in equal time, the object is said to have zero acceleration.



VELOCITY-TIME GRAPHS FOR AN ACCELERATED MOTION

(i) **Zero Acceleration** In case of zero acceleration, the velocity of the object does not change with time.

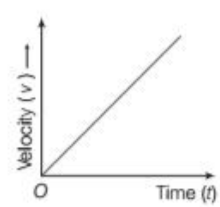


Note

The area under v-t graph gives displacement.

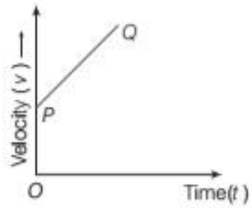
(ii) **Positive Acceleration**

(a) If the object is moving with positive acceleration having zero initial velocity, then **the velocity-time graph is a straight line starting from origin.**

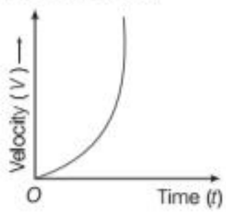


In case of positive acceleration, the velocity of the object goes on increasing with time.

(b) If the object is moving with positive acceleration having some initial velocity, then **the velocity- time graph is a straight line starting from *P*.**



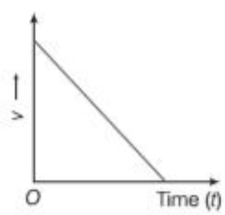
(iii) **Increasing Acceleration** In case of increasing acceleration, the velocity of the object goes on increasing exponentially (non-linearly). If the object is moving with increasing acceleration having zero initial velocity. The slope of *v*-*t* graph gives the **instantaneous acceleration.**



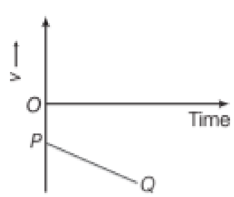
(iv) **Negative Acceleration**

(a) In case of negative acceleration, the velocity of the object decreases linearly with time.

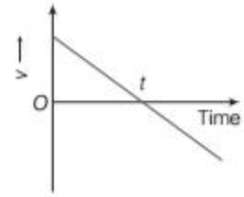
If the object is moving with negative acceleration, have some positive initial velocity, then **the velocity-time graph is a straight line having negative slope.**



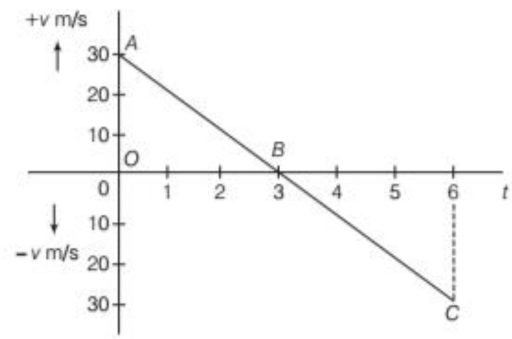
(b) If the object is moving with negative acceleration have some negative initial velocity, then **the velocity-time graph is a straight line starting from *P* as point of negative velocity.**



(c) If the object is moving with negative acceleration have some positive initial velocity, the direction of its motion can change at time (*t*). **The slope of velocity-time graph will remain constant for uniform acceleration.**



EXAMPLE |6| Analysis of Velocity-Time Graph



With the help of above velocity-time graph, find the

(i) displacement in first three seconds and

(ii) acceleration for the above graph.

Area under the *v*-*t* graph gives the measurement of displacement. Thus, displacement = Area under *v*-*t* graph.

Sol. (i) Displacement in first three seconds = Area of ΔOAB

$$= \frac{1}{2}(OB)\times(OA) = \frac{1}{2}(3)\times(+30) = +45\,\text{m}$$

(ii) Acceleration = Slope of *v*-*t* graph

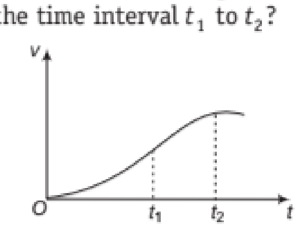
As, *v*-*t* graph is a straight line. So, consider the slope of line *AB*.

$\therefore$ Slope of line $AB = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-30}{3} = -10\,\text{m/s}^2$

So, the acceleration is negative.

EXAMPLE |7| Uniform and Non-uniform Accelerated Motion of a Particle

The velocity-time graph of a particle in one-dimensional motion is shown in figure. Which of the following formulae are correct for describing the motion of the particle over the time interval t_1 to t_2?



(i) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + \frac{1}{2}a\,(t_2 - t_1)^2$

(ii) $v(t_2) = v(t_1) + a\,(t_2 - t_1)$

(iii) $v_{av} = \left[\frac{x(t_2) - x(t_1)}{(t_2 - t_1)}\right]$ (iv) $a_{av} = \frac{[v(t_2) - v(t_1)]}{(t_2 - t_1)}$

(v) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + \frac{1}{2}a_{av}(t_2 - t_1)^2$

(vi) $x(t_2) - x(t_1)$ = Area under v-t curve bounded by the t-axis and the dotted line shown. **[NCERT]**

Sol. The slope of the given graph over the time interval t_1 to t_2 is not constant and is not uniform. It means acceleration is not constant or uniform, therefore relations (*i*), (*ii*) and (*v*) are not correct which is uniform accelerated motion, but relations (*iii*), (*iv*) and (*vi*) are correct, because these relations are true for both uniform or non-uniform accelerated motion.

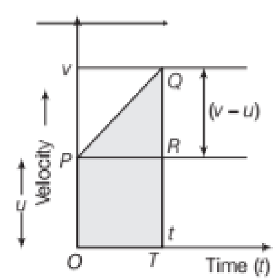
KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

If the change in velocity of an object in each unit of time is constant, then object is said to be moving with constant acceleration and such a motion is called **uniformly accelerated motion.**

If an object moves along a straight line with a constant acceleration a. Let u be the initial velocity at $t = 0$ and v be the final velocity of the object after time (t).

(i) Velocity-Time Relation

The slope of velocity-time graph gives the acceleration of the object.



$\therefore \quad a$ = Slope of PQ

$$a = \frac{QR}{PR} = \frac{v-u}{t}$$

$$\Rightarrow \quad a = \frac{v-u}{t}$$

$$\Rightarrow \quad v - u = at$$

Velocity-Time relation, $v = u + at$

(ii) Position-Time Relation

The area under the velocity-time graph gives the displacement.

Displacement of an object in time interval t is given by

x = Area of ΔPQR + Area of rectangle $OPRT$

$$= \frac{1}{2}QR \times PR + OP \times PR$$

$$= \frac{1}{2}(v-u)t + ut$$

$$= \frac{1}{2}(at)t + ut \qquad [\because v - u = at]$$

$$= \frac{1}{2}at^2 + ut$$

$$\Rightarrow \quad x = \frac{1}{2}at^2 + ut$$

Position-time relation $x = ut + \frac{1}{2}at^2$

(iii) Position-Velocity Relation

Again the displacement in time interval (t) is given by

x = area of trapezium $OPQT$.

$$= \frac{1}{2}(OP + QT) \times OT$$

$$x = \frac{1}{2}(u+v) \times t \qquad \ldots(i)$$

From $\quad v - u = at$

$$t = \frac{v-u}{a} \qquad \ldots(ii)$$

On substituting the value of t in Eq. (i), we get

$$x = \frac{1}{2}(u+v) \times \frac{(v-u)}{a}$$

$$\Rightarrow \quad v^2 - u^2 = 2ax$$

$\Rightarrow$ Position-velocity relation, $v^2 = u^2 + 2ax$

Displacement of the Particle in nth Second

Let an object is moving with initial velocity u under constant acceleration a.

To find the displacement in nth second, we subtract the position of particle at $(n-1)$th second from the position of the particle at nth second.

Displacement in nth second $= s_n - s_{n-1}$

$$s\,(n\text{th}) = \left[un + \frac{1}{2}a(n^2)\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n-1)^2$$

$$= u + \frac{a}{2}[n^2 - (n-1)^2]$$

$$= u + \frac{a}{2}(n^2 - n^2 - 1 + 2n)$$

Displacement in the nth second, $s(n\text{th}) = u + \frac{a}{2}(2n - 1)$

Above expression shows displacement in the nth second.

Steps to Solve Problems Based on Accelerated Motion

The following steps are recommended for solving problem involving accelerated motion.

Step I Make sure all the units in the problem are consistent. That is, if distances are measured in meters, be sure that velocities have units of meters per second and accelerations have units of meters per second square.

Step II Make a list of all the quantities given in the problem and a separate list of those to be determined.

Step III Think about what is going on physically in the problem and then select from the list of kinematic equations the one or ones that will enable you to determine the unknowns.

EXAMPLE |8| Mixture of Acceleration and Retardation

A motor car starts from rest and accelerates uniformly for 10s to a velocity of 20 m/s. After that car runs at a constant speed and is finally brought to rest in 40m with a constant aceleration. Total distance covered is 640 m. Find the value of acceleration, retardation and total time taken.

Sol. Let x_1, x_2 and x_3 be distances covered in three parts of the motion.

For first part of the motion, we have

$$u = 0,\ t = 10\text{s},\ v = 20 \text{ m/s}$$

As $v = u + at$

$\therefore$ $20 = 0 + a \times 10$

Acceleration, $a = 2 \text{ m/s}^2$

Distance, $x_1 = ut + \frac{1}{2}at^2$

$$= 0 \times 10 + \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ m}$$

For second part of the motion, we have

$$x_1 = 100\text{m},\ x_3 = 40\text{m}$$

As $x_1 + x_2 + x_3 = 640$

$\therefore$ $100 + x_2 + 40 = 640$

or $x_2 = 500\text{m}$

This distance is covered with a uniform speed of 20 m/s.

$\therefore$ Time taken $= \frac{500}{20} = 25$ s

For third part of the motion, we have

$$u = 20 \text{ m/s},\ v = 0,\ x = x_3 = 40\text{m}$$

As $v^2 - u^2 = 2ax$

$\therefore$ $(0)^2 - (20)^2 = 2 \times a \times 40$

or $a = -\frac{400}{80} = -5\text{m/s}^2$

$\therefore$ Retardation $= 5\text{m/s}^2$

Time taken, $t = \frac{v - u}{a} = \frac{0 - 20}{-5} = 4\text{s}$

Total time taken $= 10 + 25 + 4 = 39$ s

EXAMPLE |9| Equations of Motion by Calculus Method

Obtain equation of motion for constant acceleration using method of calculus. **[NCERT]**

Sol. From the definition of average acceleration,

$$a = \frac{dv}{dt} \Rightarrow dv = a\ dt$$

Integrating both sides and taking the limit for velocity u to v and for time 0 to t.

$$\int_u^v dv = \int_0^t a\ dt = a\int_0^t dt = a[t]_0^t \qquad [\because a \text{ is constant}]$$

$$v - u = at$$

$$v = u + at$$

Now, from the definition of velocity, $v = \frac{dx}{dt} \Rightarrow dx = v dt$

Integrating both sides and taking the limit for displacement x_0 to x and for time 0 to t.

$$\int_{x_0}^x dx = \int_0^t v\,dt = \int_0^t (u + at)dt = v_0[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$x - x_0 = ut + \frac{1}{2}at^2$$

$$x = x_0 + ut + \frac{1}{2}at^2$$

Now, we can write

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

or $v\ dv = a\ dx$

Integrating both sides and taking the limit for velocity u to v and for displacement x_0 to x.

$$\int_u^v v\ dv = \int_{x_0}^x a\ dx$$

$$\frac{v^2 - u^2}{2} = a(x - x_0)$$

$$v^2 = u^2 + 2a(x - x_0)$$

This method is also be used for motion with non-uniform acceleration.

Non-uniformly Accelerated Motion

When acceleration of particle is not constant or acceleration is a function of time, then following relations hold for one-dimensional motion.

(i) $v = \frac{ds}{dt}$ (ii) $a = \frac{dv}{dt} = v\frac{dv}{dx}$

(iii) $ds = v\,dt$ and (iv) $dv = a\,dt$ or $vdv = adx$

EXAMPLE |10| Non-uniformaly Accelerated Motion

Two particles move along x-axis. The position of particle 1 is given by $x = 6.00t^2 + 3.00t + 2.00$ (in metre and in seconds); acceleration of particle 2 is given by $a = -8.00\,t$ (in m/s^2 and seconds) and at $t = 0$, its velocity is 20 m/s. When the velocities of the particles match, find their velocities.

Sol. List all informations about particle 1 and particle 2.

For particle 1,

$$x_1 = 6.00\,t^2 + 3.00\,t + 2.00$$

For particle 2,

$$a_2 = -8.00\,t,$$
$$v_2 = 20 \text{ m/s at } t = 0 \text{ s}$$

We know that, $v(t) = \frac{dx}{dt}$

$$v(t) = 12\,t + 3 + 0$$
$$v(t) = 12\,t + 3$$

For particle 2,

We know that, $\int a(t)dt = v(t)$

$$\Rightarrow \quad v(t) = \int -8.00\,dt = -8\int t dt = \frac{-8\,t^2}{2} + C$$

$$v(t) = -4\,t^2 + C$$

If $v(t) = 20$ m/s and $t = 0$s, then

$$20\text{m/s} = -4\,(0)^2 + C$$

$$\Rightarrow \quad 20 = 0 + C \;\Rightarrow\; C = 20$$

So, $v_2(t) = -4t^2 + 20$

Since, the particles' velocities have to match, you have to set the two equations equal to each other

$$v_1(t) = v_2(t)$$
$$12\,t + 3 = -4\,t^2 + 20$$
$$4\,t^2 + 12\,t - 17 = 0$$

Since, you could not factor out, you probably know to use the quadratic formula

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{(12)^2 - 4(4)(-17)}}{2(4)}$$

$$= \frac{-12 \pm \sqrt{144 + 272}}{8}$$

$$t = \frac{-12 \pm \sqrt{416}}{8} = \frac{-12 \pm 20.3}{8}$$

$$t = -4.04 \text{ s or } t = 1.04 \text{ s}$$

As, time cannot be negative, $t = 1.04$ s.

Find the velocities of different particles.

$$v(t_1) = 12\,(1.04) + 3 = 15.48 \text{ m/s}$$
$$v(t_2) = -4\,(1.04)^2 + 20 = 15.67 \text{ m/s}$$

EQUATIONS OF MOTION FOR THE MOTION OF AN OBJECT UNDER GRAVITY

When an object is thrown upwards or fall towards the earth, then its motion is called **motion under gravity** because in these conditions (upward or downward), the acceleration of the body is equal to the acceleration due to gravity.

In case of motion under gravity, the equations of motion are given below

$$v = u + (\pm g)\,t \quad \text{...(i)}$$

$$h = ut + \frac{1}{2}(\pm g)\,t^2 \quad \text{...(ii)}$$

$$v^2 = u^2 + 2(\pm g)\,h \quad \text{...(iii)}$$

In case of upward motion, acceleration due to gravity (g) is taken as **positive** and for downward motion, g is taken as **negative**.

Feeling Weightlessness

There is a misconception that in free fall, no force acted on us since we are not able to feel our weight. Actually we feel our weight due to the reaction we get from earth. As in free fall, we are not pushing anything, so we are not getting any reaction (Newton's third law) and feeling weightlessness.

EXAMPLE |11| Free Fall

Discuss the motion of an object under free fall. Neglect air resistance. **[NCERT]**

Sol. An object released near the surface of the earth is accelerated downward under the influence of the force of gravity. The magnitude of acceleration due to gravity is represented by g. If air resistance is neglected, the object is said to be in **free fall**. If the height through which the object falls is small as compared to the earth's radius, g can be taken constant, equal to 9.8 ms^{-2}. Free fall is thus a case of motion with uniform acceleration.

We assume that the motion is in y-direction, more correctly in $(-y)$ direction because we choose upward direction as positive. Since, the acceleration due to gravity is always downward, it is in the negative direction and we have

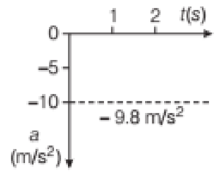
$$a = -g = -9.8 \text{ m s}^{-2}$$

The object is released from rest at $y = 0$. Therefore, $u = 0$ and the equations of motion become

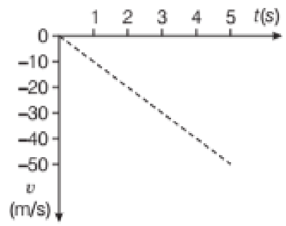
$$v = 0 - gt = -9.8t\ \text{ms}^{-1}$$
$$h = 0 - 1/2gt^2 = -4.9t^2\ \text{m}$$
$$v^2 = 0 - 2gh = -19.6h\,\text{m}^2\text{s}^{-2}$$

These equations give the velocity and the distance travelled as a function of time and also the variation of velocity with distance.

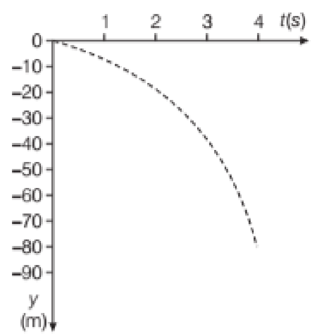
The variation of acceleration, velocity and distance with time have been plotted in
Fig. (a), (b) and (c).



(a) Variation of acceleration with time



(b) Variation of velocity with time



(c) Variation of distance with time

Note Here, magnitude of displacement and distance is equal.

EXAMPLE |12| Motion of a Ball under Gravity

A ball is thrown vertically upwards with a velocity of 20 m/s from the top of a multi storey building. The height of the point from where the ball is thrown 25 m from the ground.

(i) How high will the ball rise?

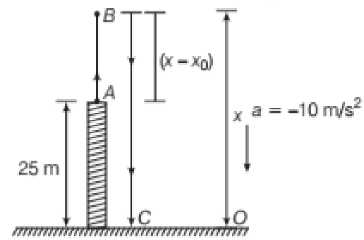
(ii) How long will it be before the ball hits the ground? Take, $g = 10\ \text{m/s}^2$ **[NCERT]**

Sol. (i) Consider vertical upward motion of ball upto highest point, we have

$$u = 20\ \text{m/s},\ v = 0,$$
$$a = -10\ \text{m/s}^2,\ t = t_1,$$
$$x = ?\ \text{(say)}$$

Using theorem, $v^2 = u^2 + 2ax$

$$x = \frac{v^2 - u^2}{2a} = \frac{0 - (20)^2}{2 \times (-10)} = 20\,\text{m}$$



(ii) As shown in figure, we have upward motion A to B in time (t_1) and downward motion (B to C) in time (t_2).

Velocity at B is zero

$$\therefore \quad v = u - gt$$
$$0 = 20 - 10\,t_1 \Rightarrow t_1 = 2\text{s}$$

From the top, the ball falls freely under acceleration due to gravity.

$$\therefore \quad x = x_0 + ut + \frac{1}{2}gt_2^2$$
$$x = 0,\ x_0 = 45\ \text{m}, u = 0 \text{ and } g = 10\ \text{m/s}^2$$
$$0 = 45 + 0 + \frac{1}{2}(-10)\,t_2^2 = 45 - 5t_2^{\ 2}$$
$$t_2^{\ 2} = 9 \Rightarrow t_2 = 3\text{s}$$

$\therefore$ Total time taken by the ball to hit the ground

$$t = t_1 + t_2 = 2 + 3 = 5\ \text{s}$$

Method II

In this method we shall consider net displacement = Final position − Initial position = 25m.

Using the formula $\quad x = ut + \frac{1}{2}gt^2$

$$\Rightarrow \quad -25 = +20t - \frac{1}{2}gt^2 = +20t - 5t^2$$
$$\Rightarrow \quad t^2 - 4t - 5 = 0$$
$$(t - 5)(t + 1) = 0 \Rightarrow t = 5\ \text{s}$$

Galileo's Law of Odd Numbers

Galileo was the first to make quantitative studies of free fall. This law states that the distances traversed by a freely falling body during equal intervals of time stand to one another in the same ratio as the odd numbers beginning with unity, i.e. 1 : 3 : 5 : 7.

EXAMPLE |13| Prove Galileo's Law of Odd Numbers

Sol. Let us divide the time interval of motion of an object under free fall into many equal intervals τ and find out the distances traversed during successive intervals of time. Since $u = 0$, then $y = -\frac{1}{2}gt^2$

To calculate the position of the object after different time intervals 0, τ, 2τ, 3τ, ... which are given in the second column of the table shown below.

t	y	y in Terms of $y_0 = (-1/2)g\tau^2$	Distance Traversed in Successive Intervals	Ratio of Distances Traversed
0	0	0		
τ	$(-1/2)g\tau^2$	y_0	y_0	1
2τ	$-4(1/2)g\tau^2$	$4y_0$	$3y_0$	3
3τ	$-9(1/2)g\tau^2$	$9y_0$	$5y_0$	5
4τ	$-16(1/2)g\tau^2$	$16y_0$	$7y_0$	7
5τ	$-25(1/2)g\tau^2$	$25y_0$	$9y_0$	9
6τ	$-36(1/2)g\tau^2$	$36y_0$	$11y_0$	11

The third column gives the positions in the unit of y_0. The fourth column shows the distances traversed in successive τs. At last, the fifth column find the distances are in simple ratio $1:3:5:7$. **Hence proved**

Stopping Distance for a Vehicle

When brakes are applied to a moving vehicle, the distance it travels before coming to halt is called **stopping distance**.

It is also an important factor for road safety. Its value depends upon the speed at which the vehicle is running and the efficiency of the braking system.

Let the distance travelled by the vehicle before it stops be d_s.

Using the equation of motion,

$$v^2 = u^2 + 2ax$$

If $v = 0$ and $x = d_s$, then

$$\text{Stopping distance, } d_s = \frac{-u^2}{2a}$$

Thus, the stopping distance is proportional to the square of the initial velocity. If we double the initial velocity, its stopping distance increase by a factor of 4 for same deceleration.

EXAMPLE |14| Halt a Car

A car moving with a speed of 50 km/h can be stopped by brakes after covering 6 m. What will be the minimum stopping distance, if the same car is moving at a speed of 100 km/h?

Sol. **In first case**,

$$u = 50 \text{ km/h} = 50 \times \frac{5}{18}$$

$$= \frac{125}{9} \text{ m/s}, \; v = 0, \; x = 6 \text{ m}$$

Using the relation, $v^2 - u^2 = 2ax$

$$0 - \left(\frac{125}{9}\right)^2 = 2a \times 6$$

$$\Rightarrow \quad a = \frac{-125 \times 125}{81 \times 2 \times 6} = -16.27 \text{ m/s}^2 \approx 16 \text{ m/s}$$

In second case,

$$u = 100 \text{ km/h} = 100 \times \frac{5}{18} = \frac{250}{9} \text{ m/s}$$

$$v = 0, \; a = -16 \text{ m/s}^2 \text{ and } x = ?$$

Using the relation, $v^2 - u^2 = 2ax$

$$0 - \left(\frac{250}{9}\right)^2 = 2 \times (-16)\, x$$

$$\Rightarrow \quad x = \frac{250 \times 250}{81 \times 2 \times 16} = 24.1 \text{ m}$$

Reaction Time

When a situation demands our immediate action, it takes sometime before we really respond. **Reaction time** is defined as the time a person takes to observe, think and act.

e.g. If a person is driving a car and suddenly a boy appears on the road, then the time elapsed before he applies the brakes of the car is the reaction time.

Reaction time depends on

(i) an individual presence of mind
(ii) the complexity of the situation

Calculation of Reaction Time

We can find reaction time by the following experiment. Suppose your friend is holding a long rod vertically in the gap between the thumb and fore-finger of your right hand. He has to drop that rod and you have to catch and hold the falling rod. You will see that rod is caught after moving a distance x.

Let this distance x be 0.21 m. If t is the reaction time, then $u = 0$, $a = 9.8 \text{ m/s}^2$, $x = 0.21$ m, $t = ?$

Using the relation, $x = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad 0.21 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2 \times 0.21}{9.8}} = 0.2 \text{ s}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. The velocity of a particle moving along a straight line is described by equation

$$v = 12 - 3t^2$$

where, x is in metre and t in s. The retardation of the particle when its velocity becomes zero, is

(a) $24\ \text{ms}^{-2}$ (b) zero (c) $6\ \text{ms}^{-2}$ (d) $12\ \text{ms}^{-2}$

Sol. (d) We know acceleration $a = \dfrac{dv}{dt}$

So, $v = 12 - 3t^2$ and $a = -6t$

At $t = 2$ s, $v = 0$ and $a = -6 \times 2$

$a = 12\ \text{ms}^{-2}$

So, retardation of the particle $= 12\ \text{ms}^{-2}$.

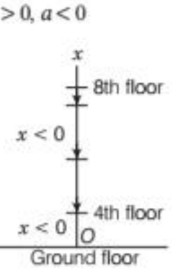
2. A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct? **[NCERT Exemplar]**

(a) $x < 0, v < 0, a > 0$ (b) $x > 0, v < 0, a < 0$

(c) $x > 0, v < 0, a > 0$ (d) $x > 0, v > 0, a < 0$

Sol. (a) As the lift is coming in downward directions displacement will be negative. We have to see whether the motion is accelerating or retarding.

When the lift reaches 4th floor is about to stop hence, motion is retarding in nature hence, $x < 0$; $a > 0$.



As displacement is in negative direction, velocity will also be negative *i.e.*, $v < 0$.

This can be shown on the adjacent graph.

3. The object is released from rest under gravity at $y = 0$. The equation of motion which correctly expresses the above situation is

(a) $v = -9.8\ t\ \text{ms}^{-1}$

(b) $v = (9.8 - 9.8\ t)$ m/s

(c) $v^2 = -19.6\ y^2\ \text{m}^2\text{s}^{-2}$

(d) $v^2 = (v_0^2 + 29.6\ y)\ \text{m}^2/\text{s}^2$

Sol. (a) For free fall, $v_0 = 0$, $a = -g = -9.8\ \text{ms}^{-2}$

The equations of motion are

$v = -9.8\ t\ \text{ms}^{-1}$ (using $v = v_0 + at$)

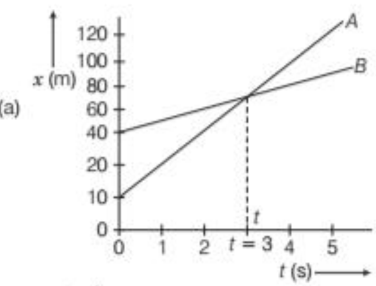
$v^2 = 2 \times (-9.8) \times y$ (using $v^2 = v_0^2 + 2ay$)

$= -19.6 y\ \text{m}^2\text{s}^{-2}$

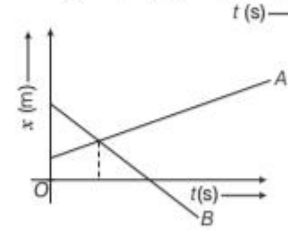
4. The average velocities of the objects A and B are v_A and v_B, respectively. The velocities are related such that $v_A > v_B$.

The position-time graph for this situation can be represented as

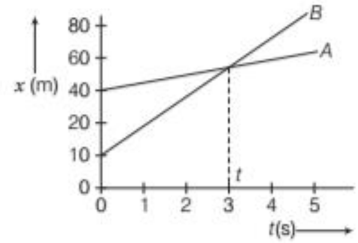
(a)



(b)



(c)



(d) None of the above

Sol. (a) Since, the velocities of the particles are positive, the slope of the straight line in $(x\text{-}t)$ graph must be positive. Since, $v_A > v_B$, the slope of straight line representing A must be greater than the slope of the straight line representing B *i.e.*, graph representing A is more steeper. Even though, A starts with lower value of position coordinate than B, it overtakes, B at $t = 3$ s.

VERY SHORT ANSWER Type Questions

5. If position of a particle at instant t is given by $x = 2t^3$, find the acceleration of the particle.

Sol. Given, $x = 2t^3$, velocity, $v = \dfrac{dx}{dt} = \dfrac{d(2t^3)}{dt} = 6t^2$

$\therefore$ Acceleration, $a = \dfrac{dv}{dt} = \dfrac{d(6t^2)}{dt} = 12\,t$

6. When a body accelerates by βt, what is the velocity after time t, when it starts from rest?

Sol. Given, acceleration $a = \beta t$

It can be written as $\int dv = \int \beta t dt$

On integrating, we get $v = \frac{\beta t^2}{2} + C = \frac{\beta t^2}{2}$ $[\because C = 0]$

7. Give an example of uniformly accelerated linear motion.

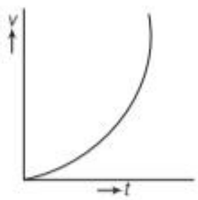
Sol. Motion of a body under gravity.

8. Constant acceleration means that x-t graph will have constant slope? Yes/No

Sol. Acceleration means that velocity is non-uniform. So, x-t graph will be curved.

9. Draw v-t graph for non-uniform accelerated motion.

Sol.



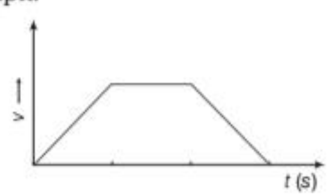
Here, acceleration is increasing.

10. Consider that the acceleration of a moving body varies with time. What does the area under acceleration-time graph for any time interval represent?

Sol. The area under acceleration-time graph for any time interval represents the change of velocity of the body during that time interval.

11. A car starts accelerating from rest for sometime, maintains the velocity for sometime and then comes to rest with uniform deceleration. Draw v-t graph.

Sol.



12. Write an expression for distance covered in nth second for a uniformly accelerated motion.

Sol. If a is the uniform acceleration, then

$$s(n\text{th}) = u + \frac{1}{2} a(2n - 1)$$

where, s (nth) is the distance covered in nth second, u is the initial velocity.

13. Find the acceleration and velocity of a ball at the instant it reaches its highest point if it was thrown up with velocity v.

Sol. Acceleration is 9.8 m/s^2 (downwards) and velocity is zero at the highest point.

SHORT ANSWER Type Questions

14. The displacement of a particle is given by at^2. What is dependency of acceleration on time?

Sol. Let x be the displacement. Then, $x = at^2$

$\therefore$ Velocity of the object, $v = \frac{dx}{dt} = 2at$

Acceleration of the object, $a = \frac{dv}{dt} = 2a$

It means that a is constant.

15. A bus starting from rest moves with a uniform acceleration of 0.1 m / s^2 for 2 min. Find (i) the speed acquired and (ii) the distance travelled.

Sol. $u = 0$, $a = 0.1$ m/s^2 and $t = 2$ min 120 s

(i) $v = u + at = 0 + 0.1 \times 120 = 12$ m/s

(ii) $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 0.1 \times (120)^2$

$= \frac{1}{2} \times 01 \times 120 \times 120 = 720$ m

16. Points P, Q and R are in a vertical line such that $PQ = QR$. A ball at P is allowed to fall freely. What is the ratio of the times of descent through PQ and QR?

Sol. Let t_1 and t_2 be the times of descent through PQ and QR, respectively.

Let $PQ = QR = h$

Then, $h = \frac{1}{2} g t_1^2$ and $2h = \frac{1}{2} g (t_1 + t_2)^2$

By dividing, we get

$$\frac{1}{2} = \frac{t_1^2}{(t_1 + t_2)^2}$$

or $$\frac{1}{\sqrt{2}} = \frac{t_1}{t_1 + t_2}$$

Hence, $t_1 : t_2 = 1 : (\sqrt{2} - 1)$

17. Which of the following is true for displacement?

(i) It cannot be zero.

(ii) Its magnitude is greater than the distance travelled by the object.

Sol. Both these statements are not true, because

(i) Its magnitude can be zero.

(ii) Its magnitude is either less than or equal to the distance travelled by the object.

18. What are uses of a velocity-time graph?

Sol. From a velocity-time graph, we can find out

(i) The velocity of a body at any instant.
(ii) The acceleration of the body and
(iii) The net displacement of the body in a given time-interval.

19. The velocity of a particle is given by equation

$$v = 4 + 2(C_1 + C_2 t)$$

where, C_1 and C_2 are constant. Find the initial velocity and acceleration of the particle.

Sol. The given equation is $v = 4 + 2(C_1 + C_2 t)$

$\Rightarrow \qquad v = (4 + 2C_1) + 2C_2 t$

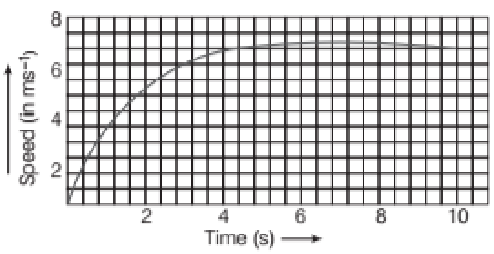
Comparing the above equations with equation of motion

$$v = u + at$$

Initial velocity, $u = 4 + 2C_1$

Acceleration of the particle $= 2C_2$

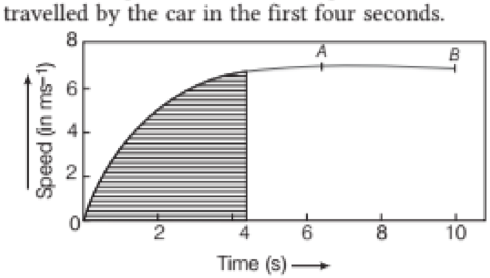
20. The speed-time graph for a car is shown in figure below.



(i) Find how far the car travels in the first 4 s? Shade the area on the graph that represents the distance travelled by the car during the period.

(ii) Which part of the graph represent uniform motion of the car?

Sol. (i) The shaded portion of the car represents the distance travelled by the car in the first four seconds.

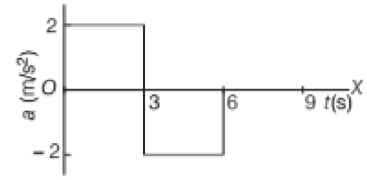


The car travels with a non-uniform speed which is accelerated in nature.

(ii) The straight line portion of the graph represents the uniform motion of the car i.e. from point A to B.

21. At $t = 0$, a particle is at rest at origin. Its acceleration is $2\ m/s^2$ for the first 3s and $-2\ m/s^2$ for next 3s.
Plot the acceleration *versus* time and velocity *versus* time graph.

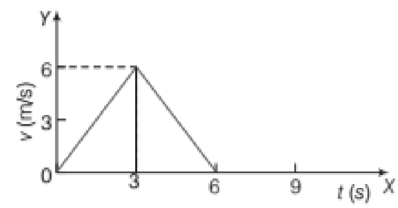
Sol. The acceleration-time graph is



The area enclosed between a-t curve gives change in velocity for the corresponding interval.

At $t = 0$, $v = 0$, hence final velocity at $t = 3$ s will increase to 6 m/s. In next 3 s, the velocity will decrease to zero.

Thus, the velocity-time graph is



22. The distance travelled by a body is proportional to the square of time. What type of motion this body has?

Sol. Let x be the distance travelled in time t. Then,

$x \propto t^2$ [given]

$x = kt^2$ [here, k = constant of proportionality]

We know that velocity is given

$$v = \frac{dx}{dt} = 2kt$$

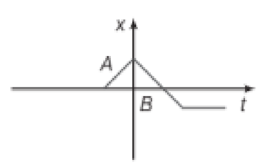
and acceleration is given by

$$a = \frac{dv}{dt} = 2k \qquad \text{[constant]}$$

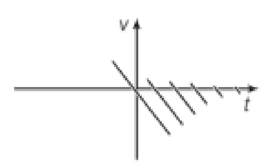
Thus, the body has uniform accelerated motion.

LONG ANSWER Type I Questions

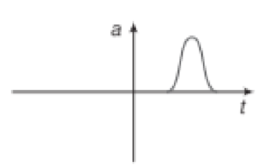
23. Suggest a suitable physical situation for each of the following graph. **[NCERT]**



(a)



(b)



(c)

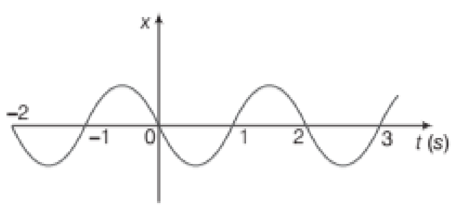
Sol. (i) In Fig. (a), the x-t graph shows that initially x is zero i.e. at rest, then it increases with time, attains a constant value and again reduces to zero with time, then it increases in opposite direction till it again attains a constant value i.e. comes to rest.

The similar physical situation arises when a ball resting on a smooth floor is kicked which rebounds from a wall with reduced speed. It then moves to the opposite wall which stops it.

(ii) In Fig. (b), the velocity changes sign again and again with passage of time and every time some speed is lost. The similar physical situation arises when a ball is thrown up with some velocity, returns back and falls freely. On striking the ground, it rebounds with reduced speed each time it strikes against the ground.

(iii) In Fig. (c), initially the body moves with uniform velocity. Its acceleration increases for a short duration and then falls to zero and thereafter the body moves with a constant velocity. The similar physical situation arises when a cricket ball moving with a uniform speed is hit with a bat for a very short interval of time.

24. Figure gives the x-t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particles at $t = 0.3$ s, 1.2 s, – 1.2 s. **[NCERT]**



In SHM, the acceleration $a = -\omega^2 x$ i.e. acceleration is directly proportional to the displacement and opposite in direction.

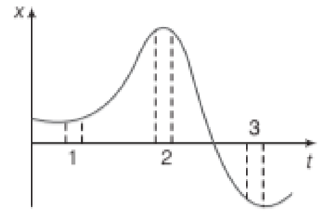
Sol. In the SHM, acceleration $a = -\omega^2 x$, where ω (i.e. angular frequency) is constant.

(i) At time $t = 0.3$ s, x is negative, the slope of x-t plot is negative, hence **position and velocity are negative**. Since $a = -\omega^2 x$, hence acceleration is positive.

(ii) At time $t = 1.2$s, x is positive, the slope of x-t plot is also positive, hence position and velocity are positive. Since $a = -\omega^2 x$, hence **acceleration is negative**

(iii) At $t = -1.2$s, x is negative, the slope of x-t plot is also negative. But since both x and t are negative here, hence **velocity is positive**. Finally, acceleration $\boldsymbol{a}$ **is also positive**.

25. Figure shows the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval the average speed is greatest and in which it is the least? Give the sign of average speed for each interval. **[NCERT]**

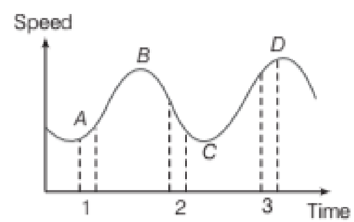


The slope of x-t graph represents the average speed in that interval of time i.e. higher the slope of x-t graph, higher is the average speed.

Sol. We know that average speed in a small interval of time is equal to the slope of x-t graph in that interval of time. **The average speed is the greatest in the interval 3 because** slope is greatest and the average speed is least in interval 2 because slope is least there.

The average speed is positive in intervals 1 and 2 because slope of x-t is positive there and **average speed is negative in interval** 3 because the slope of x-t is negative.

26. Figure gives a speed-time graph of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion, give the signs of u and a in the three intervals. What are the accelerations at the points A, B, C and D? **[NCERT]**



The slope of v-t graph represents the acceleration i.e. higher the slope of v-t graph, higher the acceleration.

Sol. We know that average acceleration in a small interval of time is equal to slope of velocity-time graph in that interval. As the slope of velocity-time graph is maximum in interval 2 as compared to other intervals 1 and 3, hence the magnitude of average acceleration is greatest in interval 2. The **average speed is greatest in interval 3** for obvious reasons.

In interval 1, the slope of velocity-time graph is positive, hence **acceleration $\boldsymbol{a}$ is positive**. The speed u is positive in this interval due to obvious reasons.

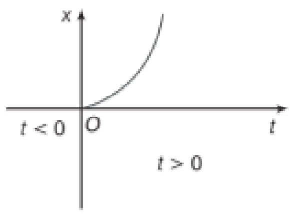
In interval 2, the slope of velocity-time graph is negative, hence **acceleration $\boldsymbol{a}$ is negative**. The speed u is positive in this interval due to obvious reasons.

In interval 3, the velocity-time graph is parallel to time axis, therefore **acceleration a is zero** in this interval but v **is positive** due to obvious reasons.

At points A, B, C and D, the velocity-time graph is parallel to time axis. Therefore, acceleration a **is zero at all the four points**.

27. Figure shows x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$?

If not, suggest a suitable physical context for this graph. **[NCERT]**



Sol. No, because the x-t graph does not represent the trajectory of the path followed by a particle. From the graph, it is noted that at $t = 0$, $x = 0$.

Context The above graph can represent the motion of a body falling freely from a tower under gravity.

28. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them? **[NCERT]**

Sol. For a train A, $u = 72 \text{ kmh}^{-1}$

$$= \frac{72 \times 1000}{60 \times 60} = 20 \text{ ms}^{-1}$$

$$t = 50 \text{ s}, a = 0, x = x_A$$

As, $x = ut + \frac{1}{2}at^2$

$$\therefore \; x_A = 20 \times 50 + \frac{1}{2} \times 0 \times 50^2 = 1000 \text{ m}$$

For train B, $u = 72 \text{ km/h} = 20 \text{ ms}^{-1}$,

$$a = 1 \text{ m/s}^2, t = 50\text{s}, x = x_B$$

As, $$x = ut + \frac{1}{2}at^2$$

$$\therefore \quad x_B = 20 \times 50 + \frac{1}{2} \times 1 \times 50^2 = 2250 \text{m}$$

Taking the guard of the train B in the last compartment of the train B, it follows that original distance between the two trains = length of train A + length of train B.

$$= 800$$

or original distance between the two trains is given by

$$= x_B - x_A = 2250 - 1000 = 1250$$

or original distance between the two trains

$$= 1250 - 800 = 450 \text{ m}$$

29. A jet plane beginning its take off moves down the runway at a constant acceleration of 4.00 m/s^2.

(i) Find the position and velocity of the plane 5.00 s after it begins to move.

(ii) If a speed of 70.0 m/s is required for the plane to leave the ground, how long a runway is required?

Sol. Because the acceleration is constant, we can apply the equations of motion derived above.

(i) We take the origin of the x-axis to be the initial position of the plane, so that $x_0 = 0$.

It is useful to begin by listing all the data given in the problem.

$$a = 4.00 \text{ m / s}^2$$

$$v = 0, \; x = 0$$

The problem may be stated in terms of the symbols as follows

Find x and v at $t = 5.00$ s

When x and u are zero, these two equations reduce to $\quad v = at \quad$ and $\quad x = \frac{1}{2}at^2$

At $\;t = 5.00$ s

$$v = (4.00 \text{ m/s}^2)(5.00\text{s}) = 20.0 \text{ m/s}$$

$$x = \frac{1}{2}(4.00 \text{ m/s}^2)(5.00\text{s})^2$$

$$x = 50.0 \text{ m}$$

(ii) The problem here may be stated as

Find x when $v = 70.0$ m/s

It contains the single unknown x, as well as a and v, which are known with $u = 0$, $v_x^2 = 2a_x x$

Solving for x, we obtain

$$x = \frac{v^2}{2a} = \frac{(70.00 \text{ m/s})^2}{2(4.00 \text{ m/s}^2)} = 613 \text{ m}$$

30. A car starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for sometime t and then decelerate at the rate $f/2$ to come to rest. If the total distance is 5s, then prove that $s = \frac{1}{2}ft^2$.

Sol. **For accelerated motion,**

$$u = 0, \; a = f, \; s = s$$

As $$v^2 - u^2 = 2as,$$

$$\therefore \quad v_1^2 - 0^2 = 2fs \Rightarrow v_1 = \sqrt{2fs},$$

Distance travelled, $s_2 = v_1 t = t\sqrt{2fs}$

For decelerated motion,

$$u = \sqrt{2fs},\ a = -f/2,\ v = 0$$

As $v^2 - u^2 = 2as,$

$\therefore\quad 0^2 - (\sqrt{2fs})^2 = 2 \times (-f/2)\, s_3$

Distance travelled, $s_3 = 2s$

Given, $s + s_2 + s_3 = 5s$

$\Rightarrow\quad s + t\sqrt{2fs} + 2s = 5s \Rightarrow t\sqrt{2fs} = 2s$

$\Rightarrow\quad s = \frac{1}{2} ft^2$

31. A player throws a ball upwards with an initial speed of $29.4\ ms^{-1}$.

(i) What is the direction of acceleration during the upward motion of the ball?

(ii) What are the velocity and acceleration of the ball at the highest point of its motion?

(iii) Choose $x = 0$ and $t = 0$ be the location and time at its highest point, vertically downward direction to be the positive direction of x-axis and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.

(iv) To what height does the ball rise and after how long does the ball return to the player's hands? (Take $g = 9.8\ ms^{-2}$ and neglect air resistance) **[NCERT]**

Sol. (i) Since, the ball is moving under the effect of gravity, the direction of acceleration due to gravity is always vertically downwards.

(ii) At the highest point, the velocity of the ball becomes zero and acceleration is equal to the acceleration due to gravity $= 9.8\ ms^{-2}$ in vertically downward direction.

(iii) When the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of x-axis and upward direction as negative direction of x-axis.

During upward motion, sign of position is negative, sign of velocity is negative and sign of acceleration is positive. During downward motion, sign of position is positive, sign of velocity is positive and sign of acceleration is also positive.

(iv) Let t be the time taken by the ball to reach the highest point where height from ground be s.

Taking vertical upward motion of the ball, we have $u = -29.4\ ms^{-1}$, $a = 9.8\ ms^{-2}$, $v = 0$, $s = S$, $t = ?$

As, $v^2 - u^2 = 2as$

$\therefore\quad 0 - (-29.4)^2 = 2 \times 9.8 \times S$

or $$S = \frac{-(29.4)^2}{2 \times 9.8} = -44.1\ \text{m}$$

Here, negative sign shows that the distance is covered in upward direction.

As, $v = u + at$

$\therefore\quad 0 = -29.4 + 9.8 \times t$

or $$t = \frac{29.4}{9.8} = 3\text{s}$$

It means time of ascent = 3 s

When an object moves under the effect of gravity alone, the time of ascent is always equal to the time of descent.

Therefore, total time after which the ball returns to the player's hand $= 3 + 3 = 6\text{s}$.

32. Read each statement below carefully and state with reasons and examples if it is true or false.

A particle in 1-D motion

(i) with zero speed at an instant may have non-zero acceleration at that instant.

(ii) with zero speed may have non-zero velocity.

(iii) with constant speed must have zero acceleration.

(iv) with positive value of acceleration must be speeding up. **[NCERT]**

Sol. (i) True, when a body is thrown vertically upwards in the space, then at the highest point, the body has zero speed but has downward acceleration equal to the acceleration due to gravity.

(ii) False, because velocity is the speed of body in a given direction. When speed is zero, the magnitude of velocity of body is zero, hence velocity is zero.

(iii) True, when a particle is moving along a straight line with a constant speed, its velocity remains constant with time. Therefore, acceleration (i.e. change in velocity/time) is zero.

(iv) False, if the initial velocity of a body is negative, then even in the case of positive acceleration, the body speeds down. A body speeds up when the acceleration acts in the direction of motion.

33. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 12s. (Take, $g = 10\ ms^{-2}$) **[NCERT]**

Sol. Taking vertical downward motion of ball from a height 90 m, we have $u = 0$, $a = 10\ \text{m/s}^2$, $s = 90$ m, $t = ?$; $u = ?$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 90}{10}} = 3\sqrt{2}\ \text{s} = 4.24\ \text{s}$$

$$u = \sqrt{2as} = \sqrt{2 \times 10 \times 90} = 30\sqrt{2}\ \text{m/s}$$

Rebound velocity of ball,

$$u' = \frac{9}{10} u = \frac{9}{10} \times 30\sqrt{2} = 27\sqrt{2}\ \text{m/s}$$

Time to reach the highest point is

$$t' = \frac{u'}{a} = \frac{27\sqrt{2}}{10} = 2.7\sqrt{2} = 2.7\sqrt{2} = 3.81 \text{ s}$$

Total time $= t + t' = 4.24 + 3.81 = 8.05$s

The ball will take further 3.81 s to fall back to floor, where its velocity before striking the floor $= 27\sqrt{2}$ m/s

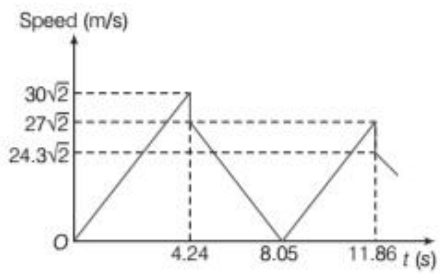
Velocity of ball after striking the floor

$$= \frac{9}{10} \times 27\sqrt{2} = 24.3\sqrt{2} \text{ m/s}$$

Total time elapsed before upward motion of ball

$$= 8.05 + 3.81 = 11.86\text{s}$$

Thus, the speed-time graph of this motion is shown in the figure.



34. State which of the following situations are possible and give an example for each of these?

(i) An object with a constant acceleration but with zero velocity.

(ii) An object moving in a certain direction with acceleration in the perpendicular direction.

Sol. Both the situations are possible

(i) When an object is projected upwards, its velocity at the top-most point is zero even though the acceleration on itis $9.8 \text{ m/s}^2 (g)$.

(ii) When a stone tied to a string is whirled in a circular path, the acceleration acting on it is always at right angles i.e. perpendicular to the direction of motion of stone (we will study about it in chapter 'motion in a plane').

35. A body is projected vertically upwards from A, the top of a tower it reaches the ground in it t_1 second. If it is projected vertically downwards from A with the same velocity it reaches the ground in t_2 second. If it falls freely, from A, prove that it would reach the ground in $\sqrt{t_1 t_2}$ second.

Sol. Using relations

Consider upwards as negative and downwards as positive.

$$h = -u t_1 + \frac{1}{2} g t_1^2 \quad \text{...(i)}$$

and $$h = u t_2 + \frac{1}{2} g t_2^2 \quad \text{...(ii)}$$

On subtracting Eqs. (i) from (ii), we get

or $$0 = u(t_2 + t_1) + \frac{1}{2} g t_2^2 - \frac{1}{2} g t_1^2$$

or $$u(t_2 + t_1) + \frac{1}{2} g (t_2 + t_1)(t_2 - t_1) = 0$$

or $$u + \frac{1}{2} g (t_2 + t_1) = 0$$

or $$u = -\frac{g}{2}(t_2 - t_1) \quad \text{...(iii)}$$

From Eqs. (i) and (iii), we get

Now, $$h = \frac{g t_1}{2}(t_2 - t_1) + \frac{1}{2} g t_1^2 = \frac{1}{2} g t_1 t_2 \quad \text{...(iv)}$$

Again, when the body falls freely.

$$h = \frac{1}{2} g t^2; \ \frac{1}{2} g t_1 t_2 = \frac{1}{2} g t^2 \quad \text{[from Eq. (iv)]}$$

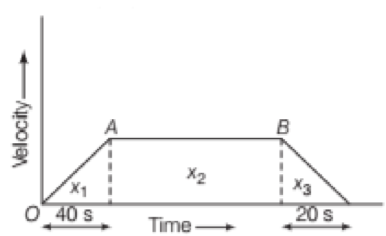
or $$t = \sqrt{t_1 t_2}$$ **Hence proved**

LONG ANSWER Type II Questions

36. A train takes 4 min to go between stations 2.25 km apart starting and finishing at rest. The acceleration is uniform for the first 40 s and the deceleration is uniform for the last 20 s.

Assuming the velocity to be constant for the remaining time, calculate the maximum speed, acceleration and retardation, use only the graphical method.

Sol. The velocity-time graph of the train's motion is shown in the following figure



Let v represents the maximum speed of the train if x_1 be the distance covered during the first 40 s, then

$$\frac{v}{2} \times 40 = x_1 \text{ or } x_1 = 20\,v$$

Since total time is 4 min, i.e. 240 s therefore, the time corresponding to velocity-time graph AB is $(240 - 40 - 20)$ s i.e. 180 s. If x_2 be the distance covered during this time, then $x_2 = 180\,v$.]

If x_3 be the distance covered during the last 20 s, then

$$x_3 = \frac{v}{2} \times 20 = 10\,v$$

Now, $$x_1 + x_2 + x_3 = 20\,v + 180\,v + 10\,v$$

or $$2250 = 210\,v$$

or $$v = \frac{225}{21} \text{ ms}^{-1} = 10.7 \text{ ms}^{-1}$$

$$\text{Acceleration} = \frac{v}{40} = \frac{10.7}{40}\ \text{ms}^{-2} = 0.2675\ \text{ms}^{-2}$$

$$\text{Retardation} = \frac{v}{20} = \frac{10.7}{20}\ \text{m s}^{-2} = 0.535\ \text{ms}^{-2}$$

37. A train passes a station A at 40 kmh^{-1} and maintains its speed for 7 km and is then uniformly retarded, stopping at B which is 8.5 km from A. A second train starts from A at the instant the first train passes and being accelerated some part of the journey and uniformly retarded for the rest, stops at B at the same times as the first train. Calculate the maximum speed of the second train, use only the graphical method.

Sol. Area $AEFG = AE \times AG \Rightarrow 7 = 40 \times AG$

or $$AG = \frac{7}{40}\ \text{h}$$

Area FGB gives the distance covered under retardation, it is $(8.5 - 7)$ km $= 1.5$ km

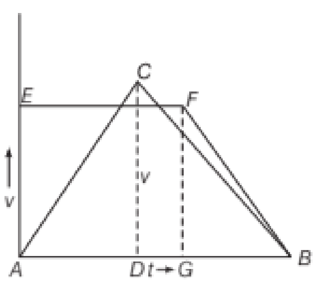
$$\text{Area of } \Delta FGB = \frac{1}{2} GB \times FG \Rightarrow GB = \frac{2 \times 1.5}{40}\ \text{h} = \frac{3}{40}\ \text{h}$$

$$\text{Total time} = \left(\frac{7}{40} + \frac{3}{40}\right)\text{h} = \frac{1}{4}\ \text{h}$$

$$\text{Area of } \Delta ACB = \frac{1}{2} AB \times CD$$

$$8.5 = \frac{1}{2} \times \frac{1}{4} \times v$$

$$v = 8.5 \times 8\ \text{kmh}^{-1} = 68\ \text{kmh}^{-1}$$



38. A three wheeler starts from rest, accelerates uniformly with $1\ \text{ms}^{-2}$ on a straight road for 10 s, and then moves with uniform velocity. Plot a graph between the distance covered by the vehicle during the nth second $(n = 1, 2, 3, \ldots)$ *versus n*. What do you expect the plot to be during accelerated motion a straight line or a parabola? **[NCERT]**

Sol. Here, $u = 0$, $a = 1\ \text{ms}^{-2}$

Distance covered in nth second is

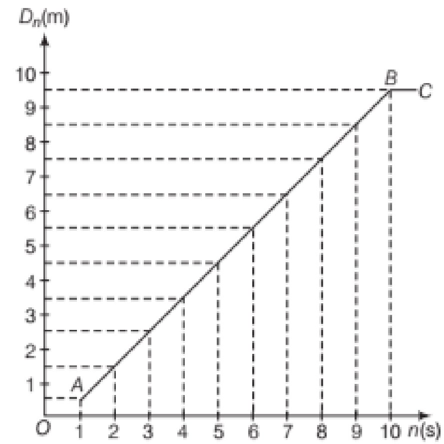
$$D_n = u + \frac{a}{2}(2n - 1)$$

$$D_n = 0 + \frac{1}{2}(2n - 1) = 0.5\,(2n - 1) \qquad \ldots\text{(i)}$$

Putting $n = 1, 2, 3, \ldots$, we can find the value of D_n.

The various values of n and corresponding values of D_n are shown below

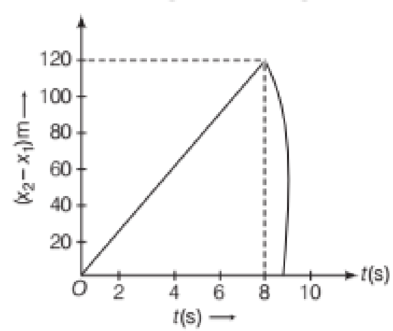
n	1	2	3	4	5	6	7	8	9	10
D_n	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5



On plotting a graph between D_n and n, we get a straight line AB as shown in figure. From Eq. (i), $D_n \propto n$ so the graph is a straight line. After 10 s, the graph is a straight line BC parallel to time axis.

39. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the graph shown in figure, correctly represents the time variation of the relative position of the second stone with respect to the first.

Neglect the air resistance and assume that the stones do not rebound after hitting the ground. Take, $g = 10\ \text{ms}^{-2}$. Give the equations for the linear and curved part of the plot. **[NCERT]**



Sol. Taking vertical upward motion of the first stone for time t, we have

$$x_0 = 200\ \text{m},\ u = 15\ \text{m/s},$$
$$a = -10\ \text{m}/\text{s}^2,\ t = t,\ x = x_1$$

As, $x = x_0 + ut + \frac{1}{2}at^2$

$\therefore$ $x_1 = 200 + 15t + \frac{1}{2}(-10)t^2$

or $x_1 = 200 + 15t - 5t^2$ ...(i)

Taking vertical upward motion of the second stone for time t, we have

$$x_0 = 200 \text{ m}, u = 30 \text{ ms}^{-1},$$
$$a = -10 \text{ ms}^{-2}, t = t, x = x_2$$

Then, $x_2 = 200 + 30t - \frac{1}{2} \times 10t^2$

$= 200 + 30t - 5t^2$ ...(ii)

When the first stone hits the ground, $x_1 = 0$, from Eq (i).

So, $t^2 - 3t - 40 = 0$

or $(t-8)(t+5) = 0$

$\therefore$ Either $t = 8$ s or -5 s

Since, $t = 0$ corresponds to the instant, when the stone was projected. Hence, negative time has no meaning in this case. So, $t = 8$ s.

When the second stone hits the ground, $x_2 = 0$, from Eq (ii).

So, $0 = 200 + 30t - 5t^2$ or $t^2 - 6t - 40 = 0$

or $(t-10)(t+4) = 0$

Therefore, either $t = 10$ s, or $t = -4$ s

Since, $t = -4$ s is meaningless, so $t = 10$ s.

Relative position of second stone w.r.t. first is

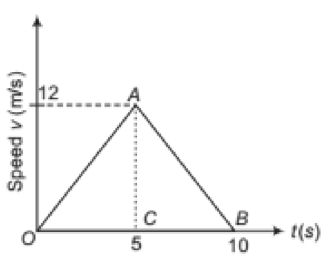
$= x_2 - x_1 = 15t$ ...(iii)

Since, $(x_2 - x_1)$ and t are linearly related, therefore, the graph is a straight line till $t = 8$ s.

For maximum separation, $t = 8$ s, so maximum separation $= 15 \times 8 = 120$ m

After 8 s only, second stone would be in motion for 2 s, so the graph is in accordance with the quadratic equation, $x_2 = 200 + 30t - 5t^2$ for the interval of time 8 s to 10 s.

40. The speed-time graph of a particle moving along a fixed direction is shown below in figure. Obtain the distance travelled by the particle between (i) $t = 0$ s to 10 s (ii) $t = 2$ s to 6 s. What is the average speed of the particle over the intervals in (i) and (ii)? **[NCERT]**



Sol. (i) Distance travelled by the particle between time interval $t = 0$ s to $t = 10$ s

$= \text{Area of triangle } OAB$

$= \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times OB \times AC$

$= \frac{1}{2} \times 10 \times 12 = 60 \text{ m}$

Average speed of the particle for this time interval

$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{60}{10} = 6 \text{ m/s}$

(ii) From the part OA of the given graph (in accelerated motion)

When $t = 0, u = 0, t = 5$ s, $v = 12$ m/s

Using equation of motion,

$$v = u + at$$
$$12 = 0 + a \times 5$$

or $a = \frac{12}{5} \text{ m/s} = 2.4 \text{ m/s}^2$

Speed of the particle at the end of $t = 2$ s

$$v' = u + at = 0 + 2.4 \times 2 = 4.8 \text{ m/s}$$

Now, distance travelled in accelerated motion from $t = 2$ s to $t = 5$ s i.e. in 3 s, for which initial speed is $u = v' = 4.8$ m/s

Using equation of motion,

$$s_1 = ut + \frac{1}{2}at^2$$
$$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times (3)^2 = 14.4 + 10.8$$
$$s_1 = 25.2 \text{ m}$$

From the part AB of the given graph (retarded motion)

Initial speed at $t = 5$ s, $u = 12$ m/s

Time taken, $t = (10 - 5) = 5$ s

Final speed at $t = 10$ s, $v = 0$

Using equation, $v = u + at$

$$0 = 12 + a \times 5$$

or $a = -\frac{12}{5} = -2.4 \text{ m/s}^2$

Now, distance travelled by the particle in retarded motion from $t = 5$ s to $t = 6$ s i.e. in 1 s,

$$s_2 = ut + \frac{1}{2}at^2$$
$$= 12 \times 1 + \frac{1}{2}(-2.4) \times 1$$
$$= 12 - 1.2 = 10.8 \text{ m}$$

$\therefore$ Total distance travelled by the particle from $t = 2$ s to $t = 6$ s

$$s = s_1 + s_2$$

$$= 25.2 + 10.8 = 36.0 \text{ m}$$

Average speed of the particle for this time interval

$$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{36}{(6-2)} = \frac{36}{4} = 9 \text{ m/s}$$

41. A motor car moving at a speed of 72 km/h cannot come to a stop is less than 3.0 s while for a truck time interval is 5.0 s. On a highway, the car is behind the truck both moving at 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bumb onto (collide with) the truck? Human response time is 0.5 s. **[NCERT Exemplar]**

Sol. Given, speed of the car as well as truck = 72 km/h

$$= 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$$

Retarded motion for truck

$$v = u + a_t t$$

$$0 = 20 + a_t \times 5$$

or $$a_t = -4 \text{ m/s}^2$$

Retarded motion for the car

$$v = u + a_c t$$

$$0 = 20 + a_c \times 3$$

or $$a_c = -\frac{20}{3} \text{ m/s}^2$$

Let car be at a distance x from truck, when truck gives the signal and t be the time taken to cover this distance.

As human response time is 0.5 s, therefore time of retarded motion of car is $(t - 0.5)$ s.

Velocity of car after time t,

$$v_c = u - at = 20 - \left(\frac{20}{3}\right)(t - 0.5)$$

Velocity of truck after time t,

$$v_t = 20 - 4t$$

To avoid the car bumb onto the truck, $v_c = v_t$

$$20 - \frac{20}{3}(t - 0.5) = 20 - 4t$$

or $$4t = \frac{20}{3}(t - 0.5)$$

or $$t = \frac{5}{3}(t - 0.5)$$

or $$3t = 5t - 2.5$$

or $$t = \frac{2.5}{2} = \frac{5}{4} \text{ s}$$

Distance travelled by the truck in time t,

$$s_t = u_t t + \frac{1}{2} a_t t^2$$

$$= 20 \times \frac{5}{4} + \frac{1}{2} \times (-4) \times \left(\frac{5}{4}\right)^2$$

$$s_t = 25 - 3.125 = 21.875 \text{ m}$$

Distance travelled by the car in time t

= Distance travelled by the car in 0.5 s (without retardation) + Distance travelled by car in $(t - 0.5)$ s (with retardation)

$$s_c = (20 \times 0.5) + 20\left(\frac{5}{4} - 0.5\right) - \frac{1}{2}\left(\frac{20}{3}\right)\left(\frac{5}{4} - 0.5\right)^2$$

$$= 23.125 \text{ m}$$

$$\therefore s_c - s_t = 23.125 - 21.875$$

$$= 1.250 \text{ m}$$

Therefore, to avoid the bumb onto the truck, the car must maintain a distance from the truck more than 1.250 m.

42. A ball is thrown upward with an initial velocity of 100 m/s. After how much time will it return? Draw velocity-time graph for the ball and find from the graph.

(i) Maximum height attained by ball and

(ii) Height of the ball after 15 s. Take, $g = 10 \text{ ms}^{-2}$

Sol. Here, $u = 100 \text{ ms}^{-1}$, $g = -10 \text{ ms}^{-1}$

At highest point, $v = 0$

As $v = u + gt \Rightarrow 0 = 100 - 10 \times t$

$\therefore$ Time taken to reach highest point

$$t = \frac{100}{10} = 10 \text{ s}$$

The ball will return to the ground at $t = 20$ s.

Velocities of the ball at different instants of time will be as follows.

At $t = 0$, $v = 100 - 10 \times 0 = 100 \text{ ms}^{-1}$

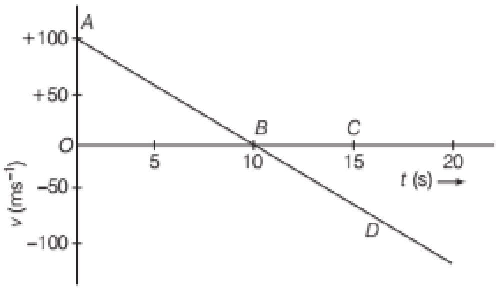
At $t = 5$ s, $v = 100 - 10 \times 5 = 50 \text{ ms}^{-1}$

At $t = 10$ s, $v = 100 - 10 \times 10 = 0$

At $t = 15$ s, $v = 100 - 10 \times 15 = -50 \text{ ms}^{-1}$

At $t = 20$ s, $v = 100 - 10 \times 20 = -100 \text{ ms}^{-1}$

The velocity time-graph will be as shown in figure.



(i) Maximum height attained by ball

$= \text{Area of } \Delta AOB$

$= \frac{1}{2} \times 10\text{s} \times 100 \text{ ms}^{-1} = 500 \text{ m}$

(ii) Height attained after 15 s

$= \text{Area of } \Delta AOB + \text{Area of } \Delta BCD$

$= 500 + \frac{1}{2}(15 - 10) \times (-50)$

$= 500 - 125 = 375 \text{ m}$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions|

1. The slope of the straight line connecting the points corresponding to (v_2, t_2) and (v_1, t_1) on a plot of velocity *versus* time gives

(a) average velocity (b) average acceleration
(c) instantaneous velocity (d) None of these

2. The kinematic equations of rectilinear motion for constant acceleration for a general situation, where the position coordinate at $t = 0$ is non-zero, say x_0 is

(a) $v = v_0 + at$

(b) $x = x_0 + v_0 t + \frac{1}{2} at^2$

(c) $v^2 = v_0^2 + 2a(x - x_0)$

(d) All of the above

3. A car is moving with a velocity of 30 ms^{-1}. On applying the brakes, the velocity decreases to 15 ms^{-1} in 2 s. The acceleration of the car is

(a) $+7.5 \text{ ms}^{-2}$

(b) -7.7 ms^{-2}

(c) -7.5 ms^{-2}

(d) $+15 \text{ ms}^{-2}$

4. An object starts from rest and moves with uniform acceleration a. The final velocity of the particle in terms of the distance x covered by it is given as

(a) $\sqrt{2ax}$ (b) $2ax$ (c) $\sqrt{\frac{ax}{2}}$ (d) $\sqrt{ax}$

Answer

1. (b)	2. (d)	3. (c)	4. (a)

VERY SHORT ANSWER Type Questions

5. Can a body have zero velocity and finite acceleration?

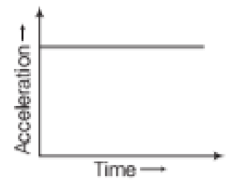
6. Is it possible for a body to be accelerated without speeding up or slowing down? If it is so, give example.

7. Plot a graph of velocity-time, for the condition if an object is moving with increasing acceleration and having zero initial velocity.

SHORT ANSWER Type Questions

8. A particle in one dimensional motion with positive value of acceleration must be speeding up. Is it true? Explain.

9. The acceleration-time graph for a body is shown in the adjoining. Plot the corresponding velocity-time graph.



10. Establish a kinematic equation $s = ut + \frac{1}{2}at^2$ from velocity-time graph for a uniformly accelerated motion.

LONG ANSWER Type I Questions

11. State which of the following situations are possible and give an example for each of these?

(i) An object with a constant acceleration but with zero velocity.

(ii) An object moving in a certain direction with an acceleration in the perpendicular direction.

12. A passenger is standing d metres away from a bus. The bus begins to move with constant acceleration (a). To catch the bus, the passenger runs at a constant speed (v) towards the bus. What must be the minimum speed of the passenger so that he may catch the bus?

[**Ans.** $\sqrt{2ad}$]

13. A particle starts moving from position of rest under a constant acceleration. If it travels a distance x in t second, what distance will it travel in next t second? [**Ans.** $3x$]

LONG ANSWER Type II Questions

14. The two ends of a moving train with a constant acceleration passes a certain point with velocities u and v. Show that the velocity with which the middle point of the train passes the same point is $\sqrt{\frac{u^2+v^2}{2}}$.

15. A car starts from rest and accelerates uniformly for 10 s to a velocity of 8 m/s, then it runs at a constant velocity and is finally brought to rest in 64 m with a constant retardation. The total distance covered by the car is 584 m. Find the values of acceleration, retardation and total time taken.

[**Ans.** 0.8 m / s^2, 0.5 m/s^2, 86 s]

16. (i) Draw position-time graphs for

(a) Accelerated motion (b) Retarded motion

(ii) A juggler throws balls into air. He throws one whenever the previous one is at its highest point. How high do the balls rise if he throws n balls in each second? Take acceleration due to gravity as g .

[**Ans.** $\frac{g}{2n^2}$]

SUMMARY

- **Motion** is the change in position of an object with time.
- If the object size is much smaller than the distance it moves in a reasonable time, then it is called **point object**.
- **Displacement** is the measure of change in position of an object with time in a particular direction $\Delta x = x_2 - x_1$
- **Velocity** is the rate of change in position or displacement of an object with time. Its SI unit is m/s.
- **Speed** is the ratio of the path length or the distance covered by an object to the time taken.
- **Acceleration** is the rate of change of velocity with time.
- **Kinematics equations for uniformly accelerated motion** (Symbols have their usual meaning)

  1. $v = u + at$ 2. $x = x_0 + ut + \frac{1}{2}at^2$ 3. $v^2 = u^2 + 2ax$
- Displacement of a particle in the nth second is given by $S_n = u + \frac{a}{2}(2n-1)$
- Equation of motion under gravity is given by (Symbols have their usual meaning)

  1. $v = u + (\pm g)t$ 2. $h = ut + \frac{1}{2}(\pm g)t^2$ 3. $v^2 = u^2 + 2(\pm g)h$
- **Stopping distance** for a vehicle is given as $d_s = \frac{-u}{2a}$
- If a body travels equal distance in equal interval of time along a straight line, then the body is said to be in uniform motion in a straight line.
- If a body travels equal distance in unequal intervals of time then it is said to be in non-uniform motion.
- For a non-uniformly accelerated motion, $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$
- Average velocity is given by, $v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$
- **Average speed** is the total distance travelled divided by the total time taken.
- **Instantaneous speed** is the limit of the average speed as the time interval becomes infinitesimally small or approaches to zero. It is given by $s_{in} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$
- **Average acceleration** is the ratio of change in velocity of the object to the time interval. It is given by, $a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$
- **Instantanous acceleration** is the acceleration of a body at a certain instant or the limiting value of average acceleration when time interval tends to zero. It is given by, $a_{in} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it in 1s is

(a) $3A + 7B$ (b) $\frac{3}{2}A + \frac{7}{3}B$

(c) $\frac{A}{2} + \frac{B}{3}$ (d) $\frac{3}{2}A + 4B$

2. At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be **[NCERT Exemplar]**

(a) $(t_1 + t_2)/2$ (b) $t_1 t_2/(t_2 - t_1)$

(c) $t_1 t_2/(t_2 + t_1)$ (d) $t_1 - t_2$

3. In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$. **[NCERT Exemplar]**

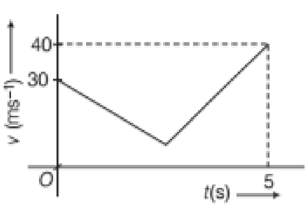
(a) The displacement in time T must always take non-negative values

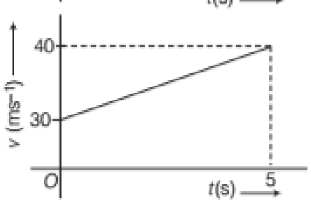
(b) The displacement x in time T satisfies $-v_0 T < x < v_0 T$

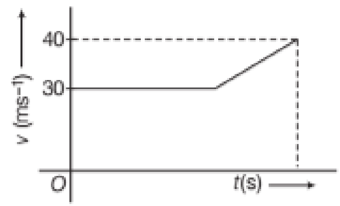
(c) The acceleration is always a non-negative number

(d) The motion has no turning points

4. An object is moving with an initial velocity of $30\ \text{ms}^{-1}$ with uniform acceleration. The velocity of object increases to $40\ \text{ms}^{-1}$ in next 5 s. The v-t graph which least represents this situation is

(a)


(b)


(c)


(d) None of the above

ASSERTION AND REASON

Direction (Q. Nos 5-9) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

5. **Assertion** In real life, in a good number of situations, the object is treated as a point object.
Reason The object is treated as point object, so far as the size of the object is much smaller than the distance it moves in a reasonable duration of time.

6. **Assertion** For motion along a straight line and in the same direction, the magnitude of average velocity is equal to the average speed.

Reason For motion along a straight line and in the same direction, the magnitude of displacement is equal to the path length.

7. **Assertion** For uniform motion, velocity is the same as the average velocity at all instants.

Reason In uniform motion along a straight line, the object covers equal distances in equal intervals of time.

8 **Assertion** In realistic situation, the x-t, v-t and a-t graphs will be smooth. This means physically that acceleration and velocity cannot change values abruptly at an instant.

Reason Changes are always continuous.

9. **Assertion** A body may be accelerated even when it is moving uniformly.

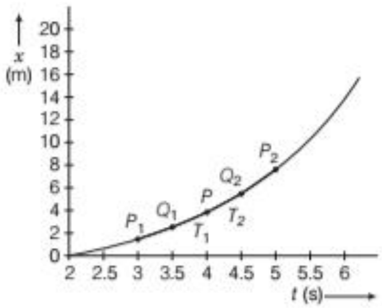
Reason When direction of motion of the body is changing, then body may have acceleration.

CASE BASED QUESTIONS

Direction (Q. Nos. 10) *This question is case study based question. Attempt any 4 sub-parts from given question.*

10. x-t **Graph**

The x-t graph represents the motion of a car at different time instants. With reference to the below graph, choose the correct for the question given below.



(i) The slope of the line P_1P_2 for the interval $t = 3$ s to $t = 5$ s gives

(a) average velocity
(b) instantaneous velocity
(c) average speed
(d) Either (a) or (c)

(ii) The slope of the line P_1P_2 for the int

I. For the time interval $t = 3.5$ s to $t = 4.5$ s, the value of average velocity is given by the slope of the line Q_1Q_2.

II. The slope of the tangent (T_1T_2) at point P gives the velocity of the car at time instant $t = 4$ s.

III. The slope of the tangent at point P gives the instantaneous velocity of the car.

(a) Both I and II (b) Only I
(c) Only III (d) I, II and III

(iii) Which of the following statements is correct with reference to the above graph as the time interval Δt approaches zero, *i.e.*, $\Delta t \to 0$?

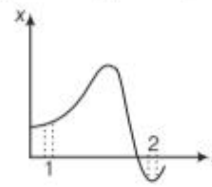
I. The line P_1P_2 becomes tangent to the position-time curve at the point P.

II. The slope of the tangent (T_1T_2) at point P gives the velocity of the car at time instant $t = 4$ s.

III. The slope of the tangent at point P gives the instantaneous velocity of the car.

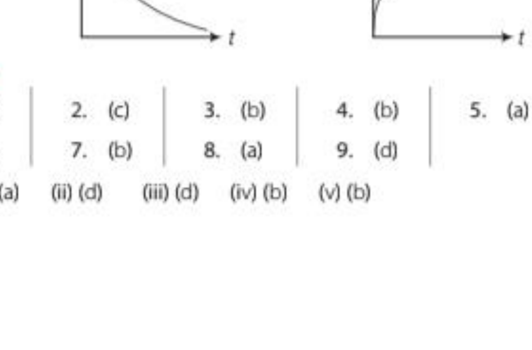
(a) Both I and II
(b) Only I
(c) Only III
(d) I, II and III

(iv) Figure shows the $(x$-$t)$ plot of a particle in one-dimensional motion. Two different equal intervals of time are speed in time intervals 1 and 2 respectively. Then,



(a) $v_1 > v_2$
(b) $v_2 > v_1$
(c) $v_1 = v_2$
(d) Data insufficient

(v) Among the four graphs shown in the figure, there is only one graph for which average velocity over the time interval $(0, T)$ can vanish for a suitably chosen T. Which one is it?



Answer

1. (c)	2. (c)	3. (b)	4. (b)	5. (a)
6. (a)	7. (b)	8. (a)	9. (d)	

10. (i) (a) (ii) (d) (iii) (d) (iv) (b) (v) (b)

VERY SHORT ANSWER Type Questions

11. Can earth be regarded as a point object when it describes its yearly journey around the sun?

12. Write the expression for distance covered in *n*th second by a uniformly accelerated body.

13. Why does a parachute descend slowly?

14. What does speedometer record: the average speed or the instantaneous speed?

15. Draw displacement-time graph for a uniformly accelerated motion. What is its shape?

SHORT ANSWER Type Questions

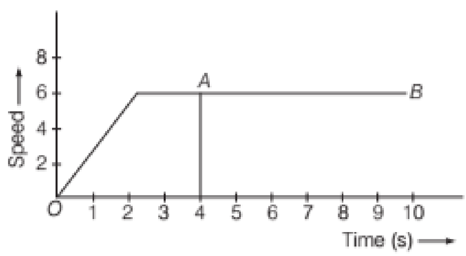
16. Two bodies of different masses m_1 and m_2 are dropped from two different heights *a* and *b*. What is the ratio of time taken by the two bodies to drop through these distances?

[**Ans.** $\sqrt{a} : \sqrt{b}$]

17. If the velocity of a particle is given by $v = \sqrt{180 - 16x}$ m/s, what will be its acceleration?

[**Ans.** -8 m/s^2]

18. The speed-time graph for a car is shown in figure below



(i) Find how far does the car travel in the first 4s. Shade the area on the graph that represents the distance travelled by the car during the period. [**Ans.** 18 m]

(ii) Which part of the graph represents uniform motion of the car?

19. A body moving with a uniform acceleration describes 12 m in 3rd s of its motion and 20 m in the 5ths. Find the velocity after 10 s.

[**Ans.** $v = 42$ m/s]

20. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If *t* is total time elapsed, then calculate

(i) the maximum velocity attained by the car

(ii) the total distance travelled by the car

[**Ans.** (a) $\frac{\alpha\beta t}{\alpha + \beta}$, (b) $\frac{1}{2}\frac{\alpha\beta t^2}{(\alpha + \beta)}$]

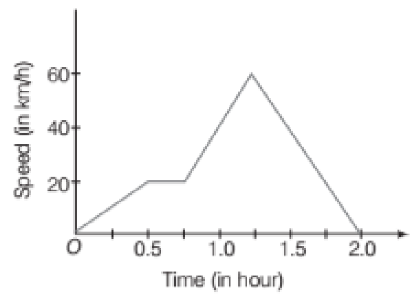
LONG ANSWER Type I Questions

21. Discuss the motion of an object under free fall and draw (i) acceleration-time (ii) velocity-time and (iii) position-time graph for this motion.

22. A car moving with a speed of 50 km/h can be stopped by brakes after at least 6 m. What will be the minimum stopping distance, if the same car is moving at a speed of 100km/h? [**Ans.** 24 m]

23. Derive an expression for stopping distance of a vehicle in terms of initial velocity v_0 and deceleration *a*.

24. A train moves from one station to another in 2 h time. Its speed during the motion is shown in graph. Determine the maximum acceleration during the journey. Also, calculate the distance covered during the time interval from 0.75 h to 1 h.



[**Ans.** $a = 160$ km/h^2, $s = 10$ km]

25. Establish the relation $x(t) = ut + \frac{1}{2}at^2$ by calculus method.

26. The distance covered by an object between timings t_1 and t_2 is given by area under the *v-t* graph between t_1 and t_2. Prove that the above statement also holds when an object is moving with negative acceleration has velocities positive at time t_1 and negative at t_2.

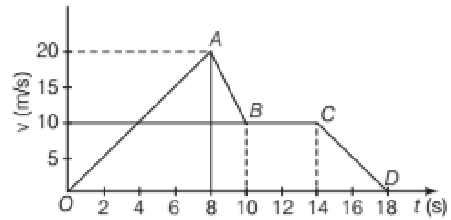
LONG ANSWER Type II Questions

27. Derive the three equations of motion by calculus method. Express the conditions under which they can be used.

28. A ball is dropped from a height of 100 m on a floor. At each collision with floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ s and $t = 12$ s.

29. A parachutist bails out from an airplane and after dropping through a distance of 40 m, he opens the parachute and decelerates at 2 m/s^2. If he reaches the ground with a speed of 2 m/s, how long does he float in the air? At what height did he bail out from the plane? [**Ans.** 15.86 s, 235m]

30. The velocity-time graph of a particle moving along a straight line as shown in figure by curve *OABCD*. Calculate the distance covered by the particle between (i) $t = 0$ to 18 s (ii) $t = 2$ s to $t = 12$ s and the maximum value of acceleration during this interval.



[**Ans.** (i) 170 m, (ii) 125 m, – 5 m/s^2]