* Laplace Transform *

- Let $f(t)$ be a function defined for $t > 0$. The laplace transform of $f(t)$ denoted by $L\{f(t)\}$ OR $\bar{f}(s)$ and defined as $\boxed{L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\,dt}$ where s is a parameter [real or complex].

Laplace transform of elementary function

(i) $f(t) = t^n$; $n > -1$

Sol^n: By definition,

$$L\{t^n\} = \bar{f}(s) = \int_0^\infty e^{-st} \cdot t^n\,dt$$

Let $st = x \Rightarrow s\,dt = dx \quad \therefore dt = \frac{dx}{s}$

$$= \int_0^\infty e^{-x} \cdot \left(\frac{x}{s}\right)^n \cdot \frac{dx}{s}$$

$$= \int_0^\infty \frac{e^{-x} \cdot x^n}{s^{n+1}} \cdot dx$$

$$L\{t^n\} = \frac{n!}{s^{n+1}} \quad ; n = 0, 1, 2, 3, \ldots$$

OR

$$L\{t^n\} = \frac{\Gamma(n+1)}{s^{n+1}} \quad ; n \text{ is in fraction},\ n > -1$$

NOTE:-

GAMMA FUNCTION

$$\Gamma(n+1) = \int_0^\infty e^{-x} \cdot x^{n+1-1}\,dx$$

$$\Gamma(n) = \int_0^\infty e^{-x} \cdot x^{n-1}\,dx$$

$$\Gamma(n+1) = n\,\Gamma(n)$$

Eg:- $\Gamma(3/2) = \Gamma(1/2+1) = \frac{1}{2}\Gamma(1/2) = \frac{\sqrt{\pi}}{2}$

(2) $f(t) = e^{at}$

$$L\{e^{at}\} = \int_0^\infty e^{-st} \cdot e^{at} \cdot dt$$

$$= \int_0^\infty e^{-(s-a)t} \cdot dt$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty = \frac{e^{-\infty}}{-(s-a)} - \frac{1}{-(s-a)} = \underline{\frac{1}{s-a}}$$

(3) $f(t) = \sin at$

$$L\{\sin at\} = \int_0^\infty e^{-st} \cdot \sin at \, dt$$

$$\left[\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]\right]$$

(4) $f(t) = \cos at$

$$L\{\cos at\} = \frac{s}{s^2+a^2}$$

(5) $f(t) = \sin h(at)$

$$L\{\sin h(at)\} = \frac{a}{s^2-a^2}$$

(6) $f(t) = \cosh(at)$

$$L\{\cosh(at)\} = \frac{s}{s^2-a^2}$$

$$\sin hat = \frac{e^{at} - e^{-at}}{2}$$

$$\cosh at = \frac{e^{at} + e^{-at}}{2}$$

Q:- $L\{\sin^3 t\}$

$\left[\cos 3x = 4\cos^3 x - 3\cos x\right]$

$\left[\sin 3x = 3\sin x - 4\sin^3 x\right]$

$\sin 3t = 3\sin t - 4\sin^3 t$

$4\sin^3 t = 3\sin t - \sin 3t$

$\sin^3 t = \frac{3}{4}\sin t - \frac{1}{4}\sin 3t$

$L\{\sin^3 t\} = L\left\{\frac{3}{4}\sin t\right\} - L\left\{\frac{1}{4}\sin 3t\right\}$

$= \frac{3}{4}\left(\frac{1}{s^2+1}\right) - \frac{1}{4}\cdot\left(\frac{3}{s^2+9}\right)$

Q:- $L\{1\}$

$= \int_0^\infty e^{-st}\cdot 1\, dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{1}{s}$

Q:- $L\{t\}$

$= \frac{1}{s^2}$

Q:- $L\{t^5\}$

$= \frac{5!}{s^6} = \frac{120}{s^6}$

Q:- $L\{t^{1/2}\}$

$= \frac{\Gamma(1/2+1)}{s^{1/2+1}} = \frac{\Gamma(3/2)}{s^{3/2}} = \frac{\frac{1}{2}\sqrt{\pi}}{s^{3/2}} = \frac{\sqrt{\pi}}{2s^{3/2}}$

Q:- $L\{t^{-1/2}\}$

$= \frac{\Gamma(-1/2+1)}{s^{-1/2+1}} = \frac{\Gamma(1/2)}{s^{1/2}} = \frac{\sqrt{\pi}}{s^{1/2}} = \sqrt{\frac{\pi}{s}}$

Q:- $L\{t^{-1}\}$

-1 it should be >-1

$\therefore$ Does not exists

Q:- $L\{\sin^2 t\}$

$$L\{1-\cos^2 t\}$$

$$L\{1\} - L\{\cos^2 t\}$$

$$L\{1\} - L\left\{\frac{1+\cos 2t}{2}\right\}$$

$$L\left\{1 - \frac{1}{2} - \frac{\cos 2t}{2}\right\}$$

$$L\left\{\frac{1}{2} - \frac{\cos 2t}{2}\right\}$$

$$= \frac{1}{2s} - \frac{1}{2} \cdot \frac{s}{(s^2+4)}$$

$$= \frac{s^2+4-s^2}{2(s^2+4)} = \frac{2}{s^2+4}$$

* Properties of Laplace Transform

(1) Shifting property

- If $L\{f(t)\} = \bar{f}(s)$ then $\boxed{L\{e^{at} \cdot f(t)\} = \bar{f}(s-a)}$

Q:- $L\{e^{2t} \cdot \sin 3t\}$

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{e^{2t} \cdot \sin 3t\} = \frac{3}{(s-2)^2+9} = \frac{3}{s^2-4s+13}$$

Q:- $L\{e^{-t} \cdot t^5\}$

$$L\{t^5\} = \frac{5!}{s^6} = \frac{120}{s^6}$$

$$L\{e^{-t} \cdot t^5\} = \frac{120}{(s+1)^6}$$

Q:- $L\{e^{-3t} \cdot \sin t\}$

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\{e^{-3t} \cdot \sin t\} = \frac{1}{(s+3)^2+1} = \frac{1}{s^2+6s+10}$$

(2) Multiply by t property

If $L\{f(t)\} = \bar{f}(s)$ then $L\{t^n \cdot f(t)\} = (-1)^n \cdot \frac{d}{ds} \bar{f}(s)$

Q:- $L\{t\sin 3t\}$

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{t\sin 3t\} = (-1) \frac{d}{ds} \left(\frac{3}{s^2+9}\right)$$

$$= (-1) \frac{-3(2s)}{(s^2+9)^2}$$

$$L\{t\sin 3t\} = \frac{+6s}{(s^2+9)^2}$$

Q:- $L\{t^2 \sin 3t\}$

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{t^2 \sin 3t\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{3}{s^2+9}\right)$$

$$= 1 \frac{d}{ds}\left(\frac{3(-2s)}{(s^2+9)^2}\right)$$

$$= -6 \frac{(s^2+9)^2 \cdot 1 - s\,2(s^2+9)(2s)}{(s^2+9)^4}$$

$$= -6\left(\frac{(s^2+9) - 4s^2}{(s^2+9)^3}\right)$$

$$= -6\left(\frac{-3s^2+9}{(s^2+9)^3}\right)$$

$$L\{t^2 \sin 3t\} = \frac{18s^2-54}{(s^2+9)^3}$$

Q:- Evaluate $L\{e^{-2t} \cdot t \cdot \cos t\}$

$$L\{\cos t\} = \frac{s}{s^2+1}$$

$$L\{t\cos t\} = (-1)\frac{d}{ds}\left(\frac{s}{s^2+1}\right)$$

$$L\{t\cos t\} = (-1)\frac{(s^2+1)\cdot 1 - s(2s)}{(s^2+1)^2} = -1\frac{(-s^2+1)}{(s^2+1)^2}$$

$$L\{\cos t \cdot t\} = \frac{s^2-1}{(s^2+1)^2}$$

$$L\{e^{-2t} \cdot t\cos t\} = \frac{(s+2)^2-1}{((s+2)^2+1)^2} = \frac{s^2+4s+3}{(s^2+4s+5)^2}$$

3] Divided by t parameter

$L\{f(t)\} = \bar{f}(s)$

then $\boxed{L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds}$

Q:- $L\left\{\frac{e^{-t} \cdot \sin t}{t}\right\}$

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds$$

$$= \int_s^\infty \frac{1}{s^2+1}\,ds$$

$$= \left[\tan^{-1}(s)\right]_s^\infty = \tan^{-1}\infty - \tan^{-1}s = \pi/2 - \tan^{-1}s = \cot^{-1}s$$

$$L\left\{e^{-t} \cdot \frac{\sin t}{t}\right\} = \cot^{-1}(s+1)$$

4] Laplace Transform of derivative

- If $L\{f(t)\} = \bar{f}(s)$ then $\boxed{L\left\{\frac{d}{dt}f(t)\right\} = s \cdot \bar{f}(s) - f(0)}$

where $f(0) = \lim_{t \to 0} f(t)$

5] Laplace transform of integral

$$L\left\{\int_0^t f(t)\,dt\right\} = \frac{\bar{f}(s)}{s}$$

Q:- $L\left\{\int_0^t \frac{e^{-t}\cdot \sin t}{t}\,dt\right\}$

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{s^2+1} = \left[\tan^{-1}(s)\right]_s^\infty = \pi/2 - \tan^{-1}s = \cot^{-1}s$$

$$L\left\{\frac{e^{-t}\cdot \sin t}{t}\right\} = \cot^{-1}(s+1)$$

$$L\left\{\int_0^t \frac{e^{-t}\cdot \sin t}{t}\,dt\right\} = \frac{\cot^{-1}(s+1)}{s}$$

Q:- $L\left\{t\int_0^t \frac{e^{t}\cdot \sin t}{t}\,dt\right\}$

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{s^2+1}\,ds = \left[\tan^{-1}s\right]_s^\infty = \pi/2 - \tan^{-1}s = \cot^{-1}s$$

$$L\left\{\frac{e^{t}\cdot \sin t}{t}\right\} = \cot^{-1}(s-1)$$

$$L\left\{\int_0^t \frac{e^{t}\cdot \sin t}{t}\,dt\right\} = \frac{\cot^{-1}(s-1)}{s}$$

$$L\left\{t\cdot\int_0^t \frac{e^{t}\sin t}{t}\,dt\right\} = (-1)\frac{d}{ds}\left(\frac{\cot^{-1}(s-1)}{s}\right)$$

$$= -1 \quad \frac{s\cdot\frac{-1}{(s-1)^2+1} - \cot^{-1}(s-1)\cdot 1}{s^2}$$

$$= \frac{1}{((s-1)^2+1)s} + \frac{\cot^{-1}(s-1)}{s^2}$$

Q:- $\int_0^\infty e^{-3t} \cdot \sin t \, dt$

$$L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

$$= \int_0^\infty e^{-3t} \cdot f(t)\, dt$$

$s = 3$

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\{e^{-3t} \cdot \sin t\} = \frac{1}{s^2+1}\bigg|_{s=3} = \underline{\underline{\frac{1}{10}}}$$

* Inverse Laplace Transform

- If $f(t)$ is given $t > 0$ then Laplace transform is defined as $L\{f(t)\} = \bar{f}(s) = \int_0^\infty e^{-st} \cdot f(t)\, dt$

then $L^{-1}\{\bar{f}(s)\} = f(t)$

Eg:- $L\{t^n\} = \frac{n!}{s^{n+1}}$

$$L\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$$

$$L\{\cos at\} = \frac{s}{s^2+a^2}$$

$$L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$$

$$L\{\sin at\} = \frac{a}{s^2+a^2}$$

$$L^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{1}{a}\sin at$$

Q:- $L^{-1}\left\{\frac{1}{2s-5}\right\}$

$$L^{-1}\left\{\frac{1}{2(s-5/2)}\right\} = \frac{1}{2}e^{5/2 t}$$

Q:- $L^{-1}\left\{\frac{1}{s(s+1)}\right\}$

$$L^{-1}\left\{\frac{1}{s}-\frac{1}{s+1}\right\} = t-e^{-t}$$

☆☆ Q:- $L^{-1}\left\{\frac{1}{(s+1)^2}\right\}$ ☆☆

- For perfect square we use shifting theorem,

$$e^{-t}L^{-1}\left\{\frac{1}{s^2}\right\} = e^{-t}\cdot t$$

☆☆ Q:- $L^{-1}\left\{\frac{s}{(s+a)^2}\right\}$ ☆☆

$$\frac{s}{(s+a)^2} = \frac{A}{s+a}+\frac{B}{(s+a)^2}$$

$$s = A(s+a)+B$$

At $s=-a$

$$\boxed{-a = B}$$

At $s=0$

$$0 = Aa - a$$

$$\boxed{A=1}$$

$$L^{-1}\left\{\frac{1}{s+a}-\frac{a}{(s+a)^2}\right\} = e^{-at}-ae^{-at}\cdot t$$

$$= e^{-at}(1-at)$$

Q:- $L^{-1}\left\{\frac{s+23}{s^2+4s+13}\right\}$

$$L^{-1}\left\{\frac{s+23}{(s+2)^2+(3)^2}\right\}$$

$$L^{-1}\left\{\frac{s+2}{(s+2)^2+(3)^2}+\frac{21}{(s+2)^2+(3)^2}\right\}$$

$$e^{-2t}\cdot\cos 3t+\frac{\cancel{21}^{\,7}}{\cancel{3}}\sin 3t\, e^{-2t}$$

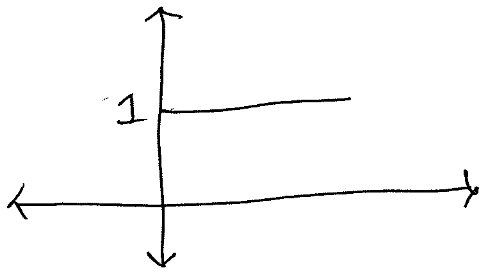
$$= e^{-2t}\left[\cos 3t+7\sin 3t\right]$$

*Laplace Transform of Unit step function

$H(t)$ or $u(t) = 1 \quad , t > 0$

OR $\qquad\qquad = 0 \quad , t < 0$

Heavy side unit function



$$\boxed{L\{u(t)\} = \frac{1}{s}}$$

$L\{u(t-a)\} = ?$

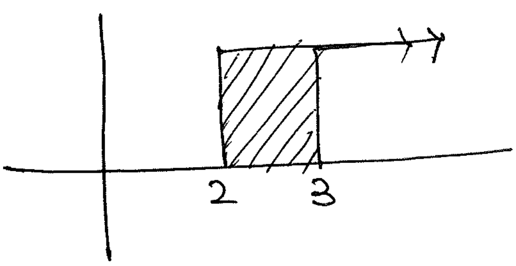
$L\{u(t)\} = 1/s$

$L\{u(t-a)\} = e^{-as} \cdot \frac{1}{s}$

$$\int_0^\infty e^{-st} \cdot u(t-a)\, dt$$

$$\int_a^\infty e^{-st} \cdot 1 \cdot dt = \left[\frac{e^{-st}}{-s}\right]_a^\infty = \frac{e^{-\infty}}{-s} - \frac{e^{-as}}{-s} = \frac{e^{-as}}{s}$$

Q:- Function is defined in graph what is the laplace transform.



Sol^n:- $L\{u(t-2) - u(t-3)\}$

$$= \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s} = \frac{e^{-2s} - e^{-3s}}{s}$$

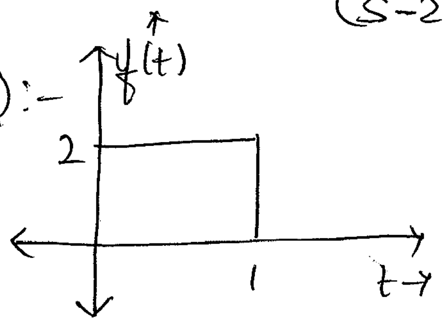
Q:- $f(t) = e^{2t} \sin 5t \cdot u(t)$

$L\{f(t)\} = ?$

$$L\{\sin 5t\} = \frac{5}{s^2+25}$$

$$L\{e^{2t} \sin 5t\} = \frac{5}{(s-2)^2+25} = \frac{5}{s^2-4s+29}$$

Q:-



Find laplace transform.

$= 2u(t) - 2u(t-1)$

$= 2L\{u(t) - u(t-1)\}$

$= 2\left[\frac{1}{s} - \frac{e^{-s}}{s}\right] = 2\left(\frac{1-e^{-s}}{s}\right)$

Q:- The laplace transform of e^{i5t} where $i = \sqrt{-1}$

$e^{\pm it} = \cos it \pm i \sin it$

$e^{i5t} = \cos 5t + i \sin 5t$

$$L\{\cos 5t\} = \frac{s}{s^2+25}$$

$$L\{i \sin 5t\} = \frac{i \cdot 5}{s^2+25}$$

$$L\{e^{i5t}\} = \frac{s+i5}{s^2+25}$$ //

* Application of Laplace Transform

1] To solve Ordinary Differential Equations

Q:- $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t$; $y(0) = 2$, $y'(0) = 1$

Step:1 Take L.T. on both sides

$$L\left\{\frac{d^2y}{dt^2}\right\} - 2L\left\{\frac{dy}{dt}\right\} + L\{y\} = L\{e^t\}$$

$$= \frac{1}{s-1}$$

we know that $L\{y'\} = s\bar{y}(s) - y(0)$

$$L\{y''\} = s^2\bar{y}(s) - sy(0) - y'(0)$$

$$s^2\bar{y}(s) - sy(0) - y'(0) - 2[s\bar{y}(s) - y(0)] + \bar{y}(s) = \frac{1}{s-1}$$

$$\bar{y}(s)[s^2 - 2s + 1] - 2s - 1 + 4 = \frac{1}{s-1}$$

$$\bar{y}(s)[s^2 - 2s + 1] - 2s + 3 = \frac{1}{s-1}$$

$$\bar{y}(s)[s^2 - 2s + 1] = \frac{1}{s-1} + 2s - 3$$

$$\bar{y}(s)[s^2 - 2s + 1] = \frac{1 + (2s-3)(s-1)}{s-1}$$

$$\bar{y}(s) = \frac{1 + 2s^2 - 2s - 3s + 3}{(s-1)(s^2 - 2s + 1)}$$

$$\bar{y}(s) = \frac{2s^2 - 5s + 4}{(s-1)(s^2 - 2s + 1)}$$

$$\bar{y}(s) = \frac{2s^2 - 5s + 4}{(s-1)(s-1)^2}$$

$$\cancel{\frac{2s^2-5s+4}{(s-1)^3}} = \cancel{\frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)^3}}$$

Step: II

$$L\{f(t)\} = \bar{y}(s)$$

$$L^{-1}\{\bar{y}(s)\} = f(t)$$

$$f(t) = L^{-1}\left\{\frac{2s^2-5s+4}{(s-1)^3}\right\}$$

$$\frac{2s^2-5s+4}{(s-1)^3} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)^3}$$

$$2s^2-5s+4 = A(s-1)^2 + B(s-1) + C$$

At $s = 1$

$2 - 5 + 4 = C$

$\boxed{C = 1}$

$2s^2 = As^2$

$\boxed{A = 2}$

$4 = A + (-B) + C$

$4 = 2 - B + 1$

$\boxed{B = -2}$

$$\frac{2s^2-5s+4}{(s-1)^3} = \frac{2}{s-1} - \frac{2}{(s-1)^2} + \frac{1}{(s-1)^3}$$

$$= L^{-1}\left\{\frac{2}{s-1}\right\} - L^{-1}\left\{\frac{1}{(s-1)^2}\right\} + L^{-1}\left\{\frac{1}{(s-1)^3}\right\}$$

$$L^{-1}\left\{\frac{s^2-5s+4}{(s-1)^3}\right\} = 2e^t - e^t \cdot t + \frac{t^2 \cdot e^t}{2}$$

$$\boxed{f(t) = e^t\left(\frac{t^2}{2} - t + 2\right)}$$

Q:- EC-14 With initial value $y(0) = y'(0) = 1$ the solution of differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ at $x = 1$ is ___

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

$$s^2 Y(s) - s y(0) - y'(0) + 4[sY(s) - y(0)] + 4Y(s) = 0$$

$$Y(s)[s^2 + 4s + 4] - s - 1 - 4 = 0$$

$$Y(s) = \frac{s+5}{s^2+4s+4}$$

$$Y(s) = \frac{s+5}{(s+2)^2}$$

Taking inverse L.T.

$$\frac{s+5}{(s+2)^2} = \frac{A}{s+2} + \frac{B}{(s+2)^2}$$

$$s+5 = A(s+2) + B$$

At $s = -2$

$$\boxed{3 = B}$$

At $s = 0$

$$5 = 2A + B$$

$$2 = 2A$$

$$\boxed{A = 1}$$

$$f(t) = L^{-1}\left\{\frac{1}{s+2} + \frac{3}{(s+2)^2}\right\}$$

$$f(t) = e^{-2x} + 3x \cdot e^{-2x}$$

$$= e^{-2} + 3e^{-2}$$

$$= e^{-2}(1+3) = 4e^{-2}$$

$$= 0.54$$

Q:- $L^{-1}\left\{\frac{3s+1}{s^3+4s^2+(k-3)s}\right]$ if $\lim_{t\to\infty} f(t) = 1$, the value of k is,

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} \cancel{s}\cdot \frac{3s+1}{\cancel{s}((s^2+4s+k-3))}$$

$$1 = \frac{1}{k-3}$$

$$k-3 = 1$$

$$\boxed{k=4}$$

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$ ⇒ Final value Theorem

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$ ⇒ Initial value Theorem

Q:- $y'' - y = t$. Solve by Laplace $y(0) = 1, y'(0) = 1$

$$s^2 Y(s) - sy(0) - y'(0) - Y(s) = \frac{1}{s^2}$$

$$y(s)[s^2-1] - s\cdot 1 - 1 = \frac{1}{s^2}$$

$$y(s)(s^2-1) = \frac{1}{s^2} + s + 1$$

$$y(s) = \frac{1+s^3+s^2}{s^2(s^2-1)}$$

$$g(s) = \frac{s^3+s^2+1}{s^2(s-1)(s+1)}$$

$$L^{-1}\left\{\frac{s^3+s^2+1}{s^2(s-1)(s+1)}\right\} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{s+1}$$

$$s^3+s^2+1 = As\cdot(s^2-1) + B(s-1)(s+1) + C(s+1)s^2 + Ds^2(s-1)$$

At $s=1$

$3 = C(2)$

$\boxed{C = 3/2}$

At $s=-1$

$1 = D(1)(-2)$

$\boxed{D = \frac{-1}{2}}$

At $s=0$

$1 = B(-1)$

$\boxed{B=-1}$

$1 = A+C+D$

$1 = A + \frac{3}{2} - \frac{1}{2}$

$\boxed{A=0}$

$$L^{-1}\left\{\frac{-1}{s^2} + \frac{3}{2(s-1)} - \frac{1}{2(s+1)}\right\}$$

$$f(t) = -t + \frac{3}{2}e^{t} - \frac{1}{2}e^{-t}$$

Q:- The Laplace Transform of function $f(t) = \frac{1}{s^2(s+1)}$. The function $f(t)$ is ____.

Sol^n:- $$\frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$$1 = As(s+1) + B(s+1) + Cs^2$$

At $s=-1$

$\boxed{1 = C}$

At $s=0$

$\boxed{1 = B}$

$A + C = 0$

$\boxed{A=-1}$

$$f(t) = L^{-1}\left\{\frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}\right\} = -1 + t + e^{-t}$$