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Alternating Current

TOPIC 1 Alternating Current, Voltage and Power

1. An alternating voltage $v(t) = 220 \sin 100$Àt volt is applied to a purely resistive load of 50Ω. The time taken for the current to rise from half of the peak value to the peak value is : **[8 April 2019 I]**

(a) 5 ms (b) 2.2 ms (c) 7.2 ms (d) 3.3 ms

2. A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $I = I_o \cos(\omega t)$. The emf induced in the smaller inner loop is nearly : **[Online April 8, 2017]**

(a) $\frac{\pi\mu_0 I_o}{2} \cdot \frac{a^2}{b} \omega \sin(\omega t)$ (b) $\frac{\pi\mu_0 I_o}{2} \cdot \frac{a^2}{b} \omega \cos(\omega t)$

(c) $\pi\mu_0 I_o \frac{a^2}{b} \omega \sin(\omega t)$ (d) $\frac{\pi\mu_0 I_o b^2}{a} \omega \cos(\omega t)$

3. A sinusoidal voltage $V(t) = 100 \sin(500t)$ is applied across a pure inductance of $L = 0.02$ H. The current through the coil is: **[Online April 12, 2014]**

(a) $10 \cos(500\,t)$ (b) $-10 \cos(500t)$

(c) $10 \sin(500t)$ (d) $-10 \sin(500t)$

4. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by **[2007]**

(a) $P = \sqrt{2} E_0 I_0$ (b) $P = \frac{E_0 I_0}{\sqrt{2}}$

(c) $P =$ zero (d) $P = \frac{E_0 I_0}{2}$

5. In a uniform magnetic field of induction B a wire in the form of a semicircle of radius r rotates about the diameter of the circle with an angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, the mean power generated per period of rotation is **[2004]**

(a) $\frac{(B\pi r\omega)^2}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$

(c) $\frac{B\pi r^2\omega}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

6. Alternating current can not be measured by D.C. ammeter because **[2004]**

(a) Average value of current for complete cycle is zero

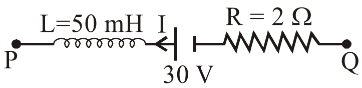
(b) A.C. Changes direction

(c) A.C. can not pass through D.C. Ammeter

(d) D.C. Ammeter will get damaged.

TOPIC 2 AC Circuit, LCR Circuit, Quality and Power Factor

7. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is ______. **[NA Sep. 06, 2020 (I)]**



8. An AC circuit has $R = 100\ \Omega$, $C = 2\ \mu F$ and $L = 80$ mH, connected in series. The quality factor of the circuit is : **[Sep. 06, 2020 (I)]**

(a) 2 (b) 0.5

(c) 20 (d) 400

9. In a series LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value C as $\left(\frac{n}{3\pi}\right) \mu F$, then value of n is ______. **[NA Sep. 06, 2020 (II)]**

10. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at $t = 0$, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is : **[Sep. 04, 2020 (II)]**

(a) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$ (b) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$

(c) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$ (d) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$

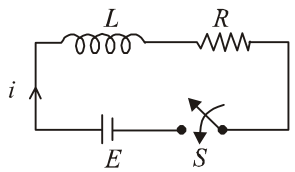
11. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2 J/°C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to : **[Sep. 03, 2020 (I)]**

(a) 418 s (b) 245 s
(c) 365 s (d) 348 s

12. An inductance coil has a reactance of 100 Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is : **[Sep. 02, 2020 (II)]**

(a) 1.1×10^{-2} H (b) 1.1×10^{-1} H
(c) 5.5×10^{-5} H (d) 6.7×10^{-7} H

13. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t = \frac{L}{R}$ is : **[12 April 2019 II]**



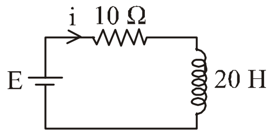
(a) $\frac{2.7EL}{R^2}$ (b) $\frac{EL}{2.7R^2}$

(c) $\frac{7.3EL}{R^2}$ (d) $\frac{EL}{7.3R^2}$

14. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [take $ln\,5 = 1.6$] **[10 April 2019 II]**

(a) 0.324 s (b) 0.103 s
(c) 0.002 s (d) 0.016 s

15. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is : **[8 April 2019 I]**



(a) $\frac{2}{ln\,2}$ (b) $\frac{1}{2}ln\,2$
(c) $2\,ln\,2$ (d) $ln\,2$

16. A circuit connected to an *ac* source of *emf* $e = e_0\sin(100t)$ with *t* in seconds, gives a phase difference of $\frac{\pi}{4}$ between the *emf e* and current *i*. Which of the following circuits will exhibit this ? **[8 April 2019 II]**

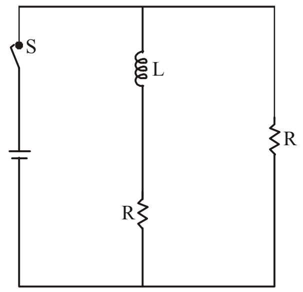
(a) RL circuit with R = 1 kΩ and L = 10 mH

(b) RL circuit with R = 1 kΩ and L = 1 mH

(d) RC circuit with R = 1 kΩ and C = 1 μF

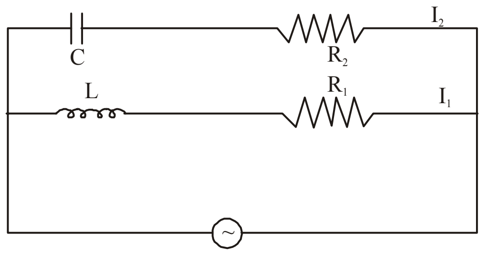
(d) RC circuit with R = 1 kΩ and C = 10 μF.

17. In the figure shown, a circuit contains two identical resistors with resistance R = 5 Ω and an inductance with L = 2 mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed? **[12 Jan. 2019 I]**



(a) 5.5 A (b) 7.5 A
(c) 3 A (d) 6 A

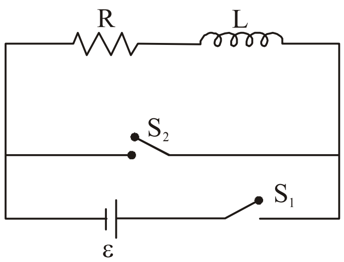
18.



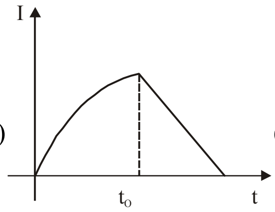
In the above circuit, $C = \frac{\sqrt{3}}{2}\mu F$, $R_2 = 20\,\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\,\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of A.C source is given by, $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is : **[12 Jan. 2019 II]**

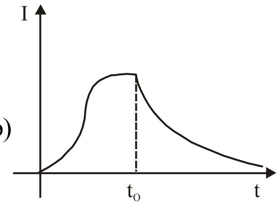
(a) 60° (b) 30°
(c) 90° (d) 0

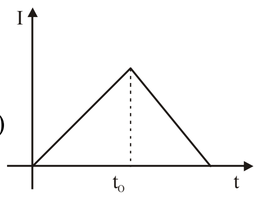
19. In the circuit shown,

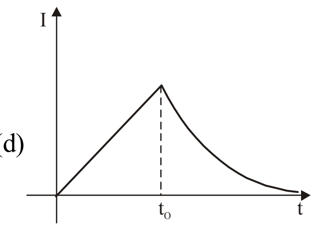


the switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. the behaviour of the current I as a function of time 't' is given by: **[11 Jan. 2019 II]**

(a)


(b)


(c)


(d)


20. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is: **[9 Jan. 2019 I]**

(a) 5.65×10^2 J (b) 2.26×10^3 J

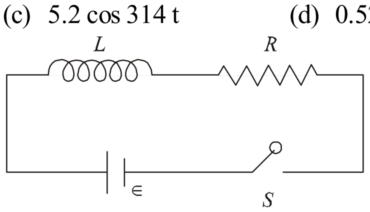
(c) 5.17×10^2 J (d) 3.39×10^3 J

21. In LC circuit the inductance L = 40 mH and capacitance C = 100 μF. If a voltage V(t) = 10 sin(314 t) is applied to the circuit, the current in the circuit is given as: **[9 Jan. 2019 II]**

(a) 0.52 cos 314 t (b) 10 cos 314 t

(c) 5.2 cos 314 t (d) 0.52 sin 314 t

22.



As shown in the figure, a battery of emf ∈ is connected to an inductor L and resistance R in series. The switch is closed at $t = 0$. The total charge that flows from the battery, between $t = 0$ and $t = t_c$ (t_c is the time constant of the circuit) is: **[8 Jan. 2020 II]**

(a) $\frac{\in R}{eL^2}$ (b) $\frac{\in L}{R^2}\left(1-\frac{1}{e}\right)$

(c) $\frac{\in L}{R^2}$ (d) $\frac{\in R}{eL^2}$

23. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be: **[7 Jan. 2020 I]**

(a) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(b) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

(c) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

(d) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

24. An emf of 20 V is applied at time $t = 0$ to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time $t = \infty$ and at $t = 40$ s is close to:

(Take $e^2 = 7.389$) **[7 Jan. 2020 II]**

(a) 1.06 (b) 1.15

(c) 1.46 (d) 0.84

25. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30\,t$

$i = 20 \sin\left(30\text{t} - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively: **[2018]**

(a) 50W, 10A (b) $\frac{1000}{\sqrt{2}}$ W, 10A

(c) $\frac{50}{\sqrt{2}}$ W, 0 (d) 50W, 0

26. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by: **[2018]**

(a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$ (c) $\frac{R}{(\omega_0 C)}$ (d) $\frac{CR}{\omega_0}$

27. A sinusoidal voltage of peak value 283 V and angular frequency 320/s is applied to a series LCR circuit. Given that R = 5 Ω, L = 25 mH and C = 1000 μF. The total impedance, and phase difference between the voltage across the source and the current will respectively be : **[Online April 9, 2017]**

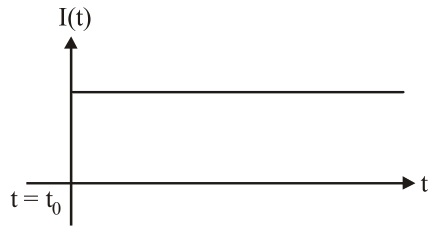
(a) 10 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$ (b) 7 Ω and 45°

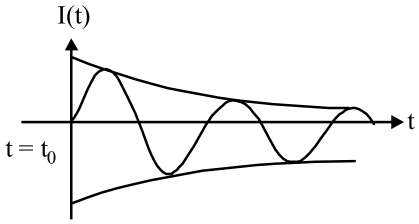
(c) 10 Ω and $\tan^{-1}\left(\frac{8}{3}\right)$ (d) 7 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$

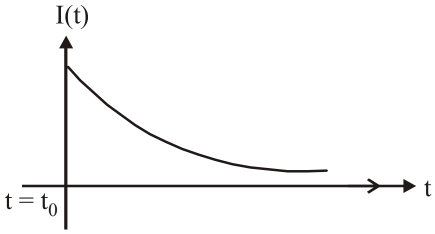
28. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to : **[2016]**

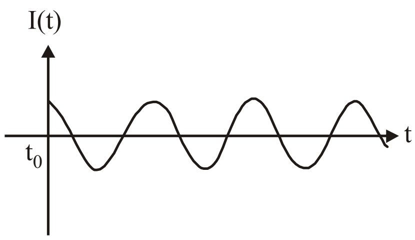
(a) 0.044 H (b) 0.065 H
(c) 80 H (d) 0.08 H

29. A series LR circuit is connected to a voltage source with $V(t) = V_0 \sin\omega t$. After very large time, current I(t) behaves as $\left(t_0 >> \frac{L}{R}\right)$: **[Online April 9, 2016]**

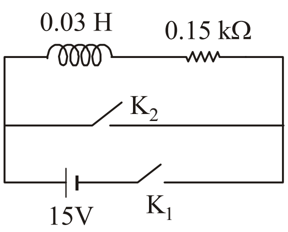
(a)


(b)


(c)


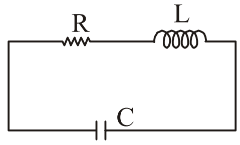
(d)


30. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V emf in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be : $\left(e^5 \cong 150\right)$ **[2015]**

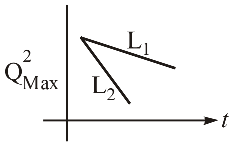


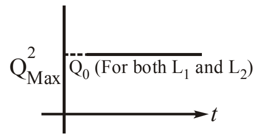
(a) 6.7 mA (b) 0.67 mA
(c) 100 mA (d) 67 mA

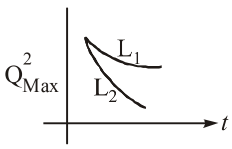
31. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :

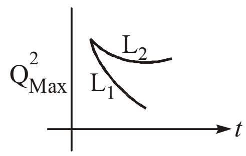


If a student plots graphs of the square of maximum charge $\left(Q^2_{Max}\right)$ on the capacitor with time(t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (*plots are schematic and not drawn to scale*) **[2015]**

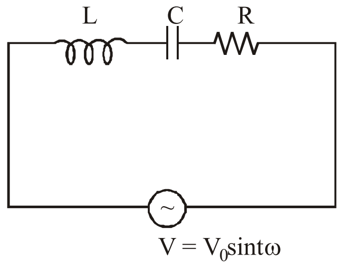
(a)


(b)


(c)


(d)


32. For the LCR circuit, shown here, the current is observed to lead the applied voltage. An additional capacitor C', when joined with the capacitor C present in the circuit, makes the power factor of the circuit unity. The capacitor C', must have been connected in : **[Online April 11, 2015]**



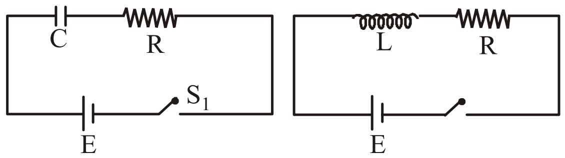
(a) series with C and has a magnitude $\frac{C}{(\omega^2 LC - 1)}$

(b) series with C and has a magnitude $\frac{1 - \omega^2 LC}{\omega^2 L}$

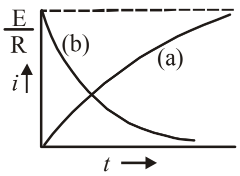
(c) parallel with C and has a magnitude $\frac{1 - \omega^2 LC}{\omega^2 L}$

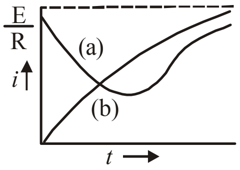
(d) parallel with C and has a magnitude $\frac{C}{(\omega^2 LC - 1)}$

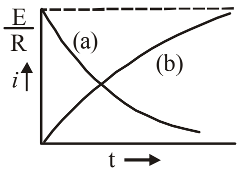
33. In the circuits (a) and (b) switches S_1 and S_2 are closed at $t = 0$ and are kept closed for a long time. The variation of current in the two circuits for $t \geq 0$ are roughly shown by figure (figures are schematic and not drawn to scale) : **[Online April 10, 2015]**

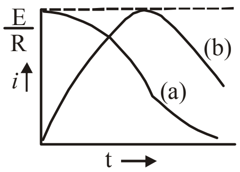


(a) (b)

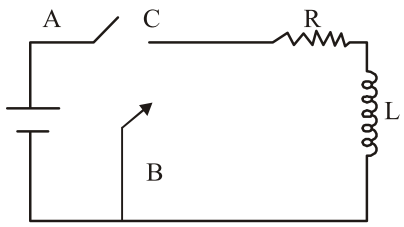
(a) 

(b) 

(c) 

(d) 

34. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to: **[2014]**

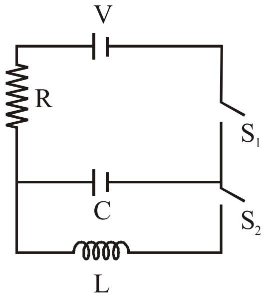


(a) $\frac{e}{1-e}$ (b) 1 (c) –1 (d) $\frac{1-e}{e}$

35. When the rms voltages V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio $V_L : V_C : V_R = 1 : 2 : 3$. If the rms voltage of the AC sources is 100 V, the V_R is close to: **[Online April 9, 2014]**

(a) 50 V (b) 70 V (c) 90 V (d) 100 V

36. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is Capacitive time constant). Which of the following statement is correct ? **[2013]**



(a) Work done by the battery is half of the energy dissipated in the resistor

(b) At, $t = \tau$, $q = CV/2$

(c) At, $t = 2\tau$, $q = CV(1 - e^{-2})$

(d) At, $t = 2\tau$, $q = CV(1 - e^{-1})$

37. A series LR circuit is connected to an ac source of frequency ω and the inductive reactance is equal to 2R. A capacitance of capacitive reactance equal to R is added in series with L and R. The ratio of the new power factor to the old one is : **[Online April 25, 2013]**

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{2}{5}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{5}{2}}$

38. When resonance is produced in a series LCR circuit, then which of the following is not correct ? **[Online April 25, 2013]**

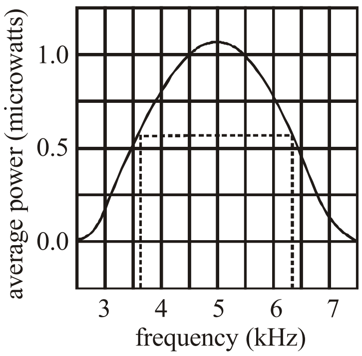
(a) Current in the circuit is in phase with the applied voltage.

(b) Inductive and capacitive reactances are equal.

(c) If R is reduced, the voltage across capacitor will increase.

(d) Impedance of the circuit is maximum.

39. The plot given below is of the average power delivered to an LRC circuit versus frequency. The quality factor of the circuit is : **[Online April 23, 2013]**

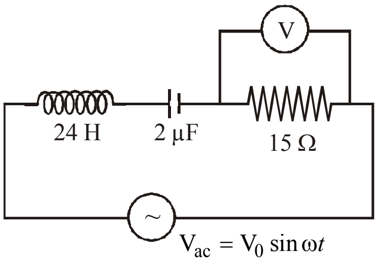


(a) 5.0 (b) 2.0 (c) 2.5 (d) 0.4

40. In a series L-C-R circuit, $C = 10^{-11}$ Farad, $L = 10^{-5}$ Henry and $R = 100$ Ohm, when a constant D.C. voltage E is applied to the circuit, the capacitor acquires a charge 10^{-9} C. The D.C. source is replaced by a sinusoidal voltage source in which the peak voltage E_0 is equal to the constant D.C. voltage E. At resonance the peak value of the charge acquired by the capacitor will be : **[Online April 22, 2013]**

(a) 10^{-15} C (b) 10^{-6}C (c) 10^{-10}C (d) 10^{-8} C

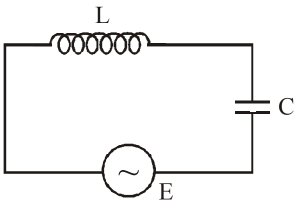
41. An LCR circuit as shown in the figure is connected to a voltage source V_{ac} whose frequency can be varied.



The frequency, at which the voltage across the resistor is maximum, is : **[Online April 22, 2013]**

(a) 902 Hz (b) 143 Hz (c) 23 Hz (d) 345 Hz

42. In the circuit shown here, the voltage across E and C are respectively 300 V and 400 V. The voltage E of the ac source is : **[Online April 9, 2013]**



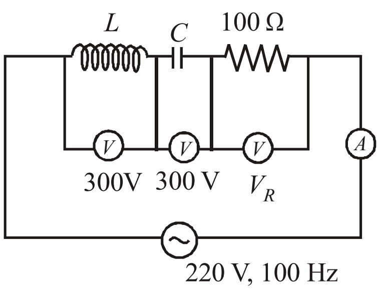
(a) 400 Volt (b) 500 Volt (c) 100 Volt (d) 700 Volt

43. A resistance R and a capacitance C are connected in series to a battery of negligible internal resistance through a key. The key is closed at $t = 0$. If after t sec the voltage across the capacitance was seven times the voltage across R, the value of t is **[Online May 12, 2012]**

(a) $3RC\,\ell n\,2$ (b) $2\,RC\,\ell n\,2$

(c) $2\,RC\,\ell n\,7$ (d) $3\,RC\,\ell n\,7$

44. In an LCR circuit shown in the following figure, what will be the readings of the voltmeter across the resistor and ammeter if an $a.c.$ source of 220V and 100 Hz is connected to it as shown? **[Online May 7, 2012]**



(a) 800 V, 8 A (b) 110 V, 1.1 A

(c) 300 V, 3 A (d) 220V, 2.2 A

45. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is: **[2011]**

(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$

(c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

46. A resistor 'R' and $2\mu F$ capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10} 2.5 = 0.4$) **[2011]**

(a) $1.7 \times 10^5\,\Omega$ (b) $2.7 \times 10^6\,\Omega$

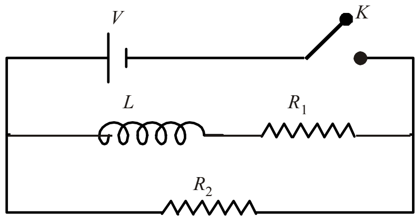
(c) $3.3 \times 10^7\,\Omega$ (d) $1.3 \times 10^4\,\Omega$

47. Combination of two identical capacitors, *a* resistor R and *a dc* voltage source of voltage 6V is used in an experiment on a (C-R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time for needed for reducing the voltage of the fully charged series combination by half is **[2011 RS]**

(a) 10 second (b) 5 second

(c) 2.5 second (d) 20 second

48. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is **[2010]**



(a) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

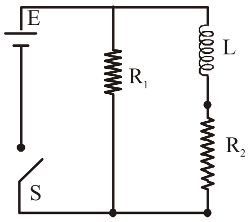
(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t = \infty$

(d) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

49. In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is **[2010]**

(a) 305 W (b) 210 W (c) Zero W (d) 242 W

50. 

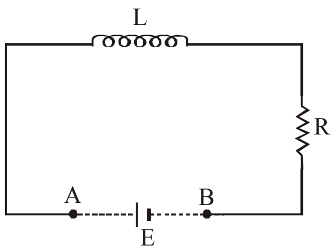
An inductor of inductance L = 400 mH and resistors of resistance $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[2009]**

(a) $\frac{12}{t}e^{-3t}$ V (b) $6\left(1-e^{-t/0.2}\right)$ V
(c) $12e^{-5t}$ V (d) $6e^{-5t}$ V

51. In a series resonant LCR circuit, the voltage across R is 100 volts and R = 1 kΩ with C = 2μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is **[2006]**

(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

52. An inductor (L = 100 mH), a resistor (R = 100 Ω) and a battery (E = 100 V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is **[2006]**



(a) 1/eA (b) eA (c) 0.1 A (d) 1 A

53. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is **[2006]**

(a) N.A.B.R.ω (b) N.A.B
(c) N.A.B.R. (d) N.A.B.ω

54. The phase difference between the alternating current and emf is $\frac{\pi}{2}$. Which of the following cannot be the constituent of the circuit? **[2005]**

(a) R, L (b) C alone (c) L alone (d) L, C

55. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be **[2005]**

(a) 0.4 (b) 0.8 (c) 0.125 (d) 1.25

56. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2V. The current reaches half of its steady state value in **[2005]**

(a) 0.1 s (b) 0.05 s (c) 0.3 s (d) 0.15 s

57. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of **[2005]**

(a) 8μF (b) 4μF (c) 2μF (d) 1μF

58. In an LCR series a.c. circuit, the voltage across each of the components, L, C and R is 50V. The voltage across the LC combination will be **[2004]**

(a) 100 V (b) $50\sqrt{2}$ V
(c) 50 V (d) 0 V (zero)

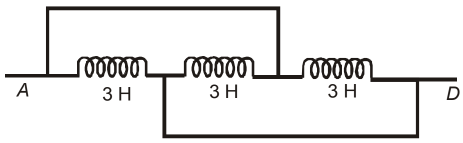
59. In a LCR circuit capacitance is changed from C to 2 C. For the resonant frequency to remain unchanged, the inductance should be changed from L to **[2004]**

(a) $L/2$ (b) $2L$ (c) $4L$ (d) $L/4$

60. The power factor of an AC circuit having resistance (R) and inductance (L) connected in series and an angular velocity ω is **[2002]**

(a) $R/\omega L$ (b) $R/(R^2 + \omega^2L^2)^{1/2}$
(c) $\omega L/R$ (d) $R/(R^2 - \omega^2L^2)^{1/2}$

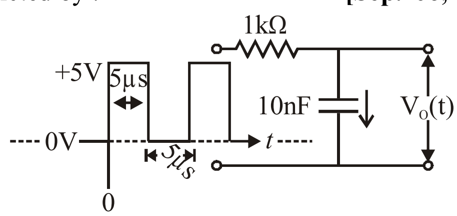
61. The inductance between A and D is **[2002]**

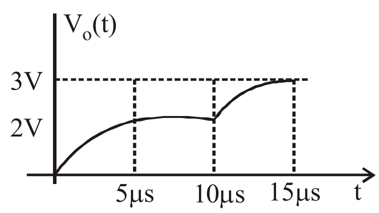


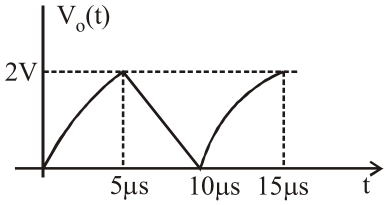
(a) 3.66 H (b) 9 H
(c) 0.66 H (d) 1 H

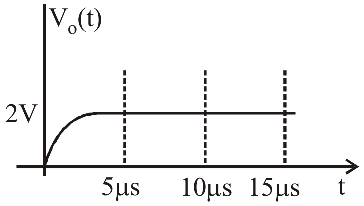
TOPIC 3 Transformers and LC Oscillations

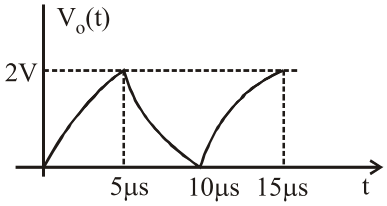
62. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_o(t)$, across the capacitor is correctly depicted by : **[Sep. 06, 2020 (I)]**



(a) 

(b)


(c)


(d)


63. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are : **[10 April 2019 I]**

(a) 220 V and 20 A (b) 440 V and 20 A
(c) 440 V and 5 A (d) 220 V and 10 A

64. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be: **[9 Jan. 2019 II]**

(a) 50 A (b) 45 A (c) 35 A (d) 25 A

65. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns, giving the output power at 230 V. If the current in the primary of the transformer is 5 A, and its efficiency is 90%, the output current would be: **[Online April 16, 2018]**

(a) 20 A (b) 40 A (c) 45 A (d) 25 A

66. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is **[2003]**

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q

67. The core of any transformer is laminated so as to **[2003]**

(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage

68. In a transformer, number of turns in the primary coil are 140 and that in the secondary coil are 280. If current in primary coil is 4 A, then that in the secondary coil is **[2002]**

(a) 4 A (b) 2 A (c) 6 A (d) 10 A.

Hints & Solutions

1. (d) As $V(t) = 220 \sin 100\,\pi t$

so, $I(t) = \frac{220}{50} \sin 100\,\pi t$

i.e., $I = I_m = \sin(100\,\pi t)$

For $I = I_m$

$$t_1 = \frac{\pi}{2} \times \frac{1}{100\pi} = \frac{1}{200} \text{ sec.}$$

and for $I = \frac{I_m}{2}$

$$\Rightarrow \frac{I_m}{2} = I_m \sin(100\,\pi t_2) \Rightarrow \frac{\pi}{6} = 100\,\pi t_2$$

$$\Rightarrow t_2 = \frac{1}{600} s$$

$$\therefore t_{\text{req}} = \frac{1}{200} - \frac{1}{600} = \frac{2}{600} = \frac{1}{300} s = 3.3 \text{ ms}$$

2. (a) For two concentric circular coil,

Mutual Inductance $M = \frac{\mu_0 \pi N_1 N_2 a^2}{2b}$

here, $N_1 = N_2 = 1$

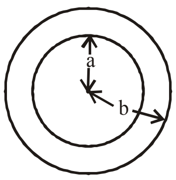
Hence, $M = \frac{\mu_0 \pi a^2}{2b}$ ..... (i)

and given $I = I_0 \cos \omega t$ ..... (ii)

Now according to Faraday's second law induced emf

$$e = -M\frac{dI}{dt}$$

From eq. (ii),



$$e = \frac{-\mu_0 \pi a^2}{2b} \frac{d}{dt}(I_0 \cos \omega t)$$

$$e = \frac{\mu_0 \pi a^2}{2b} I_0 \sin \omega t\,(\omega)$$

$$e = \frac{\pi \mu_0 I_0}{2} \cdot \frac{a^2}{b} \omega \sin \omega t$$

3. (b) In a pure inductive circuit current always lags behind the emf by $\frac{\pi}{2}$.

If $v(t) = v_0 \sin \omega t$

then $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

Now, given $v(t) = 100 \sin(500\,t)$

and $I_0 = \frac{E_0}{\omega L} = \frac{100}{500 \times 0.02}$ $[\because L = 0.02\text{H}]$

$$I_0 = 10 \sin\left(500t - \frac{\pi}{2}\right)$$

$$I_0 = -10\cos(500t)$$

4. (c) We know that power consumed in a.c. circuit is given by,

$P = E_{\text{rms}} . I_{\text{rms}} \cos \phi$

Here, $E = E_0 \sin \omega t$

$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

This means the phase difference, is $\phi = \frac{\pi}{2}$

$$\because \cos \phi = \cos\frac{\pi}{2} = 2$$

$$\therefore P = E_{rms} . I_{rms} . \cos\frac{\pi}{2} = 0$$

5. (b) $\phi = \vec{B}.\vec{A}\,;\ \phi = BA\cos\omega t$

$$\varepsilon = -\frac{d\phi}{dt} = \omega BA \sin\omega t\,;\ i = \frac{\omega BA}{R}\sin\omega t$$

$$P_{inst} = i^2 R = \left(\frac{\omega BA}{R}\right)^2 \times R\sin^2 \omega t$$

$$P_{avg} = \frac{\int_0^T P_{inst} \times dt}{\int_0^T dt} = \frac{(\omega BA)^2}{R} \frac{\int_0^T \sin^2 \omega t dt}{\int_0^T dt} = \frac{1}{2}\frac{(\omega BA)^2}{R}$$

$$\therefore P_{avg} = \frac{(\omega B \pi r^2)^2}{8R} \qquad \left[A = \frac{\pi r^2}{2}\right]$$

6. (a) D.C. ammeter measure average value of current. In AC current, average value of current in complete cycle is zero. Hence reading will be zero.

7. (33)

Here, $L = 50\text{ mH} = 50 \times 10^{-3}$ H; $I = 1$ A, $R = 2\Omega$

$$V_P - L\frac{dl}{dt} - 30 + RI = V_Q$$

$$\Rightarrow V_P - V_Q = 50 \times 10^{-3} \times 10^2 + 30 - 1 \times 2$$

$$= 5 + 30 - 2 = 33 \text{ V.}$$

8. **(a)** Quality factor,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{80\times10^{-3}}{2\times10^{-6}}}$$

$$= \frac{1}{100}\sqrt{40\times10^{3}} = \frac{200}{100} = 2$$

9. **(400)**

Given: Power $P = 400$ W, Voltage $V = 250$ V

$$P = V_m \cdot I_{\text{rms}} \cdot \cos\phi$$

$$\Rightarrow 400 = 250 \times I_{\text{rms}} \times 0.8 \Rightarrow I_{\text{rms}} = 2\,\text{A}$$

Using $P = I_{\text{rms}}^2 R$

$$(I_{\text{rms}})^2 \cdot R = P \Rightarrow 4 \times R = 400$$

$$\Rightarrow R = 100\Omega$$

Power factor is,

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}}$$

$$\Rightarrow 0.8 = \frac{100}{\sqrt{100^2 + X_L^2}} \Rightarrow 100^2 + X_L^2 = \left(\frac{100}{0.8}\right)^2$$

$$\Rightarrow X_L = \sqrt{-100^2 + \left(\frac{100}{0.8}\right)^2} \Rightarrow X_L = 75\Omega$$

When power factor is unity,

$$X_C = X_L = 75 \Rightarrow \frac{1}{\omega C} = 75$$

$$\Rightarrow C = \frac{1}{75\times2\pi\times50} = \frac{1}{7500\pi}F$$

$$= \left(\frac{10^6}{2500}\times\frac{1}{3\pi}\right)\mu F = \frac{400}{3\pi}\mu F$$

$N = 400$

10. **(a)** Potential energy stored in the inductor

$$U = \frac{1}{2}LI^2$$

During growth of current,

$$i = I_{\max}\left(1 - e^{-Rt/L}\right)$$

For U to be $\frac{U_{\max}}{n}$; i has to be $\frac{I_{\max}}{\sqrt{n}}$

$$\therefore \quad \frac{I_{\max}}{\sqrt{n}} = I_{\max}(1 - e^{-Rt/L})$$

$$\Rightarrow e^{-Rt/L} = 1 - \frac{1}{\sqrt{n}} = \frac{\sqrt{n}-1}{\sqrt{n}}$$

$$\Rightarrow -\frac{Rt}{L} = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

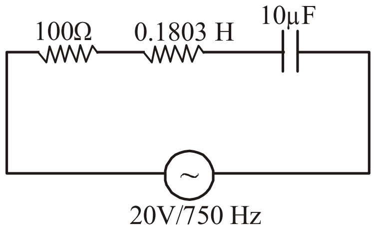
$$\Rightarrow t = \frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$$

11. **(d)** Here, $R = 100, X_L = L\omega = 0.1803 \times 750 \times 2\pi = 850\Omega$,

$$X_C = \frac{1}{C\omega} = \frac{1}{10^{-5}\times2\pi\times750} = 21.23\Omega$$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{100^2 + (850 - 21.23)^2} = 834.77 \simeq 835$$



$$H = i_{\text{rms}}^2 Rt = \left(\frac{V_{\text{rms}}}{|Z|}\right)^2 RT = (ms)\Delta t$$

$$\Rightarrow \frac{20}{835}\times\frac{20}{835}\times100t = (2)\times10$$

$\because V_{\text{rms}} = 20$ V and $\Delta t = 10°$C

$\therefore$ Time, $t = 348.61$ s.

12. **(a)** Given,

Reactance of inductance coil, $Z = 100\Omega$

Frequency of AC signal, $v = 1000$ Hz

Phase angle, $\phi = 45°$

$$\tan\phi = \frac{X_L}{R} = \tan 45° = 1$$

$$\Rightarrow X_L = R$$

Reactance, $Z = 100 = \sqrt{X_L^2 + R^2}$

$$\Rightarrow 100 = \sqrt{R^2 + R^2}$$

$$\Rightarrow \sqrt{2}R = 100 \Rightarrow R = 50\sqrt{2}$$

$$\therefore X_L = 50\sqrt{2}$$

$$\Rightarrow L\omega = 50\sqrt{2} \qquad (\because X_L = \omega L)$$

$$\Rightarrow L = \frac{50\sqrt{2}}{2\pi\times1000} \qquad (\because \omega = 2\pi v)$$

$$= \frac{25\sqrt{2}}{\pi}\text{ mH}$$

$$= 1.1\times10^{-2}\text{ H}$$

13. **(b)** We have, $i = i_0(1 - e^{-t/c}) = \dfrac{\varepsilon}{R}(1 - e^{-t/c})$

Charge, $q = \int_0^{\tau} i\,dt$

$$= \frac{\varepsilon}{R}\int_0^{\tau}(1 - e^{-t/\tau})dt = \frac{E}{R}\frac{\tau}{e} = \frac{E}{R}\times\frac{(L/R)}{e} = \frac{EL}{2.7R^2}$$

14. **(d)** $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$ Here $R = R_L + r = 1\Omega$

$$0.8I_0 = I_0\left(1 - e^{-\frac{t}{.01}}\right)$$

$\Rightarrow 0.8 = 1 - e^{-100t}$

$\Rightarrow e^{-100t} = 0.2 = \left(\dfrac{1}{5}\right)$

$\Rightarrow 100t = \ln 5 \Rightarrow t = \dfrac{1}{100}\ln 5 = 0.016$ s

15. **(c)** $i^2 R = \left(\tau\dfrac{di}{dt}\right)i$

$\Rightarrow \dfrac{di}{dt} = \dfrac{i}{\tau}$

$\Rightarrow t = \tau\, l n2 = 2 l n2 \left[\text{as}\,\tau = \dfrac{L}{R} = \dfrac{20}{10} = 2\right]$

16. **(d)** $\omega = 100$ rad/s

We know that

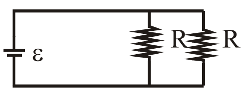
$$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR}$$

or $\tan 45^\circ = \dfrac{1}{\omega CR}$

or $\omega CR = 1$

LHS: $\omega CR = 10 \times 10 \times 10^{-6} \times 10^3 = 1$

17. **(d)** Long time after switch is closed, the inductor will be idle so, the equivalent diagram will be as below



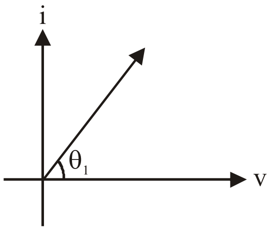
$$I = \frac{\varepsilon}{\left(\frac{R\times R}{R+R}\right)} = \frac{2\varepsilon}{R} = \frac{2\times 15}{5} = 6\,A$$

18. **(Bonus)**

Capacitive reactance,

$$X_c = \frac{1}{\omega C} = \frac{4}{10^{-6}\times\sqrt{3}\times 100} = \frac{2\times 10^4}{\sqrt{3}}$$

$\tan\theta_1 = \dfrac{X_C}{R_2} = \dfrac{10^3}{\sqrt{3}}$

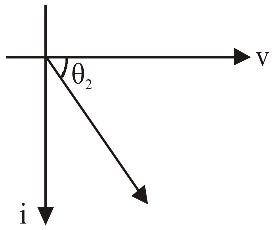


θ_1 is close to 90°

For L-R circuit

$X_L = \omega L = 100\times\dfrac{\sqrt{3}}{10} = 10\sqrt{3}$

$R_1 = 10$



$\tan\theta_2 = \dfrac{X_L}{R_1}$

$\tan\theta_2 = \sqrt{3} \Rightarrow \theta_2 = \tan^{-1}(\sqrt{3})$

$\theta_2 = 60°$

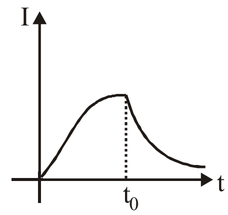
So, phase difference comes out 90° + 60° = 150°

If R_2 is 20 KΩ

then phase difference comes out to be 60 + 30 = 90°.

Therefore Ans. is **Bonus**

19. **(b)**



The current will grow for the time t = 0 to $t = t_0$ and after that decay of current takes place.

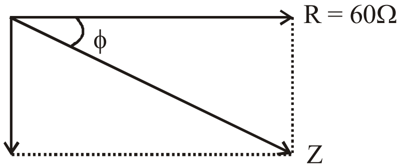
20. **(c)** Given: $R = 60\Omega$, $f = 50$ Hz, $\omega = 2\pi f = 100\pi$ and $v = 24v$

$C = 120\,\mu f = 120\times 10^{-6} f$

$$x_C = \frac{1}{\omega C} = \frac{1}{100\pi\times 120\times 10^{-6}} = 26.52\Omega$$

$x_L = \omega L = 100\pi\times 20\times 10^{-3} = 2\pi\Omega$

$x_C - x_L = 20.24 \approx 20$



$$z = \sqrt{R^2 + (x_C - x_L)^2}$$

$z = 20\sqrt{10}\Omega$

$$\cos\phi = \frac{R}{z} = \frac{60}{20\sqrt{10}} = \frac{3}{\sqrt{10}}$$

$$P_{avg} = VI\cos\phi, I = \frac{v}{z} = \frac{v^2}{z}\cos\phi = 8.64\text{ watt}$$

Energy dissipated (Q) in time t = 60s is

$Q = P.t = 8.64\times 60 = 5.17\times 10^2 J$

21. **(a)** Given, Inductance, $L = 40$ mH

Capacitance, $C = 100\,\mu F$

Impedance, $Z = X_C - X_L$

$$\Rightarrow \quad Z = \frac{1}{\omega C} - \omega L \qquad \left(\because X_c = \frac{1}{\omega C} \text{ and } X_L = \omega L\right)$$

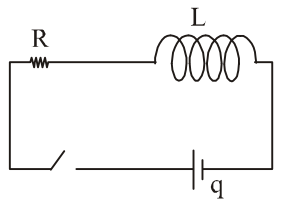
$$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$$

$= 19.28\Omega$

Current, $i = \frac{V_0}{Z}\sin(\omega t + \pi/2)$

$$\Rightarrow \quad i = \frac{10}{19.28}\cos\omega t = 0.52\cos(314\,t)$$

22. **(a)** For series connection of a resistor and inductor, time variation of current is $I = I_0(1 - e^{-t/T_c})$



Here, $T_C = \frac{L}{R}$

$$q = \int_0^{T_c} i\,dt$$

$$\Rightarrow \int dq = \int \frac{E}{R}\left(1 - e^{-t/t_c}\right)dt$$

$$\Rightarrow q = \frac{\in}{R}\left[t + t_C e^{-t/t_c}\right]_0^{t_c}$$

$$\Rightarrow q = \frac{\in}{R}\left[t_C + \frac{t_C}{e} - t_C\right]$$

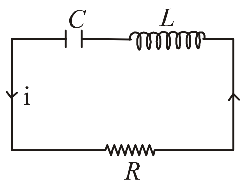
$$\Rightarrow q = \frac{\in}{R}\frac{L}{\text{Re}}$$

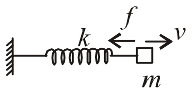
$$\therefore q = \frac{\in L}{R^2 e}$$

23. **(d)** In damped harmonic oscillation,

$$\frac{md^2x}{dt^2} = -kx - bv$$

$$\Rightarrow \quad \frac{md^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad \text{....(i)}$$





In LCR circuit, $\frac{-q}{C} - iR - \frac{Ldi}{dt} = 0$

$$L\frac{d^2}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \quad \text{...(ii)}$$

Comparing equations (i) & (ii)

$$L \leftrightarrow m,\ C \leftrightarrow \frac{1}{k},\ R \leftrightarrow b$$

24. **(a)** The current (I) in LR series circuit is given by

$$I = \frac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

At $t = \infty$,

$$I_\infty = \frac{20}{5}\left(I - e^{\frac{-\infty}{L/R}}\right) = 4 \qquad \text{...(i)}$$

At $t = 40$s,

$$\left(1 - e^{\frac{-40 \times 5}{10 \times 10^{-3}}}\right) = 4(1 - e^{-20,000}) \qquad \text{...(ii)}$$

Dividing (i) by (ii) we get

$$\Rightarrow \quad \frac{I_\infty}{I_{40}} = \frac{1}{1 - e^{-20,000}},$$

25. **(b)** As we know, average power $P_{avg} = V_{rms} I_{rms} \cos\theta$

$$= \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\theta = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45^\circ \quad (\because \theta = 45^\circ)$$

$$P_{avg} = \frac{1000}{\sqrt{2}} \text{ watt}$$

Wattless current $I = I_{rms}\sin\theta$

$$= \frac{I_0}{\sqrt{2}}\sin\theta = \frac{20}{\sqrt{2}}\sin 45^\circ = 10\text{A}$$

26. **(a)** Quality factor $Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$

27. **(b)** Given,

$V_0 = 283$ volt, $\omega = 320$, $R = 5\,\Omega$, $L = 25$ mH, $C = 1000\,\mu$F

$x_L = \omega L = 320 \times 25 \times 10^{-3} = 8\,\Omega$

$$x_C = \frac{1}{\omega C} = \frac{1}{320 \times 1000 \times 10^{-6}} = 3.1\,\Omega$$

Total impedance of the circuit :

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{25 + (4.9)^2} = 7\,\Omega$$

Phase difference between the voltage and current

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{4.9}{5} \approx 1 \Rightarrow \phi = 45^\circ$$

28. **(b)** Here

$$i = \frac{e}{\sqrt{R^2 + X_L^2}} = \frac{e}{\sqrt{R^2 + \omega^2 L^2}} = \frac{e}{\sqrt{R^2 + 4\pi^2 v^2 L^2}}$$

$$10 = \frac{220}{\sqrt{64 + 4\pi^2(50)^2 L}}$$

$$\left[\because R = \frac{V}{I} = \frac{80}{10} = 8\right]$$

On solving we get

$L = 0.065$ H

29. **(d)**

30. **(b)** $I(0)=\frac{15\times 100}{0.15\times 10^3}=0.1A$

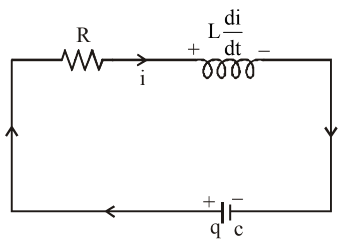
$I(\infty)=0$

$I(t)=[I(0)-I(\infty)]\;e^{\frac{-t}{L/R}}+i(\infty)$

$I(t)=0.1\;e^{\frac{-t}{L/R}}=0.1\,e^{\frac{R}{L}}$

$I(t)=0.1\;e^{\frac{0.15\times 1000}{0.03}}=0.67mA$

31. **(c)** From KVL at any time t



$\frac{q}{c}-iR-L\frac{di}{dt}=0$

$i=-\frac{dq}{dt}\Rightarrow\frac{q}{c}+\frac{dq}{dt}R+\frac{Ld^2q}{dt^2}=0$

$\frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{q}{Lc}=0$

From damped harmonic oscillator, the amplitude is given by $A\;=A_o e-\frac{dt}{2m}$

Double differential equation

$\frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+\frac{k}{m}x=0$

$Q_{\max}=Q_o e^{-\frac{Rt}{2L}}\Rightarrow Q_{\max}^2=Q_o^2 e^{-\frac{Rt}{L}}$

Hence damping will be faster for lesser self inductance.

32. **(c)** Power factor

$$\cos\phi=\frac{R}{\sqrt{R^2+\left[\omega L-\frac{1}{\omega(C+C')}\right]^2}}=1$$

On solving we get,

$\omega L=\frac{1}{\omega(C+C')}$

$C'=\frac{1-\omega^2 LC}{\omega^2 L}$

Hence option (c) is the correct answer.

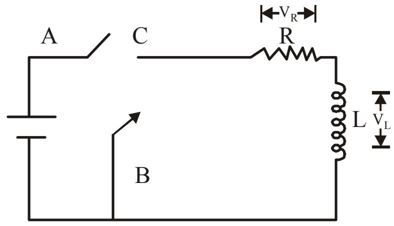
33. **(c)** For capacitor circuit, $i\;=\;i_0 e^{-t/RC}$

For inductor circuit, $i=i_0\left(1-e^{-\frac{Rt}{L}}\right)$

Hence graph (c) correctly depicts i versus t graph.

34. **(c)** Applying Kirchhoff's law of voltage in closed loop

$-V_R-V_C=0\;\Rightarrow\;\frac{V_R}{V_C}=-1$



35. **(c)** Given, $V_L:V_C:V_R=1:2:3$

$V=100\,V$

$V_R=?$

As we know,

$V=\sqrt{V_R^2+(V_L-V_C)^2}$

Solving we get, $V_R\;\simeq 90V$

36. **(c)** Charge on he capacitor at any time t is given by $q=CV\,(1-e^{t/\tau})$

at $t=2\tau$

$q=CV\,(1-e^{-2})$

37. **(d)** Power factor $_{(old)}$

$=\frac{R}{\sqrt{R^2+X_L{}^2}}=\frac{R}{\sqrt{R^2+(2R)^2}}=\frac{R}{\sqrt{5}R}$

Power factor$_{(new)}$

$=\frac{R}{\sqrt{R^2+(X_L-X_C)^2}}=\frac{R}{\sqrt{R^2+(2R-R)^2}}=\frac{R}{\sqrt{2}R}$

$\therefore\;\frac{\text{New power factor}}{\text{Old power factor}}=\frac{\frac{R}{\sqrt{2}R}}{\frac{R}{\sqrt{5}R}}=\sqrt{\frac{5}{2}}$

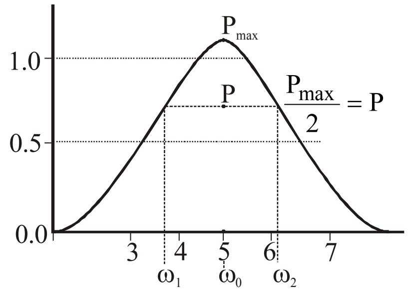
38. **(d)** Impedance (Z) of the series LCR circuit is

$Z=\sqrt{R^2+(X_L-X_C)^2}$

At resonance, $X_L\;=X_C$

Therefore, $Z_{minimum}=R$

39. **(b)**



Quality factor of the circuit

$=\frac{\omega_0}{\omega_2-\omega_1}=\frac{5}{2.5}=2.0$

40. **(d)**

41. **(c)** Frequency $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.4\sqrt{24\times 2\times 10^{-6}}}$

$\simeq 23$Hz

42. **(c)** Voltage E of the ac source

$E = V_C - V_L = 400\text{ V} - 300\text{ V} = 100\text{ V}$

43. **(a)** $t = 3\,RC\,ln\,2$

44. **(d)** In case of series RLC circuit,

Equation of voltage is given by

$V^2 = V_R^2 + (V_L - V_C)^2$

Here, $V = 220$ V; $V_L = V_C = 300$ V

$\therefore V_R = \sqrt{V^2} = 220\text{V}$

Current $i = \frac{V}{R} = \frac{220}{100} = 2.2\text{A}$

45. **(a)** Energy stored in magnetic field $= \frac{1}{2}\text{Li}^2$

Energy stored in electric field $= \frac{1}{2}\frac{q^2}{C}$

Energy will be equal when

$\therefore \frac{1}{2}Li^2 = \frac{1}{2}\frac{q^2}{C}$

$\tan\omega t = 1$

$q = q_0\cos\omega t$

$\Rightarrow \frac{1}{2}L(\omega q_0\sin\omega t)^2 = \frac{(q_0\cos\omega t)^2}{2C}$

$\Rightarrow \omega = \frac{1}{\sqrt{LC}} \quad \Rightarrow \omega t = \frac{\pi}{4}$

$\Rightarrow t = \frac{\pi}{4}\sqrt{LC}$

46. **(b)** We have, $V = V_0(1 - e^{-t/RC})$

$\Rightarrow 120 = 200(1 - e^{-t/RC})$

$e^{-t/r} = \frac{200 - 120}{200} = \frac{80}{200}$

$t = \log_e(2.5)$

$\Rightarrow t = RC\,\text{in}\,(2.5) \qquad [\because r = RC]$

$\Rightarrow R = 2.71\times 10^6\,\Omega$

47. **(c)** Time constant for parallel combination $= 2RC$

Time constant for series combination $= \frac{RC}{2}$

In first case :

$V = V_0\left(1 - e^{-\frac{t}{CR}}\right) \Rightarrow \frac{V_0}{2} = V_0 - V_0e^{-\frac{t}{CR}}$

$V = V_0e^{-\frac{t_1}{2RC}} = \frac{V_0}{2}$ ...(1)

In second case :

In series grouping, equivalent capacitance $= \frac{C}{2}$

$V = V_0e^{-\frac{t_2}{(RC/2)}} = \frac{V_0}{2}$ ....(2)

From (1) and (2)

$\frac{t_1}{2RC} = \frac{t_2}{(RC/2)}$

$\Rightarrow t_2 = \frac{t_1}{4} = \frac{10}{4} = 2.5$ sec.

48. **(c)** At $t = 0$, no current will flow through L and R_1 as inductor will offer infinite resistance.

$\therefore$ Current through battery, $i = \frac{V}{R_2}$

At $t = \infty$, inductor behave as conducting wire

Effective resistance, $R_{eff} = \frac{R_1R_2}{R_1 + R_2}$

$\therefore$ Current through battery $= \frac{V}{R_{eff}} = \frac{V(R_1 + R_2)}{R_1R_2}$

49. **(d)** When only the capacitance is removed phase difference between current and voltage is

$\tan\phi = \frac{X_L}{R}$

$\Rightarrow \tan\phi = \frac{\omega L}{R}$

$\Rightarrow \omega L = R\tan\phi = 200\times\frac{1}{\sqrt{3}} = \frac{200}{\sqrt{3}}$

When only inductor is removed, phase difference between current and voltage is

$\therefore \quad \tan\phi = \frac{1}{\omega CR}$

$\Rightarrow \frac{1}{\omega C} = R\tan\phi = 200\times\frac{1}{\sqrt{3}} = \frac{200}{\sqrt{3}}$

Impedance of the circuit,

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

$$= \sqrt{(200)^2 + \left(\frac{200}{\sqrt{3}} - \frac{200}{\sqrt{3}}\right)^2} = 200\,\Omega$$

Power dissipated in the circuit $= V_{rms}I_{rms}\cos\phi$

$$= V_{rms}\cdot\frac{V_{rms}}{Z}\cdot\frac{R}{Z}\left(\because \cos\phi = \frac{R}{Z}\right) = \frac{V^2_{rms}R}{Z^2}$$

$$= \frac{(220)^2\times 200}{(200)^2} = \frac{220\times 220}{200} = 242\text{ W}$$

50. **(c)** Growth in current in branch containing L and R_2 when switch is closed is given by

$i = \frac{E}{R_2}[1 - e^{-R_2t/L}]$

$\Rightarrow \frac{di}{dt} = \frac{E}{R_2}\cdot\frac{R_2}{L}\cdot e^{-R_2t/L} = \frac{E}{L}e^{-\frac{R_2t}{L}}$

Hence, potential drop across L

$$V_L = \frac{Ldi}{dt} = \left(\frac{E}{L}e^{-R_2 t/L}\right)L$$

$$= Ee^{-R_2 t/L} = 12e^{-\frac{2t}{400\times10^{-3}}} = 12e^{-5t}\text{V}$$

51. **(c)** Across resistor, $I = \frac{V}{R} = \frac{100}{1000} = 0.1\ A$

At resonance,

$$X_L = X_C = \frac{1}{\omega C} = \frac{1}{200\times 2\times 10^{-6}} = 2500$$

Voltage across L is

$$I X_L = 0.1\times 2500 = 250\,\text{V}$$

52. **(a)** Initially, when steady state is achieved,

$$i = \frac{E}{R}$$

Let E is short circuited at $t = 0$. Then

At $t = 0$

Maximum current, $i_0 = \frac{E}{R} = \frac{100}{100} = 1A$

Let during decay of current at any time the current flowing

is $-L\frac{di}{dt} - iR = 0$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

$$\Rightarrow \int_{i_0}^{i}\frac{di}{i} = \int_0^t -\frac{R}{L}dt$$

$$\Rightarrow \log_e \frac{i}{i_0} = -\frac{R}{L}t$$

$$\Rightarrow i = i_0\, e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{E}{R}e^{-\frac{R}{L}t} = 1\times e^{\frac{-100\times10^{-3}}{100\times10^{-3}}} = \frac{1}{e}$$

53. **(d)** $e = -\frac{d\phi}{dt} = -\frac{d(N\vec{B}.\vec{A})}{dt}$

$$= -N\frac{d}{dt}(BA\cos\omega t) = NBA\omega\sin\omega t$$

$$\Rightarrow e_{max} = NBA\omega$$

54. **(a)** Phase difference for R–L circuit lies between

$\left(0, \frac{\pi}{2}\right)$ but 0 or $\pi/2$

55. **(b)** Given, Resistance of circuit, $R = 12\,\Omega$

Imedance of circuit, $Z = 15\,\Omega$

Power factor $= \cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$

56. **(a)** Current in inductor circuit is given by,

$$i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\frac{i_0}{2} = i_0(1 - e^{-\frac{Rt}{L}}) \Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2}$$

Taking log on both the sides,

$$-\frac{Rt}{L} = \log 1 - \log 2$$

$$\Rightarrow t = \frac{L}{R}\log 2 = \frac{300\times10^{-3}}{2}\times 0.69$$

$\Rightarrow t = 0.1$ sec.

57. **(d)** For maximum power, $X_L = X_C$, which yields

$$C = \frac{1}{(2\pi n)^2 L} = \frac{1}{4\pi^2\times 50\times 50\times 10}$$

$$\therefore C = 0.1\times10^{-5}F = 1\mu F$$

58. **(d)** In a series LCR circuit voltage across the inductor and capacitor are in opposite phase

$\therefore$ Net voltage difference across

$LC = 50 - 50 = 0$

59. **(a)** Resonant frequency, $F_r = \frac{1}{2\pi\sqrt{LC}}$

For resonant frequency to remain same

LC = constant

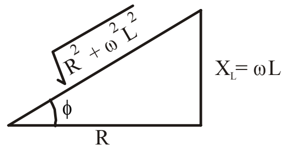
$\therefore LC = L'C'$

$\Rightarrow LC = L'\times 2C$

$$\Rightarrow L' = \frac{L}{2}$$

60. **(b)** Resistance of the inductor, $X_L = \omega L$

The impedance triangle for resistance (R) and inductor (L) connected in series is shown in the figure.



Net impedance of circuit $Z = \sqrt{X_L^2 + R^2}$

Power factor, $\cos\phi = \frac{R}{Z}$

$$\Rightarrow \cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

61. **(d)** All three inductors are connected in parallel. The equivalent inductance L_p is given by

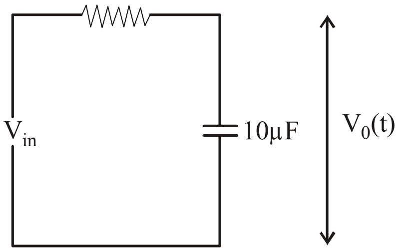
$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

$\therefore L_p = 1$

62. **(a)** When first pulse is applied, the potential across capacitor

$$V_0(t) = V_{in}\left(1 - e^{\frac{1}{RC}}\right)$$

At $t = 5\mu s = 5\times10^{-6}\,s$



$$V_0(t) = 5\left(1 - e^{\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right) = 5(1 - e^{-0.5}) = 2V$$

When no pulse is applied, capacitor will discharge.
Now, $V_{in} = 0$ means discharging.

$$V_0(t) = 2e^{\frac{1}{RC}} = 2e^{-0.5} = 1.21 \text{ V}$$

Now for next 5 μs

$$V_0(t) = 5 - 3.79e^{\frac{1}{RC}}$$

After 5 μs again, $V_0(t) = 2.79$ Volt ≈ 3 V

Hence, graph (a) correctly depicts.

63. **(c)** Power output $(V_2I_2) = 2.2$ kW

$$\therefore\ V_2 = \frac{2.2\text{kW}}{(10\text{A})} = 220 \text{ volts}$$

$\therefore$ Input voltage for step-down transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = 2$$

$$V_{input} = 2 \times V_{output} = 2 \times 220$$
$$= 440 \text{ V}$$

Also $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

$$\therefore\quad I_1 = \frac{1}{2} \times 10 = 5\text{A}$$

64. **(b)** Efficiency, $\eta = \frac{P_{out}}{P_{in}} = \frac{V_sI_s}{V_pI_p}$

$$\Rightarrow 0.9 = \frac{230 \times I_s}{2300 \times 5}$$

$$\Rightarrow I_s = 0.9 \times 50 = 45\text{A}$$

Output current = 45A

65. **(c)** Given : $V_P = 2300$ V, $V_s = 230$ V, $I_P = 5$ A, $n = 90\% = 0.9$

Efficiency $n = 0.9 = \frac{P_s}{P_P} \Rightarrow P_s = 0.9\ P_p$

$V_sI_s = 0.9 \times V_PI_P \quad (\because P = VI)$

$$I_s = \frac{0.9 \times 2300 \times 5}{230} = 45\text{A}$$

66. **(c)** When the capacitor is completely charged, the total energy in the LC circuit is with the capacitor and that energy is given by

$$U_{max} = \frac{1}{2}\frac{Q^2}{C}$$

When half energy is with the capacitor in the form of electric field between the plates of the capacitor we get

$$\frac{U_{max}}{2} = \frac{1}{2}\frac{q'^2}{C}$$

Here q' is the charge on the plate of capacitor when energy is shared equally.

$$\therefore\ \frac{1}{2} \times \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{q'^2}{C} \Rightarrow q' = \frac{Q}{\sqrt{2}}$$

67. **(a)** Laminated core provide less area of cross-section for the current to flow. Because of this, resistance of the core increases and current decreases there by decreasing the energy loss due to eddy current.

68. **(b)** Number of turns in primary

$N_p = 140$

Number of turns in secondary $N_s = 280$, $I_p = 4A$, $I_s = ?$

Using transformation ratio for a transformer $\frac{I_s}{I_p} = \frac{N_p}{N_s}$

$$\Rightarrow \frac{I_s}{4} = \frac{140}{280}$$

$$\Rightarrow I_s = 2\,A$$