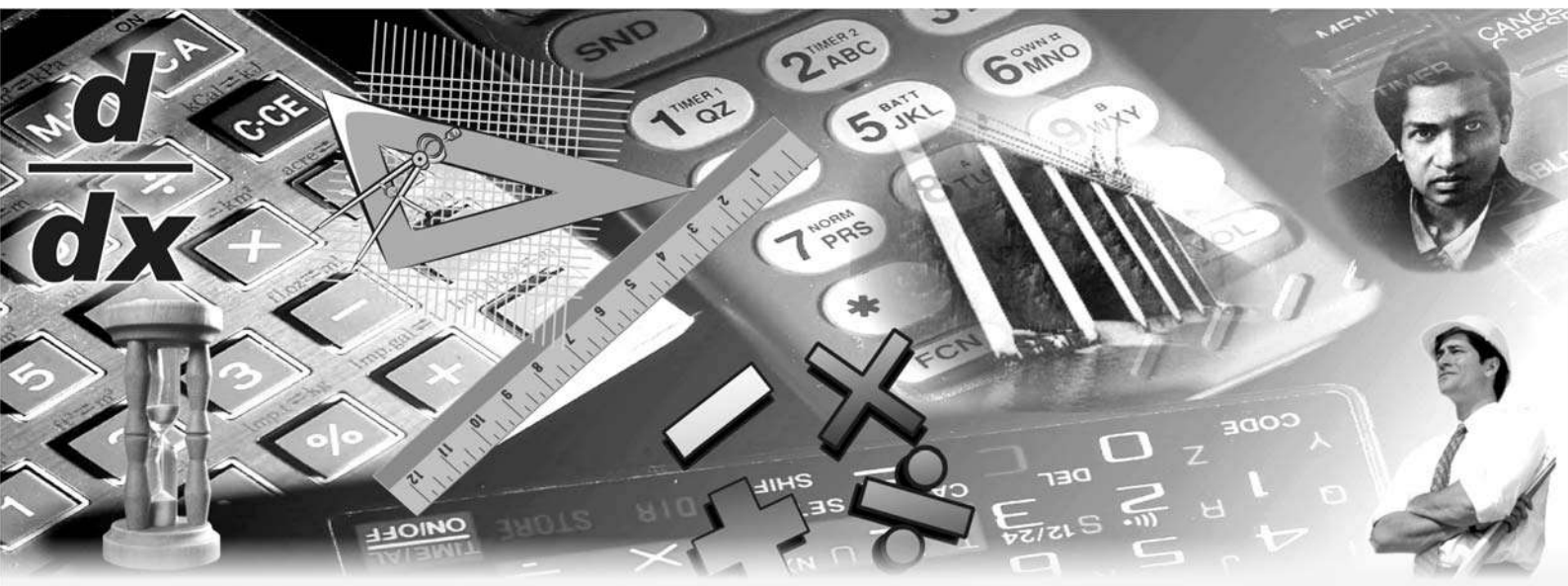




Chapter 12

प्रतिलोम त्रिकोणमितीय फलन

फलन $f: A \rightarrow B$ व्युत्क्रमणीय होगा यदि f एकैकी आच्छादक हो तथा यह इस प्रकार परिभाषित है, कि $f(x) = y \Rightarrow f^{-1}(y) = x$
प्रतिलोम त्रिकोणमितीय फलनों के ग्राफ
(Graphs of inverse trigonometric functions)

(Domain and range of inverse trigonometric functions)

फलन	डोमेन (D)	रेंज (R)
$\sin^{-1} x$	$-1 \leq x \leq 1$ या $[-1, 1]$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ या $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$-1 \leq x \leq 1$ या $[-1, 1]$	$0 \leq \theta \leq \pi$ या $[0, \pi]$
$\tan^{-1} x$	$-\infty < x < \infty$ अर्थात् $x \in R$ या $(-\infty, \infty)$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ या $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	$-\infty < x < \infty$ अर्थात् $x \in R$ या $(-\infty, \infty)$	$0 < \theta < \pi$ या $(0, \pi)$
$\sec^{-1} x$	$x \leq -1, x \geq 1$ या $(-\infty, -1] \cup [1, \infty)$	$\theta \neq \frac{\pi}{2}, 0 \leq \theta \leq \pi$ या $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
$\operatorname{cosec}^{-1} x$	$x \leq -1, x \geq 1$ या $(-\infty, -1] \cup [1, \infty)$	$\theta \neq 0, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ या $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

प्रतिलोम त्रिकोणमितीय फलनों के गुणधर्म
(Graphs of inverse trigonometric functions)

(i) $\sin^{-1}(\sin \theta) = \theta$, जबकि $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$,

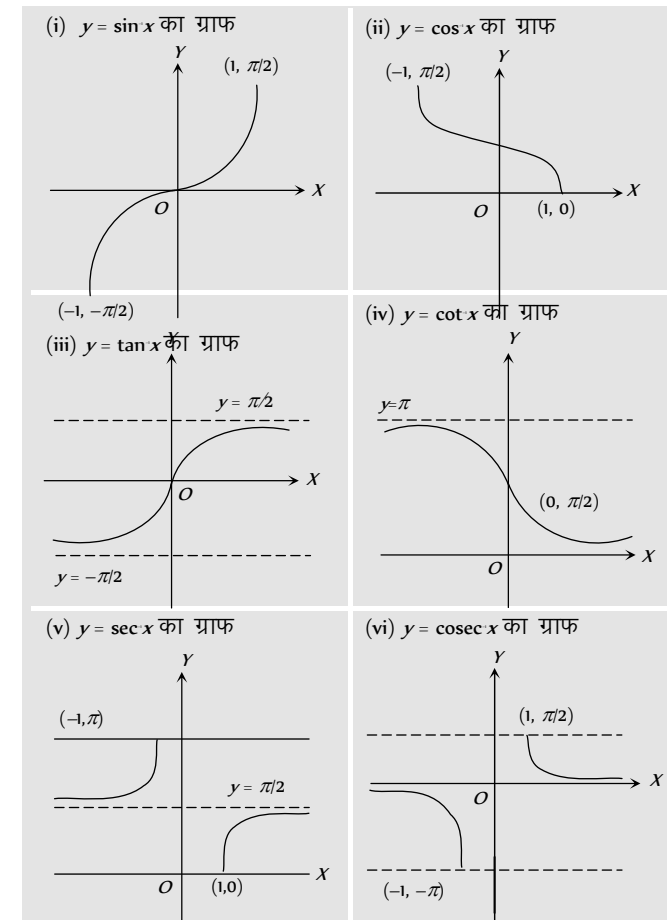
$\cos^{-1}(\cos \theta) = \theta$, जबकि $0 \leq \theta \leq \pi$

$\tan^{-1}(\tan \theta) = \theta$, जबकि $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$,

$\cot^{-1}(\cot \theta) = \theta$, जबकि $0 < \theta < \pi$

$\sec^{-1}(\sec \theta) = \theta$, जबकि $0 \leq \theta < \frac{\pi}{2}$ या $\frac{\pi}{2} < \theta \leq \pi$

$\operatorname{cosec}^{-1}(\operatorname{cosec} \theta) = \theta$, जबकि $-\frac{\pi}{2} \leq \theta < 0$ या $0 < \theta \leq \frac{\pi}{2}$



प्रतिलोम फलनों के प्रान्त एवं परिसर

- (2) $\sin(\sin^{-1} x) = x$, जबकि $-1 \leq x \leq 1$
 $\cos(\cos^{-1} x) = x$, जबकि $-1 \leq x \leq 1$
 $\tan(\tan^{-1} x) = x$, जबकि $-\infty < x < \infty$
 $\cot(\cot^{-1} x) = x$, जबकि $-\infty < x < \infty$
 $\sec(\sec^{-1} x) = x$, जबकि $-\infty < x \leq -1$ या $1 \leq x < \infty$
 $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x$, जबकि $-\infty < x \leq -1$ या $1 \leq x < \infty$

- (3) $\sin^{-1}(-x) = -\sin^{-1} x$, $\cos^{-1}(-x) = \pi - \cos^{-1} x$
 $\tan^{-1}(-x) = -\tan^{-1} x$, $\cot^{-1}(-x) = \pi - \cot^{-1} x$
 $\sec^{-1}(-x) = \pi - \sec^{-1} x$, $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

(4) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, $x \in [-1, 1]$

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, $x \in \mathbb{R}$

$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$, $x \in (-\infty, -1] \cup [1, \infty)$

(5) वृत्तीय प्रतिलोम फलनों के मुख्य मान

सारणी : 12.1

$x \geq 0$ के लिए मुख्य मान	$x < 0$ के लिए मुख्य मान
$0 \leq \sin^{-1} x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \sin^{-1} x < 0$
$0 \leq \cos^{-1} x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cos^{-1} x \leq \pi$
$0 \leq \tan^{-1} x < \frac{\pi}{2}$	$-\frac{\pi}{2} < \tan^{-1} x < 0$
$0 < \cot^{-1} x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cot^{-1} x < \pi$
$0 \leq \sec^{-1} x < \frac{\pi}{2}$	$\frac{\pi}{2} < \sec^{-1} x \leq \pi$
$0 < \operatorname{cosec}^{-1} x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \operatorname{cosec}^{-1} x < 0$

(6) रूपान्तरणीय गुणधर्म (Conversion property)

माना $\sin^{-1} x = y \Rightarrow x = \sin y$

$\Rightarrow \operatorname{cosec} y = \left(\frac{1}{x}\right) \Rightarrow y = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

$\sin^{-1} x = \cos^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$

$= \cot^{-1} \frac{\sqrt{1-x^2}}{x} = \sec^{-1} \left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

$\cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \left(\frac{\sqrt{1-x^2}}{x}\right)$

$= \sec^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} \left(\frac{1}{\sqrt{1-x^2}}\right) = \cot^{-1} \left(\frac{x}{\sqrt{1-x^2}}\right)$

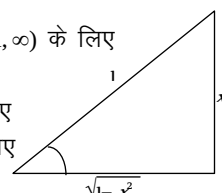
$\tan^{-1} x = \sin^{-1} \left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1} \left(\frac{1}{\sqrt{1+x^2}}\right)$

$= \cot^{-1} \left(\frac{1}{x}\right) = \sec^{-1} \sqrt{1+x^2} = \operatorname{cosec}^{-1} \left(\frac{\sqrt{1+x^2}}{x}\right)$

$\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$, सभी $x \in (-\infty, -1] \cup [1, \infty)$ के लिए

$\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$, सभी $x \in (-\infty, -1] \cup [1, \infty)$ के लिए

$\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x, & x > 0 \text{ के लिए} \\ -\pi + \cot^{-1} x, & x < 0 \text{ के लिए} \end{cases}$



(7) वृत्तीय प्रतिलोम फलनों का व्यापक हल : हम जानते हैं कि यदि α न्यूनतम कोण हो जिसका $\sin \alpha = x$ है, तब x को $n\pi + (-1)^n \alpha$, जहाँ $n = 0, 1, 2, \dots$ लिखते हैं। अतः $\sin^{-1} x$ का व्यापक मान $n\pi + (-1)^n \alpha$ है।

$\sin^{-1} x = n\pi + (-1)^n \alpha$, $-1 \leq x \leq 1$, यदि $\sin \alpha = x$ तथा $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$

इस प्रकार शेष वृत्तीय फलनों का व्यापक मान निम्न प्रकार से है :

$\cos^{-1} x = 2n\pi \pm \alpha$, $-1 \leq x \leq 1$	यदि $\cos \alpha = x$, $0 \leq \alpha \leq \pi$
$\tan^{-1} x = n\pi + \alpha$, $x \in \mathbb{R}$;	यदि $\tan \alpha = x$, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$
$\cot^{-1} x = n\pi + \alpha$, $x \in \mathbb{R}$;	यदि $\cot \alpha = x$, $0 \leq \alpha \leq \pi$
$\sec^{-1} x = 2n\pi \pm \alpha$, $x \geq 1$ या $x \leq -1$	यदि $\sec \alpha = x$, $0 \leq \alpha \leq \pi$ तथा $\alpha \neq \frac{\pi}{2}$
$\operatorname{cosec}^{-1} x = n\pi + (-1)^n \alpha$, $x \geq 1$ या $x \leq -1$	यदि $\operatorname{cosec} \alpha = x$, $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ तथा $x \neq 0$

प्रतिलोम त्रिकोणमितीय फलनों के योग व अन्तर पर आधारित सूत्र (Formulae for sum and difference of inverse trigonometric function)

(1) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy}\right)$;

यदि $x > 0, y > 0$ एवं $xy < 1$

(2) $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \left(\frac{x+y}{1-xy}\right)$;

यदि $x > 0, y > 0$ एवं $xy > 1$

(3) $\tan^{-1} x + \tan^{-1} y = -\pi + \tan^{-1} \left(\frac{x+y}{1-xy}\right)$;

यदि $x < 0, y < 0$ एवं $xy > 1$

(4) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy}\right)$;

यदि $xy > -1$

(5) $\tan^{-1} x - \tan^{-1} y = \pi + \tan^{-1} \left(\frac{x-y}{1+xy}\right)$;

यदि $x > 0, y < 0$ एवं $xy < -1$

(6) $\tan^{-1} x - \tan^{-1} y = -\pi + \tan^{-1} \left(\frac{x-y}{1+xy}\right)$;

यदि $x < 0, y > 0$ एवं $xy < -1$

$$(7) \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \left[\frac{x + y + z - xyz}{1 - xy - yz - zx} \right]$$

$$(8) \tan^{-1} x_1 + \tan^{-1} x_2 + \dots + \tan^{-1} x_n \\ = \tan^{-1} \left[\frac{S_1 - S_3 + S_5 - \dots}{1 - S_2 + S_4 - S_6 + \dots} \right]$$

जहाँ $S_k; x_1, x_2, \dots, x_n$ के गुणनफल का योग प्रदर्शित करता है।

$$(9) \cot^{-1} x + \cot^{-1} y = \cot^{-1} \frac{xy - 1}{y + x}$$

$$(10) \cot^{-1} x - \cot^{-1} y = \cot^{-1} \frac{xy + 1}{y - x}$$

$$(11) \sin^{-1} x + \sin^{-1} y = \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\};$$

यदि $-1 \leq x, y \leq 1$ एवं $x^2 + y^2 \leq 1$ या यदि $xy < 0$ एवं $x^2 + y^2 > 1$

$$(12) \sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\},$$

यदि $0 < x, y \leq 1$ एवं $x^2 + y^2 > 1$

$$(13) \sin^{-1} x + \sin^{-1} y = -\pi - \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\},$$

यदि $-1 \leq x, y < 0$ एवं $x^2 + y^2 > 1$

$$(14) \sin^{-1} x - \sin^{-1} y = \sin^{-1} \{x\sqrt{1-y^2} - y\sqrt{1-x^2}\},$$

यदि $-1 \leq x, y \leq 1$ एवं $x^2 + y^2 \leq 1$ यदि या $xy > 0$ एवं $x^2 + y^2 > 1$.

$$(15) \sin^{-1} x - \sin^{-1} y = \pi - \sin^{-1} \{x\sqrt{1-y^2} - y\sqrt{1-x^2}\},$$

यदि $0 < x \leq 1, -1 \leq y < 0$ एवं $x^2 + y^2 > 1$.

$$(16) \sin^{-1} x - \sin^{-1} y = -\pi - \sin^{-1} \{x\sqrt{1-y^2} - y\sqrt{1-x^2}\},$$

यदि $-1 \leq x < 0, 0 < y \leq 1$ एवं $x^2 + y^2 > 1$.

$$(17) \cos^{-1} x + \cos^{-1} y = \cos^{-1} \{xy - \sqrt{1-x^2} \cdot \sqrt{1-y^2}\},$$

यदि $-1 \leq x, y \leq 1$ एवं $x + y \geq 0$.

$$(18) \cos^{-1} x + \cos^{-1} y = 2\pi - \cos^{-1} \{xy - \sqrt{1-x^2} \cdot \sqrt{1-y^2}\},$$

यदि $-1 \leq x, y \leq 1$ एवं $x + y \leq 0$

$$(19) \cos^{-1} x - \cos^{-1} y = \cos^{-1} \{xy + \sqrt{1-x^2} \cdot \sqrt{1-y^2}\},$$

यदि $-1 \leq x, y \leq 1$, एवं $x \leq y$.

$$(20) \cos^{-1} x - \cos^{-1} y = -\cos^{-1} \{xy + \sqrt{1-x^2} \cdot \sqrt{1-y^2}\},$$

यदि $-1 \leq y \leq 0, 0 < x \leq 1$ एवं $x \geq y$.

$$(4) 3 \sin^{-1} x = \sin^{-1}(3x - 4x^3), \quad \text{यदि } -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$(5) 3 \sin^{-1} x = \pi - \sin^{-1}(3x - 4x^3), \quad \text{यदि } \frac{1}{2} < x \leq 1$$

$$(6) 3 \sin^{-1} x = -\pi - \sin^{-1}(3x - 4x^3), \quad \text{यदि } -1 \leq x < -\frac{1}{2}$$

$$(7) 2 \cos^{-1} x = \cos^{-1}(2x^2 - 1), \quad \text{यदि } 0 \leq x \leq 1$$

$$(8) 2 \cos^{-1} x = 2\pi - \cos^{-1}(2x^2 - 1), \quad \text{यदि } -1 \leq x \leq 0$$

$$(9) 3 \cos^{-1} x = \cos^{-1}(4x^3 - 3x), \quad \text{यदि } \frac{1}{2} \leq x \leq 1$$

$$(10) 3 \cos^{-1} x = 2\pi - \cos^{-1}(4x^3 - 3x), \quad \text{यदि } -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$(11) 3 \cos^{-1} x = 2\pi + \cos^{-1}(4x^3 - 3x), \quad \text{यदि } -1 \leq x < -\frac{1}{2}$$

$$(12) 2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right), \quad \text{यदि } -1 < x \leq 1$$

$$(13) 2 \tan^{-1} x = \pi + \tan^{-1} \left(\frac{2x}{1-x^2} \right), \quad \text{यदि } x > 1$$

$$(14) 2 \tan^{-1} x = -\pi + \tan^{-1} \left(\frac{2x}{1-x^2} \right), \quad \text{यदि } x < -1$$

$$(15) 2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right), \quad \text{यदि } -1 \leq x \leq 1$$

$$(16) 2 \tan^{-1} x = \pi - \sin^{-1} \left(\frac{2x}{1+x^2} \right), \quad \text{यदि } x > 1$$

$$(17) 2 \tan^{-1} x = -\pi - \sin^{-1} \left(\frac{2x}{1+x^2} \right), \quad \text{यदि } x < -1$$

$$(18) 2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right), \quad \text{यदि } 0 \leq x < \infty$$

$$(19) 2 \tan^{-1} x = -\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right), \quad \text{यदि } -\infty < x \leq 0$$

$$(20) 3 \tan^{-1} x = \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right), \quad \text{यदि } \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

$$(21) 3 \tan^{-1} x = \pi + \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right), \quad \text{यदि } x > \frac{1}{\sqrt{3}}$$

$$(22) 3 \tan^{-1} x = -\pi + \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right), \quad \text{यदि } x < -\frac{1}{\sqrt{3}}$$

$$(23) \tan^{-1} \left[\frac{x}{\sqrt{a^2 - x^2}} \right] = \sin^{-1} \frac{x}{a}$$

$$(24) \tan^{-1} \left[\frac{3a^2x - x^3}{a(a^2 - 3x^2)} \right] = 3 \tan^{-1} \frac{x}{a}$$

$$(25) \tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$$

अपवर्त्य कोणों के प्रतिलोम त्रिकोणमितीय अनुपात (Inverse trigonometric ratios of multiple angles)

$$(1) 2 \sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2}), \quad \text{यदि } -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$$(2) 2 \sin^{-1} x = \pi - \sin^{-1} 2x\sqrt{1-x^2}, \quad \text{यदि } \frac{1}{\sqrt{2}} \leq x \leq 1$$

$$(3) 2 \sin^{-1} x = -\pi - \sin^{-1}(2x\sqrt{1-x^2}), \quad \text{यदि } -1 \leq x \leq -\frac{1}{\sqrt{2}}$$

$$(26) \tan^{-1} \sqrt{\frac{1-x}{1+x}} = \frac{1}{2} \cos^{-1} x$$

Tips & Tricks

- ✎ I चतुर्थांश सभी प्रतिलोम फलन में उभयनिष्ठ होता है।
- ✎ III चतुर्थांश का प्रतिलोम फलन में उपयोग नहीं किया जाता है।
- ✎ IV चतुर्थांश का उपयोग दक्षिणावर्त दिशा में होता है, अर्थात् $-\frac{\pi}{2} \leq y \leq 0$.
- ✎ $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$ को क्रमशः $\arcsin x, \arccos x$ एवं $\arctan x$ भी लिखते हैं।
- ✎ यह ध्यान देना चाहिए यदि कुछ नहीं कहा गया है तब वृत्तीय प्रतिलोम फलनों का मुख्य मान ही लेते हैं।
- ✎ यदि $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$ हो, तो $xy + yz + zx = 1$.
- ✎ यदि $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$ हो, तो $x + y + z = xyz$.
- ✎ यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$ हो, तो $x^2 + y^2 + z^2 + 2xyz = 1$.
- ✎ यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$ हो, तो $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.
- ✎ यदि $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$ हो, तो $xy + yz + zx = 3$.
- ✎ यदि $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ हो, तो $x^2 + y^2 + z^2 + 2xyz = 1$.
- ✎ यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ हो, तो $xy + yz + zx = 3$.
- ✎ यदि $\sin^{-1} x + \sin^{-1} y = \theta$ हो, तो $\cos^{-1} x + \cos^{-1} y = \pi - \theta$.
- ✎ यदि $\cos^{-1} x + \cos^{-1} y = \theta$ हो, तो $\sin^{-1} x + \sin^{-1} y = \pi - \theta$.
- ✎ यदि $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{2}$ हो, तो $xy = 1$.
- ✎ यदि $\cot^{-1} x + \cot^{-1} y = \frac{\pi}{2}$ हो, तो $xy = 1$.
- ✎ यदि $\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \theta$ हो, तो

$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \theta + \frac{y^2}{b^2} = \sin^2 \theta.$$

Ordinary Thinking

Objective Questions

प्रतिलोम त्रिकोणमितीय फलन

1. यदि $\cos^{-1}\left(\frac{1}{x}\right) = \theta$, तो $\tan \theta =$

[MNR 1978; MP PET 1989]

(a) $\frac{1}{\sqrt{x^2-1}}$ (b) $\sqrt{x^2+1}$
(c) $\sqrt{1-x^2}$ (d) $\sqrt{x^2-1}$
2. $\sin(\cot^{-1} x) =$ [MNR 1987; MP PET 2001; DCE 2002]

(a) $\sqrt{1+x^2}$ (b) x
(c) $(1+x^2)^{-3/2}$ (d) $(1+x^2)^{-1/2}$
3. $\cos\left(\sin^{-1} \frac{5}{13}\right) =$

(a) $\frac{12}{13}$ (b) $-\frac{12}{13}$
(c) $\frac{5}{12}$ (d) इनमें से कोई नहीं
4. $\cot^{-1}(-\sqrt{3}) =$

(a) $-\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$
5. $1 + \cot^2(\sin^{-1} x) =$

(a) $\frac{1}{2x}$ (b) x^2
(c) $\frac{1}{x^2}$ (d) $\frac{2}{x}$
6. यदि $\sin^{-1} \frac{1}{2} = \tan^{-1} x$, तो $x =$

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{2}}$ (d) इनमें से कोई नहीं
7. $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) =$

(a) $\tan^{-1} x$ (b) $\frac{1}{2} \tan^{-1} x$
(c) $2 \tan^{-1} x$ (d) इनमें से कोई नहीं
8. $\tan^{-1} \frac{x}{\sqrt{a^2-x^2}} =$

(a) $\frac{1}{a} \sin^{-1}\left(\frac{x}{a}\right)$ (b) $a \sin^{-1}\left(\frac{x}{a}\right)$
(c) $\sin^{-1}\left(\frac{x}{a}\right)$ (d) $\sin^{-1}\left(\frac{a}{x}\right)$

9. $\cos(\tan^{-1} x) =$ [MP PET 1988; MNR 1981]
 (a) $\sqrt{1+x^2}$ (b) $\frac{1}{\sqrt{1+x^2}}$
 (c) $1+x^2$ (d) इनमें से कोई नहीं
10. $\tan\left[\sec^{-1}\sqrt{1+x^2}\right] =$
 (a) $\frac{1}{x}$ (b) x
 (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$
11. $\sec^{-1}[\sec(-30^\circ)] =$ [MP PET 1992]
 (a) -60° (b) -30°
 (c) 30° (d) 150°
12. $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right] =$
 (a) $\frac{\pi}{4} - \frac{x}{2}$ (b) $\frac{\pi}{4} + \frac{x}{2}$
 (c) $\frac{x}{2}$ (d) $\frac{\pi}{4} - x$
13. $\tan^{-1}\frac{1}{\sqrt{x^2-1}} =$
 (a) $\frac{\pi}{2} + \operatorname{cosec}^{-1}x$ (b) $\frac{\pi}{2} + \sec^{-1}x$
 (c) $\operatorname{cosec}^{-1}x$ (d) $\sec^{-1}x$
14. $\sin^{-1}\left(-\frac{1}{2}\right)$ का मुख्य मान है
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$
 (c) $-\frac{\pi}{3}$ (d) $-\frac{\pi}{6}$
15. $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3) =$ [EAMCET 2001]
 (a) 5 (b) 13
 (c) 15 (d) 6
16. $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right] =$
 (a) $\sin^{-1}x + \sin^{-1}\sqrt{x}$ (b) $\sin^{-1}x - \sin^{-1}\sqrt{x}$
 (c) $\sin^{-1}\sqrt{x} - \sin^{-1}x$ (d) इनमें से कोई नहीं
17. यदि $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$, तो $x =$
 (a) 1 (b) $\sqrt{3}$
 (c) $\frac{1}{\sqrt{3}}$ (d) इनमें से कोई नहीं
18. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) =$
 (a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$
 (c) $\frac{\pi}{6}$ (d) इनमें से कोई नहीं
19. $\sin \cot^{-1} \tan \cos^{-1} x$ का मान है [Bihar CEE 1974]
 (a) x (b) $\frac{\pi}{2}$
 (c) 1 (d) इनमें से कोई नहीं
20. $\sin^{-1}\frac{\sqrt{x}}{\sqrt{x+a}} =$
 (a) $\cos^{-1}\sqrt{\frac{x}{a}}$ (b) $\operatorname{cosec}^{-1}\sqrt{\frac{x}{a}}$
 (c) $\tan^{-1}\sqrt{\frac{x}{a}}$ (d) इनमें से कोई नहीं
21. यदि $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, तो $x =$ [MNR 1994; Kerala (Engg.) 2005]
 (a) 1 (b) 0
 (c) $\frac{4}{5}$ (d) $\frac{1}{5}$
22. यदि $\sin^{-1}x = \theta + \beta$ तथा $\sin^{-1}y = \theta - \beta$, तो $1+xy =$
 (a) $\sin^2\theta + \sin^2\beta$ (b) $\sin^2\theta + \cos^2\beta$
 (c) $\cos^2\theta + \cos^2\beta$ (d) $\cos^2\theta + \sin^2\beta$
23. यदि $\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{2}{3} = \sin^{-1}x$, तो $x =$ [Roorkee 1995]
 (a) 0 (b) $\frac{\sqrt{5}-4\sqrt{2}}{9}$
 (c) $\frac{\sqrt{5}+4\sqrt{2}}{9}$ (d) $\frac{\pi}{2}$
24. $\tan(\cos^{-1}x) =$ [IIT 1993]
 (a) $\frac{\sqrt{1-x^2}}{x}$ (b) $\frac{x}{1+x^2}$
 (c) $\frac{\sqrt{1+x^2}}{x}$ (d) $\sqrt{1-x^2}$
25. $\sin^{-1}x$ का प्रान्त है [Roorkee 1993]
 (a) $(-\pi, \pi)$ (b) $[-1, 1]$
 (c) $(0, 2\pi)$ (d) $(-\infty, \infty)$
26. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ का मुख्य मान है [Roorkee 1992]
 (a) $-\frac{2\pi}{3}$ (b) $-\frac{\pi}{3}$
 (c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$
27. $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right] =$ [Karnataka CET 1994]
 (a) $\frac{25}{24}$ (b) $\frac{25}{7}$
 (c) $\frac{24}{25}$ (d) इनमें से कोई नहीं

28. यदि $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$, तब $\sin^{-1}(\sin x)$ बराबर है
 (a) x (b) $-x$
 (c) $\pi + x$ (d) $\pi - x$
29. यदि $\pi \leq x \leq 2\pi$, तब $\cos^{-1}(\cos x)$ बराबर है
 (a) x (b) $-x$
 (c) $2\pi + x$ (d) $2\pi - x$
30. $\sin^{-1}(\sin 10)$ का मान है
 (a) 10 (b) $10 - 3\pi$
 (c) $3\pi - 10$ (d) इनमें से कोई नहीं
31. $\tan^{-1}\left(\frac{1-x}{1+x}\right)$, $0 \leq x \leq 1$, के अधिकतम व न्यूनतम मान हैं
 (a) 0, π (b) $0, \frac{\pi}{4}$
 (c) $-\frac{\pi}{4}, \frac{\pi}{4}$ (d) $\frac{\pi}{4}, \frac{\pi}{2}$
32. यदि x अधनात्मक सुग्राह (Permissible) मान रखता है, तब $\sin^{-1} x =$
 (a) $\cos^{-1} \sqrt{1-x^2}$ (b) $-\cos^{-1} \sqrt{1-x^2}$
 (c) $\cos^{-1} \sqrt{x^2-1}$ (d) $\pi - \cos^{-1} \sqrt{1-x^2}$
33. $\left[\sin\left(\tan^{-1} \frac{3}{4}\right) \right]^2 =$ [EAMCET 1983]
 (a) $\frac{3}{5}$ (b) $\frac{5}{3}$
 (c) $\frac{9}{25}$ (d) $\frac{25}{9}$
34. $\sin^{-1}\left(\sin \frac{5\pi}{3}\right)$ का मुख्य मान है [MP PET 1996]
 (a) $\frac{5\pi}{3}$ (b) $-\frac{5\pi}{3}$
 (c) $-\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$
35. x का वह मान, जो समीकरण $\tan^{-1} x = \sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ को संतुष्ट करता है, होगा [Pb. CET 1999]
 (a) 3 (b) -3
 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$
36. $\sec(\operatorname{cosec}^{-1} x) =$ [Kurukshetra CEE 2001]
 (a) $\operatorname{cosec}(\sec^{-1} x)$ (b) $\cot x$
 (c) π (d) इनमें से कोई नहीं
37. $\tan^{-1} x$ का परिसर (रेंज) होगा [DCE 2002]
 (a) $\left(\pi, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
 (c) $(-\pi, \pi)$ (d) $(0, \pi)$
38. यदि $\theta = \sin^{-1}[\sin(-600^\circ)]$, तब θ का एक संभावित मान होगा [Kerala (Engg.) 2002]
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
 (c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$
39. समीकरण $\sin^{-1} x = 2 \tan^{-1} x$ का हल समुच्चय होगा [AMU 2002]
 (a) $\{1, 2\}$ (b) $\{-1, 2\}$
 (c) $\{-1, 1, 0\}$ (d) $\{1, 1/2, 0\}$
40. $\cos(\tan^{-1}(\tan 2))$ का मान होगा [AMU 2002]
 (a) $\frac{1}{\sqrt{5}}$ (b) $-\frac{1}{\sqrt{5}}$
 (c) $\cos 2$ (d) $-\cos 2$
41. यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$ तब $x^2 + y^2 + z^2 + 2xyz$ का मान है [Pb. CET 2002]
 (a) 0 (b) 1
 (c) 2 (d) 3
42. $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] =$
 (a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$
 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
43. $\sin[\cot^{-1}(\cos \tan^{-1} x)] =$
 (a) $\frac{x}{\sqrt{x^2+2}}$ (b) $\frac{x}{\sqrt{x^2+1}}$
 (c) $\frac{1}{\sqrt{x^2+2}}$ (d) $\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}$
44. यदि $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1} x)$, तब $x =$ [IIT Screening 2004]
 (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
 (c) 0 (d) $\frac{9}{4}$
45. $\cos^{-1} \frac{4}{5} + \tan^{-1} \frac{3}{5} =$
 (a) $\tan^{-1} \frac{27}{11}$ (b) $\sin^{-1} \frac{11}{27}$
 (c) $\cos^{-1} \frac{11}{27}$ (d) इनमें से कोई नहीं
46. $\sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x} =$
 (a) π (b) $\frac{\pi}{2}$
 (c) $\frac{3\pi}{2}$ (d) इनमें से कोई नहीं
47. $2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} =$
 (a) 90° (b) 60°
 (c) 45° (d) $\tan^{-1} 2$
48. $\tan\left(90^\circ - \cot^{-1} \frac{1}{3}\right) =$
 (a) 3 (b) $\frac{2}{3}$
 (c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{10}}$

49. $\tan\left[\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right] =$ [IIT 1983; EAMCET 1988; MP PET 1990; MNR 1992]
 (a) 6/17 (b) 17/6
 (c) 7/16 (d) 16/7
50. $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 =$
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
 (c) 0 (d) इनमें से कोई नहीं
51. $\cot^{-1}\frac{3}{4} + \sin^{-1}\frac{5}{13} =$
 (a) $\sin^{-1}\frac{63}{65}$ (b) $\sin^{-1}\frac{12}{13}$
 (c) $\sin^{-1}\frac{65}{68}$ (d) $\sin^{-1}\frac{5}{12}$
52. यदि $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, तो [Roorkee 1994]
 (a) $x^2 + y^2 + z^2 + xyz = 0$
 (b) $x^2 + y^2 + z^2 + 2xyz = 0$
 (c) $x^2 + y^2 + z^2 + xyz = 1$
 (d) $x^2 + y^2 + z^2 + 2xyz = 1$
53. यदि $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$, तो $A =$ [MP PET 1988]
 (a) $x - y$ (b) $x + y$
 (c) $\frac{x - y}{1 + xy}$ (d) $\frac{x + y}{1 - xy}$
54. यदि $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, तो [Karnataka CET 1996]
 (a) $x + y + z - xyz = 0$ (b) $x + y + z + xyz = 0$
 (c) $xy + yz + zx + 1 = 0$ (d) $xy + yz + zx - 1 = 0$
55. यदि $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, तो $x =$
 (a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$
 (c) $\pm\sqrt{\frac{5}{2}}$ (d) $\pm\frac{1}{2}$
56. $\cos^{-1}\sqrt{1-x} + \sin^{-1}\sqrt{1-x} =$
 (a) π (b) $\frac{\pi}{2}$
 (c) 1 (d) 0
57. $\cos\left[2\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5}\right] =$ [IIT 1981]
 (a) $\frac{2\sqrt{6}}{5}$ (b) $-\frac{2\sqrt{6}}{5}$
 (c) $\frac{1}{5}$ (d) $-\frac{1}{5}$
58. $\tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc} =$
 (a) $\tan^{-1}a - \tan^{-1}b$ (b) $\tan^{-1}a - \tan^{-1}c$
 (c) $\tan^{-1}b - \tan^{-1}c$ (d) $\tan^{-1}c - \tan^{-1}a$
59. यदि $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$, तो $x =$ [Roorkee 1978, 80; MNR 1986; Pb. CET 2001; Karnataka CET 2002]
 (a) -1 (b) $\frac{1}{6}$
 (c) $-1, \frac{1}{6}$ (d) इनमें से कोई नहीं
60. यदि $\cot^{-1}x + \tan^{-1}3 = \frac{\pi}{2}$, तो $x =$
 (a) 1/3 (b) 1/4
 (c) 3 (d) 4
61. $2\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{24}{25} =$
 (a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$
 (c) $\frac{5\pi}{3}$ (d) इनमें से कोई नहीं
62. $\cos\left[\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right] =$ [MP PET 1991; MNR 1990]
 (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
 (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$
63. $\tan^{-1}x + \cot^{-1}(x+1) =$
 (a) $\tan^{-1}(x^2+1)$ (b) $\tan^{-1}(x^2+x)$
 (c) $\tan^{-1}(x+1)$ (d) $\tan^{-1}(x^2+x+1)$
64. $\cot^{-1}\frac{xy+1}{x-y} + \cot^{-1}\frac{yz+1}{y-z} + \cot^{-1}\frac{zx+1}{z-x} =$
 (a) 0 (b) 1
 (c) $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z$ (d) इनमें से कोई नहीं
65. यदि $\tan^{-1}\frac{a+x}{a} + \tan^{-1}\frac{a-x}{a} = \frac{\pi}{6}$, तो $x^2 =$
 (a) $2\sqrt{3}a$ (b) $\sqrt{3}a$
 (c) $2\sqrt{3}a^2$ (d) इनमें से कोई नहीं
66. यदि $\cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$, तो $x =$ [AMU 1978]
 (a) 0 (b) 1
 (c) -1 (d) 2
67. $\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5} =$
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
 (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$
68. $\tan^{-1}\frac{1-x^2}{2x} + \cos^{-1}\frac{1-x^2}{1+x^2} =$
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
 (c) π (d) 0

69. यदि $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$, तो $x =$

- (a) $\pm \frac{1}{2}$ (b) $0, \frac{1}{2}$
(c) $0, -\frac{1}{2}$ (d) $0, \pm \frac{1}{2}$

70. यदि $\cos^{-1}x + \cos^{-1}y = 2\pi$, तो $\sin^{-1}x + \sin^{-1}y =$

- (a) π (b) $-\pi$
(c) $\frac{\pi}{2}$ (d) इनमें से कोई नहीं

71. $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3 =$

[MP PET 1993; Karnataka CET 1995]

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

72. यदि $\cot^{-1}\alpha + \cot^{-1}\beta = \cot^{-1}x$, तो $x =$

[MP PET 1992]

- (a) $\alpha + \beta$ (b) $\alpha - \beta$
(c) $\frac{1+\alpha\beta}{\alpha+\beta}$ (d) $\frac{\alpha\beta-1}{\alpha+\beta}$

73. यदि $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1}x$, तो $x =$

[MNR 1984; UPSEAT 1999; Pb. CET 2004]

- (a) $\frac{a-b}{1+ab}$ (b) $\frac{b}{1+ab}$
(c) $\frac{b}{1-ab}$ (d) $\frac{a+b}{1-ab}$

74. $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} =$

[MP PET 1998; UPSEAT 2004]

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

75. $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} =$

[AMU 1976, 77]

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) इनमें से कोई नहीं

76. $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} =$

[Roorkee 1981]

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) इनमें से कोई नहीं

77. यदि $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, तब $\cos^{-1}x + \cos^{-1}y =$

[EAMCET 1994]

- (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) π

78. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) =$

[EAMCET 1994]

- (a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$
(c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

79. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) =$

[EAMCET 1992]

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{4}$ या $-\frac{3\pi}{4}$

80. यदि $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, तो $x =$

[EAMCET 1983; Karnataka CET 2004]

- (a) 4 (b) 5
(c) 1 (d) 3

81. $2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) =$

[EAMCET 1983]

- (a) $\tan^{-1}\left(\frac{49}{29}\right)$ (b) $\frac{\pi}{2}$
(c) 0 (d) $\frac{\pi}{4}$

82. $\cos^{-1}\left(\frac{15}{17}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) =$

[EAMCET 1981]

- (a) $\frac{\pi}{2}$ (b) $\cos^{-1}\left(\frac{171}{221}\right)$
(c) $\frac{\pi}{4}$ (d) इनमें से कोई नहीं

83. $\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) =$

[Karnataka CET 1994]

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\cos^{-1}\left(\frac{4}{5}\right)$ (d) π

84. समीकरण $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$ का हल है

[Karnataka CET 1993]

- (a) $x = 1$ (b) $x = -1$
(c) $x = 0$ (d) $x = \pi$

85. यदि $x^2 + y^2 + z^2 = r^2$, तब

$$\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{zx}{yr}\right) =$$

- (a) π (b) $\frac{\pi}{2}$
(c) 0 (d) इनमें से कोई नहीं

86. $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ के अधिकतम व न्यूनतम मान हैं
 (a) $-\frac{\pi}{2}, \frac{\pi}{2}$ (b) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$
 (c) $\frac{7\pi^3}{8}, \frac{\pi^3}{32}$ (d) इनमें से कोई नहीं
87. यदि $a < \frac{1}{32}$, $(\sin^{-1} x)^3 + (\cos^{-1} x)^3 = a\pi^3$ के हलों की संख्या है
 (a) 0 (b) 1
 (c) 2 (d) अनन्त
88. यदि $k \leq \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \leq K$, तब
 (a) $k = 0, K = \pi$ (b) $k = 0, K = \frac{\pi}{2}$
 (c) $k = \frac{\pi}{2}, K = \pi$ (d) इनमें से कोई नहीं
89. यदि $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, तब x बराबर है
 (a) -1 (b) 1
 (c) 0 (d) इनमें से कोई नहीं
90. यदि $\tan(x+y) = 33$ तथा $x = \tan^{-1} 3$, तब y होगा
 (a) 0.3 (b) $\tan^{-1}(1.3)$
 (c) $\tan^{-1}(0.3)$ (d) $\tan^{-1}\left(\frac{1}{18}\right)$
91. यदि $\tan^{-1} \frac{x-1}{x+1} + \tan^{-1} \frac{2x-1}{2x+1} = \tan^{-1} \frac{23}{36}$, तो $x =$
 [ISM Dhanbad 1973]
 (a) $\frac{3}{4}, \frac{-3}{8}$ (b) $\frac{3}{4}, \frac{3}{8}$
 (c) $\frac{4}{3}, \frac{3}{8}$ (d) इनमें से कोई नहीं
92. $\tan^{-1} \frac{c_1 x - y}{c_1 y + x} + \tan^{-1} \frac{c_2 - c_1}{1 + c_2 c_1} +$
 $\tan^{-1} \frac{c_3 - c_2}{1 + c_3 c_2} + \dots + \tan^{-1} \frac{1}{c_n} =$
 (a) $\tan^{-1} \frac{y}{x}$ (b) $\tan^{-1} yx$
 (c) $\tan^{-1} \frac{x}{y}$ (d) $\tan^{-1}(x-y)$
93. $\sin\left\{\sin^{-1} \frac{1}{2} + \cos^{-1} \frac{1}{2}\right\} =$ [EAMCET 1985]
 (a) 0 (b) -1
 (c) 2 (d) 1
94. $\sin^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{1}{3} =$ [ISM Dhanbad 1971]
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
 (c) $\frac{\pi}{4}$ (d) इनमें से कोई नहीं
95. $\sin^{-1} x + \cos^{-1} x =$ [Pb. CET 1997; DCE 2002]
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
 (c) -1 (d) 1
96. $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} =$
 [MP PET 1997, 2003; UPSEAT 2000; Karnataka CET 2001; Pb. CET 2004]
 (a) 0 (b) $\pi/4$
 (c) $\pi/2$ (d) π
97. $\tan^{-1}\left(\frac{1}{11}\right) + \tan^{-1}\left(\frac{2}{12}\right) =$ [DCE 1999]
 (a) $\tan^{-1}\left(\frac{33}{132}\right)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$
 (c) $\tan^{-1}\left(\frac{132}{33}\right)$ (d) इनमें से कोई नहीं
98. यदि $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$, तो x का मान होगा
 [Karnataka CET 1999]
 (a) $\sqrt{2}$ (b) 3
 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
99. यदि $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, तो x का मान होगा
 [Roorkee 1999; Karnataka CET 1999]
 (a) 0 (b) $\frac{1}{\sqrt{5}}$
 (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$
100. यदि $\sin^{-1} a + \sin^{-1} b + \sin^{-1} c = \pi$, तब
 $a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2}$ का मान होगा
 [UPSEAT 1999]
 (a) $2abc$ (b) abc
 (c) $\frac{1}{2}abc$ (d) $\frac{1}{3}abc$
101. $\cos^{-1}\left(\cos \frac{5\pi}{3}\right) + \sin^{-1}\left(\sin \frac{5\pi}{3}\right)$ का मान होगा
 [Roorkee 2000]
 (a) 0 (b) $\frac{\pi}{2}$
 (c) $\frac{2\pi}{3}$ (d) $\frac{10\pi}{3}$
102. यदि $4 \sin^{-1} x + \cos^{-1} x = \pi$, तब x का मान होगा
 [UPSEAT 2001]
 (a) 0 (b) $\frac{1}{2}$
 (c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

103. यदि $\sin^{-1} \frac{3}{5} + \cos^{-1} \left(\frac{12}{13} \right) = \sin^{-1} C$, तब C का मान होगा

[Pb. CET 1999]

- (a) $\frac{65}{56}$ (b) $\frac{24}{65}$
(c) $\frac{16}{65}$ (d) $\frac{56}{65}$

104. $\sin \left\{ \tan^{-1} \left(\frac{1-x^2}{2x} \right) + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right\}$ का मान होगा

[Kurukshetra CEE 2001]

- (a) 0 (b) 1
(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

105. $\cos^{-1} \left(\cos \frac{5\pi}{3} \right) + \sin^{-1} \left(\cos \frac{5\pi}{3} \right)$ का मान होगा

[UPSEAT 2003]

- (a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{3}$
(c) $\frac{10\pi}{3}$ (d) 0

106. $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) - \sin^{-1} \left(\frac{1}{2} \right)$ का मान होगा

[MP PET 2003]

- (a) 45° (b) 90°
(c) 15° (d) 30°

107. यदि $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, तब $xy + yz + zx$ का मान होगा

[Karnataka CET 2003]

- (a) 0 (b) 1
(c) 3 (d) -3

108. $\cos \left[\cos^{-1} \left(\frac{-1}{7} \right) + \sin^{-1} \left(\frac{-1}{7} \right) \right] =$

[EAMCET 2003]

- (a) $-1/3$ (b) 0
(c) $1/3$ (d) $4/9$

109. $\tan \left[\sin^{-1} \left(\frac{3}{5} \right) + \cos^{-1} \left(\frac{3}{\sqrt{13}} \right) \right]$ का मान होगा

[AMU 2001]

- (a) $\frac{6}{17}$ (b) $\frac{6}{\sqrt{13}}$
(c) $\frac{\sqrt{13}}{5}$ (d) $\frac{17}{6}$

110. $\tan \left(\tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{3} \right)$ का मान होगा

[AMU 2001]

- (a) $5/6$ (b) $7/6$
(c) $1/6$ (d) $1/7$

111. यदि $\cos^{-1} \sqrt{p} + \cos^{-1} \sqrt{1-p} + \cos^{-1} \sqrt{1-q} = \frac{3\pi}{4}$, तब q का मान होगा

[Karnataka CET 2002; Pb. CET 2000]

- (a) 1 (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

112. $\cot^{-1}[(\cos \alpha)^{1/2}] - \tan^{-1}[(\cos \alpha)^{1/2}] = x$, तब $\sin x$ का मान होगा

[AIEEE 2002]

- (a) $\tan^2 \left(\frac{\alpha}{2} \right)$ (b) $\cot^2 \left(\frac{\alpha}{2} \right)$
(c) $\tan \alpha$ (d) $\cot \left(\frac{\alpha}{2} \right)$

113. यदि $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, तो $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} =$

[MP PET 1991]

- (a) 0 (b) 1
(c) $\frac{1}{xyz}$ (d) xyz

114. $\tan \left[\frac{1}{2} \sin^{-1} \left(\frac{2a}{1+a^2} \right) + \frac{1}{2} \cos^{-1} \left(\frac{1-a^2}{1+a^2} \right) \right] =$

- (a) $\frac{2a}{1+a^2}$ (b) $\frac{1-a^2}{1+a^2}$
(c) $\frac{2a}{1-a^2}$ (d) इनमें से कोई नहीं

115. यदि $A = \tan^{-1} x$, तो $\sin 2A =$

[MNR 1988; UPSEAT 2000]

- (a) $\frac{2x}{\sqrt{1-x^2}}$ (b) $\frac{2x}{1-x^2}$
(c) $\frac{2x}{1+x^2}$ (d) इनमें से कोई नहीं

116. यदि $\cos(2 \sin^{-1} x) = \frac{1}{9}$, तो x का मान होगा

[Roorkee 1975]

- (a) केवल $\frac{2}{3}$ (b) केवल $-\frac{2}{3}$
(c) $\frac{2}{3}$, $-\frac{2}{3}$ (d) न तो $\frac{2}{3}$ और न ही $-\frac{2}{3}$

117. यदि $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$, तो $x =$

- (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) इनमें से कोई नहीं

118. $\tan \left(2 \cos^{-1} \frac{3}{5} \right) =$

- (a) $\frac{7}{25}$ (b) $\frac{24}{25}$
(c) $-\frac{24}{7}$ (d) $\frac{8}{3}$

119. $\tan \left[2 \tan^{-1} \left(\frac{1}{5} \right) - \frac{\pi}{4} \right] =$

[IIT 1984]

- (a) $\frac{17}{7}$ (b) $-\frac{17}{7}$
(c) $\frac{7}{17}$ (d) $-\frac{7}{17}$

120. यदि $2 \cos^{-1} \sqrt{\frac{1+x}{2}} = \frac{\pi}{2}$, तो $x =$

- (a) 1 (b) 0
(c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

121. $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right] =$ [Roorkee 1986]
 (a) $\frac{3-\sqrt{5}}{2}$ (b) $\frac{3+\sqrt{5}}{2}$
 (c) $\frac{2}{3-\sqrt{5}}$ (d) $\frac{2}{3+\sqrt{5}}$
122. $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) =$
 (a) $\cot^{-1}\sqrt{x}$ (b) $\tan^{-1}\sqrt{x}$
 (c) $\tan^{-1}x$ (d) $\cot^{-1}x$
123. $\sin\left(4\tan^{-1}\frac{1}{3}\right) =$
 (a) $\frac{12}{25}$ (b) $\frac{24}{25}$
 (c) $\frac{1}{5}$ (d) इनमें से कोई नहीं
124. $3\tan^{-1}a =$ [MP PET 1993]
 (a) $\tan^{-1}\frac{3a+a^3}{1+3a^2}$ (b) $\tan^{-1}\frac{3a-a^3}{1+3a^2}$
 (c) $\tan^{-1}\frac{3a+a^3}{1-3a^2}$ (d) $\tan^{-1}\frac{3a-a^3}{1-3a^2}$
125. $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239} =$ [MNR 1995]
 (a) π (b) $\frac{\pi}{2}$
 (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$
126. यदि $3\sin^{-1}\frac{2x}{1-x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$ तो $x =$
 (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
 (c) 1 (d) इनमें से कोई नहीं
127. $\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right) + \cos(\tan^{-1}2\sqrt{2})$ का मान होगा [AMU 1999]
 (a) $\frac{16}{15}$ (b) $\frac{14}{15}$
 (c) $\frac{12}{15}$ (d) $\frac{11}{15}$
128. $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$ का मान होगा [Karnataka CET 2003]
 (a) $\frac{1}{\sqrt{10}}$ (b) $-\frac{1}{\sqrt{10}}$
 (c) $\frac{1}{10}$ (d) $-\frac{1}{10}$
129. $\cos^{-1}(\cos 12) - \sin^{-1}(\sin 14)$ का मान है [J & K 2005]
 (a) -2 (b) $8\pi - 26$
 (c) $4\pi + 2$ (d) इनमें से कोई नहीं
130. $\cos^{-1}\left(\frac{3+5\cos x}{5+3\cos x}\right)$ का मान है [Kerala (Engg.) 2005]
 (a) $\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$ (b) $2\tan^{-1}\left(2\tan\frac{x}{2}\right)$
 (c) $\frac{1}{2}\tan^{-1}\left(2\tan\frac{x}{2}\right)$ (d) $2\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$
 (e) $\tan^{-1}\left(\tan\frac{x}{2}\right)$
131. यदि $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, हो तो $4x^2 - 4xy\cos\alpha + y^2 =$ [AIEEE 2005]
 (a) $4\sin^2\alpha$ (b) $-4\sin^2\alpha$
 (c) $2\sin 2\alpha$ (d) 4
132. यदि $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$ है, तो [Karnataka CET 2005]
 (a) $x+y+xy=1$ (b) $x+y-xy=1$
 (c) $x+y+xy+1=0$ (d) $x+y-xy+1=0$
133. यदि $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, तो x का मान है [Orissa JEE 2005]
 (a) $\left(0, -\frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, 0\right)$
 (c) $\{0\}$ (d) $(-1, 0)$
134. यदि $\triangle ABC$ में $\angle A = 90^\circ$ हो, तो $\tan^{-1}\left(\frac{c}{a+b}\right) + \tan^{-1}\left(\frac{b}{a+c}\right) =$ [Kerala (Engg.) 2005]
 (a) 0 (b) 1
 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$
 (e) $\frac{\pi}{8}$
135. $\sin^{-1}x - \sin^{-1}2x = \pm\frac{\pi}{3}$ का हल है [Karnataka CET 2005]
 (a) $\pm\frac{1}{3}$ (b) $\pm\frac{1}{4}$
 (c) $\pm\frac{\sqrt{3}}{2}$ (d) $\pm\frac{1}{2}$
136. समीकरण $\cos^{-1}x + \cos^{-1}2x + \pi = 0$ के वास्तविक हलों की संख्या है [Orissa JEE 2005]
 (a) 1 (b) 2
 (c) 0 (d) ∞
137. $\sin\left[3\sin^{-1}\left(\frac{1}{5}\right)\right] =$ [Kerala (Engg.) 2005]
 (a) $\frac{71}{125}$ (b) $\frac{74}{125}$
 (c) $\frac{3}{5}$ (d) $\frac{1}{2}$
 (e) $-\frac{3}{5}$

Critical Thinking

Objective Questions

1. $\cot^{-1} \left[\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}} \right] =$ [MNR 1986]

- (a) $\pi - x$ (b) $2\pi - x$
(c) $\frac{x}{2}$ (d) $\pi - \frac{x}{2}$

2. $\sin^{-1} \left[\sin \left(\frac{2\pi}{3} \right) \right]$ का मुख्य मान है [IIT 1986]

- (a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) $\frac{4\pi}{3}$ (d) इनमें से कोई नहीं

3. यदि $\theta = \tan^{-1} a, \phi = \tan^{-1} b$ तथा $ab = -1$, तो $\theta - \phi =$

- (a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) इनमें से कोई नहीं

4. यदि $\tan(\cos^{-1} x) = \sin \left(\cot^{-1} \frac{1}{2} \right)$, तो $x =$

- (a) $\pm \frac{5}{3}$ (b) $\pm \frac{\sqrt{5}}{3}$
(c) $\pm \frac{5}{\sqrt{3}}$ (d) इनमें से कोई नहीं

5. यदि किसी $x \in (-1, 1)$ के लिये $\sin^{-1} x = \frac{\pi}{5}$, तब $\cos^{-1} x =$

[IIT 1992]

- (a) $\frac{3\pi}{10}$ (b) $\frac{5\pi}{10}$
(c) $\frac{7\pi}{10}$ (d) $\frac{9\pi}{10}$

6. यदि $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = \pi$ तो $p^2 + q^2 + r^2 + 2pqr =$

[Karnataka CET 2004]

- (a) 3 (b) 1
(c) 2 (d) -1

7. $\tan \left[\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b} \right] + \tan \left[\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b} \right] =$

[MP PET 1999]

- (a) $\frac{2a}{b}$ (b) $\frac{2b}{a}$
(c) $\frac{a}{b}$ (d) $\frac{b}{a}$

8. $\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] =$

- (a) $\frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$ (b) $\frac{\pi}{4} + \cos^{-1} x^2$
(c) $\frac{\pi}{4} + \frac{1}{2} \cos^{-1} x$ (d) $\frac{\pi}{4} - \frac{1}{2} \cos^{-1} x^2$

9. समीकरण $\sin^{-1} x - \cos^{-1} x = \cos^{-1} \left(\frac{\sqrt{3}}{2} \right)$ रखता है

- (a) कोई हल नहीं (b) अद्वितीय हल
(c) अनन्त हल (d) इनमें से कोई नहीं

10. $\sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4 + m^2 + 2} \right)$ बराबर है

- (a) $\tan^{-1} \left(\frac{n^2 + n}{n^2 + n + 2} \right)$ (b) $\tan^{-1} \left(\frac{n^2 - n}{n^2 - n + 2} \right)$
(c) $\tan^{-1} \left(\frac{n^2 + n + 2}{n^2 + n} \right)$ (d) इनमें से कोई नहीं

11. $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$ के वास्तविक हलों की संख्या है [IIT 1999]

- (a) शून्य (b) एक
(c) दो (d) अनन्त

12. समीकरण $2 \cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$ के हल होंगे [AMU 1999]

- (a) कोई हल नहीं (b) केवल एक हल
(c) दो हल (d) तीन हल

13. यदि $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, तब $x + y + z =$

[Kerala (Engg.) 2002]

- (a) xyz (b) 0
(c) 1 (d) $2xyz$

14. $2 \tan^{-1} \left[\sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right] =$

[Dhanbad Engg. 1976]

- (a) $\cos^{-1} \left(\frac{a \cos \theta + b}{a + b \cos \theta} \right)$
(b) $\cos^{-1} \left(\frac{a + b \cos \theta}{a \cos \theta + b} \right)$
(c) $\cos^{-1} \left(\frac{a \cos \theta}{a + b \cos \theta} \right)$
(d) $\cos^{-1} \left(\frac{b \cos \theta}{a \cos \theta + b} \right)$

15. यदि $2 \tan^{-1}(\cos x) = \tan^{-1}(\operatorname{cosec}^2 x)$, तब x का मान होगा

[UPSEAT 2002]

- (a) $\frac{\pi}{2}$ (b) π
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

Answers

प्रतिलोम त्रिकोणमितीय फलन

1	d	2	d	3	a	4	b	5	c
6	b	7	b	8	c	9	b	10	b
11	c	12	a	13	c	14	d	15	c
16	b	17	c	18	b	19	a	20	c
21	d	22	b	23	c	24	a	25	b
26	b	27	d	28	d	29	d	30	c
31	b	32	b	33	c	34	c	35	a
36	a	37	b	38	a	39	c	40	c
41	b	42	c	43	d	44	a	45	a
46	a	47	d	48	c	49	b	50	d
51	a	52	d	53	c	54	d	55	c
56	b	57	b	58	b	59	b	60	c
61	a	62	a	63	d	64	a	65	c
66	b	67	b	68	b	69	d	70	b
71	b	72	d	73	d	74	d	75	a
76	c	77	b	78	a,d	79	c	80	d
81	d	82	d	83	a	84	c	85	b
86	c	87	a	88	a	89	a	90	c
91	d	92	c	93	d	94	a	95	b
96	b	97	d	98	c	99	b	100	a
101	a	102	b	103	d	104	b	105	a
106	d	107	c	108	b	109	d	110	d
111	d	112	a	113	b	114	c	115	c
116	c	117	b	118	c	119	d	120	b
121	a,d	122	b	123	b	124	d	125	d
126	b	127	b	128	a	129	a	130	d
131	a	132	b	133	c	134	c	135	d
136	c	137	a						

Critical Thinking Questions

1	d	2	d	3	c	4	b	5	a
6	b	7	b	8	a	9	b	10	a
11	c	12	a	13	a	14	a	15	d

AS Answers and Solutions

प्रतिलोम त्रिकोणमितीय फलन

- (d) दिया गया है कि, $\cos^{-1}\left(\frac{1}{x}\right) = \theta \Rightarrow \cos \theta = \frac{1}{x}$

अब, $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{1 - (1/x)^2}}{1/x} = \sqrt{x^2 - 1}$.
- (d) माना $\cot^{-1} x = \theta \Rightarrow x = \cot \theta$

अब $\operatorname{cosec} \theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + x^2}$

$\therefore \sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\sqrt{1 + x^2}} \Rightarrow \theta = \sin^{-1} \frac{1}{\sqrt{1 + x^2}}$

$\therefore \sin(\cot^{-1} x) = \sin\left(\sin^{-1} \frac{1}{\sqrt{1 + x^2}}\right)$

$= \frac{1}{\sqrt{1 + x^2}} = (1 + x^2)^{-1/2}$.
- (a) माना $\sin^{-1} \frac{5}{13} = x \Rightarrow \sin x = \frac{5}{13}$

$\Rightarrow \cos x = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$

$\Rightarrow \cos\left(\sin^{-1} \frac{5}{13}\right) = \cos\left(\cos^{-1} \frac{12}{13}\right) = \frac{12}{13}$.
- (b) $\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3}) = \frac{5\pi}{6}$.
- (c) माना $\sin^{-1} x = \theta \Rightarrow \sin \theta = x$

अब $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta = \frac{1}{x^2}$

अतः $1 + \cot^2(\sin^{-1} x) = \frac{1}{x^2}$.
- (b) दिया गया है कि, $\sin^{-1} \frac{1}{2} = \tan^{-1} x$

$\Rightarrow \tan^{-1} x = \frac{\pi}{6} \Rightarrow x = \tan 30^\circ = \frac{1}{\sqrt{3}}$.
- (b) $\tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right) = \tan^{-1}\left[\frac{\sqrt{1 + \tan^2 \theta} - 1}{\tan \theta}\right]$

($x = \tan \theta$ रखने पर)

$= \tan^{-1}\left[\frac{\sec \theta - 1}{\tan \theta}\right] = \tan^{-1}\left[\frac{1 - \cos \theta}{\sin \theta}\right]$

$= \tan^{-1}\left[\frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}\right] = \tan^{-1} \tan \frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$.

$$8. (c) \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} = \tan^{-1} \left(\frac{a \sin \theta}{a \cos \theta} \right) \quad (x = a \sin \theta \text{ रखने पर})$$

$$= \tan^{-1}(\tan \theta) = \theta = \sin^{-1} \left(\frac{x}{a} \right).$$

$$9. (b) \text{ माना } \theta = \tan^{-1} x \Rightarrow x = \tan \theta$$

$$\therefore \cos \theta = \frac{1}{\sqrt{1 + \tan^2 \theta}} = \frac{1}{\sqrt{1 + x^2}}$$

$$\text{अतः } \cos \theta = \cos(\tan^{-1} x) = \frac{1}{\sqrt{1 + x^2}}.$$

$$10. (b) \tan \left(\sec^{-1} \sqrt{1 + x^2} \right) = \tan \left(\sec^{-1} \sqrt{1 + \tan^2 \theta} \right)$$

$$(x = \tan \theta \text{ रखने पर})$$

$$= \tan(\sec^{-1} \sec \theta) = \tan \theta = x.$$

$$11. (c) \sec^{-1}[\sec(-30^\circ)] = \sec^{-1}(\sec 30^\circ) = 30^\circ.$$

$$12. (a) \tan^{-1} \left[\frac{\cos x}{1 + \sin x} \right] = \tan^{-1} \left[\frac{\sin(\pi/2 - x)}{1 + \cos(\pi/2 - x)} \right]$$

$$= \tan^{-1} \left[\frac{2 \sin(\pi/4 - x/2) \cos(\pi/4 - x/2)}{2 \cos^2(\pi/4 - x/2)} \right]$$

$$= \tan^{-1} \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) = \frac{\pi}{4} - \frac{x}{2}.$$

$$13. (c) \tan^{-1} \frac{1}{\sqrt{x^2 - 1}} = \tan^{-1} \frac{1}{\sqrt{\operatorname{cosec}^2 \theta - 1}}$$

$$(x = \operatorname{cosec} \theta \text{ रखने पर})$$

$$= \tan^{-1} \frac{1}{\cot \theta} = \theta = \operatorname{cosec}^{-1} x.$$

$$14. (d) \sin^{-1} \left(-\frac{1}{2} \right) \text{ का मुख्य मान } = \sin^{-1} \sin(-30^\circ) = -\frac{\pi}{6}.$$

$$15. (c) \text{ माना } \tan^{-1} 2 = \alpha \Rightarrow \tan \alpha = 2$$

$$\text{और } \cot^{-1} 3 = \beta \Rightarrow \cot \beta = 3$$

$$\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$$

$$= \sec^2 \alpha + \operatorname{cosec}^2 \beta = 1 + \tan^2 \alpha + 1 + \cot^2 \alpha$$

$$= 2 + (2)^2 + (3)^2 = 15.$$

$$16. (b) \text{ माना } x = \sin \theta \text{ तथा } \sqrt{x} = \sin \phi$$

$$\text{अतः } \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$$

$$= \sin^{-1}(\sin \theta \sqrt{1 - \sin^2 \phi} - \sin \phi \sqrt{1 - \sin^2 \theta})$$

$$= \sin^{-1}(\sin \theta \cos \phi - \sin \phi \cos \theta) = \sin^{-1} \sin(\theta - \phi)$$

$$= \theta - \phi = \sin^{-1}(x) - \sin^{-1}(\sqrt{x}).$$

$$17. (c) \text{ हमें ज्ञात है कि, } \tan^{-1} \frac{1-x}{1+x} = \frac{1}{2} \tan^{-1} x$$

$$\Rightarrow \tan^{-1} \left[\frac{1 - \tan \theta}{1 + \tan \theta} \right] = \frac{1}{2} \theta \quad (x = \tan \theta \text{ रखने पर})$$

$$\Rightarrow \tan^{-1} \left[\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta} \right] = \frac{\theta}{2}$$

$$\Rightarrow \tan^{-1} \tan \left(\frac{\pi}{4} - \theta \right) = \frac{\theta}{2} \Rightarrow \frac{\pi}{4} - \theta = \frac{\theta}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6} = \tan^{-1} x \Rightarrow x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}.$$

$$18. (b) \cos^{-1} \left(\cos \frac{7\pi}{6} \right) = \cos^{-1} \left\{ \cos \left(\pi + \frac{\pi}{6} \right) \right\}$$

$$= \cos^{-1} \left(-\cos \frac{\pi}{6} \right) = \pi - \cos^{-1} \cos \frac{\pi}{6} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

$$19. (a) \text{ माना } \cos^{-1} x = \theta \Rightarrow x = \cos \theta \Rightarrow \sec \theta = \frac{1}{x}$$

$$\Rightarrow \tan \theta = \sqrt{\sec^2 \theta - 1} = \sqrt{\frac{1}{x^2} - 1} = \frac{1}{x} \sqrt{1 - x^2}$$

$$\text{अब } \sin \cot^{-1} \tan \theta = \sin \cot^{-1} \left(\frac{1}{x} \sqrt{1 - x^2} \right)$$

$$\text{साथ ही, } x = \sin \theta \text{ रखने पर,}$$

$$\sin \cot^{-1} \left(\frac{1}{x} \sqrt{1 - x^2} \right) = \sin \cot^{-1} \left(\frac{\sqrt{1 - \sin^2 \theta}}{\sin \theta} \right)$$

$$= \sin \cot^{-1}(\cot \theta) = \sin \theta = x.$$

$$20. (c) x = a \tan^2 \theta \text{ रखने पर}$$

$$\sin^{-1} \frac{\sqrt{x}}{\sqrt{x+a}} = \sin^{-1} \frac{\sqrt{a} \sqrt{\tan^2 \theta}}{\sqrt{a \tan^2 \theta + a}} = \sin^{-1} \frac{\sqrt{a} \tan \theta}{\sqrt{a} \sec \theta}$$

$$= \sin^{-1} \sin \theta = \theta = \tan^{-1} \left(\sqrt{\frac{x}{a}} \right).$$

$$21. (d) \sin^{-1} \frac{1}{5} + \cos^{-1} x = \frac{\pi}{2}$$

$$\therefore \sin^{-1} \frac{1}{5} = \frac{\pi}{2} - \cos^{-1} x = \sin^{-1} x$$

$$\therefore x = \frac{1}{5}.$$

$$22. (b) \text{ स्पष्टतः } x = \sin(\theta + \beta) \text{ और } y = \sin(\theta - \beta)$$

$$\therefore 1 + xy = 1 + \sin(\theta + \beta) \sin(\theta - \beta)$$

$$= 1 + \sin^2 \theta - \sin^2 \beta = \sin^2 \theta + \cos^2 \beta.$$

$$23. (c) \sin^{-1} \frac{1}{3} + \sin^{-1} \frac{2}{3}$$

$$= \sin^{-1} \left[\frac{1}{3} \sqrt{1 - \frac{4}{9}} + \frac{2}{3} \sqrt{1 - \frac{1}{9}} \right] = \sin^{-1} \left[\frac{\sqrt{5} + 4\sqrt{2}}{9} \right]$$

$$\text{अतः } x = \frac{\sqrt{5} + 4\sqrt{2}}{9}.$$

$$24. (a) \text{ माना } \cos^{-1} x = \theta \text{ तब } x = \cos \theta$$

$$\Rightarrow \tan \theta = \sqrt{\sec^2 \theta - 1} = \sqrt{\frac{1}{x^2} - 1} = \sqrt{\frac{1 - x^2}{x^2}}$$

$$\therefore \tan(\cos^{-1} x) = \tan \theta = \frac{\sqrt{1 - x^2}}{x}.$$

25. (b) यह स्पष्ट है।

26. (b) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

27. (d) $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right] = \cot\left[\cot^{-1}\left(\frac{7}{24}\right)\right] = \frac{7}{24}$.

28. (d) हमें ज्ञात है कि, $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$
 $\Rightarrow \frac{-\pi}{2} \leq x - \pi \leq \frac{\pi}{2} \Rightarrow \frac{-\pi}{2} \leq \pi - x \leq \frac{\pi}{2}$
 $\Rightarrow \sin^{-1}\{\sin(\pi - x)\} = \pi - x$.

29. (d) यह दिया गया है कि, $\pi \leq x \leq 2\pi$
 $\Rightarrow -\pi \geq -x \geq -2\pi \Rightarrow \pi \geq 2\pi - x \geq 0$
 $\Rightarrow \cos^{-1}\{\cos(2\pi - x)\} = 2\pi - x$.

30. (c) चूँकि $3\pi < 10 < 3\pi + \frac{\pi}{2} \Rightarrow 0 < 10 - 3\pi < \frac{\pi}{2}$
 $\Rightarrow \frac{-\pi}{2} < 3\pi - 10 < 0 \Rightarrow \sin^{-1}\{\sin(3\pi - 10)\} = 3\pi - 10$.

31. (b) हमें ज्ञात है कि
 $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}1 - \tan^{-1}x = \frac{\pi}{4} - \tan^{-1}x$
 चूँकि $0 \leq x \leq 1 \Rightarrow 0 \leq \tan^{-1}x \leq \frac{\pi}{4}$
 $\Rightarrow 0 \geq -\tan^{-1}x \geq \frac{-\pi}{4} \Rightarrow \frac{\pi}{4} \geq \frac{\pi}{4} - \tan^{-1}x \geq 0$
 $\Rightarrow \frac{\pi}{4} \geq \tan^{-1}\left(\frac{1-x}{1+x}\right) \geq 0$.

32. (b) माना $\sin^{-1}x = y$ तब $x = \sin y$
 चूँकि $-1 \leq x \leq 0$, अतः $\frac{-\pi}{2} \leq \sin^{-1}x \leq 0$

अतः $\frac{-\pi}{2} \leq y \leq 0$

हमें ज्ञात है कि, $\cos y = \sqrt{1 - \sin^2 y}$
 $\Rightarrow \cos y = \sqrt{1 - x^2}$, $0 \leq y \leq \pi$ के लिए(i)

अब $-\frac{\pi}{2} \leq y \leq 0 \Rightarrow \frac{\pi}{2} \geq -y \geq 0$
 $\Rightarrow \cos(-y) = \sqrt{1 - x^2}$ {(i) से}
 $\Rightarrow -y = \cos^{-1}\sqrt{1 - x^2} \Rightarrow y = -\cos^{-1}\sqrt{1 - x^2}$.

33. (c) $\left[\sin\left(\tan^{-1}\frac{3}{4}\right)\right]^2 = \left[\sin\left(\sin^{-1}\frac{3}{5}\right)\right]^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$.

34. (c) $\sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

35. (a) दिया गया है, $\tan^{-1}x = \sin^{-1}\left[\frac{3}{\sqrt{10}}\right]$
 $\Rightarrow x = \tan\left\{\sin^{-1}\left[\frac{3}{\sqrt{10}}\right]\right\} = \tan\{\tan^{-1}3\} \Rightarrow x = 3$.

36. (a) हम जानते हैं कि $\sec(\operatorname{cosec} x) = \operatorname{cosec}(\sec x)$
 $= \frac{|x|}{\sqrt{x^2 - 1}}$, $|x| > 1$ के

लिये।

37. (b) यह स्पष्ट है।

38. (a) $\theta = \sin^{-1}[\sin(-600^\circ)]$
 $\Rightarrow \theta = \sin^{-1}[-\sin(240^\circ)] = \sin^{-1}[-\sin(180^\circ + 60^\circ)]$
 $\Rightarrow \theta = \sin^{-1}\sin 60^\circ = \sin^{-1}\left[\sin\left(\frac{\pi}{3}\right)\right] = \frac{\pi}{3}$.

39. (c) $\sin^{-1}x = 2\tan^{-1}x \Rightarrow \sin^{-1}x = \sin^{-1}\frac{2x}{1+x^2}$
 $\Rightarrow \frac{2x}{1+x^2} = x \Rightarrow x^3 - x = 0$
 $\Rightarrow x(x+1)(x-1) = 0 \Rightarrow x = \{-1, 1, 0\}$.

40. (c) $\cos[\tan^{-1}(\tan 2)] = \cos 2$.

41. (b) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2}$
 $\sin^{-1}x = \alpha$, $\sin^{-1}y = \beta$, $\sin^{-1}z = \gamma$ रखने पर
 $\therefore \alpha + \beta + \gamma = \frac{\pi}{2}$, (दिया है)

या $\alpha + \beta = \frac{\pi}{2} - \gamma$ या $\cos(\alpha + \beta) = \cos\left(\frac{\pi}{2} - \gamma\right)$
 $\cos \alpha \cos \beta - \sin \alpha \sin \beta = \sin \gamma$ (i)

और हमें ज्ञात है कि, $\sin \alpha = x \Rightarrow \cos \alpha = \sqrt{1 - x^2}$
 इसी तरह $\cos \beta = \sqrt{1 - y^2}$
 $\therefore$ समीकरण (i) से हमें $\sqrt{1 - x^2} \cdot \sqrt{1 - y^2} = xy + z$ प्राप्त होता है।
 दोनों पक्षों का वर्ग करने पर, $x^2 + y^2 + z^2 + 2xyz = 1$.

42. (c) $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \cos \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
 $= \cos \cos^{-1}\sqrt{1 - \frac{3}{4}} = \frac{1}{2}$.

43. (d) $\sin[\cot^{-1}(\cos \tan^{-1}x)]$
 $= \sin\left[\cot^{-1}\left(\cos \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right]$
 $= \sin\left[\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right] = \sin\left[\sin^{-1}\sqrt{\frac{1+x^2}{2+x^2}}\right] = \sqrt{\frac{1+x^2}{2+x^2}}$.

44. (a) $\sin[\cot^{-1}(x+1)] = \sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right)$
 $= \frac{1}{\sqrt{x^2+2x+2}}$

$\cos(\tan^{-1}x) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$

अतः $\frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}}$

$$\Rightarrow x^2 + 2x + 2 = 1 + x^2 \Rightarrow x = -\frac{1}{2}.$$

$$45. \quad (a) \quad \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{3}{5} = \tan^{-1} \left[\frac{\sqrt{1 - \frac{16}{25}}}{\frac{4}{5}} \right] + \tan^{-1} \frac{3}{5}$$

$$\left[\begin{array}{l} \text{चूँकि } \cos^{-1} x = \tan^{-1} \frac{\sqrt{1-x^2}}{x} \end{array} \right]$$

$$= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5} = \tan^{-1} \left(\frac{\frac{3}{4} + \frac{3}{5}}{1 - \frac{3}{4} \cdot \frac{3}{5}} \right) = \tan^{-1} \left(\frac{27}{11} \right).$$

$$46. \quad (a) \quad \sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x}$$

$$= \{ \sin^{-1}(x) + \cos^{-1}(x) \} + \left\{ \sin^{-1} \left(\frac{1}{x} \right) + \cos^{-1} \left(\frac{1}{x} \right) \right\}$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi.$$

$$47. \quad (d) \quad 2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} = \tan^{-1} \left(\frac{\frac{2}{3}}{1 - \frac{1}{9}} \right) + \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \tan^{-1} \left(\frac{\frac{2}{3}}{\frac{8}{9}} \right) + \tan^{-1} \left(\frac{1}{2} \right) = \tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \tan^{-1} \left(\frac{\frac{1}{2} + \frac{3}{4}}{1 - \frac{1}{2} \times \frac{3}{4}} \right) = \tan^{-1}(2).$$

$$48. \quad (c) \quad \tan \left(90^\circ - \cot^{-1} \frac{1}{3} \right) = \cot \cdot \cot^{-1} \frac{1}{3} = \frac{1}{3}.$$

$$49. \quad (b) \quad \tan \left[\cos^{-1} \frac{4}{5} + \tan^{-1} \frac{2}{3} \right]$$

$$= \tan \left[\tan^{-1} \frac{\sqrt{1 - \frac{16}{25}}}{\frac{4}{5}} + \tan^{-1} \frac{2}{3} \right]$$

$$= \tan \left[\tan^{-1} \left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \cdot \frac{2}{3}} \right) \right] = \tan \cdot \tan^{-1} \frac{17}{6} = \frac{17}{6}.$$

$$50. \quad (d) \quad \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$$

$$= \tan^{-1} 1 + \pi + \tan^{-1} \left(\frac{5}{-5} \right) = \tan^{-1} 1 + \pi - \tan^{-1} 1 = \pi.$$

$$51. \quad (a) \quad \text{माना } \cot^{-1} \frac{3}{4} = \theta \Rightarrow \cot \theta = \frac{3}{4}$$

$$\text{तथा } \sin \theta = \frac{1}{\sqrt{1 + \cot^2 \theta}} = \frac{1}{\sqrt{1 + (9/16)}} = \frac{4}{5}$$

$$\text{अतः } \cot^{-1} \frac{3}{4} + \sin^{-1} \frac{5}{13} = \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13}$$

$$= \sin^{-1} \left[\frac{4}{5} \cdot \sqrt{1 - \frac{25}{169}} + \frac{5}{13} \cdot \sqrt{1 - \frac{16}{25}} \right]$$

$$= \sin^{-1} \left[\frac{4}{5} \cdot \frac{12}{13} + \frac{5}{13} \cdot \frac{3}{5} \right] = \sin^{-1} \left[\frac{48 + 15}{65} \right] = \sin^{-1} \frac{63}{65}.$$

$$52. \quad (d) \quad \text{दिया गया है कि, } \cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$$

$$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = \cos^{-1}(-1)$$

$$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}(-1) - \cos^{-1}(z)$$

$$\Rightarrow \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \cos^{-1}\{(-1)(z)\}$$

$$\Rightarrow xy - \sqrt{(1-x^2)(1-y^2)} = -z$$

$$\Rightarrow (xy + z) = \sqrt{(1-x^2)(1-y^2)}$$

$$\text{दोनों पक्षों का वर्ग करने पर, } x^2 + y^2 + z^2 + 2xyz = 1.$$

$$\text{ट्रिक : } x = y = z = \frac{1}{2} \text{ रखने पर,}$$

$$\cos^{-1} \frac{1}{2} + \cos^{-1} \frac{1}{2} + \cos^{-1} \frac{1}{2} = \pi$$

स्पष्ट है कि x, y, z के इन मानों के लिये विकल्प (d) सही है।

$$53. \quad (c) \quad \text{दिया गया है कि } \tan^{-1} x - \tan^{-1} y = \tan^{-1} A$$

$$\Rightarrow \tan^{-1} \left(\frac{x-y}{1+xy} \right) = \tan^{-1} A.$$

$$\text{अतः } A = \frac{x-y}{1+xy}.$$

$$54. \quad (d) \quad \text{दिया गया है कि, } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} \left[\frac{x+y+z-xyz}{1-xy-yz-zx} \right] = \frac{\pi}{2}$$

$$\Rightarrow \left[\frac{x+y+z-xyz}{1-xy-yz-zx} \right] = \tan \frac{\pi}{2} = \frac{1}{0}$$

$$\text{अतः } xy + yz + zx - 1 = 0.$$

$$\text{ट्रिक : } x = y = z = \frac{1}{\sqrt{3}},$$

$$\text{अतः } \tan^{-1} \frac{1}{\sqrt{3}} + \tan^{-1} \frac{1}{\sqrt{3}} + \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{2}$$

स्पष्ट है कि x, y, z के इन मानों के लिये विकल्प (d) सही है।

$$55. \quad (c) \quad \text{हमें ज्ञात है कि } \tan^{-1} \frac{x-1}{x+2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{x-1}{x+2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x+2} \right) \left(\frac{x+1}{x+2} \right)} \right] = \frac{\pi}{4}$$

$$\Rightarrow \left[\frac{2x(x+2)}{x^2 + 4 + 4x - x^2 - 1} \right] = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{2x(x+2)}{4x+5} = \tan \frac{\pi}{4} = 1$$

$$\Rightarrow 2x^2 + 4x = 4x + 5 \Rightarrow x = \pm \sqrt{\frac{5}{2}}.$$

$$56. \quad (b) \quad \cos^{-1} \sqrt{1-x} + \sin^{-1} \sqrt{1-x} = \sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x} = \frac{\pi}{2}.$$

$$57. \quad (b) \quad \cos \left(\cos^{-1} \frac{1}{5} + \sin^{-1} \frac{1}{5} + \cos^{-1} \frac{1}{5} \right) = \cos \left(\frac{\pi}{2} + \cos^{-1} \frac{1}{5} \right) \\ = -\sin \left(\cos^{-1} \frac{1}{5} \right) = -\sin \left(\sin^{-1} \sqrt{\frac{24}{25}} \right) = -\frac{2\sqrt{6}}{5}.$$

$$58. \quad (b) \quad \tan^{-1} \left(\frac{a-b}{1+ab} \right) + \tan^{-1} \left(\frac{b-c}{1+bc} \right) \\ = \tan^{-1}(a) - \tan^{-1}(b) + \tan^{-1}(b) - \tan^{-1}(c) \\ = \tan^{-1}(a) - \tan^{-1}(c).$$

$$59. \quad (b) \quad \tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4} \Rightarrow \tan^{-1} \left(\frac{2x+3x}{1-(2x)(3x)} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{5x}{1-6x^2} \right) = \tan^{-1}(1) \Rightarrow \frac{5x}{1-6x^2} = 1$$

$$\Rightarrow 1-6x^2 = 5x \Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow (x+1) \left(x - \frac{1}{6} \right) = 0 \Rightarrow x = -1, \frac{1}{6}$$

किन्तु -1 संतुष्ट नहीं करता है।

ट्रिक : विकल्प की जाँच करें। स्पष्टतः $x = 1/6$ के लिये समीकरण संतुष्ट करता है किन्तु $x = -1$ के लिये नहीं।

$$60. \quad (c) \quad \text{हमें ज्ञात है कि, } \cot^{-1} x + \tan^{-1} 3 = \frac{\pi}{2}$$

$$\Rightarrow \cot^{-1} x + \tan^{-1} 3 = \frac{\pi}{2} \Rightarrow \tan^{-1} \frac{1}{x} + \tan^{-1} 3 = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{1}{x} + 3}{1 - \frac{1}{x} \cdot 3} \right) = \tan^{-1} \left(\frac{1}{0} \right)$$

$$\Rightarrow \frac{3x+1}{x-3} = \frac{1}{0} \Rightarrow x = 3$$

वैकल्पिक: जैसा कि हम जानते हैं कि

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \text{ अतः दिये गये प्रश्न के लिये}$$

$x = 3$ होना चाहिए।

$$61. \quad (a) \quad 2 \sin^{-1} \frac{3}{5} + \cos^{-1} \frac{24}{25} = \sin^{-1} 2 \times \frac{3}{5} \sqrt{1 - \frac{9}{25}} + \cos^{-1} \frac{24}{25} \\ = \sin^{-1} \frac{24}{25} + \cos^{-1} \frac{24}{25} = \frac{\pi}{2}.$$

$$62. \quad (a) \quad \cos \left[\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} \right] = \cos \left[\tan^{-1} \left(\frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \times \frac{1}{2}} \right) \right] \\ = \cos \{ \tan^{-1}(1) \} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

$$63. \quad (d) \quad \tan^{-1} x + \cot^{-1}(x+1) = \tan^{-1} x + \tan^{-1} \left(\frac{1}{x+1} \right)$$

$$= \tan^{-1} \left[\frac{x + \frac{1}{x+1}}{1 - \frac{x}{x+1}} \right] = \tan^{-1} (x^2 + x + 1).$$

$$64. \quad (a) \quad \cot^{-1} \frac{xy+1}{x-y} + \cot^{-1} \frac{yz+1}{y-z} + \cot^{-1} \frac{zx+1}{z-x} \\ = \cot^{-1} y - \cot^{-1} x + \cot^{-1} z - \cot^{-1} y \\ + \cot^{-1} x - \cot^{-1} z = 0.$$

नोट : विद्यार्थी इसे सूत्र की तरह याद रखें।

$$65. \quad (c) \quad \text{दिया गया समीकरण } \tan^{-1} \frac{a+x}{a} + \tan^{-1} \frac{a-x}{a} = \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{a+x}{a} + \frac{a-x}{a}}{1 - \frac{a+x}{a} \cdot \frac{a-x}{a}} \right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{2a^2}{x^2} = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow x^2 = 2\sqrt{3}a^2.$$

$$66. \quad (b) \quad \cos^{-1} \frac{3}{5} - \sin^{-1} \frac{4}{5} = \cos^{-1} x$$

$$\Rightarrow \cos^{-1} \frac{3}{5} - \cos^{-1} \sqrt{1 - \frac{16}{25}} = \cos^{-1} x$$

$$\Rightarrow \cos^{-1} \frac{3}{5} - \cos^{-1} \frac{3}{5} = \cos^{-1} x$$

$$\Rightarrow \cos^{-1} x = 0 \Rightarrow x = 1.$$

$$67. \quad (b) \quad \cot^{-1} 3 + \operatorname{cosec}^{-1} \sqrt{5} = \cot^{-1} 3 + \cot^{-1} 2$$

$$= \cot^{-1} \left(\frac{3 \times 2 - 1}{3 + 2} \right) = \cot^{-1}(1) = \frac{\pi}{4}.$$

$$68. \quad (b) \quad x = \tan \theta \text{ रखने पर,}$$

$$\tan^{-1} \frac{1-x^2}{2x} + \cos^{-1} \frac{1-x^2}{1+x^2}$$

$$= \tan^{-1} \left(\frac{1 - \tan^2 \theta}{2 \tan \theta} \right) + \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$= \tan^{-1}(\cot 2\theta) + \cos^{-1}(\cos 2\theta) = \frac{\pi}{2} - 2\theta + 2\theta = \frac{\pi}{2}.$$

$$69. \quad (d) \quad \tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1} 3x$$

$$\Rightarrow \tan^{-1}(x-1) + \tan^{-1}(x) = \tan^{-1} 3x - \tan^{-1}(x+1)$$

$$\Rightarrow \tan^{-1} \left[\frac{(x-1)+x}{1-(x-1)(x)} \right] = \tan^{-1} \left[\frac{3x-(x+1)}{1+3x(x+1)} \right]$$

$$\Rightarrow \frac{2x-1}{1-x^2+x} = \frac{2x-1}{1+3x^2+3x}$$

$$\Rightarrow (1-x^2+x)(2x-1) = (1+3x^2+3x)(2x-1)$$

$$\text{हल करने के पश्चात् } x = 0, \pm \frac{1}{2}.$$

$$70. \quad (b) \quad \cos^{-1} x + \cos^{-1} y = 2\pi$$

$$\Rightarrow \frac{\pi}{2} - \sin^{-1} x + \frac{\pi}{2} - \sin^{-1} y = 2\pi$$

$$\Rightarrow \pi - (\sin^{-1} x + \sin^{-1} y) = 2\pi$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = -\pi$$

$$\begin{aligned} 71. \quad (b) \quad \sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3 &= \cot^{-1} \left(\frac{\sqrt{1 - \frac{1}{5}}}{\frac{1}{\sqrt{5}}} \right) + \cot^{-1} 3 \\ &= \cot^{-1}(2) + \cot^{-1}(3) = \cot^{-1} \left(\frac{2 \times 3 - 1}{3 + 2} \right) = \cot^{-1}(1) = \frac{\pi}{4}. \end{aligned}$$

$$\begin{aligned} 72. \quad (d) \quad \text{दिया गया है कि, } \cot^{-1} \alpha + \cot^{-1} \beta &= \cot^{-1} x \\ \Rightarrow \cot^{-1} \left(\frac{\alpha\beta - 1}{\alpha + \beta} \right) &= \cot^{-1} x \Rightarrow x = \frac{\alpha\beta - 1}{\alpha + \beta}. \end{aligned}$$

$$\begin{aligned} 73. \quad (d) \quad \sin^{-1} \left(\frac{2a}{1+a^2} \right) + \sin^{-1} \left(\frac{2b}{1+b^2} \right) &= 2 \tan^{-1} x \\ a = \tan \theta \text{ और } b = \tan \phi \text{ रखने पर} \\ \text{अतः } \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) + \sin^{-1} \left(\frac{2 \tan \phi}{1 + \tan^2 \phi} \right) &= 2 \tan^{-1} x \\ \Rightarrow \sin^{-1} \sin(2\theta) + \sin^{-1} \sin(2\phi) &= 2 \tan^{-1} x \\ \Rightarrow 2(\theta + \phi) = 2 \tan^{-1} x \\ \text{अतः } x = \tan(\theta + \phi) \Rightarrow x &= \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} \\ \text{इन मानों को प्रतिस्थापित करने पर, } x &= \frac{a+b}{1-ab}. \end{aligned}$$

$$74. \quad (d) \quad \cos^{-1} \frac{1}{2} + 2 \sin^{-1} \frac{1}{2} = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{2\pi}{3}.$$

$$\begin{aligned} 75. \quad (a) \quad \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5} - \tan^{-1} \frac{8}{19} \\ = \tan^{-1} \left[\frac{\frac{3}{4} + \frac{3}{5}}{1 - \frac{3}{4} \times \frac{3}{5}} \right] - \tan^{-1} \frac{8}{19} = \tan^{-1} \frac{27}{11} - \tan^{-1} \frac{8}{19} \\ = \tan^{-1} \left[\frac{\frac{27}{11} - \frac{8}{19}}{1 + \frac{27}{11} \times \frac{8}{19}} \right] = \tan^{-1} \left(\frac{425}{425} \right) = \tan^{-1}(1) = \frac{\pi}{4}. \end{aligned}$$

$$\begin{aligned} 76. \quad (c) \quad 4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} \\ = 2 \tan^{-1} \left[\frac{\frac{2}{5}}{1 - \frac{1}{25}} \right] - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} \\ = 2 \tan^{-1} \left(\frac{5}{12} \right) - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} \\ = \tan^{-1} \left[\frac{\frac{5}{6}}{1 - \frac{25}{144}} \right] - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} \\ = \tan^{-1} \left(\frac{120}{119} \right) - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} \end{aligned}$$

$$\begin{aligned} &= \tan^{-1} \left(\frac{120}{119} \right) + \tan^{-1} \left[\frac{\frac{1}{99} - \frac{1}{70}}{1 + \frac{1}{99} \times \frac{1}{70}} \right] \\ &= \tan^{-1} \left(\frac{120}{119} \right) + \tan^{-1} \left(\frac{-29}{6931} \right) \\ &= \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{29}{6931} = \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{1}{239} \\ &= \tan^{-1} \left[\frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \times \frac{1}{239}} \right] = \tan^{-1}(1) = \frac{\pi}{4}. \end{aligned}$$

$$\begin{aligned} 77. \quad (b) \quad \sin^{-1} x + \sin^{-1} y &= \frac{2\pi}{3} \\ \Rightarrow \frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y &= \frac{2\pi}{3} \\ \Rightarrow \cos^{-1} x + \cos^{-1} y &= \pi - \frac{2\pi}{3} = \frac{\pi}{3}. \end{aligned}$$

$$\begin{aligned} 78. \quad (a,d) \quad \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} &= \tan^{-1} \left(\frac{(1/4) + (2/9)}{1 - (1/4) \times (2/9)} \right) \\ &= \tan^{-1} \left(\frac{1}{2} \right) = \frac{1}{2} \cdot 2 \tan^{-1} \left(\frac{1}{2} \right) = \frac{1}{2} \tan^{-1} \frac{2(1/2)}{1 - (1/4)} \\ &= \frac{1}{2} \tan^{-1} \frac{4}{3} = \frac{1}{2} \cos^{-1} \frac{3}{5}. \end{aligned}$$

$$\begin{aligned} 79. \quad (c) \quad \tan^{-1} \frac{x}{y} - \tan^{-1} \left(\frac{x-y}{x+y} \right) &= \tan^{-1} \frac{x}{y} - \tan^{-1} \left(\frac{1-y/x}{1+y/x} \right) \\ &= \tan^{-1} \frac{x}{y} - \left(\tan^{-1} 1 - \tan^{-1} \frac{y}{x} \right) \\ &= \tan^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x} - \frac{\pi}{4} \\ &= \tan^{-1} \frac{x}{y} + \cot^{-1} \frac{x}{y} - \frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}. \end{aligned}$$

$$\begin{aligned} 80. \quad (d) \quad \sin^{-1} \frac{x}{5} + \operatorname{cosec}^{-1} \frac{5}{4} &= \frac{\pi}{2} \Rightarrow \sin^{-1} \frac{x}{5} = \frac{\pi}{2} - \operatorname{cosec}^{-1} \frac{5}{4} \\ \Rightarrow \sin^{-1} \frac{x}{5} &= \frac{\pi}{2} - \sin^{-1} \frac{4}{5} \Rightarrow \sin^{-1} \frac{x}{5} = \cos^{-1} \frac{4}{5} \\ \Rightarrow \sin^{-1} \left(\frac{x}{5} \right) &= \sin^{-1} \frac{3}{5} \Rightarrow x = 3. \end{aligned}$$

$$\begin{aligned} 81. \quad (d) \quad 2 \tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \left(\frac{1}{7} \right) &= \tan^{-1} \left(\frac{2(1/3)}{1 - (1/9)} \right) + \tan^{-1} \left(\frac{1}{7} \right) \\ &= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} = \tan^{-1} \left(\frac{(3/4) + (1/7)}{1 - (3/4) \times (1/7)} \right) \\ &= \tan^{-1} \left(\frac{25}{25} \right) = \frac{\pi}{4}. \end{aligned}$$

$$\begin{aligned} 82. \quad (d) \quad \cos^{-1} \left(\frac{15}{17} \right) + 2 \tan^{-1} \left(\frac{1}{5} \right) \\ = \cos^{-1} \left(\frac{15}{17} \right) + \cos^{-1} \left(\frac{1 - 1/25}{1 + 1/25} \right) \\ = \cos^{-1} \left(\frac{15}{17} \right) + \cos^{-1} \left(\frac{12}{13} \right) \end{aligned}$$

$$= \cos^{-1} \left(\frac{15}{17} \times \frac{12}{13} - \sqrt{1 - \left(\frac{15}{17} \right)^2} \sqrt{1 - \left(\frac{12}{13} \right)^2} \right)$$

$$= \cos^{-1} \left(\frac{140}{221} \right).$$

83. (a) $\sin^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7}$

$$= \tan^{-1} \left(\frac{(3/4) + (1/7)}{1 - (3/4) \times (1/7)} \right) = \tan^{-1} \left(\frac{25}{25} \right) = \tan^{-1} 1 = \frac{\pi}{4}.$$

84. (c) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1}(1+x) = \frac{\pi}{2} - \tan^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \cot^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \tan^{-1} \left(\frac{1}{1-x} \right)$$

$$\Rightarrow 1+x = \frac{1}{1-x} \Rightarrow 1-x^2 = 1 \Rightarrow x = 0.$$

85. (b) $\tan^{-1} \left(\frac{xy}{zr} \right) + \tan^{-1} \left(\frac{yz}{xr} \right) + \tan^{-1} \left(\frac{xz}{yr} \right)$

$$= \tan^{-1} \left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \left(\frac{x^2 + y^2 + z^2}{r^2} \right)} \right] = \tan^{-1} \infty = \frac{\pi}{2}.$$

86. (c) हमें ज्ञात है कि, $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$$= (\sin^{-1} x + \cos^{-1} x)^3 - 3 \sin^{-1} x \cos^{-1} x (\sin^{-1} x + \cos^{-1} x)$$

$$= \frac{\pi^3}{8} - 3(\sin^{-1} x \cos^{-1} x) \frac{\pi}{2}$$

$$= \frac{\pi^3}{8} - \frac{3\pi}{2} \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x \right)$$

$$= \frac{\pi^3}{8} - \frac{3\pi^2}{4} \sin^{-1} x + \frac{3\pi}{2} (\sin^{-1} x)^2$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[(\sin^{-1} x)^2 - \frac{\pi}{2} \sin^{-1} x \right]$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2} \left[\left(\sin^{-1} x - \frac{\pi}{4} \right)^2 \right] - \frac{3\pi^3}{32}$$

$$= \frac{\pi^3}{32} + \frac{3\pi}{2} \left(\sin^{-1} x - \frac{\pi}{4} \right)^2$$

$\therefore$ न्यूनतम मान $\frac{\pi^3}{32}$ है और चूँकि $\left(\sin^{-1} x - \frac{\pi}{4} \right)^2 \leq \left(\frac{3\pi}{4} \right)^2$

$\therefore$ महत्तम मान $\frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8}$ है।

87. (a) पिछले हल द्वारा $\frac{\pi^3}{32} \leq (\sin^{-1} x)^3 + (\cos^{-1} x)^3 \leq \frac{7\pi^3}{8}$

यहाँ $a < \frac{1}{32}$. अतः हलों की संख्या शून्य है।

88. (a) हमें ज्ञात है कि,

$$\sin^{-1} x + \cos^{-1} x + \tan^{-1} x = \frac{\pi}{2} + \tan^{-1} x$$

चूँकि $\frac{-\pi}{2} \leq \tan^{-1} x \leq \frac{\pi}{2} \Rightarrow 0 \leq \frac{\pi}{2} + \tan^{-1} x \leq \pi$

$\therefore K = \pi, k = 0$.

89. (a) $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$

$$\Rightarrow (\tan^{-1} x + \cot^{-1} x)^2 - 2 \tan^{-1} x \left(\frac{\pi}{2} - \tan^{-1} x \right) = \frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{4} - 2 \times \frac{\pi}{2} \tan^{-1} x + 2(\tan^{-1} x)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1} x)^2 - \pi \tan^{-1} x - \frac{3\pi^2}{8} = 0$$

$$\Rightarrow \tan^{-1} x = -\frac{\pi}{4}, \frac{3\pi}{4} \Rightarrow \tan^{-1} x = -\frac{\pi}{4} \Rightarrow x = -1.$$

90. (c) $x + y = \tan^{-1} 33 \Rightarrow y = \tan^{-1} 33 - \tan^{-1} 3$

$$= \tan^{-1} \frac{33-3}{1+99} = \tan^{-1} \frac{30}{100} \Rightarrow y = \tan^{-1}(0.3).$$

91. (d) $\frac{x-1}{x+1} + \frac{2x-1}{2x+1} = \frac{23}{36} \left[1 - \frac{(x-1)(2x-1)}{(x+1)(2x+1)} \right]$

$$\Rightarrow 24x^2 - 23x - 12 = 0 \Rightarrow (3x-4)(8x+3) = 0.$$

92. (c) $\tan^{-1} \left(\frac{c_1 x - y}{c_1 y + x} \right) + \tan^{-1} \left(\frac{c_2 - c_1}{1 + c_2 c_1} \right) + \tan^{-1} \left(\frac{c_3 - c_2}{1 + c_3 c_2} \right) +$

$$\dots + \tan^{-1} \frac{1}{c_n}$$

$$= \tan^{-1} \left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y} \cdot \frac{1}{c_1}} \right) + \tan^{-1} \left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1 c_2}} \right)$$

$$+ \tan^{-1} \left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2 c_3}} \right) + \dots + \tan^{-1} \frac{1}{c_n}$$

$$= \tan^{-1} \frac{x}{y} - \tan^{-1} \frac{1}{c_1} + \tan^{-1} \frac{1}{c_1} - \tan^{-1} \frac{1}{c_2} + \tan^{-1} \frac{1}{c_2}$$

$$- \tan^{-1} \frac{1}{c_3} + \dots + \tan^{-1} \frac{1}{c_{n-1}} - \tan^{-1} \frac{1}{c_n} + \tan^{-1} \frac{1}{c_n}$$

$$= \tan^{-1} \left(\frac{x}{y} \right).$$

93. (d) $\sin \left(\sin^{-1} \frac{1}{2} + \cos^{-1} \frac{1}{2} \right) = \sin \frac{\pi}{2} = 1.$

94. (a) $\sin^{-1} \frac{4}{5} = \tan^{-1} \frac{4}{3}, 2 \tan^{-1} \frac{1}{3} = \tan^{-1} \frac{3}{4} = \cot^{-1} \frac{4}{3}$

और $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}.$

95. (b) यह गुणधर्म है।

$$96. \quad (b) \quad \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} \right)$$

$$= \tan^{-1} \left(\frac{5}{5} \right) = \tan^{-1} 1 = \frac{\pi}{4}.$$

$$97. \quad (d) \quad \tan^{-1} \left(\frac{1}{11} \right) + \tan^{-1} \left(\frac{2}{12} \right)$$

$$= \tan^{-1} \left(\frac{\frac{1}{11} + \frac{2}{12}}{1 - \frac{1}{11} \times \frac{2}{12}} \right) = \tan^{-1} \left(\frac{12 + 22}{130} \right)$$

$$= \tan^{-1} \left(\frac{34}{130} \right) = \tan^{-1} \left(\frac{17}{65} \right).$$

98. (c) दिये गये समीकरण को निम्न प्रकार से लिखा जा सकता है

$$\tan^{-1} x + \cot^{-1} x + \cot^{-1} x = \frac{2\pi}{3}$$

$$\Rightarrow \cot^{-1} x = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6} \Rightarrow x = \sqrt{3}.$$

$$99. \quad (b) \quad \because \cot^{-1} \frac{1}{2} = \cos^{-1} \frac{1}{\sqrt{5}}$$

अतः दिये गये समीकरण को निम्न प्रकार से लिखा जा सकता है

$$\sin^{-1} x + \cos^{-1} \frac{1}{\sqrt{5}} = \frac{\pi}{2}$$

$$\Rightarrow x = \frac{1}{\sqrt{5}}.$$

$$100. \quad (a) \quad \text{माना} \quad \sin^{-1} a = A,$$

$$\sin^{-1} b = B,$$

$$\sin^{-1} c = C$$

$$\therefore \sin A = a, \sin B = b, \sin C = c$$

तथा $A + B + C = \pi$, तब

$$\sin 2A + \sin 2B + \sin 2C$$

$$= 4 \sin A \sin B \sin C$$

.....(i)

$$\Rightarrow \sin A \cos A + \sin B \cos B + \sin C \cos C$$

$$= 2 \sin A \sin B \sin C$$

$$\Rightarrow \sin A \sqrt{1 - \sin^2 A} + \sin B \sqrt{1 - \sin^2 B} + \sin C \sqrt{1 - \sin^2 C}$$

$$= 2 \sin A \sin B \sin C.$$

.....(ii)

$$\Rightarrow a\sqrt{1 - a^2} + b\sqrt{1 - b^2} + c\sqrt{1 - c^2} = 2abc,$$

$$\text{जबकि} \quad \sin^{-1} a + \sin^{-1} b + \sin^{-1} c = \pi.$$

$$101. \quad (a) \quad \cos^{-1} \left(\cos \frac{5\pi}{3} \right) + \sin^{-1} \left(\sin \frac{5\pi}{3} \right)$$

$$= \cos^{-1} \left[\cos \left(2\pi - \frac{\pi}{3} \right) \right] + \sin^{-1} \left[\sin \left(2\pi - \frac{\pi}{3} \right) \right]$$

$$= \frac{\pi}{3} - \frac{\pi}{3} = 0.$$

$$102. \quad (b) \quad \text{हम जानते हैं कि, } 4 \sin^{-1} x + \cos^{-1} x = \pi$$

$$\Rightarrow 3 \sin^{-1} x + \sin^{-1} x + \cos^{-1} x = \pi$$

$$\Rightarrow 3 \sin^{-1} x = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x = \pi/6 \Rightarrow x = \sin \frac{\pi}{6} = \frac{1}{2}.$$

$$103. \quad (d) \quad \text{दिया गया है कि, } \sin^{-1} C = \sin^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13}$$

$$\Rightarrow C = \sin \left(\sin^{-1} \frac{3}{5} + \cos^{-1} \frac{12}{13} \right)$$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ का प्रयोग करने पर,

$$\Rightarrow C = \frac{3}{5} \times \frac{12}{13} + \sqrt{1 - \frac{9}{25}} \sqrt{1 - \frac{144}{169}} \Rightarrow C = \frac{56}{65}.$$

$$104. \quad (b) \quad \sin \left[\tan^{-1} \left(\frac{1 - x^2}{2x} \right) + \cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right) \right]$$

$x = \tan \theta$ रखने पर,

$$\sin \left[\tan^{-1} \left(\frac{1 - \tan^2 \theta}{2 \tan \theta} \right) + \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) \right]$$

$$= \sin [\tan^{-1} (\cot 2\theta) + \cos^{-1} (\cos 2\theta)]$$

$$= \sin [\tan^{-1} \tan(\pi/2 - 2\theta) + \cos^{-1} \cos 2\theta]$$

$$= \sin \frac{\pi}{2} = 1.$$

$$105. \quad (a) \quad \cos^{-1} \left[\cos \frac{5\pi}{3} \right] + \sin^{-1} \left[\frac{\cos 5\pi}{3} \right] = \frac{\pi}{2}$$

$$(\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}).$$

$$106. \quad (d) \quad \sin^{-1} \left[\frac{\sqrt{3}}{2} \right] - \sin^{-1} \left[\frac{1}{2} \right] = 60^\circ - 30^\circ = 30^\circ.$$

$$107. \quad (c) \quad \text{दिया गया है, } \cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$$

$$\therefore 0 \leq \cos^{-1} x \leq \pi$$

$$\therefore 0 \leq \cos^{-1} y \leq \pi \quad \text{और} \quad 0 \leq \cos^{-1} z \leq \pi$$

$$\text{यहाँ} \quad \cos^{-1} x = \cos^{-1} y = \cos^{-1} z = \pi$$

$$\Rightarrow x = y = z = \cos \pi = -1$$

$$\therefore xy + yz + zx = (-1)(-1) + (-1)(-1) + (-1)(-1) \\ = 1 + 1 + 1 = 3.$$

108. (b) $\cos \left\{ \cos^{-1} \left(\frac{-1}{7} \right) + \sin^{-1} \left(\frac{-1}{7} \right) \right\} = \cos \frac{\pi}{2} = 0.$

109. (d) $\tan \left[\sin^{-1} \left(\frac{3}{5} \right) + \cos^{-1} \left(\frac{3}{\sqrt{13}} \right) \right]$
 $= \tan \left(\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{2}{3} \right) = \tan \left(\tan^{-1} \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \cdot \frac{2}{3}} \right)$
 $= \tan \left[\tan^{-1} \frac{17}{12} \times \frac{12}{6} \right] = \frac{17}{6}.$

110. (d) $\tan \left[\tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{3} \right] = \tan \left[\tan^{-1} \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{6}} \right]$
 $= \tan \tan^{-1} \left(\frac{1}{6} \times \frac{6}{7} \right) = \frac{1}{7}.$

111. (d) माना $\alpha = \cos^{-1} \sqrt{p}$; $\beta = \cos^{-1} \sqrt{1-p}$
 और $\gamma = \cos^{-1} \sqrt{1-q}$ or $\cos \alpha = \sqrt{p}$; $\cos \beta = \sqrt{1-p}$

$$\text{और } \cos \gamma = \sqrt{1-q}.$$

$$\text{अतः } \sin \alpha = \sqrt{1-p}, \sin \beta = \sqrt{p} \text{ तथा } \sin \gamma = \sqrt{q}.$$

दी गयी समीकरण को लिखा जा सकता है :

$$\alpha + \beta + \gamma = \frac{3\pi}{4} \text{ या } \alpha + \beta = \frac{3\pi}{4} - \gamma$$

$$\text{या } \cos(\alpha + \beta) = \cos \left(\frac{3\pi}{4} - \gamma \right)$$

$$\Rightarrow \cos \alpha \cos \beta - \sin \alpha \sin \beta =$$

$$\cos \left\{ \pi - \left(\frac{\pi}{4} + \gamma \right) \right\} = -\cos \left(\frac{\pi}{4} + \gamma \right)$$

$$\Rightarrow \sqrt{p} \sqrt{1-p} - \sqrt{1-p} \sqrt{p} = - \left(\frac{1}{\sqrt{2}} \sqrt{1-q} - \frac{1}{\sqrt{2}} \sqrt{q} \right)$$

$$\Rightarrow 0 = \sqrt{1-q} - \sqrt{q} \Rightarrow 1-q = q \Rightarrow q = \frac{1}{2}.$$

112. (a) $\tan^{-1} \left[\frac{1}{\sqrt{\cos \alpha}} \right] - \tan^{-1} [\sqrt{\cos \alpha}] = x$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{1}{\sqrt{\cos \alpha}} - \sqrt{\cos \alpha}}{1 + \frac{\sqrt{\cos \alpha}}{\sqrt{\cos \alpha}}} \right] = x \Rightarrow \tan x = \frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}}$$

$$\therefore \sin x = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{2 \sin^2 \frac{\alpha}{2}}{2 \cos^2 \frac{\alpha}{2}} = \tan^2 \left(\frac{\alpha}{2} \right).$$

113. (b) $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \pi$

$$\Rightarrow \tan^{-1} x + \tan^{-1} y = \pi - \tan^{-1} z$$

$$\Rightarrow \frac{x+y}{1-xy} = -z \Rightarrow x+y = -z + xyz$$

$$\Rightarrow x+y+z = xyz$$

xyz से भाग देने पर,

$$\frac{1}{yz} + \frac{1}{xz} + \frac{1}{xy} = 1.$$

नोट : विद्यार्थी इस प्रश्न को सूत्र की तरह याद रखें।

114. (c) $\tan \left[\frac{1}{2} \sin^{-1} \left(\frac{2a}{1+a^2} \right) + \frac{1}{2} \cos^{-1} \left(\frac{1-a^2}{1+a^2} \right) \right]$

$$= \tan \left[\frac{1}{2} \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) + \frac{1}{2} \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) \right]$$

(माना $a = \tan \theta$)

$$= \tan \left[\frac{1}{2} \sin^{-1}(\sin 2\theta) + \frac{1}{2} \cos^{-1}(\cos 2\theta) \right]$$

$$= \tan(2\theta) = \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2a}{1 - a^2}$$

ट्रिक : $a = 0$ रखने पर $\tan(0 + 0) = 0$ जो कि विकल्प (a) व (c) द्वारा दिया जाता है। साथ ही $a = 1$ रखने पर,

$$\tan \left(\frac{\pi}{4} + \frac{\pi}{4} \right) = \infty \text{ जो कि विकल्प (c) द्वारा दिया जाता है।}$$

115. (c) दिया गया है कि $A = \tan^{-1} x$

$$\text{अब } x = \tan A \Rightarrow \sin 2A = \frac{2 \tan A}{1 + \tan^2 A} = \frac{2x}{1 + x^2}.$$

116. (c) $\cos(2 \sin^{-1} x) = \frac{1}{9}$

$$\Rightarrow \cos(\sin^{-1} 2x \sqrt{1-x^2}) = \frac{1}{9}$$

$$\Rightarrow \cos(\cos^{-1} \sqrt{1-4x^2+4x^4}) = \frac{1}{9}$$

$$\Rightarrow 1 - 2x^2 = \frac{1}{9} \Rightarrow 2x^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\Rightarrow x^2 = \frac{4}{9} \Rightarrow x = \pm \frac{2}{3}.$$

117. (b) हमें ज्ञात है कि, $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2 \cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2 \operatorname{cosec} x)$$

$$\frac{2 \cos x}{\sin^2 x} = 2 \operatorname{cosec} x \Rightarrow 2 \cos x = 2 \sin x$$

$$\text{या } \sin x = \cos x$$

$$\Rightarrow x = \frac{\pi}{4}$$

118. (c) $\tan\left(2 \cos^{-1} \frac{3}{5}\right) = \tan\left[\cos^{-1}\left(2 \cdot \frac{9}{25} - 1\right)\right]$

$$\{\because 2 \cos^{-1} x = \cos^{-1}(2x^2 - 1)\}$$

$$= \tan \cos^{-1}\left(-\frac{7}{25}\right) = \tan \tan^{-1}\left[\frac{1 - \frac{49}{625}}{-\frac{7}{25}}\right] = -\frac{24}{7}$$

$$\text{अतः } \tan\left(2 \cos^{-1} \frac{3}{5}\right) = -\frac{24}{7}$$

119. (d) $\tan\left[2 \tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right] = \tan\left[\tan^{-1} \frac{\frac{2}{5}}{1 - \frac{1}{25}} - \tan^{-1}(1)\right]$

$$= \tan\left[\tan^{-1} \frac{5}{12} - \tan^{-1}(1)\right] = \tan \tan^{-1}\left(\frac{\frac{5}{12} - 1}{1 + \frac{5}{12}}\right) = -\frac{7}{17}$$

120. (b) दी गयी समीकरण है, $2 \cos^{-1} \sqrt{\frac{1+x}{2}} = \frac{\pi}{2}$

$$\Rightarrow \cos^{-1} \sqrt{\frac{1+x}{2}} = \frac{\pi}{4} \Rightarrow \cos \frac{\pi}{4} = \frac{\sqrt{1+x}}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{1+x}}{\sqrt{2}} \Rightarrow 1 = \sqrt{1+x} \Rightarrow x = 0$$

121. (a, d) $\tan\left[\frac{1}{2} \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$

$$\text{माना } \frac{1}{2} \cos^{-1} \frac{\sqrt{5}}{3} = \theta \Rightarrow \cos 2\theta = \frac{\sqrt{5}}{3}$$

$$\text{किन्तु } \cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \Rightarrow \frac{\sqrt{5}}{3} = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\Rightarrow \sqrt{5} + \sqrt{5} \tan^2 \theta = 3 - 3 \tan^2 \theta$$

$$\Rightarrow (\sqrt{5} + 3) \tan^2 \theta = 3 - \sqrt{5} \Rightarrow \tan^2 \theta = \frac{3 - \sqrt{5}}{3 + \sqrt{5}}$$

$$\Rightarrow \tan^2 \theta = \frac{(3 - \sqrt{5})^2}{4} \Rightarrow \tan \theta = \frac{3 - \sqrt{5}}{2}$$

परिमेयीकरण करने पर,

$$\Rightarrow \tan \theta = \frac{3 - \sqrt{5}}{2} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{2}{3 + \sqrt{5}}$$

122. (b) माना $x = \tan^2 \theta \Rightarrow \theta = \tan^{-1} \sqrt{x}$

$$\text{अब, } \frac{1}{2} \cos^{-1}\left(\frac{1-x}{1+x}\right)$$

$$= \frac{1}{2} \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$$

$$= \frac{1}{2} \cos^{-1} \cos 2\theta = \frac{2\theta}{2} = \theta = \tan^{-1} \sqrt{x}$$

123. (b) $\sin\left(4 \tan^{-1} \frac{1}{3}\right) = \sin\left[2 \tan^{-1}\left(\frac{2/3}{1 - (1/9)}\right)\right]$

$$= \sin\left[2 \tan^{-1} \frac{3}{4}\right] = \sin \sin^{-1}\left(\frac{2 \times (3/4)}{1 + (9/16)}\right)$$

$$= \frac{3}{2} \times \frac{16}{25} = \frac{24}{25} \quad \left(\because 2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2}\right)$$

124. (d) यह स्पष्ट है।

125. (d) चूँकि $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$

$$\therefore 4 \tan^{-1} \frac{1}{5} = 2 \left[2 \tan^{-1} \frac{1}{5}\right] = 2 \tan^{-1} \frac{\frac{2}{5}}{1 - \frac{1}{25}}$$

$$= 2 \tan^{-1} \frac{10}{24} = \tan^{-1} \frac{\frac{20}{24}}{1 - \frac{100}{576}} = \tan^{-1} \frac{120}{119}$$

$$4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239} = \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{1}{239}$$

$$= \tan^{-1} \frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \cdot \frac{1}{239}} = \tan^{-1} \frac{(120 \times 239) - 119}{(119 \times 239) + 120}$$

$$\Rightarrow \tan^{-1} 1 = \frac{\pi}{4}$$

126. (b) $3 \sin^{-1} \frac{2x}{1+x^2} - 4 \cos^{-1} \frac{1-x^2}{1+x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$

$x = \tan \theta$ रखने पर,

$$3 \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right) - 4 \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$$

$$+ 2 \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right) = \frac{\pi}{3}$$

$$\Rightarrow 3 \sin^{-1}(\sin 2\theta) - 4 \cos^{-1}(\cos 2\theta)$$

$$+ 2 \tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$$

$$\Rightarrow 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3} \Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{6} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \Rightarrow x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\begin{aligned} 127. \quad (b) \quad & \sin \left[2 \tan^{-1} \left(\frac{1}{3} \right) \right] + \cos [\tan^{-1} (2\sqrt{2})] \\ &= \sin \left[\tan^{-1} \frac{2/3}{1-1/9} \right] + \cos [\tan^{-1} (2\sqrt{2})] \\ &= \sin [\tan^{-1} 3/4] + \cos [\tan^{-1} 2\sqrt{2}] \\ &= \frac{3}{5} + \frac{1}{3} = \frac{14}{15}. \end{aligned}$$

$$128. \quad (a) \quad \text{माना } \cos^{-1} \frac{4}{5} = x \Rightarrow \cos x = \frac{4}{5} \quad \dots(i)$$

$$\text{अब } \sin \left(\frac{1}{2} \cos^{-1} \frac{4}{5} \right) = \sin \left(\frac{x}{2} \right) \quad \dots(ii)$$

$$\text{समीकरण (i) से, } \cos x = \frac{4}{5} \Rightarrow 1 - 2 \sin^2 \frac{x}{2} = \frac{4}{5}$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = 1 - \frac{4}{5} = \frac{1}{5} \Rightarrow \sin \frac{x}{2} = \sqrt{\frac{1}{10}}.$$

$$129. \quad (a) \quad \cos^{-1}(\cos 12) - \sin^{-1}(\sin 14) \Rightarrow 12 - 14 = -2.$$

$$130. \quad (d) \quad x = \frac{\pi}{2} \text{ लेने पर } \cos x = 0$$

$$\cos^{-1} \left(\frac{3+5 \cos x}{5+3 \cos x} \right) = \cos^{-1} \left(\frac{3}{5} \right) = \tan^{-1} \left(\frac{4}{3} \right)$$

$$x = \frac{\pi}{2}, 2 \tan^{-1} \left(\frac{1}{2} \tan \frac{x}{2} \right) \text{ में रखने पर}$$

$$\begin{aligned} & 2 \tan^{-1} \left(\frac{1}{2} \tan \frac{\pi}{4} \right) \\ &= 2 \tan^{-1} \left(\frac{1}{2} \right) = \tan^{-1} \left(\frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} \right) = \tan^{-1} \left(\frac{4}{3} \right). \end{aligned}$$

$$131. \quad (a) \quad \text{यदि } \cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \theta$$

$$\text{तब } \frac{x^2}{a^2} - \frac{2xy}{ab} \cos \theta + \frac{y^2}{b^2} = \sin^2 \theta$$

$$\text{यहाँ } \cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha;$$

$$\therefore \frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha$$

$$x^2 - xy \cos \alpha + \frac{y^2}{4} = \sin^2 \alpha$$

$$4x^2 - 4xy \cos \alpha + y^2 = 4 \sin^2 \alpha.$$

$$132. \quad (b) \quad \tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}; \quad \tan^{-1} \left(\frac{x+y}{1-xy} \right) = \tan^{-1} 1$$

$$\frac{x+y}{1-xy} = 1; \quad x+y+xy = 1.$$

$$133. \quad (c) \quad \sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}(1-x) = \left(\frac{\pi}{2} - 2 \sin^{-1} x \right)$$

$$\Rightarrow 1-x = \sin \left(\frac{\pi}{2} - 2 \sin^{-1} x \right)$$

$$\Rightarrow 1-x = \sin \frac{\pi}{2} \cos(2 \sin^{-1} x) - \cos \frac{\pi}{2} \sin(2 \sin^{-1} x)$$

$$\Rightarrow 1-x = \cos(2 \sin^{-1} x)^2$$

$$\Rightarrow 1-x = \cos \cos^{-1}(1-2x) \Rightarrow 2x^2 - x = 0$$

$x = 0$, $x = 1/2$ जोकि दिये गये समीकरण को संतुष्ट नहीं करता हैं।

$\therefore x = 0$ एकमात्र हल है।

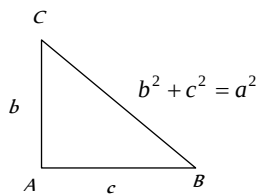
$$134. \quad (c) \quad \angle A = 90^\circ$$

$$\tan^{-1} \left(\frac{c}{a+b} \right) + \tan^{-1} \left(\frac{b}{a+c} \right)$$

$$= \tan^{-1} \left[\frac{\frac{c}{a+b} + \frac{b}{a+c}}{1 - \left(\frac{c}{a+b} \right) \left(\frac{b}{a+c} \right)} \right]$$

$$= \tan^{-1} \left[\frac{ca + c^2 + ab + b^2}{a^2 + ab + ca + bc - bc} \right]$$

$$= \tan^{-1} \left[\frac{a^2 + ab + ca}{a^2 + ab + ca} \right] = \tan^{-1}(1) = \frac{\pi}{4}.$$



$$135. \quad (d) \quad \sin^{-1} 2x = \sin^{-1} x - \sin^{-1} \frac{\sqrt{3}}{2}$$

$$\sin^{-1} 2x = \sin^{-1} \left(x \sqrt{1 - \frac{3}{4}} - \frac{\sqrt{3}}{2} \sqrt{1 - x^2} \right)$$

$$2x = \left(\frac{x}{2} - \frac{\sqrt{3}}{2} \sqrt{1 - x^2} \right)$$

$$\frac{\sqrt{3}}{2} \sqrt{1 - x^2} = \frac{x}{2} - 2x = \frac{-3x}{2}$$

$$\frac{3(1 - x^2)}{4} = \frac{9x^2}{4}$$

$$\Rightarrow 3 - 3x^2 = 9x^2 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm \frac{1}{2}.$$

$$136. \quad (c) \quad \cos^{-1} x + \cos^{-1}(2x) = -\pi \Rightarrow \cos^{-1} 2x = -\pi - \cos^{-1} x$$

$$\Rightarrow 2x = \cos(\pi + \cos^{-1} x)$$

$$\Rightarrow 2x = \cos \pi (\cos \cos^{-1} x) - \sin \pi \sin(\cos^{-1} x)$$

$$2x = -x \Rightarrow x = 0$$

किन्तु $x = 0$ दिये गये समीकरण को सन्तुष्ट नहीं करता है।

अतः कोई हल विद्यमान नहीं है।

$$\begin{aligned} 137. \quad (a) \quad \sin\left[3 \sin^{-1} \frac{1}{5}\right] &= \sin\left[\sin^{-1}\left\{3\left(\frac{1}{5}\right) - 4\left(\frac{1}{5}\right)^3\right\}\right] \\ &= \sin\left[\sin^{-1}\left\{\frac{3}{5} - \frac{4}{125}\right\}\right] = \sin\left[\sin^{-1}\left(\frac{75-4}{125}\right)\right] \\ &= \sin\left[\sin^{-1} \frac{71}{125}\right] = \frac{71}{125}. \end{aligned}$$

Critical Thinking Questions

$$\begin{aligned} 1. \quad (d) \quad \cot^{-1} \left[\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}} \right] \\ = \cot^{-1} \left[\frac{(\sqrt{1-\sin x} + \sqrt{1+\sin x}) \cdot (\sqrt{1-\sin x} + \sqrt{1+\sin x})}{(\sqrt{1-\sin x} - \sqrt{1+\sin x}) \cdot (\sqrt{1-\sin x} + \sqrt{1+\sin x})} \right] \\ = \cot^{-1} \left[\frac{(1-\sin x) + (1+\sin x) + 2\sqrt{1-\sin^2 x}}{(1-\sin x) - (1+\sin x)} \right] \\ = \cot^{-1} \left[\frac{2(1+\cos x)}{-2\sin x} \right] = \cot^{-1} \left[-\frac{2\cos^2(x/2)}{2\sin(x/2)\cos(x/2)} \right] \\ = \cot^{-1} \left(-\cot \frac{x}{2} \right) = \cot^{-1} \left[\cot \left(\pi - \frac{x}{2} \right) \right] = \pi - \frac{x}{2}. \end{aligned}$$

ट्रिक : $x = \frac{\pi}{4}$ रखने पर व्यंजक

$$\begin{aligned} \cot^{-1} \left[\frac{\sqrt{\sqrt{2}-1} + \sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1} - \sqrt{\sqrt{2}+1}} \right] &\text{ होगा।} \\ = \cot^{-1} \left[\frac{\sqrt{2}-1 + \sqrt{2}+1 + 2\sqrt{2-1}}{\sqrt{2}-1 - \sqrt{2}-1} \right] \\ = \cot^{-1} \left[\frac{2\sqrt{2}+2}{-2} \right] = \cot^{-1}(-1-\sqrt{2}) = 157.5^\circ. \end{aligned}$$

$$2. \quad (d) \quad \sin^{-1} \left[\sin \left(\pi - \frac{2\pi}{3} \right) \right] \text{ का मुख्य मान } = \sin^{-1} \sin \left(\frac{\pi}{3} \right) = \frac{\pi}{3}.$$

$$\begin{aligned} 3. \quad (c) \quad \text{दिया गया है कि } \theta = \tan^{-1} a \text{ और } \phi = \tan^{-1} b \text{ और } ab = -1. \\ \Rightarrow \tan \theta \tan \phi = -1 \Rightarrow \tan \theta = -\cot \phi \Rightarrow \theta - \phi = \frac{\pi}{2}. \end{aligned}$$

$$4. \quad (b) \quad \text{दिया गया है कि } \tan\{\cos^{-1}(x)\} = \sin\left(\cot^{-1} \frac{1}{2}\right)$$

$$\text{माना } \cot^{-1} \frac{1}{2} = \phi \Rightarrow \frac{1}{2} = \cot \phi$$

$$\Rightarrow \sin \phi = \frac{1}{\sqrt{1+\cot^2 \phi}} = \frac{2}{\sqrt{5}}$$

$$\text{माना } \cos^{-1} x = \theta \Rightarrow \sec \theta = \frac{1}{x}$$

$$\Rightarrow \tan \theta = \sqrt{\sec^2 \theta - 1}$$

$$\Rightarrow \tan \theta = \sqrt{\frac{1}{x^2} - 1}$$

$$\Rightarrow \tan \theta = \frac{\sqrt{1-x^2}}{x}$$

$$\text{अतः } \tan\{\cos^{-1}(x)\} = \sin\left(\cot^{-1} \frac{1}{2}\right)$$

$$\Rightarrow \tan\left(\tan^{-1} \frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1} \frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \sqrt{(1-x^2)5} = 2x$$

$$\text{दोनों पक्षों का वर्ग करने पर, } x = \pm \frac{\sqrt{5}}{3}.$$

$$5. \quad (a) \quad \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}.$$

$$6. \quad (b) \quad \text{ट्रिक : दिये गये प्रतिबंधानुसार } p = q = r = \frac{1}{2} \text{ रखने पर,}$$

$$\begin{aligned} p^2 + q^2 + r^2 + 2pqr \\ = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \\ = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{2}{8} = 1. \end{aligned}$$

$$7. \quad (b) \quad \tan\left[\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b}\right]$$

$$\text{माना } \frac{1}{2} \cos^{-1} \frac{a}{b} = \theta \Rightarrow \cos 2\theta = \frac{a}{b}$$

$$\text{अतः } \tan\left[\frac{\pi}{4} + \theta\right] + \tan\left[\frac{\pi}{4} - \theta\right]$$

$$= \frac{1+\tan \theta}{1-\tan \theta} + \frac{1-\tan \theta}{1+\tan \theta} = \frac{(1+\tan \theta)^2 + (1-\tan \theta)^2}{(1-\tan^2 \theta)}$$

$$= \frac{1+\tan^2 \theta + 2\tan \theta + 1 + \tan^2 \theta - 2\tan \theta}{(1+\tan^2 \theta)}$$

$$= \frac{2(1+\tan^2 \theta)}{1-\tan^2 \theta} = 2 \sec 2\theta = \frac{2}{\cos 2\theta}$$

$$= \frac{2}{a/b} = \frac{2b}{a}.$$

$$\begin{aligned} 8. \quad (a) \quad & \tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] \\ &= \tan^{-1} \left[\frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}} \right] \\ & \quad (x^2 = \cos 2\theta \text{ रखने पर } \Rightarrow \theta = \frac{1}{2} \cos^{-1} x^2) \end{aligned}$$

$$\begin{aligned} &= \tan^{-1} \left[\frac{\sqrt{2} \cos \theta + \sqrt{2} \sin \theta}{\sqrt{2} \cos \theta - \sqrt{2} \sin \theta} \right] \\ &= \tan^{-1} \left[\frac{1 + \tan \theta}{1 - \tan \theta} \right] = \tan^{-1} \left[\frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} \right] \\ &= \tan^{-1} \tan \left(\frac{\pi}{4} + \theta \right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2. \end{aligned}$$

$$9. \quad (b) \quad \text{हमें ज्ञात है कि, } \sin^{-1} x - \cos^{-1} x = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

$$\text{किन्तु } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\therefore \sin^{-1} x = \frac{\pi}{3} \text{ और } \cos^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2} \text{ अद्वितीय हल है।}$$

$$\begin{aligned} 10. \quad (a) \quad & \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4 + m^2 + 2} \right) \\ &= \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{1 + (m^2 + m + 1)(m^2 - m + 1)} \right) \\ &= \sum_{m=1}^n \tan^{-1} \left(\frac{(m^2 + m + 1) - (m^2 - m + 1)}{1 + (m^2 + m + 1)(m^2 - m + 1)} \right) \\ &= \sum_{m=1}^n [\tan^{-1}(m^2 + m + 1) - \tan^{-1}(m^2 - m + 1)] \\ &= (\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3) + \\ & \quad (\tan^{-1} 13 - \tan^{-1} 7) + \dots + [\tan^{-1}(n^2 + n + 1) \\ & \quad - \tan^{-1}(n^2 - n + 1)] \\ &= \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1 = \tan^{-1} \left(\frac{n^2 + n}{2 + n^2 + n} \right). \end{aligned}$$

$$11. \quad (c) \quad \tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

$\tan^{-1} \sqrt{x(x+1)}$ परिभाषित है जब

$$x(x+1) \geq 0 \quad \dots(i)$$

$\sin^{-1} \sqrt{x^2 + x + 1}$ परिभाषित है जब

$$0 \leq x(x+1) + 1 \leq 1 \text{ या } 0 \leq x(x+1) \leq 0 \quad \dots(ii)$$

(i) और (ii) से, $x(x+1) = 0$

या $x = 0$ और -1

अतः हलों की संख्या 2 है।

$$12. \quad (a) \quad \text{दिया गया समीकरण } 2 \cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6} \text{ है।}$$

$$\Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x = 4\pi/3$$

जोकि संभव नहीं हैं क्योंकि $\cos^{-1} x \in [0, \pi]$

$$13. \quad (a) \quad \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$$

$$\Rightarrow \tan^{-1} \left[\frac{x+y+z-xyz}{1-xy-yz-zx} \right] = \pi$$

$$\Rightarrow x+y+z-xyz = 0$$

$$\Rightarrow x+y+z = xyz.$$

$$\begin{aligned} 14. \quad (a) \quad & 2 \tan^{-1} \left[\sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right] = \cos^{-1} \left[\frac{1 - \left(\frac{a-b}{a+b} \right) \tan^2 \frac{\theta}{2}}{1 + \left(\frac{a-b}{a+b} \right) \tan^2 \frac{\theta}{2}} \right] \\ & \left(\because 2 \tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2} \right) \end{aligned}$$

$$= \cos^{-1} \left[\frac{(a+b) - (a-b) \tan^2 \frac{\theta}{2}}{(a+b) + (a-b) \tan^2 \frac{\theta}{2}} \right]$$

$$= \cos^{-1} \left[\frac{a \left(1 - \tan^2 \frac{\theta}{2} \right) + b \left(1 + \tan^2 \frac{\theta}{2} \right)}{a \left(1 + \tan^2 \frac{\theta}{2} \right) + b \left(1 - \tan^2 \frac{\theta}{2} \right)} \right]$$

$$= \cos^{-1} \left[\frac{\frac{a \left(1 - \tan^2 \frac{\theta}{2} \right)}{1 + \tan^2 \frac{\theta}{2}} + b}{a + b \left(\frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \right)} \right] = \cos^{-1} \left[\frac{a \cos \theta + b}{a + b \cos \theta} \right].$$

15. (d) $2 \tan^{-1}(\cos x) = \tan^{-1}(\operatorname{cosec}^2 x)$

$$\Rightarrow \tan^{-1} \left(\frac{2 \cos x}{1 - \cos^2 x} \right) = \tan^{-1} \left(\frac{1}{\sin^2 x} \right)$$

$$\Rightarrow \frac{2 \cos x}{\sin^2 x} = \frac{1}{\sin^2 x} \Rightarrow 2 \cos x = 1$$

$$\Rightarrow x = \frac{\pi}{3}.$$

प्रतिलोम त्रिकोणमितीय फलन

SET Self Evaluation Test -12

1. यदि $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$, $x \geq 0$, तब वह न्यूनतम अंतराल, जिसमें θ स्थित होगा, है [Orissa JEE 2003]
- (a) $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$ (b) $0 < \theta < \pi$
- (c) $-\frac{\pi}{4} \leq \theta \leq 0$ (d) $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$
2. यदि $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, तब $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$ का मान है
- (a) 0 (b) 3
- (c) -3 (d) 9
3. यदि $x + \frac{1}{x} = 2$, तब $\sin^{-1} x$ का मुख्य मान है
- (a) $\pi/4$ (b) $\pi/2$
- (c) π (d) $3\pi/2$
4. यदि $\tan(\cos^{-1} x) = \sin\left[\cot^{-1}\left(\frac{1}{2}\right)\right]$, तब x (केवल मुख्य मान लेने पर) का मान होगा [IIT 1991; AMU 2001]
- (a) $\frac{1}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}$
- (c) $\frac{3}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{3}$
5. यदि ΔABC में $A = \tan^{-1} 2$ और $B = \tan^{-1} 3$, तो कोण C का मान है
- (a) $\pi/2$ (b) $\pi/3$
- (c) $\pi/4$ (d) इनमें से कोई नहीं
6. यदि a, b, c धनात्मक वास्तविक संख्याएँ हैं, एवं $\theta = \tan^{-1} \sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1} \sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1} \sqrt{\frac{c(a+b+c)}{ab}}$, तो $\tan \theta =$ [IIT 1981]
- (a) 0 (b) 1
- (c) $\frac{a+b+c}{abc}$ (d) इनमें से कोई नहीं
7. यदि $\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \alpha$, तो $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} =$ [MP PET 1995; UPSEAT 1999]
- (a) $\sin^2 \alpha$ (b) $\cos^2 \alpha$
- (c) $\tan^2 \alpha$ (d) $\cot^2 \alpha$
8. यदि x_1, x_2, x_3, x_4 समीकरण $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos \beta - \sin \beta = 0$ के मूल हैं, तब $\tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4 =$
- (a) β (b) $\frac{\pi}{2} - \beta$
- (c) $\pi - \beta$ (d) $-\beta$
9. यदि $\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$ तथा दोनों x तथा y धनात्मक व पूर्णांक हैं, तब x तथा y के मान हैं [Roorkee 1993]
- (a) (1, 2) तथा (2, 7) (b) (1, 2) तथा (1, 7)
- (c) (1, 7) तथा (2, 7) (d) (1, 7) तथा (2, 1)
10. यदि $a_1, a_2, a_3, \dots, a_n$ समान्तर श्रेणी में हैं, तथा सार्वनुपात d है तो $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \dots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right] =$
- (a) $\frac{(n-1)d}{a_1 + a_n}$ (b) $\frac{(n-1)d}{1 + a_1a_n}$
- (c) $\frac{nd}{1 + a_1a_n}$ (d) $\frac{a_n - a_1}{a_n + a_1}$
11. यदि प्रतिलोम फलन के केवल मुख्य मानों का उपयोग किया जावे तो $\tan\left(\cos^{-1} \frac{1}{5\sqrt{2}} - \sin^{-1} \frac{4}{\sqrt{17}}\right)$ का मान होगा [IIT 1994]
- (a) $\sqrt{29/3}$ (b) $29/3$
- (c) $\sqrt{3/29}$ (d) $3/29$
12. यदि $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \dots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots\right) = \frac{\pi}{2}$ जबकि $0 < |x| < \sqrt{2}$, तब x का मान होगा [IIT Screening 2001]
- (a) $\frac{1}{2}$ (b) 1
- (c) $-\frac{1}{2}$ (d) -1
13. यदि $\sec^{-1} x = \operatorname{cosec}^{-1} y$, तब $\cos^{-1} \frac{1}{x} + \cos^{-1} \frac{1}{y} =$ [Orissa JEE 2002]
- (a) π (b) $\frac{\pi}{4}$
- (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$
14. $\sin(2 \sin^{-1} 0.8) =$ [MNR 1980]
- (a) 0.96 (b) 0.48
- (c) 0.64 (d) इनमें से कोई नहीं
15. यदि $\sin^{-1} \frac{2a}{1+a^2} - \cos^{-1} \frac{1-b^2}{1+b^2} = \tan^{-1} \frac{2x}{1-x^2}$, तो $x =$ [EAMCET 1981]
- (a) a (b) b
- (c) $\frac{a+b}{1-ab}$ (d) $\frac{a-b}{1+ab}$

AS Answers and Solutions

(SET - 12)

1. (b) $\therefore \theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x = \frac{\pi}{2} - \tan^{-1} x$

हम जानते हैं कि, $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

$$\Rightarrow \frac{\pi}{2} > -\tan^{-1} x > -\frac{\pi}{2}$$

$$\therefore 0 < \frac{\pi}{2} - \tan^{-1} x < \pi.$$

2. (a) जैसा कि हम जानते हैं कि $\sin^{-1} x$, $\frac{\pi}{2}$ से बड़ा नहीं हो सकता है।

$$\text{अतः } \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$$

$$\text{अतः } x = y = z = 1$$

$$\text{इन मानों को व्यंजक में रखने पर, } 1+1+1-\frac{9}{1+1+1}=0.$$

3. (b) $x + \frac{1}{x} = 2 \Rightarrow x = 1$

$$\text{अतः } \sin^{-1} x \text{ का मुख्य मान } \frac{\pi}{2} \text{ है।}$$

4. (d) $\cot^{-1}\left(\frac{1}{2}\right) = \theta \Rightarrow \cot \theta = \frac{1}{2}$ रखने पर,

$$\therefore \sin \theta = \frac{2}{\sqrt{5}}. \quad \cos^{-1} x = \theta \text{ रखने पर, } \therefore x = \cos \theta$$

$$\text{साथ ही, } \tan \theta = \frac{2}{\sqrt{5}}, \quad \therefore x = \cos \theta = \frac{\sqrt{5}}{3}.$$

5. (c) दिया गया है कि, $\angle A = \tan^{-1} 2$, $\angle B = \tan^{-1} 3$

हम जानते हैं कि, $\angle A + \angle B + \angle C = \pi$

$$\Rightarrow \tan^{-1} 2 + \tan^{-1} 3 + \angle C = \pi$$

$$\Rightarrow \tan^{-1}\left(\frac{2+3}{1-2 \times 3}\right) + \angle C = \pi \Rightarrow \tan^{-1}(-1) + \angle C = \pi$$

$$\Rightarrow \frac{3\pi}{4} + \angle C = \pi \Rightarrow \angle C = \frac{\pi}{4}.$$

6. (a) $\theta = \tan^{-1} \sqrt{\frac{a(a+b+c)}{bc}}$

$$+ \tan^{-1} \sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1} \sqrt{\frac{c(a+b+c)}{ab}}$$

$$\text{माना } s^2 = \frac{a+b+c}{abc}$$

$$\text{अतः } \theta = \tan^{-1} \sqrt{a^2 s^2} + \tan^{-1} \sqrt{b^2 s^2} + \tan^{-1} \sqrt{c^2 s^2}$$

$$= \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs)$$

$$= \tan^{-1} \left[\frac{as + bs + cs - abcs^3}{1 - abs^2 - acs^2 - bcs^2} \right]$$

$$\text{अतः } \tan \theta = \left[s \frac{(a+b+c) - abcs^2}{1 - (ab+bc+ca)s^2} \right]$$

$$= \left[\frac{s[(a+b+c) - (a+b+c)]}{1 - s^2(ab+bc+ca)} \right] = 0,$$

$$[\text{चूँकि } s^2 abc = (a+b+c)]$$

ट्रिक : यह एक सर्वसमिका है अतः यह a, b, c के किसी भी मान के लिए सत्य होगी। माना $a = b = c = 1$ तब

$$\theta = \tan^{-1} \sqrt{3} + \tan^{-1} \sqrt{3} + \tan^{-1} \sqrt{3} = \pi \Rightarrow \tan \theta = 0.$$

7. (a) हमें ज्ञात है कि, $\cos^{-1} \left[\frac{x}{a} \cdot \frac{y}{b} - \sqrt{\left(1 - \frac{x^2}{a^2}\right)} \sqrt{\left(1 - \frac{y^2}{b^2}\right)} \right] = \alpha$

$$\Rightarrow \frac{xy}{ab} - \sqrt{\left(1 - \frac{x^2}{a^2}\right)} \sqrt{\left(1 - \frac{y^2}{b^2}\right)} = \cos \alpha$$

$$\therefore \left(\frac{xy}{ab} - \cos \alpha \right)^2 = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2 y^2}{a^2 b^2}$$

$$\Rightarrow \frac{x^2 y^2}{a^2 b^2} + \cos^2 \alpha - \frac{2xy}{ab} \cos \alpha = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2 y^2}{a^2 b^2}$$

$$\Rightarrow \frac{x^2}{y^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = 1 - \cos^2 \alpha = \sin^2 \alpha.$$

8. (b) हमें ज्ञात है कि,

$$\Sigma x_1 = \sin 2\beta, \Sigma x_1 x_2 = \cos 2\beta, \Sigma x_1 x_2 x_3 = \cos \beta$$

$$\text{तथा } x_1 x_2 x_3 x_4 = -\sin \beta$$

$$\therefore \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$$

$$= \tan^{-1} \left(\frac{\Sigma x_1 - \Sigma x_1 x_2 x_3}{1 - \Sigma x_1 x_2 + x_1 x_2 x_3 x_4} \right)$$

$$= \tan^{-1} \left(\frac{\sin 2\beta - \cos \beta}{1 - \cos 2\beta - \sin \beta} \right)$$

$$= \tan^{-1} \left(\frac{(2 \sin \beta - 1) \cos \beta}{\sin \beta (2 \sin \beta - 1)} \right) = \tan^{-1} (\cot \beta)$$

$$= \tan^{-1} \left[\tan \left(\frac{\pi}{2} - \beta \right) \right] = \frac{\pi}{2} - \beta.$$

9. (a) $\tan^{-1} x + \tan^{-1} \frac{1}{y} = \tan^{-1} 3$

$\therefore xy + 1 = 3(y - x)$

अतः (1, 2) तथा (2, 7) चूँकि x, y धनात्मक व पूर्णांक हैं।

10. (b) हमें ज्ञात है कि, $\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \dots$

$\dots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)$

$= \tan^{-1}\left(\frac{a_2 - a_1}{1 + a_1a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1 + a_2a_3}\right) + \dots$

$\dots + \tan^{-1}\left(\frac{a_n - a_{n-1}}{1 + a_{n-1}a_n}\right)$

$= (\tan^{-1} a_2 - \tan^{-1} a_1) + (\tan^{-1} a_3 - \tan^{-1} a_2) + \dots$

$\dots + (\tan^{-1} a_n - \tan^{-1} a_{n-1})$

$= \tan^{-1} a_n - \tan^{-1} a_1 = \tan^{-1}\left(\frac{a_n - a_1}{1 + a_n a_1}\right)$

$= \tan^{-1}\left(\frac{(n-1)d}{1 + a_1 a_n}\right)$

11. (d) $\tan\left(\cos^{-1} \frac{1}{5\sqrt{2}} - \sin^{-1} \frac{4}{\sqrt{17}}\right)$

$= \tan(\tan^{-1} 7 - \tan^{-1} 4) = \tan\left[\tan^{-1}\left(\frac{7-4}{1+28}\right)\right] = \frac{3}{29}$

$\therefore$ प्रश्नानुसार,

$x - \frac{x^2}{2} + \frac{x^3}{4} - \dots = x^2 - \frac{x^4}{2} + \frac{x^6}{4} \dots$

$\Rightarrow \frac{x}{1 + \frac{x}{2}} = \frac{x^2}{1 + \frac{x^2}{2}}, \quad (\because 0 < |x| < \sqrt{2})$

$\Rightarrow \frac{x}{2+x} = \frac{x^2}{2+x^2} \Rightarrow 2x + x^3 = 2x^2 + x^3 \Rightarrow x = x^2$

$\therefore x - x^2 = 0 \Rightarrow x(1-x) = 0 \Rightarrow x = 0$ तथा $x = 1$,

किन्तु $x \neq 0$. अतः $x = 1$.

13. (d) दिया गया है, $\sec^{-1} x = \operatorname{cosec}^{-1} y$

$\cos^{-1}\left(\frac{1}{x}\right) = \sin^{-1}\left(\frac{1}{y}\right) \Rightarrow \cos^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{y}\right)$

$\Rightarrow \cos^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{y}\right) = \frac{\pi}{2}$

14. (a) $\sin(2 \sin^{-1} 0.8) = \sin\left(2 \sin^{-1} \frac{4}{5}\right)$

$= \sin\left(\sin^{-1} 2 \cdot \frac{4}{5} \cdot \sqrt{1 - \frac{16}{25}}\right) = \frac{8}{5} \cdot \frac{3}{5} = \frac{24}{25} = 0.96$

15. (d) $a = \tan \theta, b = \tan \phi$ तथा $x = \tan \psi$ रखने पर

$\sin^{-1}(\sin 2\theta) - \cos^{-1}(\cos 2\phi) = \tan^{-1}(\tan 2\psi)$

$\Rightarrow 2\theta - 2\phi = 2\psi \Rightarrow \theta - \phi = \psi$

दोनों पक्षों का 'tan' लेने पर $\tan(\theta - \phi) = \tan \psi$

$\Rightarrow \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi} = \tan \psi$

इन मानों को प्रतिस्थापित करने पर, $\frac{a-b}{1+ab} = x$.

* * *

12. (b) हम जानते हैं कि, $\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2}, |y| \leq 1$