Short Answer Questions-I (PYQ)

[2 Mark]

Q.1. If $A=\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix}$, then find the value of *k* if |2*A*| = *k*|*A*|.

Ans.

$\because \quad A=\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix} \quad \Rightarrow \quad 2A=\begin{bmatrix}2 & 4\\ 8 & 4\end{bmatrix}$

Given, $\quad |2A| = k|A|$

$\Rightarrow \quad \begin{vmatrix}2 & 4\\ 8 & 4\end{vmatrix} = k\begin{vmatrix}1 & 2\\ 4 & 2\end{vmatrix}$

$\Rightarrow \quad 8-32 = k\{2-8\} \quad \Rightarrow \quad -24 = -6k \quad \Rightarrow \quad k = 4$

Q.2. What is the value of the following determinant?

Ans.

$$\Delta = \begin{vmatrix}4 & a & b+c\\ 4 & b & c+a\\ 4 & c & a+b\end{vmatrix}$$

Q.3. If $\begin{vmatrix}x & x\\ 1 & x\end{vmatrix} = \begin{vmatrix}3 & 4\\ 1 & 2\end{vmatrix}$, then write the positive value of *x*.

Ans.

We have $\begin{vmatrix}x & x\\ 1 & x\end{vmatrix} = \begin{vmatrix}3 & 4\\ 1 & 2\end{vmatrix}$

$\Rightarrow \quad x^2 - x = 6 - 4 \quad \Rightarrow \quad x^2 - x - 2 = 0$

$\Rightarrow \quad x^2 - 2x + x - 2 = 0 \quad \Rightarrow \quad x(x-2) + 1(x-2) = 0$

$\Rightarrow \quad (x-2)\,(x+1) = 0 \quad \Rightarrow \quad x = 2 \quad \text{or} \quad x = -1 \quad$ (Not accepted)

$\Rightarrow \quad x = 2$

Q.4. Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$.

Ans.

Here, $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2$, we get

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

Taking $(x+y+z)$ common from R_1, we get

$$= (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

Applying $R_3 \to R_3 + 3R_1$, we get

$$= (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 0 & 0 & 0 \end{vmatrix}$$

$= 0$ $\quad [\because R_3 \text{ is zero}]$

Short Answer Questions-I (OIQ)

[2 Mark]

Q.1. Evaluate the determinant:

$$\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$$

Ans.

Let $\Delta = \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

$$= (x+1)\begin{vmatrix} x^2 - x + 1 & x - 1 \\ 1 & 1 \end{vmatrix}$$ [Taking out $(x + 1)$ common from R_2]

$$= (x + 1)\{x^2 - x + 1 - x + 1\} = (x + 1)\ (x^2 - 2x + 2)$$

$$= x^3 - 2x^2 + 2x + x^2 - 2x + 2 = x^3 - x^2 + 2$$

Q.2. Evaluate: $\begin{vmatrix} a & b & c \\ a + 2x & b + 2y & c + 2z \\ x & y & z \end{vmatrix}$

Ans.

$$\begin{vmatrix} a & b & c \\ a + 2x & b + 2y & c + 2z \\ x & y & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$$

$$= 0 + 2\begin{vmatrix} a & b & c \\ x & y & z \\ x & y & z \end{vmatrix}$$ [$\because$ Two rows are same, so determinant is zero]

$$= 0 + 2 \times 0 = 0$$

Q.3. Evaluate: $\begin{vmatrix} 1 & a & b + c \\ 1 & b & c + a \\ 1 & c & a + b \end{vmatrix}$

Ans.

$$\text{Let } \Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

Applying $C_2 \to C_2 + C_3$, we get

$$= \begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & c+a \\ 1 & a+b+c & a+b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$$

$= (a + b + c) \cdot 0$ [∵ Two columns are same, so determinant is zero]

$= 0$

Q.4. What is the value of the determinant given below?

$$\begin{vmatrix} 6 & a & b+c \\ 6 & b & c+a \\ 6 & c & a+b \end{vmatrix}$$

Ans.

$$\text{Let } \Delta = \begin{vmatrix} 6 & a & b+c \\ 6 & b & c+a \\ 6 & c & a+b \end{vmatrix}$$

Applying $C_2 \to C_2 + C_3$, we get

$$= \begin{vmatrix} 6 & a+b+c & b+c \\ 6 & a+b+c & c+a \\ 6 & a+b+c & a+b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 6 & 1 & b+c \\ 6 & 1 & c+a \\ 6 & 1 & a+b \end{vmatrix} = 6(a+b+c) \begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & (a+b) \end{vmatrix}$$

$= 6\ (a + b + c) \cdot 0 = 0$ [∵ Two columns are same, so determinant is zero]

Q.5. Show that points *A* (*a*, *b* + *c*), *B* (*b*, *c* + *a*) and *C* (*c*, *a* + *b*) are collinear.

Ans.

Obviously, if the area of DABC formed by points A, B and C is zero then A, B, C will be collinear.

Now, Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix} \qquad [\text{Applying } C_1 \to C_1 + C_2]$$

$$= \frac{a+b+c}{2}\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix} = \frac{a+b+c}{2} \times 0 = 0 \qquad [\because C_1 = C_3]$$

Q.6. Evaluate: $\begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$

Ans.

$$\begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$$

$$= 0 - \sin\alpha \{0 - \sin\beta . \cos\alpha\} - \cos\alpha \{\sin\alpha \sin\beta - 0\}$$

$$= \sin\alpha . \cos\alpha \sin\beta - \sin\alpha . \sin\beta . \cos\alpha$$

$$= 0$$

Q.7. Find the area of the triangle with vertices at the points (2, 7), (1, 1), (10, 8).

Ans.

We know that area of a triangle with vertices at (x_1, y_1), (x_2, y_2), (x_3, y_3) is absolute value of

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \text{Required area} = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix} = \frac{1}{2}[2(1-8) - 7(1-10) + 1(8-10)]$$

$$= \frac{1}{2}[2 \times (-7) - 7 \times (-9) + (-2)]$$

$$= \frac{1}{2}[-14 + 63 - 2] = \frac{47}{2} \text{ sq unit.}$$

Q.8. Find the value of *k*, if area of a triangle is 4 sq unit when its vertices are (*k*, 0), (4, 0) and (0, 2).

Ans.

$$\text{We know that area of triangle} = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow \quad = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 4 \qquad \text{[Given]}$$

$$\Rightarrow \quad = \frac{1}{2}[k(0-2) - 0 + 1(8-0)] = \pm 4 \quad \Rightarrow \quad \frac{1}{2}[-2k+8] = \pm 4$$

$$\Rightarrow \quad [-k+4] = \pm 4 \quad \Rightarrow \quad -k+4 = 4 \quad \text{or} \quad -k+4 = -4$$

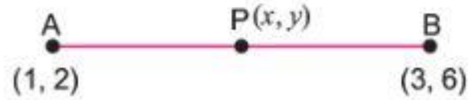
i.e., $k = 0$ or $k = 8$

Q.9. Find equation of line joining (1, 2) and (3, 6) using determinants.

Ans.

Let $P(x, y)$ be the general point on the line joining $A(1, 2)$ and $B(3, 6)$.

From figure it is obvious that the area of ΔAPB is zero.



$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0 \qquad \Rightarrow \quad \begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad x(2-6) - y(1-3) + 1(6-6) = 0 \quad \Rightarrow \quad -4x + 2y = 0$$

It is required equation of line.

Q.10. Without expanding evaluate the determinant:

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (a^y + a^{-y})^2 & (a^y - a^{-y})^2 & 1 \\ (a^z + a^{-z})^2 & (a^z + a^{-z})^2 & 1 \end{vmatrix}, \text{ where } a > 0 \text{ and } x, y, z \in R.$$

Ans.

Let Δ be the given determinant. Applying $C_1 \rightarrow C_1 - C_2$, we get

$$\Delta = \begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (a^y - a^{-y})^2 & 1 \\ 4 & (a^z - a^{-z})^2 & 1 \end{vmatrix} \qquad [\text{Using } (a+b)^2 - (a-b)^2 = 4ab]$$

Taking out 4 from C_1, we get

$$\Delta = 4\begin{vmatrix} 1 & (a^x - a^{-x})^2 & 1 \\ 1 & (a^y - a^{-y})^2 & 1 \\ 1 & (a^z - a^{-z})^2 & 1 \end{vmatrix} \quad \Rightarrow \quad \Delta = 4 \times 0 = 0. \qquad [\because C_1 \text{ and } C_2 \text{ are identical}]$$