

Trigonometric Functions



TOPIC 1

Circular System, Trigonometric Ratios, Domain and Range of Trigonometric Functions, Trigonometric Ratios of Allied Angles



1. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:

[Jan. 9, 2019 (I)]

- (a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
 (b) $13 - 4\cos^6\theta$
 (c) $13 - 4\cos^2\theta + 6\cos^4\theta$
 (d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

2. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$.

Then $f_4(x) - f_6(x)$ equals

[2014]

- (a) $\frac{1}{4}$ (b) $\frac{1}{12}$
 (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

3. If $2\cos\theta + \sin\theta = 1$ $\left(\theta \neq \frac{\pi}{2}\right)$,

then $7\cos\theta + 6\sin\theta$ is equal to: [Online April 11, 2014]

- (a) $\frac{1}{2}$ (b) 2
 (c) $\frac{11}{2}$ (d) $\frac{46}{5}$

4. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$

can be written as :

[2013]

- (a) $\sin A \cos A + 1$ (b) $\sec A \csc A + 1$
 (c) $\tan A + \cot A$ (d) $\sec A + \csc A$

5. The value of $\cos 255^\circ + \sin 195^\circ$ is [Online May 26, 2012]

- (a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}-1}{\sqrt{2}}$
 (c) $-\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$ (d) $\frac{\sqrt{3}+1}{\sqrt{2}}$

6. Let $f(x) = \sin x$, $g(x) = x$.

Statement 1: $f(x) \leq g(x)$ for x in $(0, \infty)$

Statement 2: $f(x) \leq 1$ for x in $(0, \infty)$ but $g(x) \rightarrow \infty$ as $x \rightarrow \infty$.

[Online May 7, 2012]

- (a) Statement 1 is true, Statement 2 is false.
 (b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
 (c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
 (d) Statement 1 is false, Statement 2 is true.

7. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x . The maximum area enclosed by the park is

[2006]

- (a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$
 (c) $\frac{1}{2}x^2$ (d) πx^2

TOPIC 2

 Trigonometric Identities,
Conditional Trigonometric
Identities, Greatest and Least
Value of Trigonometric Expressions


8. The value of $\cos^3\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) \cdot \sin\left(\frac{3\pi}{8}\right)$ is

[Jan. 9, 2020 (I)]

- (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

9. If $\frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$,
 $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _____.

[Jan. 8, 2020 (II)]

10. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and

$$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right), \text{ then : } \quad \text{[Sep. 05, 2020 (II)]}$$

- (a) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$ (b) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4} \cos \frac{\pi}{8}$
(c) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4} \cos \frac{\pi}{8}$ (d) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$

11. The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin \theta, \cos \theta)$ lie on the same side of the line $x + y = 1$ is :

[Sep. 02, 2020 (II)]

- (a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
(c) $\left(0, \frac{3\pi}{4}\right)$ (d) $\left(0, \frac{\pi}{4}\right)$

12. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30° , then the distance (in m) of the foot of the tower from the point A is:

[April 12, 2019 (II)]

- (a) $15(3 + \sqrt{3})$ (b) $15(5 - \sqrt{3})$
(c) $15(3 - \sqrt{3})$ (d) $15(1 + \sqrt{3})$

13. The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is : [April 9, 2019 (II)]

- (a) $\frac{3}{4} + \cos 20^\circ$ (b) $\frac{3}{4}$
(c) $\frac{3}{2}(1 + \cos 20^\circ)$ (d) $\frac{3}{2}$

14. Two poles standing on a horizontal ground are of heights 5m and 10m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is: [April. 09, 2019 (II)]

- (a) $5(2 + \sqrt{3})$ (b) $5(\sqrt{3} + 1)$
(c) $\frac{5}{2}(2 + \sqrt{3})$ (d) $10(\sqrt{3} - 1)$

15. The value of $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$ is:

[April. 09, 2019 (II)]

- (a) $\frac{1}{16}$ (b) $\frac{1}{32}$
(c) $\frac{1}{18}$ (d) $\frac{1}{36}$

16. If $\cos(\alpha + \beta) = \frac{3}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to : [April 8, 2019 (I)]

- (a) $\frac{63}{52}$ (b) $\frac{63}{16}$
(c) $\frac{21}{16}$ (d) $\frac{33}{52}$

17. If $\sin^4 \alpha + 4 \cos^4 \beta + 2 = 4 \sqrt{2} \sin \alpha \cos \beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to : [Jan. 12, 2019 (II)]

- (a) 0 (b) -1
(c) $\sqrt{2}$ (d) $-\sqrt{2}$

18. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \dots$. Then for all $x \in \mathbb{R}$, the value of $f_4(x) - f_6(x)$ is equal to :

[Jan. 11, 2019 (I)]

- (a) $\frac{1}{12}$ (b) $\frac{1}{4}$
(c) $-\frac{1}{12}$ (d) $\frac{5}{12}$

19. The value of [Jan. 10, 2019 (II)]

$$\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \cdot \dots \cdot \cos \frac{\pi}{2^{10}} \cdot \sin \frac{\pi}{2^{10}}$$

- (a) $\frac{1}{512}$ (b) $\frac{1}{1024}$
(c) $\frac{1}{256}$ (d) $\frac{1}{2}$

20. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is :

[2017]

- (a) $-\frac{7}{9}$ (b) $-\frac{3}{5}$
(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

21. If m and M are the minimum and the maximum values of $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x$, $x \in \mathbb{R}$, then $M - m$ is equal to :

[Online April 9, 2016]

- (a) $\frac{9}{4}$ (b) $\frac{15}{4}$
(c) $\frac{7}{4}$ (d) $\frac{1}{4}$

22. If $\cos \alpha + \cos \beta = \frac{3}{2}$ and $\sin \alpha + \sin \beta = \frac{1}{2}$ and θ is the arithmetic mean of α and β , then $\sin 2\theta + \cos 2\theta$ is equal to :

[Online April 11, 2015]

- (a) $\frac{3}{5}$ (b) $\frac{7}{5}$
(c) $\frac{4}{5}$ (d) $\frac{8}{5}$

23. If $\operatorname{cosec} \theta = \frac{p+q}{p-q}$ ($p \neq q \neq 0$), then $\left| \cot \left(\frac{\pi}{4} + \frac{\theta}{2} \right) \right|$ is equal to:

[Online April 9, 2014]

- (a) $\sqrt{\frac{p}{q}}$ (b) $\sqrt{\frac{q}{p}}$
(c) $\sqrt{pq}$ (d) pq

24. If $A = \sin^2 x + \cos^4 x$, then for all real x :

[2011]

- (a) $\frac{13}{16} \leq A \leq 1$ (b) $1 \leq A \leq 2$
(c) $\frac{3}{4} \leq A \leq \frac{13}{16}$ (d) $\frac{3}{4} \leq A \leq 1$

25. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$,

where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha =$

[2010]

- (a) $\frac{56}{33}$ (b) $\frac{19}{12}$
(c) $\frac{20}{7}$ (d) $\frac{25}{16}$

26. Let **A** and **B** denote the statements

$$\mathbf{A} : \cos \alpha + \cos \beta + \cos \gamma = 0$$

$$\mathbf{B} : \sin \alpha + \sin \beta + \sin \gamma = 0$$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then :

[2009]

- (a) **A** is false and **B** is true
(b) both **A** and **B** are true
(c) both **A** and **B** are false
(d) **A** is true and **B** is false

27. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is

[2007]

- (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) 2

28. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is

[2006]

- (a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$
(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

29. If $u = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$ then the difference between the maximum and minimum values of u^2 is given by

[2004]

- (a) $(a-b)^2$ (b) $2\sqrt{a^2 + b^2}$
(c) $(a+b)^2$ (d) $2(a^2 + b^2)$

30. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.

If $\sin \alpha + \sin \beta = -\frac{21}{65}$ and $\cos \alpha + \cos \beta = -\frac{27}{65}$, then the

value of $\cos \frac{\alpha - \beta}{2}$

[2004]

- (a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$
(c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

31. The function $f(x) = \log(x + \sqrt{x^2 + 1})$, is

[2003]

- (a) neither an even nor an odd function
(b) an even function
(c) an odd function
(d) a periodic function.

32. The period of $\sin^2 \theta$ is

[2002]

- (a) π^2 (b) π
(c) 2π (d) $\pi/2$

33. Which one is not periodic? [2002]
- (a) $|\sin 3x| + \sin^2 x$ (b) $\cos \sqrt{x} + \cos^2 x$
 (c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$

TOPIC 3 Solutions of Trigonometric Equations



34. If the equation $\cos^4 \theta + \sin^4 \theta + \lambda = 0$ has real solutions for θ , then λ lies in the interval : [Sep. 02, 2020 (II)]

- (a) $\left[-\frac{5}{4}, -1\right]$ (b) $\left[-1, -\frac{1}{2}\right]$
 (c) $\left[-\frac{1}{2}, -\frac{1}{4}\right]$ (d) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

35. The number of distinct solutions of the equation, $\log_{1/2} |\sin x| = 2 - \log_{1/2} |\cos x|$ in the interval $[0, 2\pi]$, is _____. [Jan. 9, 2020 (I)]

36. The number of solutions of the equation

$$1 + \sin^4 x = \cos^2 3x, x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right] \text{ is : [April 12, 2019 (I)]}$$

- (a) 3 (b) 5
 (c) 7 (d) 4

37. Let S be the set of all $\alpha \in \mathbb{R}$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to :

[April 12, 2019 (II)]

- (a) $\mathbb{R}$ (b) $[1, 4]$
 (c) $[3, 7]$ (d) $[2, 6]$

38. If $[x]$ denotes the greatest integer $\leq x$, then the system of linear equations

$$[\sin \theta] x + [-\cos \theta] y = 0$$

$$[\cot \theta] x + y = 0$$

[April 12, 2019 (II)]

- (a) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and

has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

- (b) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

- (c) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have

infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

- (d) have infinitely many solutions if

$$\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$$

39. Let $S = \{\theta \in [-2\pi, 2\pi] : 2 \cos^2 \theta + 3 \sin \theta = 0\}$.

Then the sum of the elements of S is: [April 9, 2019 (I)]

- (a) $\frac{13\pi}{6}$ (b) $\frac{5\pi}{3}$
 (c) 2π (d) π

40. If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which

$\sin x - \sin 2x + \sin 3x = 0$, is: [Jan. 09, 2019 (II)]

- (a) 3 (b) 1
 (c) 4 (d) 2

41. The number of solutions of $\sin 3x = \cos 2x$, in the interval

$\left(\frac{\pi}{2}, \pi\right)$ is [Online April 15, 2018]

- (a) 3 (b) 4
 (c) 2 (d) 1

42. If sum of all the solutions of the equation

$$8 \cos x \cdot \left(\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) - 1 \text{ in } [0, \pi] \text{ is } k\pi,$$

then k is equal to :

[2018]

- (a) $\frac{13}{9}$ (b) $\frac{8}{9}$
 (c) $\frac{20}{9}$ (d) $\frac{2}{3}$

43. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation

$$\cos x + \cos 2x + \cos 3x + \cos 4x = 0 \text{ is:}$$

[2016]

- (a) 7 (b) 9
 (c) 3 (d) 5

44. The number of $x \in [0, 2\pi]$ for which

$$\left| \sqrt{2 \sin^4 x + 18 \cos^2 x} - \sqrt{2 \cos^4 x + 18 \sin^2 x} \right| = 1 \text{ is}$$

[Online April 9, 2016]

- (a) 2 (b) 6
 (c) 4 (d) 8

45. The number of values of α in $[0, 2\pi]$ for which

$$2 \sin^3 \alpha - 7 \sin^2 \alpha + 7 \sin \alpha = 2, \text{ is: [Online April 9, 2014]}$$

- (a) 6 (b) 4
 (c) 3 (d) 1

46. Let $A = \{\theta : \sin(\theta) = \tan(\theta)\}$ and $B = \{\theta : \cos(\theta) = 1\}$ be two sets. Then :

[Online April 25, 2013]

- (a) $A = B$
 (b) $A \subset B$
 (c) $B \subset A$
 (d) $A \subset B$ and $B - A \neq \emptyset$

47. The number of solutions of the equation $\sin 2x - 2 \cos x + 4 \sin x = 4$ in the interval $[0, 5\pi]$ is :

[Online April 23, 2013]

- (a) 3 (b) 5
(c) 4 (d) 6

48. **Statement-1:** The number of common solutions of the trigonometric equations $2 \sin^2 \theta - \cos 2\theta = 0$ and $2 \cos^2 \theta - 3 \sin \theta = 0$ in the interval $[0, 2\pi]$ is two.

Statement-2: The number of solutions of the equation, $2 \cos^2 \theta - 3 \sin \theta = 0$ in the interval $[0, \pi]$ is two.

[Online April 22, 2013]

- (a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.
(c) Statement-1 is false; Statement-2 is true.
(d) Statement-1 is true; Statement-2 is false.

49. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : [2012]

- (a) infinite number of real roots
(b) no real roots
(c) exactly one real root
(d) exactly four real roots

50. The possible values of $\theta \in (0, \pi)$ such that

$$\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0 \text{ are} \quad [2011RS]$$

(a) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(b) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(c) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(d) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

51. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2 \sin^2 x + 5 \sin x - 3 = 0$ is [2006]

- (a) 4 (b) 6
(c) 1 (d) 2

52. The number of solution of $\tan x + \sec x = 2 \cos x$ in $[0, 2\pi)$ is [2002]

- (a) 2 (b) 3
(c) 0 (d) 1



Hints & Solutions



$$\begin{aligned}
 1. \quad (b) \quad & 3(\sin \theta - \cos \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4\sin^6 \theta \\
 &= 3(1 - 2\sin \theta \cos \theta)^2 + 6(1 + 2\sin \theta \cos \theta) + 4\sin^6 \theta \\
 &= 3(1 + 4\sin^2 \theta \cos^2 \theta - 4\sin \theta \cos \theta) + 6 \\
 &\quad - 12\sin \theta \cos \theta + 4\sin^6 \theta \\
 &= 9 + 12\sin^2 \theta \cos^2 \theta + 4\sin^6 \theta \\
 &= 9 + 12\cos^2 \theta (1 - \cos^2 \theta) + 4(1 - \cos^2 \theta)^3 \\
 &= 9 + 12\cos^2 \theta - 12\cos^4 \theta + 4(1 - \cos^6 \theta - 3\cos^2 \theta + 3\cos^4 \theta) \\
 &= 9 + 4 - 4\cos^6 \theta \\
 &= 13 - 4\cos^6 \theta
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (b) \quad & \text{Let } f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x) \\
 \text{Consider } f_4(x) - f_6(x) &= \frac{1}{4}(\sin^4 x + \cos^4 x) \\
 &\quad - \frac{1}{6}(\sin^6 x + \cos^6 x) \\
 &= \frac{1}{4}[1 - 2\sin^2 x \cos^2 x] - \frac{1}{6}[1 - 3\sin^2 x \cos^2 x] \\
 &= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (d) \quad & \text{Given } 2\cos \theta + \sin \theta = 1 \\
 \text{Squaring both sides, we get} \\
 (2\cos \theta + \sin \theta)^2 &= 1^2 \\
 \Rightarrow 4\cos^2 \theta + \sin^2 \theta + 4\sin \theta \cos \theta &= 1 \\
 \Rightarrow 3\cos^2 \theta + (\cos^2 \theta + \sin^2 \theta) + 4\sin \theta \cos \theta &= 1 \\
 \Rightarrow 3\cos^2 \theta + 1 + 4\sin \theta \cos \theta &= 1 \\
 \Rightarrow 3\cos^2 \theta + 4\sin \theta \cos \theta &= 0 \\
 \Rightarrow \cos \theta(3\cos \theta + 4\sin \theta) &= 0 \\
 \Rightarrow 3\cos \theta + 4\sin \theta = 0 \Rightarrow 3\cos \theta &= -4\sin \theta \\
 \Rightarrow \frac{-3}{4} = \tan \theta = \sqrt{\sec^2 \theta - 1} &= \frac{-3}{4} \\
 \left(\because \tan \theta = \sqrt{\sec^2 \theta - 1} \right)
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \sec^2 \theta - 1 &= \left(\frac{-3}{4} \right)^2 = \frac{9}{16} \\
 \Rightarrow \sec^2 \theta &= \frac{9}{16} + 1 = \frac{25}{16} \Rightarrow \sec \theta = \frac{5}{4} \\
 \text{or } \boxed{\cos \theta = \frac{4}{5}} \quad & \dots(1)
 \end{aligned}$$

$$\text{Now, } \sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta + \left(\frac{4}{5} \right)^2 = 1$$

$$\sin^2 \theta + \frac{16}{25} = 1 \Rightarrow \sin^2 \theta = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\sin \theta = \pm \frac{3}{5} \quad \dots(2)$$

Taking $\left(\sin \theta = +\frac{3}{5} \right)$ because $\left(\sin \theta = -\frac{3}{5} \right)$ cannot satisfy the given equation.

Therefore; $7\cos \theta + 6\sin \theta$

$$= 7 \times \frac{4}{5} + 6 \times \frac{3}{5} = \frac{28}{5} + \frac{18}{5} = \frac{46}{5}$$

4. (b) Given expression can be written as

$$\begin{aligned}
 & \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A} \\
 & \left(\because \tan A = \frac{\sin A}{\cos A} \text{ and } \cot A = \frac{\cos A}{\sin A} \right)
 \end{aligned}$$

$$= \frac{1}{\sin A - \cos A} \left\{ \frac{\sin^3 A - \cos^3 A}{\cos A \sin A} \right\}$$

$$\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$= \frac{\sin^2 A + \sin A \cos A + \cos^2 A}{\sin A \cos A}$$

$$= 1 + \sec A \csc A$$

$$\begin{aligned}
 5. \quad (c) \quad & \text{Consider } \cos 255^\circ + \sin 195^\circ \\
 &= \cos (270^\circ - 15^\circ) + \sin (180^\circ + 15^\circ) \\
 &= -\sin 15^\circ - \sin 15^\circ
 \end{aligned}$$

$$= -2 \sin 15^\circ = -2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right) = -\left(\frac{\sqrt{3}-1}{\sqrt{2}} \right)$$

$$\begin{aligned}
 6. \quad (c) \quad & \text{Let } f(x) = \sin x \text{ and } g(x) = x \\
 \text{Statement-1: } & f(x) \leq g(x) \forall x \in (0, \infty)
 \end{aligned}$$

$$\text{i.e., } \sin x \leq x \forall x \in (0, \infty)$$

which is true

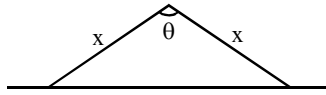
$$\text{Statement-2: } f(x) \leq 1 \forall x \in (0, \infty)$$

$$\text{i.e., } \sin x \leq 1 \forall x \in (0, \infty)$$

It is true and

$g(x) = x \rightarrow \infty$ as $x \rightarrow \infty$ also true.

7. (c) $\text{Area} = \frac{1}{2}x^2 \sin \theta$



Maximum value of $\sin \theta$ is 1 at $\theta = \frac{\pi}{2}$

$$A_{\max} = \frac{1}{2}x^2$$

8. (b) $\cos^3 \frac{\pi}{8} \left[4\cos^3 \frac{\pi}{8} - 3\cos \frac{\pi}{8} \right]$
 $+ \sin^3 \frac{\pi}{8} \left[3\sin \frac{\pi}{8} - 4\sin^3 \frac{\pi}{8} \right]$

$$= 4\cos^6 \frac{\pi}{8} - 4\sin^6 \frac{\pi}{8} - 3\cos^4 \frac{\pi}{8} + 3\sin^4 \frac{\pi}{8}$$

$$= 4 \left[\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right]$$

$$\left[\left(\sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right) \right]$$

$$- 3 \left[\left(\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right) \left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right) \right]$$

$$= \cos \frac{\pi}{4} \left[4 \left(1 - \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right) - 3 \right]$$

$$= \frac{1}{\sqrt{2}} \left[1 - \frac{1}{2} \right] = \frac{1}{2\sqrt{2}}$$

9. (1) $\frac{\sqrt{2} \sin \alpha}{\sqrt{2} \cos \alpha} = \frac{1}{7}$ and $\sqrt{\frac{1 - \cos^2 \beta}{2}} = \frac{1}{10}$

$$\Rightarrow \frac{\sqrt{2} \sin \beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$$

$$\therefore \tan \alpha = \frac{1}{7} \text{ and } \sin \beta = \frac{1}{\sqrt{10}}$$

$$\tan \beta = \frac{1}{3}$$

$$\therefore \tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \cdot \frac{1}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta}$$

$$= \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$$

10. (d) $L + M = 1 - 2 \sin^2 \frac{\pi}{8} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad \dots(i)$

and $L - M = -\cos \frac{\pi}{8} \quad \dots(ii)$

From equations (i) and (ii),

$$L = \frac{1}{2} \left(\frac{1}{\sqrt{2}} - \cos \frac{\pi}{8} \right) = \frac{1}{2\sqrt{2}} - \frac{1}{2} \cos \frac{\pi}{8} \text{ and}$$

$$M = \frac{1}{2} \left(\frac{1}{\sqrt{2}} + \cos \frac{\pi}{8} \right) = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

11. (a) Let $f(x, y) = x + y - 1$

Given $(1, 2)$ and $(\sin \theta, \cos \theta)$ are lies on same side.

$$\therefore f(1, 2) \cdot f(\sin \theta, \cos \theta) > 0$$

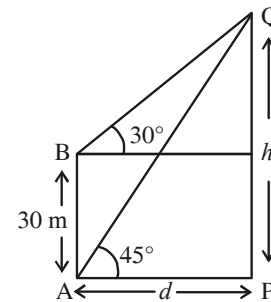
$$\Rightarrow 2[\sin \theta + \cos \theta - 1] > 0$$

$$\Rightarrow \sin \theta + \cos \theta > 1 \Rightarrow \sin \left(\theta + \frac{\pi}{4} \right) > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta + \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{3\pi}{4} \right) \Rightarrow \theta \in \left(0, \frac{\pi}{2} \right)$$

12. (a) Let the height of the tower be h and distance of the foot of the tower from the point A is d .

By the diagram,



$$\tan 45^\circ = \frac{h}{d} = 1$$

$$h = d \quad \dots(i)$$

$$\tan 30^\circ = \frac{h - 30}{d}$$

$$\sqrt{3}(h - 30) = d \quad \dots(ii)$$

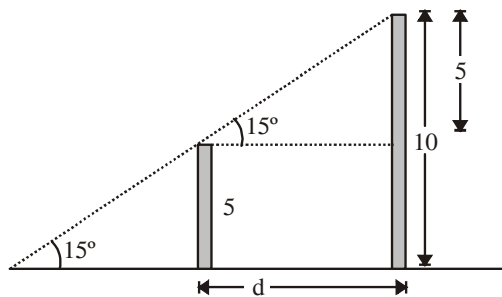
Put the value of h from (i) to (ii),

$$\sqrt{3}d = d + 30\sqrt{3}$$

$$d = \frac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}(\sqrt{3}+1) = 15(3+\sqrt{3})$$

$$\begin{aligned}
 13. \quad (b) \quad & \cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ \\
 &= \left(\frac{1 + \cos 20^\circ}{2} \right) + \left(\frac{1 + \cos 100^\circ}{2} \right) - \frac{1}{2} (2 \cos 10^\circ \cos 50^\circ) \\
 &= 1 + \frac{1}{2} (\cos 20^\circ + \cos 100^\circ) - \frac{1}{2} [\cos 60^\circ + \cos 40^\circ] \\
 &= \left(1 - \frac{1}{4} \right) + \frac{1}{2} [\cos 20^\circ + \cos 100^\circ - \cos 40^\circ] \\
 &= \frac{3}{4} + \frac{1}{2} [2 \cos 60^\circ \times \cos 40^\circ - \cos 40^\circ] \\
 &= \frac{3}{4}
 \end{aligned}$$

14. (a)



By the diagram,

$$\begin{aligned}
 \tan 15^\circ = \frac{5}{d} &\Rightarrow d = \frac{5}{\tan 15^\circ} = \frac{5(\sqrt{3}+1)}{\sqrt{3}-1} \\
 &= \frac{5(4+2\sqrt{3})}{2} = 5(2+\sqrt{3})
 \end{aligned}$$

$$15. \quad (a) \quad \because \sin(60^\circ + A) \cdot \sin(60^\circ - A) \sin A = \frac{1}{4} \sin 3A$$

$$\therefore \sin 10^\circ \sin 50^\circ \sin 70^\circ = \sin 10^\circ \sin(60^\circ - 10^\circ)$$

$$\sin(60^\circ + 10^\circ) = \frac{1}{4} \sin 30^\circ$$

$$\Rightarrow \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{4} \sin^2 30^\circ = \frac{1}{16}$$

 16. (b) $\because \alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$$\cos(\alpha + \beta) = \frac{3}{5}, \text{ then } \sin(\alpha + \beta) = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\tan(\alpha + \beta) = \frac{4}{3}$$

$$\text{And } \sin(\alpha - \beta) = \frac{5}{13}, \text{ then}$$

$$\cos(\alpha - \beta) = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{12}{13}$$

$$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$$

$$\text{Now, } \tan 2\alpha = \tan((\alpha + \beta) + (\alpha - \beta))$$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} = \frac{63}{16}$$

 17. (d) $\because$ The given equation is

$$\sin^4 \alpha + 4 \cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cdot \cos \beta, \alpha, \beta \in [0, \pi]$$

Then, by A.M., G.M. inequality;

$$\text{A.M.} \geq \text{G.M.}$$

$$\frac{\sin^4 \alpha + 4 \cos^4 \beta + 1 + 1}{4} \geq (\sin^4 \alpha \cdot 4 \cos^4 \beta \cdot 1 \cdot 1)^{\frac{1}{4}}$$

$$\sin^4 \alpha + 4 \cos^4 \beta + 1 + 1 \geq 4\sqrt{2} \sin \alpha \cdot \cos \beta$$

 Inequality still holds when $\cos \beta < 0$ but L.H.S. is positive than $\cos \beta > 0$, then

$$\text{L.H.S.} = \text{R.H.S.}$$

$$\therefore \sin^4 \alpha = 1 \text{ and } \cos^4 \beta = \frac{1}{4}$$

$$\Rightarrow \alpha = \frac{\pi}{2} \text{ and } \beta = \frac{\pi}{4}$$

$$\therefore \cos(\alpha + \beta) - \cos(\alpha - \beta)$$

$$= \cos\left(\frac{\pi}{2} + \beta\right) - \cos\left(\frac{\pi}{2} - \beta\right)$$

$$= -\sin \beta - \sin \beta = -2 \sin \frac{\pi}{4} = -\sqrt{2}$$

$$18. \quad (a) \quad f_k(x) = \frac{1}{k} (\sin^k x + \cos^k x)$$

$$f_4(x) = \frac{1}{4} [\sin^4 x + \cos^4 x]$$

$$= \frac{1}{4} \left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2} \right]$$

$$= \frac{1}{4} \left[1 - \frac{(\sin 2x)^2}{2} \right]$$

$$f_6(x) = \frac{1}{6} [\sin^6 x + \cos^6 x]$$

$$= \frac{1}{6} \left[(\sin^2 x + \cos^2 x) - \frac{3}{4} (\sin^2 x)^2 \right]$$

$$= \frac{1}{6} \left[1 - \frac{3}{4} (\sin 2x)^2 \right]$$

$$\begin{aligned} \text{Now } f_4(x) - f_6(x) &= \frac{1}{4} - \frac{1}{6} - \frac{(\sin 2x)^2}{8} + \frac{1}{8} (\sin 2x)^2 \\ &= \frac{1}{12} \end{aligned}$$

$$\begin{aligned} 19. \quad (a) \quad A &= \cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \dots \cos \frac{\pi}{2^{10}} \cdot \sin \frac{\pi}{2^{10}} \\ &= \frac{1}{2} \left(\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \dots \cos \frac{\pi}{2^9} \sin \frac{\pi}{2^9} \right) \\ &= \frac{1}{2^8} \left(\cos \frac{\pi}{2^2} \cdot \sin \frac{\pi}{2^2} \right) = \frac{1}{2^9} \sin \frac{\pi}{2} \\ &= \frac{1}{512} \end{aligned}$$

$$\begin{aligned} 20. \quad (a) \quad \text{We have} \\ 5 \tan^2 x - 5 \cos^2 x &= 2(2 \cos^2 x - 1) + 9 \\ \Rightarrow 5 \tan^2 x - 5 \cos^2 x &= 4 \cos^2 x - 2 + 9 \\ \Rightarrow 5 \tan^2 x &= 9 \cos^2 x + 7 \\ \Rightarrow 5(\sec^2 x - 1) &= 9 \cos^2 x + 7 \\ \text{Let } \cos^2 x &= t \\ \Rightarrow \frac{5}{t} - 9t - 12 &= 0 \\ \Rightarrow 9t^2 + 12t - 5 &= 0 \\ \Rightarrow 9t^2 + 15t - 3t - 5 &= 0 \\ \Rightarrow (3t - 1)(3t + 5) &= 0 \\ \Rightarrow t &= \frac{1}{3} \text{ as } t \neq -\frac{5}{3} \end{aligned}$$

$$\cos 2x = 2 \cos^2 x - 1 = 2 \left(\frac{1}{3} \right) - 1 = -\frac{1}{3}$$

$$\cos 4x = 2 \cos^2 2x - 1 = 2 \left(-\frac{1}{3} \right)^2 - 1 = -\frac{7}{9}$$

$$\begin{aligned} 21. \quad (b) \quad &4 + \frac{1}{2} \sin^2 2x - 2 \cos^4 x \\ &4 + 2(1 - \cos^2 x) \cos^2 x - 2 \cos^4 x \\ &-4 \left\{ \cos^4 x - \frac{\cos^2 x}{2} - 1 + \frac{1}{16} - \frac{1}{16} \right\} \\ &-4 \left\{ \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \right\} \\ &0 \leq \cos^2 x \leq 1 \\ &-\frac{1}{4} \leq \cos^2 x - \frac{1}{4} \leq \frac{3}{4} \\ &0 \leq \left(\cos^2 x - \frac{1}{4} \right)^2 \leq \frac{9}{16} \end{aligned}$$

$$-\frac{17}{16} \leq \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \leq \frac{9}{16} - \frac{17}{16}$$

$$\frac{17}{4} \geq -4 \left\{ \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \right\} \geq \frac{1}{2}$$

$$M = \frac{17}{4}$$

$$m = \frac{1}{2}$$

$$M - m = \frac{17}{4} - \frac{2}{4} = \frac{15}{4}$$

$$22. \quad (b) \quad \text{Let } \cos \alpha + \cos \beta = \frac{3}{2}$$

$$\Rightarrow 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = \frac{3}{2} \quad \dots(i)$$

$$\text{and } \sin \alpha + \sin \beta = \frac{1}{2}$$

$$\Rightarrow 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = \frac{1}{2} \quad \dots(ii)$$

On dividing (ii) by (i), we get

$$\tan \left(\frac{\alpha + \beta}{2} \right) = \frac{1}{3}$$

$$\text{Given : } \theta = \frac{\alpha + \beta}{2} \Rightarrow 2\theta = \alpha + \beta$$

$$\text{Consider } \sin 2\theta + \cos 2\theta = \sin(\alpha + \beta) + \cos(\alpha + \beta)$$

$$= \frac{\frac{2}{3}}{1 + \frac{1}{9}} + \frac{1 - \frac{1}{9}}{1 + \frac{1}{9}} = \frac{6}{10} + \frac{8}{10} = \frac{7}{5}$$

$$23. \quad (b) \quad \operatorname{cosec} \theta = \frac{p+q}{p-q}, \quad \sin \theta = \frac{p-q}{p+q}$$

$$\cos \theta = \pm \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left(\frac{p-q}{p+q} \right)^2} = \frac{2\sqrt{pq}}{(p+q)}$$

$$\left| \cot \left(\frac{\pi}{4} + \frac{\theta}{2} \right) \right| = \left| \frac{\cot \frac{\pi}{4} \cot \frac{\theta}{2} - 1}{\cot \frac{\pi}{4} + \cot \frac{\theta}{2}} \right| = \left| \frac{\cot \frac{\theta}{2} - 1}{\cot \frac{\theta}{2} + 1} \right|$$

$$= \left| \frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}} \right|$$

On rationalizing denominator, we get

$$\begin{aligned} & \left| \left(\frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}} \right) \left(\frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}} \right) \right| \\ &= \left| \frac{\cos \theta}{\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} + 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right| \\ &= \left| \frac{\cos \theta}{1 + \sin \theta} \right| = \left| \frac{2\sqrt{pq} / (p+q)}{1 + \frac{(p-q)}{p+q}} \right| = \frac{\sqrt{pq}}{p} = \sqrt{\frac{q}{p}} \end{aligned}$$

24. (d) $A = \sin^2 x + \cos^4 x$
 $= \sin^2 x + \cos^2 x(1 - \sin^2 x)$
 $= \sin^2 x + \cos^2 x - \frac{1}{4}(2 \sin x \cdot \cos x)^2$
 $= 1 - \frac{1}{4} \sin^2(2x)$
 $\therefore -1 \leq \sin 2x \leq 1$
 $\Rightarrow 0 \leq \sin^2(2x) \leq 1$
 $\Rightarrow 0 \geq -\frac{1}{4} \sin^2(2x) \geq -\frac{1}{4}$
 $\Rightarrow 1 \geq 1 - \frac{1}{4} \sin^2(2x) \geq 1 - \frac{1}{4}$
 $\Rightarrow 1 \geq A \geq \frac{3}{4}$

25. (a) $\cos(\alpha + \beta) = \frac{4}{5} \Rightarrow \tan(\alpha + \beta) = \frac{3}{4}$
 $\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$
 $\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$
 $= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{56}{33}$

26. (b) Given that

$$\begin{aligned} & \cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2} \\ & \Rightarrow 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)] + 3 = 0 \\ & \Rightarrow 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)] \\ & \quad + \sin^2 \alpha + \cos^2 \alpha + \sin^2 \beta + \cos^2 \beta \\ & \quad + \sin^2 \gamma + \cos^2 \gamma = 0 \\ & \Rightarrow [\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma + 2 \sin \alpha \sin \beta \\ & \quad + 2 \sin \beta \sin \gamma + 2 \sin \gamma \sin \alpha] + [\cos^2 \alpha + \cos^2 \beta \\ & \quad + \cos^2 \gamma + 2 \cos \alpha \cos \beta + 2 \cos \beta \cos \gamma \\ & \quad + 2 \cos \gamma \cos \alpha] = 0 \\ & [\because \cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B] \end{aligned}$$

$$\begin{aligned} & \Rightarrow [\sin \alpha + \sin \beta + \sin \gamma]^2 + [\cos \alpha + \cos \beta + \cos \gamma]^2 = 0 \\ & \Rightarrow \sin \alpha + \sin \beta + \sin \gamma = 0 \text{ and } \cos \alpha + \cos \beta + \cos \gamma = 0 \\ & \therefore A \text{ and } B \text{ both are true.} \end{aligned}$$

27. (c) Given that $p^2 + q^2 = 1$

$\therefore p = \cos \theta$ and $q = \sin \theta$ satisfy the given equation

Then $p + q = \cos \theta + \sin \theta$

We know that

$$-\sqrt{a^2 + b^2} \leq a \cos \theta + b \sin \theta \leq \sqrt{a^2 + b^2}$$

$$\therefore -\sqrt{2} \leq \cos \theta + \sin \theta \leq \sqrt{2}$$

Hence max. value of $p + q$ is $\sqrt{2}$

28. (c) $\cos x + \sin x = \frac{1}{2} \Rightarrow 1 + \sin 2x = \frac{1}{4}$

$$\Rightarrow \sin 2x = -\frac{3}{4},$$

$$\therefore \pi < 2x < 2\pi$$

$$\Rightarrow \frac{\pi}{2} < x \leq \pi \quad \dots(i)$$

$$\begin{aligned} & \frac{2 \tan x}{1 + \tan^2 x} = -\frac{3}{4} \\ & \Rightarrow 3 \tan^2 x + 8 \tan x + 3 = 0 \end{aligned}$$

$$\therefore \tan x = \frac{-8 \pm \sqrt{64 - 36}}{6} = -\frac{4 \pm \sqrt{7}}{3}$$

$$\text{for } \frac{\pi}{2} < x < \pi, \tan x < 0$$

$$\therefore \tan x = \frac{-4 - \sqrt{7}}{3}$$

29. (a) $u^2 = a^2 + b^2 + 2 \sqrt{\frac{(a^4 + b^4) \cos^2 \theta \sin^2 \theta}{+ a^2 b^2 (\cos^4 \theta + \sin^4 \theta)}} \quad \dots(1)$

$$\begin{aligned} & \text{Now, } (a^4 + b^4) \cos^2 \theta \sin^2 \theta + a^2 b^2 (\cos^4 \theta + \sin^4 \theta) \\ &= (a^4 + b^4) \cos^2 \theta \sin^2 \theta + a^2 b^2 (1 - 2 \cos^2 \theta \sin^2 \theta) \\ &= (a^4 + b^4 - 2a^2 b^2) \cos^2 \theta \sin^2 \theta + a^2 b^2 \end{aligned}$$

$$= (a^2 - b^2)^2 \cdot \frac{\sin^2 2\theta}{4} + a^2 b^2 \quad \dots(2)$$

$$\therefore 0 \leq \sin^2 2\theta \leq 1$$

$$\Rightarrow 0 \leq (a^2 - b^2)^2 \frac{\sin^2 2\theta}{4} \leq \frac{(a^2 - b^2)^2}{4}$$

$$\begin{aligned} & \Rightarrow a^2 b^2 \leq (a^2 - b^2)^2 \frac{\sin^2 2\theta}{4} + a^2 b^2 \\ & \leq (a^2 - b^2)^2 \cdot \frac{1}{4} + a^2 b^2 \quad \dots(3) \end{aligned}$$

From (1)

$$a^2 + b^2 + 2\sqrt{a^2b^2} \leq u^2 \leq a^2 + b^2 + \frac{2}{2}\sqrt{(a^2 + b^2)^2}$$

$$(a+b)^2 \leq u^2 \leq 2(a^2 + b^2)$$

$\therefore$ Max. value – Min. value

$$= 2(a^2 + b^2) - (a+b)^2 = (a-b)^2$$

30. (d) $\pi < \alpha - \beta < 3\pi$

$$\Rightarrow \frac{\pi}{2} < \frac{\alpha - \beta}{2} < \frac{3\pi}{2} \Rightarrow \cos \frac{\alpha - \beta}{2} < 0 \quad \dots(1)$$

$$\sin \alpha + \sin \beta = -\frac{21}{65}$$

$$\Rightarrow 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = -\frac{21}{65} \quad \dots(2)$$

$$\cos \alpha + \cos \beta = -\frac{27}{65}$$

$$\Rightarrow 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = -\frac{27}{65} \quad \dots(3)$$

Squaring and adding (2) and (3), we get

$$4 \cos^2 \frac{\alpha - \beta}{2} = \frac{(21)^2 + (27)^2}{(65)^2} = \frac{1170}{65 \times 65}$$

$$\therefore \cos^2 \frac{\alpha - \beta}{2} = \frac{9}{130} \Rightarrow \cos \frac{\alpha - \beta}{2} = -\frac{3}{\sqrt{130}} \quad [\text{from (1)}]$$

31. (c) Given $f(x) = \log(x + \sqrt{x^2 + 1})$

$$f(-x) = \log \left\{ -x + \sqrt{x^2 + 1} \right\} = \log \left\{ \frac{x^2 - x^2 + 1}{x + \sqrt{x^2 + 1}} \right\}$$

$$= -\log(x + \sqrt{x^2 + 1}) = -f(x)$$

$\Rightarrow f(x)$ is an odd function.

32. (b) We know that $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$;

$$\text{Since period of } \cos 2\theta = \frac{2\pi}{2} = \pi$$

Hence period of $\sin^2 \theta$ is also π .

33. (b) we know that $\cos \sqrt{x}$ is non periodic

$\therefore \cos \sqrt{x} + \cos^2 x$ can not be periodic.

34. (b) $\sin^4 \theta + \cos^4 \theta = -\lambda$

$$\Rightarrow (\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cdot \cos^2 \theta = -\lambda$$

$$\Rightarrow 1 - 2\sin^2 \theta \cos^2 \theta = -\lambda$$

$$\Rightarrow \lambda = \frac{(\sin 2\theta)^2}{2} - 1$$

$$\Rightarrow \text{as } \sin^2 2\theta \in [0, 1]$$

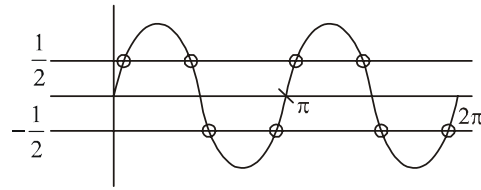
$$\Rightarrow \lambda \in \left[-1, \frac{-1}{2} \right]$$

35. (8) $\log_{1/2} |\sin x| = 2 - \log_{1/2} |\cos x|$

$$\Rightarrow \log_{1/2} |\sin x \cos x| = 2$$

$$\Rightarrow |\sin x \cos x| = \frac{1}{4}$$

$$\Rightarrow \sin 2x = \pm \frac{1}{2}$$



Hence, total number of solutions = 8.

36. (c) Consider equation, $1 + \sin^4 x = \cos^2 3x$

$$\text{L.H.S.} = 1 + \sin^4 x \text{ and R.H.S.} = \cos^2 3x$$

$$\therefore \text{L.H.S.} \geq 1 \text{ and R.H.S.} \leq 1 \Rightarrow \text{L.H.S.} = \text{R.H.S.} = 1$$

$$\sin^4 x = 0, \text{ and } \cos^2 3x = 1$$

$$\Rightarrow \sin x = 0 \text{ and } (4\cos^2 x - 3)^2 \cos^2 x = 1$$

$$\Rightarrow \sin x = 0 \text{ and } \cos^2 x = 1 \Rightarrow x = 0, \pm\pi, \pm 2\pi$$

Hence, total number of solutions is 5.

37. (d) Given equation is, $\cos 2x + \alpha \sin x = 2\alpha - 7$

$$1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7$$

$$2\sin^2 x - \alpha \sin x + (2\alpha - 8) = 0$$

$$\Rightarrow \sin x = \frac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha + 8)}}{4}$$

$$\Rightarrow \sin x = \frac{\alpha \pm (\alpha - 8)}{4} \Rightarrow \sin x = \frac{\alpha - 4}{4} \quad [\sin x = 2 \text{ (rejected)}]$$

$$\therefore \text{equation has solution, then } \frac{\alpha - 4}{4} \in [-1, 1]$$

$$\Rightarrow \alpha \in [2, 6]$$

38. (a) According to the question, there are two cases.

Case 1 : $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3} \right)$

In this interval, $[\sin \theta] = 0$, $[-\cos \theta] = 0$ and $[\cot \theta] = -1$

Then the system of equations will be ;

$$0 \cdot x + 0 \cdot y = 0 \text{ and } -x + y = 0$$

Which have infinitely many solutions.

Case 2 : $\theta \in \left(\pi, \frac{7\pi}{6} \right)$

In this interval, $[\sin \theta] = -1$ and $[-\cos \theta] = 0$,

Then the system of equations will be ;

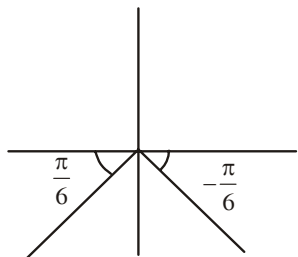
$$-x + 0 \cdot y = 0 \text{ and } [\cot \theta] x + y = 0$$

Clearly, $x = 0$ and $y = 0$ which has unique solution.

39. (c) $2\cos^2\theta + 3\sin\theta = 0$

$$(2\sin\theta + 1)(\sin\theta - 2) = 0$$

$$\Rightarrow \sin\theta = -\frac{1}{2} \text{ or } \sin\theta = 2 \rightarrow \text{Not possible}$$



The required sum of all solutions in $[-2\pi, 2\pi]$ is

$$= \left(\pi + \frac{\pi}{6}\right) + \left(2\pi - \frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right) + \left(-\pi + \frac{\pi}{6}\right) = 2\pi$$

40. (d) $\sin x - \sin 2x + \sin 3x = 0$

$$\Rightarrow \sin x - 2 \sin x \cdot \cos x + 3 \sin x - 4 \sin^3 x = 0$$

$$\Rightarrow 4 \sin x - 4 \sin^3 x - 2 \sin x \cdot \cos x = 0$$

$$\Rightarrow 2 \sin x(1 - \sin^2 x) - \sin x \cdot \cos x = 0$$

$$\Rightarrow 2 \sin x \cdot \cos^2 x - \sin x \cdot \cos x = 0$$

$$\Rightarrow \sin x \cdot \cos x(2 \cos x - 1) = 0$$

$$\therefore \sin x = 0, \cos x = 0, \cos x = \frac{1}{2}$$

$$\therefore x = 0, \frac{\pi}{3} \quad \therefore x \in \left[0, \frac{\pi}{2}\right)$$

41. (d) $\sin 3x = \cos 2x$

$$\Rightarrow 3 \sin x - 4 \sin^3 x = 1 - 2 \sin^2 x$$

$$\Rightarrow 4 \sin^3 x - 2 \sin^2 x - 3 \sin x + 1 = 0$$

$$\Rightarrow \sin x = 1, \frac{-2 \pm 2\sqrt{5}}{8}$$

$$\text{In the interval } \left(\frac{\pi}{2}, \pi\right), \sin x = \frac{-2 + 2\sqrt{5}}{8}$$

So, there is only one solution.

42. (a) $\therefore 8 \cos x \left(\cos^2 \frac{\pi}{6} - \sin^2 x - \frac{1}{2} \right) = 1$

$$\Rightarrow 8 \cos x \left(\frac{3}{4} - \frac{1}{2} - \sin^2 x \right) = 1$$

$$\Rightarrow 8 \cos x \left(\frac{1}{4} - (1 - \cos^2 x) \right) = 1$$

$$\Rightarrow 8 \cos x \left(\frac{1}{4} - 1 + \cos^2 x \right) = 1$$

$$\Rightarrow 8 \cos x \left(\cos^2 x - \frac{3}{4} \right) = 1$$

$$\Rightarrow 8 \left(\frac{4 \cos^3 x - 3 \cos x}{4} \right) = 1$$

$$\Rightarrow 2(4 \cos^3 x - 3 \cos x) = 1$$

$$\Rightarrow 2 \cos 3x = 1 \Rightarrow \cos 3x = \frac{1}{2}$$

$$\therefore 3x = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$$

$$\text{In } x \in [0, \pi]: x = \frac{\pi}{9}, \frac{2\pi}{3} + \frac{\pi}{9}, \frac{2\pi}{3} - \frac{\pi}{9}, \text{ only}$$

Sum of all the solutions of the equation

$$= \left(\frac{1}{9} + \frac{2}{3} + \frac{1}{9} + \frac{2}{3} - \frac{1}{9} \right) \pi = \frac{13}{9} \pi$$

43. (a) $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$$\Rightarrow 2 \cos 2x \cos x + 2 \cos 3x \cos x = 0$$

$$\Rightarrow 2 \cos x \left(2 \cos \frac{5x}{2} \cos \frac{x}{2} \right) = 0$$

$$\cos x = 0, \cos \frac{5x}{2} = 0, \cos \frac{x}{2} = 0$$

$$x = \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$$

44. (d) $\left| \sqrt{2 \sin^4 x + 18 \cos^2 x} - \sqrt{2 \cos^4 x + 18 \sin^2 x} \right| = 1$

$$\sqrt{2 \sin^4 x + 18 \cos^2 x} - \sqrt{2 \cos^4 x + 18 \sin^2 x} = \pm 1$$

$$\sqrt{2 \sin^4 x + 18 \cos^2 x} = \pm 1 + \sqrt{2 \cos^4 x + 18 \sin^2 x}$$

by squaring both the sides we will get 8 solutions

45. (c) $2 \sin^3 \alpha - 7 \sin^2 \alpha + 7 \sin \alpha - 2 = 0$

$$\Rightarrow 2 \sin^2 \alpha (\sin \alpha - 1) - 5 \sin \alpha (\sin \alpha - 1) + 2 (\sin \alpha - 1) = 0$$

$$\Rightarrow (\sin \alpha - 1) (2 \sin^2 \alpha - 5 \sin \alpha + 2) = 0$$

$$\Rightarrow \sin \alpha - 1 = 0 \text{ or } 2 \sin^2 \alpha - 5 \sin \alpha + 2 = 0$$

$$\sin \alpha = 1 \text{ or } \sin \alpha = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm 3}{4}$$

$$\alpha = \frac{\pi}{2} \text{ or } \sin \alpha = \frac{1}{2}, 2$$

Now, $\sin \alpha \neq 2$

$$\text{for, } \sin \alpha = \frac{1}{2}$$

$$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$$

There are three values of α between $[0, 2\pi]$

46. (b) Let $A = \{\theta : \sin \theta = \tan \theta\}$
and $B = \{\theta : \cos \theta = 1\}$

$$\text{Now, } A = \left\{ \theta : \sin \theta = \frac{\sin \theta}{\cos \theta} \right\}$$

$$= \{\theta : \sin \theta (\cos \theta - 1) = 0\}$$

$$= \{\theta = 0, \pi, 2\pi, 3\pi, \dots\}$$

$$\text{For } B : \cos \theta = 1 \Rightarrow \theta = \pi, 2\pi, 4\pi, \dots$$

This shows that A is not contained in B . i.e. $A \not\subset B$ but $B \subset A$.

47. (a) $\sin 2x - 2 \cos x + 4 \sin x = 4$
 $\Rightarrow 2 \sin x \cdot \cos x - 2 \cos x + 4 \sin x - 4 = 0$
 $\Rightarrow (\sin x - 1)(\cos x - 2) = 0$
 $\therefore \cos x - 2 \neq 0, \therefore \sin x = 1$

$$\therefore x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$$

48. (b) $2 \sin^2 \theta - \cos 2\theta = 0$
 $\Rightarrow 2 \sin^2 \theta - (1 - 2 \sin^2 \theta) = 0$
 $\Rightarrow 2 \sin^2 \theta - 1 + 2 \sin^2 \theta = 0$

$$\Rightarrow 4 \sin^2 \theta = 1 \Rightarrow \sin \theta = \pm \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \theta \in [0, 2\pi]$$

$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\begin{aligned} \text{Now } 2 \cos^2 \theta - 3 \sin \theta &= 0 \\ \Rightarrow 2(1 - \sin^2 \theta) - 3 \sin \theta &= 0 \\ \Rightarrow -2 \sin^2 \theta - 3 \sin \theta + 2 &= 0 \\ \Rightarrow -2 \sin^2 \theta - 4 \sin \theta + \sin \theta + 2 &= 0 \\ \Rightarrow 2 \sin^2 \theta - \sin \theta + 4 \sin \theta - 2 &= 0 \\ \Rightarrow \sin \theta (2 \sin \theta - 1) + 2(2 \sin \theta - 1) &= 0 \end{aligned}$$

$$\Rightarrow \sin \theta = \frac{1}{2}, -2$$

But $\sin \theta = -2$, is not possible

$$\therefore \sin \theta = \frac{1}{2}, -2 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Hence, there are two common solution, there each of the statement-1 and 2 are true but statement-2 is not a correct explanation for statement-1.

49. (b) Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$
 Put $e^{\sin x} = t$ in the given equation, we get
 $t^2 - 4t - 1 = 0$

$$\Rightarrow t = \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{4 \pm \sqrt{20}}{2}$$

$$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5} \quad (\because t = e^{\sin x})$$

$$\Rightarrow e^{\sin x} = 2 - \sqrt{5} \text{ and } e^{\sin x} = 2 + \sqrt{5}$$

$$\Rightarrow e^{\sin x} = 2 - \sqrt{5} < 0$$

$$\text{and } \sin x = \ln(2 + \sqrt{5}) > 1$$

So, rejected.

Hence, given equation has no solution.

$\therefore$ The equation has no real roots.

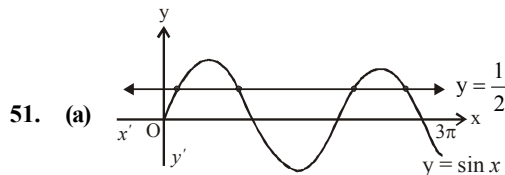
50. (d) $\sin 4\theta + 2 \sin 4\theta \cos 3\theta = 0$
 $\sin 4\theta(1 + 2 \cos 3\theta) = 0$

$$\sin 4\theta = 0 \quad \text{or} \quad \cos 3\theta = -\frac{1}{2}$$

$$4\theta = n\pi; n \in I$$

$$\text{or } 3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$$

$$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4} \quad \text{or} \quad \theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9} \quad [\because \theta \in (0, \pi)]$$



$$\begin{aligned} 2 \sin^2 x + 5 \sin x - 3 &= 0 \\ \Rightarrow (\sin x + 3)(2 \sin x - 1) &= 0 \end{aligned}$$

$$\Rightarrow \sin x = \frac{1}{2} \quad \text{and } \sin x \neq -3$$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

52. (b) $\therefore \tan x + \sec x = 2 \cos x$;
 $\Rightarrow \sin x + 1 = 2 \cos^2 x$
 $\Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$;
 $\Rightarrow 2 \sin^2 x + \sin x - 1 = 0$;
 $\Rightarrow (2 \sin x - 1)(\sin x + 1) = 0$
 $\Rightarrow \sin x = \frac{1}{2}, -1$;
 $\Rightarrow x = 30^\circ, 150^\circ, 270^\circ$.
 Number of solution = 3